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A PERFORMANCE COMPARISON OF UTILITY FUNCTIONS FOR GAME THEORY BASED WEAPON-TARGET ASSIGNMENT

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ABSTRACT. The weapon-target assignment problem has been considered as an essential issue for military applications to provide a protection for defended assets. The goal of a typical weapon-target assignment problem is to maximize the expected survivability of the valuable assets. In this study, defense of naval vessels that encounter aerial targets is considered. The vessels are assumed to have different types of weapons having various firepower and cost as well as the incoming targets may have different attack capabilities. In a typical scenario, in addition to protecting assets, it is also desirable to minimize the cost of weapons. Therefore, an asset-based static weapon-target assignment problem is considered in order to both maximize the expected survivability of the assets and minimize the weapon budget. Thus, a co-operative game theory based solution is proposed which relates the utilities of the individuals to the global utility and reach the Nash equilibrium.

1. Introduction

Weapon target assignment (WTA) has long been a fundamental issue in the military domain. Eliminating targets that attack valuable assets is a very complex problem and traditionally needs to be solved by command and control officers. However, due to the increasing number of threats and the complexity of the problem, an automated system is required to help the command and control officer make the right decision. WTA problem is closely related with the threat evaluation procedure that provides the information about the intents and capabilities of the incoming targets. The determination of which weapon to assign to incoming targets by the defense unit depends on knowledge such as the asset targeted by the threat and the destruction

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capability of the threat. Therefore, defense unit performs the WTA under the assumption of perfect information gathered by the threat evaluation [1].

Weapon assignment to incoming targets can be discussed in several aspects in military operations. In addition to the ground-based air defense including airbases, factories or valuable areas, maximizing the expected survivability of the vessels as a defended asset in a maritime environment is also considered in the literature [1]. Besides, static case considers the single assignment of weapons of the defending units to incoming threats, whereas a continuum of several stages situation, also called as shoot-look-shoot, is considered in the dynamic case. Moreover, target-based WTA models are also discussed as a special case of the asset-based models. A summary of several solutions for single or multiple objectives for the WTA problem is discussed in [2].

Employing game theoretical solution for WTA problem has several benefits. A game-theoretical vehicle-target assignment problem is discussed in [3] by aligning the individual utility functions to a global utility function with different aspects. Authors state that despite limited communication capabilities in an ambiguous environment, the vehicles act as individually logical units and can operate and decide by themselves.

WTA problem is discussed in the literature with different aspects. In [4], Kline et. al. discussed the evolution of WTA and analyzed and compared exact algorithms and heuristic algorithms such as very large-scale neighborhood search or genetic algorithm. In [5], a known solution of an optimization problem and a game theoretic approach have been compared and authors state that their game theory based solution have similar results with the first fit decreasing and best fit decreasing algorithms.
In [6], Karasakal considers the maximization of the defense of a valuable asset by a number of escorting vessels. In a recent study [7], authors discuss asset-based multi-objective WTA problem and show that multi-objective evolutionary algorithm based on decomposition has an effective performance. In [8], there is another research which uses the game theory and fuzzy logic as a hybrid system on decision making process. In the literature, there are also examples for multi-objective weapon target assignments which are solved by empirical approaches. As a comprehensive survey [9] explains several algorithms for WTA problem. In [10], Şahin and Leblebicioğlu present an approach as a fuzzy classification problem. They use a rule-based fuzzy classifier for weapon target assignment.

In this study, a static WTA problem for naval vessels is considered in terms of both maximizing the expected survivability of the defended assets and minimizing the cost of using weapons. All vessels are considered as valuable assets having different types of weapons with various firepower and cost. Besides, the capabilities of the attacking air threats are various. Therefore, an efficient weapon-target assignment is required for assets to protect both themselves and each other. The proposed solution relies on a co-operative game. Each vessel has its individual utility that needs to be maximized. Besides, these individual utility functions are linked with a global utility function by an alignment function and while each individual tries to increase its utility function, individuals also contribute to the global utility function. Therefore, Nash equilibrium is obtained when the individuals reach to the point where there is no improvement for their utility values, which is considered as the optimal assignment for each vessel. There are three different utility functions given in [3] on the performance of the game theoretical WTA assignments. As discussed in [3], vessel's utility functions are forced to align with the global utility function to reach an optimization at the global utility function. An alignment function is used to align the utilities. Since a dynamic range restriction approach is utilized in this study, a range restricted utility function is crucial. However, due to the overlapping regions of the weapons, the alignment cannot be obtained. We employ equally shared utility (ESU), wonderful life utility (WLU), and identical interest utility (IIU) functions along with the range restricted utility (RRU) function to compare the performance of the discussed utility function's combinations. The results show that not all combinations of the different utility functions yield an optimal solution.

In section 2, system model is given. In section 3, the theoretical background is explained. Simulation parameters are given in section 4. In section 5, the results of the simulations are shown. Finally, in section 6 the conclusions that we achieve are explained.

2. System Definition

In asset-based WTA problem, we assume that there are M weapons, N targets, and K assets. The probability p_{ij} is the probability that weapon i kills the target j whereas π_{jk} is the probability that target j completely destroys the asset k. The main target of the WTA problem is to increase the survivability probability of the assets. Also, we assign different values for each asset k, represented with ω_k. The objective function of the asset-based model to be maximized is as follows.

$$\sum_{k=1}^{K} \omega_k \prod_{j\in G_k}\left[1 - \pi_{jk} \prod_{i=1}^{M}\left(1 - p_{ij}\right)^{x_{ij}}\right] \tag{1}$$

Here, the set of targets that wants to destroy the asset k is represented with G_k and x_{ij} is a binary value which represents the assignment of weapon i to the target j. In our problem, we also consider the weapon costs and the value of the threat. Now, we expand the objective function given in (1) as follows.

$$\sum_{k=1}^{K} \omega_k \prod_{j \in G_k} \left[1 - \pi_{jk} . \Omega_j \prod_{i=1}^{M} \left(1 - p_{ij}\right)^{x_{ij}} - \log \varphi_i \right] \quad (2)$$

Now, in this new utility function Ω_j represents the value of the target j and φ_i is the cost of weapon i. Since WTA model aims to assign the most suitable weapon to the corresponding target, the discussed utility function utilizes the value of the target. In other words, while the utility function is increasing the probability of the survival of the assets, simultaneously the model considers the proper weapon assignment to the threats in terms of their costs.

As we mentioned our aim is to design an aerial defense system for a naval fleet. It is straightforward that the vessels on the fleet have various types of weapons and also there are various kinds of threats.

FIGURE 1 It shows a simplified naval environment having three assets and three incoming targets. The number of assets and targets may vary according to the scenario of the problem.

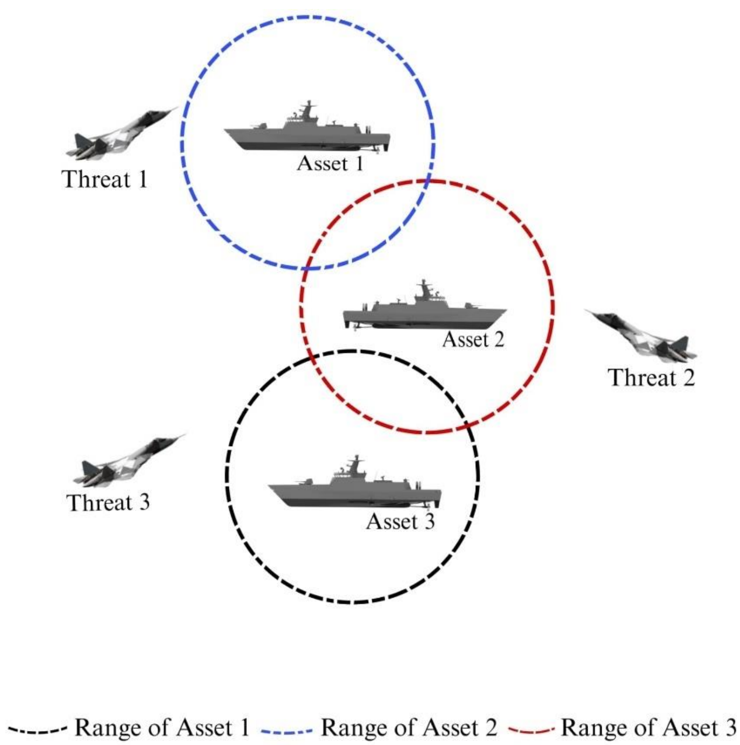

FIGURE 1 A simplified naval environment.

In Figure 1, circles define the range of weapons on each vessel. Our aim is to design an algorithm to make the choice of optimum weapon from the vessels' inventory while maximizing the survivability of assets and simultaneously minimizing the weapon costs.

3. Game Theoretical Solution

WTA problem can be considered as a M scalar optimization problem. There are solutions for this problem in the literature [1, 2, 6, 7, 11, 12]. In this study, we consider the problem in a game-theoretical approach where vessels are players and try to optimize their individual utilities. Using the game theory makes the algorithm more efficient. Game theoretical solutions will reduce communication and computational burdens, as each individual can make decisions with limited or no information from other individuals [3].

In a game-theoretical approach, each vessel is a player and tries to maximize its expected utility function. However, in an area defense scenario with more than one vessel, defending units are also expected to contribute to the global utility. For this reason, it is necessary to define a game where the vessels act in cooperation. Thus, any weapon of each vessel would be determined according to the common benefit or utility of the whole system.

Optimal assignment depends on the defensive and offensive characteristics of vessels and targets, respectively as well as the number of vessels and targets. It is straightforward that each vessel intends to optimize its self-utility function. However, alignment of vessel utilities should be taken into account to reach a maximal global utility [3]. On the other hand, alignment of individual utilities with a global utility is discussed in ordinal potential games.

Expected utility functions of the vessels are calculated according to the von Neumann-Morgenstern utility approach [13]. To align the utility functions of each vessel with the global utility function, we follow the definitions given in [14].

V_i and a_i represent the *i*-th vessel and the assignment of the corresponding vessel. The weapon-target assignment depends on whether the *j*-th target T_j is within the range of the *i*-th vessel. The set of vessels and assignments are shown as $V \coloneqq \{V_1, V_2, \dots, V_N\}$ and $a \coloneqq \{a_1, a_2, \dots, a_N\}$, respectively. Thus, each individual vessel V_i selects a proper assignment a_i to have a maximum utility function value $U_{V_i}(a)$. Players of the game decide to maximize their utility functions while providing a high global utility function, $U_g(a)$ [3].

In the range restricted utility function approach, the utility of a vessel and the global utility, $U_g(a)$ can be given as by (3).

$$U_{V_i}(a) = \sum_{T_j \in \mathcal{A}_i} U_{T_j}(a) \tag{3}$$

The basic problem with range restriction is that the restricted areas may overlap. For the overlapping regions another alignment function needed to be used in conjunction with the range restricted utility function.

The second utility function that has been defined in [3] is equally shared utility function. According to equally shared utility function, given in (4), the vessels share the utility value if they lock on the same target.

$$U_{V_i}(a) = \frac{U_{T_j}(a)}{n_{T_j}(a)}, \text{ if } a_i = \mathcal{T}_j \tag{4}$$

Equally shared utility function may not yield to optimum solution especially if one of the targets has much higher value than the others. The third utility function defined in [3] is wonderful life utility function. According to wonderful life utility function, given in (5), the vessels get as much utility as they contribute to the global utility.

$$U_{V_i}(a_i, a_{-i}) = U_{\mathcal{T}_j}(a_i, a_{-i}) - U_{\mathcal{T}_j}(\mathcal{T}_0, a_{-i}), \text{ if } a_i = \mathcal{T}_j \tag{5}$$

The last utility function to discuss is identical interest utility function. According to identical interest utility function all vessels' individual utilities equal to the global utility. In [3], authors state that identical interest utility function yields to optimum solution. On the other hand, with identical interest utility function, as shown in (6), every vessel needs to know each other's utility and the global utility. Thus, it increases communication and computational burden.

$$U_{V_i}(a) = U_g(a), \ \forall V_i \in V \tag{6}$$

In the proposed solution, if there are more than one threat for the assets, a two-step solution is considered. In the first step, the vessel will decide on which target they should be assigned and after that they decide the weapon of the corresponding vessel. If there are more than one vessel assigned to a single threat, they will play the game of the single threat case until the game reaches to the Nash equilibrium. The Nash equilibrium is the point of no regret, which is the optimum solution for the game.

To be able to have an improved computational burden, we limited the range. The range has been designed dynamically rather than a static one. On this matter, the "dualist game" has been the base of the algorithm. The dualist game is based on the dualist scenes in western movies. Two gunman approaches each other, and they try to shoot each other. The problem here is that to decide when they should fire. The solution to that game is that a gunman should fire his weapon if on the next step the other gunman's probability of shooting him is higher than his probability of shooting the other gunman on the current step.

If we apply the dualist game to our application, the vessels shall fire their weapons if there is a certain kind of threat level to the asset. The main reason for that is, when the target gets closer to the weapon, the uncertainty level begins to drop, and the probability of kill for the weapon gets higher. By limiting the range, the weapon systems do not need to check for assignment for every target, instead, they need to check for an assignment just for the targets which are in the range of them. In other words, the following equation 7 can be used to limit the range. For interested readers, computational burden of game theoretical algorithms is examined in [15]. Thus, when the situation in equation 7 happens, the vessel shall fire its weapon.

$$\prod_{k=1}^{K} \pi_{jk} > \prod_{i=1}^{M} p_{ij} \tag{7}$$

In addition to deciding the timing of the firing units, the equation 7 also provides a limit for the range. In [6], Karasakal shows a decision methodology for determining the vessel that fires first by estimating the trajectory of the target. Unlike that methodology, this study makes the decision of the vessels which are able to fire by employing adaptive limitation of the range.

By using equation 1 and 2 it is possible to calculate the utility for a single vessel. On the other hand, it is important to note that, every vessel will try to maximize its own utility and the result may not be aligned with the global utility. It is also possible that the outcome may not be optimum. To align the utilities of the vessels one shall use the potential functions described by equations 8, 9, and 10 as in [3].

In the following equations, V is the space that defines vessels, a is the assignment profile, $U_{V_i}(a)$ is the utility for vessel number i, $U_g(a)$ is the global utility, and a_{-i} is the assignment profile for all vessels except vessel number i. Then,

$$U_{V_i}(a_i', a_{-i}) - U_{V_i}(a_i'', a_{-i}) > 0 \leftrightarrow U_g(a_i', a_{-i}) - U_g(a_i'', a_{-i}) > 0 \tag{8}$$

If equation 8 is satisfied, then the utility for vessel is aligned with the global utility. Arslan et. al. defined the optimum assignment condition in [3] as,

$$a_{opt} \in argmax\, U_g(a), a \in A \tag{9}$$

To be able to align the global utility and individuals' utility, we employ potential games, also mentioned in [3] and [14]. Potential games are based on existence of a potential function. When a player changes its strategy, if the difference in the function is equal to the difference in the expected utility, then this function is a potential function for this game and the game is called as a potential game. [3] and [14] describe this type of game as,

$$U_{V_i}(a_i, a_{-i}) - U_{V_i}(a_i'', a_{-i}) = \Theta(a_i', a_{-i}) - \Theta(a_i'', a_{-i}) \tag{10}$$

The function $\Theta(\mathrm{a}): \mathrm{A} \to \mathrm{R}$ is called as the potential function. There are two different types of potential functions, which are ordinal and cardinal potential functions. Ordinal potential functions are defined in [3] and [14] as,

$$U_{V_i}(a_i', a_{-i}) - U_{V_i}(a_i'', a_i) > 0 \leftrightarrow \Theta(a_i', a_{-i}) - \Theta(a_i'', a_{-i}) > 0 \tag{11}$$

Here, the function $\Theta(\mathrm{a}): \mathrm{A} \to \mathrm{R}$ is called as the ordinal potential function.

This study takes advantage of the potential functions discussed in [3] when expected utility of the vessels and the global utility need to be increased at the same level. The first phase of the games that we have designed, as mentioned before, if there are multiple targets, then the vessels negotiate among each other by playing a game between them (with each other) to decide which vessel will be assigned to which target. In addition to that, if there are only one target and multiple vessels, the optimum vessel and the optimum weapon system to counter the attack will be decided. More on that, if there are multiple targets and only one vessel, the optimum weapon assignment to each target will be shown. If there is only one target and only one vessel, which is eligible to take the shot, then the optimum weapon system shall counter the attack. Therefore, the game is designed to find the optimum global utility, not the optimum utility for the vessels.

4. Simulation Parameters

Simulations parameters are discussed within this section. The simulation has run on a MATLAB environment. An example representation of the studied different simulation cases is given in
Figure *2*.

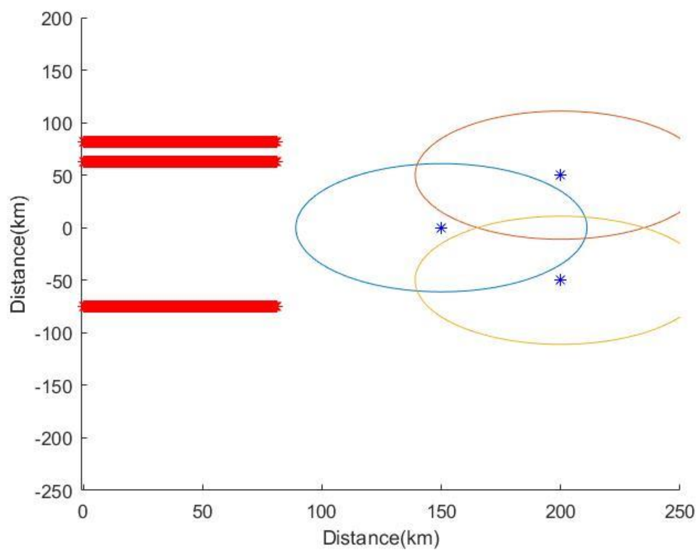

FIGURE 2 Simulation interface.

Here, the red lines represent the trajectory of the threats. Blue dots represent the vessels, the circles represent the range of the weapon systems. The threats have been distributed over the map randomly, and then they start to approach through the asset. The weapon systems counter the threats when they enter the area that can be seen on the interface.

In the simulation the vessels have multiple weapon systems. At the same time, there are many threats having different characteristics and weapon systems having different capabilities. There are multiple vessels to protect the assets.

The targets are assumed to be approaching from different directions, outside of the vessels range. Once they arrived at the range then the vessels shall react. One of the goals of the model is to cover all the targets. There shall not be unassigned target if there are enough weapons. Therefore, we used the alignment functions that we covered in previous section. Without the alignment functions, there is a possibility for the vessels to lock on the same target and there would be some targets that are unassigned to any weapons. This situation especially exists when utility of one of the targets is far larger than that of others. An alignment function aligns the individual utilities to the global utility. Therefore, the vessels have to try to increase the global utility. Once all the targets are covered, if there are still some weapons left, then they can lock on the targets that previously assigned to the other weapons.

Here, we assume that there are three weapons, three vessels, and three threats. Table 1 shows the probabilities that weapon *i* kills the threat *j*.

TABLE 1 Probability that weapon *i* kills the threat *j*, p_{ij}.

	All Vessels		
	Weapon 1	Weapon 2	Weapon 3
Threat 1	0.6	0.3	0.0001
Threat 2	0.8	0.5	0.3
Threat 3	0.9	0.6	0.5

The values in the simulation have been chosen to cover a wide range of weapons and threats. Threat #1 is clearly the strongest threat and hardest to kill. Table 1 also shows that weapon #1 is the strongest weapon and has the highest probability to kill. Weapon #3 and threat #3 added as weak weapon and threat. Weapon #2 and threat #2 are the mediocre ones. By this way, in the simulation in can be examined how the system reacts to weak and strong threats by having different weapons with different capabilities.

It is also mentioned that there are multiple objectives for the algorithm and one of them is to minimize the ammunition budget. This study assumes that the ammunitions for different weapon systems are not the same. Table 2 shows the ammunition budget of weapon *i*. It is assumed that the strongest weapon should have the highest price and the weakest should be the least expensive one.

TABLE 2 Ammunition budget of weapon *i*, φ_i.

Weapon 1	Weapon 2	Weapon 3
0.95 units	0.6 units	0.2 units

One of the parameters that we take into account is the probability that threat *j* destroys the asset k. Table 3 shows these probability values. It is assumed that the strongest threat should have the highest probability to destroy the asset and the weakest has the lowest probability.

TABLE 3 Probability that threat *j* destroys the asset *k*, π_{jk}.

Threat 1	Threat 2	Threat 3
1	0.5	0.3

Another parameter that we need to define the utility function is asset's value constant. The assets may have different importance level in a real combat zone. Therefore, we chose a most valuable asset, a least valuable asset and a mediocre one. Table 4 shows the asset values.

TABLE 4 Asset values, ω_k.

Asset 1	Asset 2	Asset 3
1	0.5	0.3

The last parameter that we need to define is a value constant for the threats. It is assumed that the strongest threat should be the most valuable one and, the weakest threat should be the least valuable one. The constant is given in Table 5.

TABLE 5 Threat values, Ω_j.

Threat 1	Threat 2	Threat 3
1	0.5	0.3

Using these values in equation 2, we are able to match these situations into numeric values. It also fits The Neumann-Morgenstern Theorem and its axioms.
A pseudo code for the simulation algorithm is given with Algorithm 1.

4. RESULTS

The results of our simulations using the weapon and threat types which have been mentioned in the previous section are given within this section. For the first trial we simulated a single threat situation. When the threat #1 gets into the range of a vessel, the utility function produces values for three weapons on it. For this trial, a single threat is assumed to be in the range. This scenario is based on the method of static games. Hence, the time is assumed to be frozen for a moment. The weapon system should fire the weapon, which has the highest utility value. According to the results given in Table 6, weapon #1 has the highest utility value, which is shown bold.

ALGORITHM 1 Pseudo code for the simulation algorithm.

```
for (Each asset)
    Define range;
end
if (Any threat in any range of any asset)
    Check if it is in the range of single or multiple assets
    if (Single asset against single threat)
        Get utility values for the weapons;
        Assign the weapon with the highest utility;
    else if (Multiple assets against single threat OR multiple assets multiple threats)
        for(Each asset has a shared threat in their shared range)
            Get utility values for the weapons;
            Assign the weapon with the highest utility;
        end
        Prepare the game matrix;
        Apply the alignment function;
        Find the Nash Equilibrium;
        Assign the weapon;
    else if (Single asset against multiple threats)
        Get utility values for the weapons;
        Assign the weapon with the highest utility;
    end
end
```

TABLE 6 Simulation results of a single threat scenario for threat #1.

Weapon	Weapon #1	Weapon #2	Weapon #3
Maximum Utility Value	**0.5050**	0.2400	0.0000

We considered the simulation results for a single threat case for the threat #2 and #3 and the results are given in Table 7 and Table 8, respectively.

TABLE 7 Simulation results of a single threat scenario for threat #2.

Weapon	Weapon #1	Weapon #2	Weapon #3
Maximum Utility Value	**0.3050**	0.1900	0.1300

TABLE 8 Simulation results of a single threat scenario, for threat #3.

Weapon	Weapon #1	Weapon #2	Weapon #3
Maximum Utility Value	0.1600	0.1200	**0.1750**

For the scenarios above, the range is limited as well. The range limitations are made by using dualist game model. Equation 7 shows how range limitation is done by using the dualist game. For this situation, the weapon will be fired when the probability to kill for the threat of the asset is greater than the probability to kill for the weapon of the threat. For this reason, the cumulative distribution functions of the weapon and of the threat have been used.

For the scenarios that have multiple threats and multiple vessels to fire the target, it has been mentioned that an alignment to global utility is needed. In [3], range restricted utility function is shown as one of the candidates as a potential function. However, when the ranges of the vessels overlap, the RRU function does not yield the optimum results. Table 9 shows such scenario with threat #1 and threat #3.

TABLE 9 Simulation results of two threats and two vessels having overlapping ranges scenario, RRU function with no alignment function.

	Vessel #2		
		Threat #1	Threat #3
Vessel #1	Threat #1	**0.5050, 0.5050**	0.5050, 0.0000
	Threat #3	0.0000, 0.5050	0.0000, 0.0000

As one can see from Table 9, threat #1 strategy is strictly dominant strategy for vessel #1 (0.5050 >0, 0.5050>0). Thus, vessel #2, with the knowledge that vessel #1 is choosing the threat #1, must choose threat #1 as well (0.5050>0). Therefore, the Nash equilibrium is at [Threat #1, Threat #1]. As equation 3 shows that the global utility is some of the vessels' individual utilities. There is only one vessel can hit the target and therefore, the maximum global utility can be 0.5050 with this result. On the other hand, the maximum global utility could be sum of each target's utility. Thus, the maximum global utility should be 1.01 and this result shows that, without an additional alignment, when the ranges of two vessels are overlapped, there is not any alignment function anymore and as a result the global utility is not the optimum one.

For the overlapping area case, another alignment is needed. The equally shared utility function in [3] has been employed along with the range restricted utility function to overcome this problem. This function makes an equal distribution of utility if two or more vessels have the same target. In [3] it is mentioned that equally shared utility function may not be optimum for some situations for the alignment. Our simulations show that especially if one of the targets' utility is high enough, even though the utilities are shared, all of the weapons still try to lock on the high value target which yields a sub-optimum results. To show why equally shared utility function failed, we can show an example trial with two threats and two vessels with overlapping range areas. When the targets were on the overlapping area both weapon systems choose their optimum weapons to counter the threat, which were the ones that maximize their utility function. Here, the method is the same with one target and one vessel case. In other words, equation 3 calculates the utilities of the weapons and assign them to the targets. For this example, the difference is that there are multiple weapon systems and multiple targets, and the system needs to decide which weapon system will counter to the incoming targets. The game theory based solution gets the best weapon values for both weapon systems to counter the threats as inputs. The inputs produce a game matrix.

TABLE 10 Simulation results of two threats and two vessels having overlapping ranges scenario, ESU function with RRU function.

		Vessel #2	
		Threat #1	Threat #3
Vessel #1	Threat #1	**0.2525, 0.2525**	0.5050, 0.1750
	Threat #3	0.1750, 0.5050	0.0875, 0.0875

As one can see from Table 10, selecting threat #1 is the strictly dominant strategy for vessel #1 (0.2525>0.1750 and 0.5050>0.0875). From this, we know that vessel #1 cannot make a profitable deviation from threat #1 strategy. Thus, we know that vessel #1 must choose threat #1. So, for the situation that vessel #1 choosing threat #1 strategy for vessel #2 again threat #1 strategy is the dominant one (0.2525>0.1750). Therefore, the Nash equilibrium is at [Threat #1, Threat #1] point. For some situations like in Table 10, the most lethal threat has a very high utility value, the vessels are inclined to lock it even though they share the utility; it is still the highest utility for them individually. We can observe from Table 10 that with using the equally shared utility function, the system's global utility may not be at its optimum. Thus, we employ wonderful life utility function that has been mentioned in [3], instead of equally shared utility function. According to wonderful life utility function, a vessel only gets utility when it contributes to the global utility. This function along with range restricted utility function has increased the global utility and solved the problems that have been encountered during the scenarios with equally shared utility function. Equation 5 explains the use of wonderful life utility function with range restricted utility function. With this potential function the vessels do not only lock to the most important target, but some of them also choose different targets as well.

To show how wonderful life utility function changes the results, we observed the same scenario, discussed in Table 10, with wonderful life utility function instead of employing equally shared utility function. Simulation results are given in Table 11.

TABLE 11 Simulation results of two threats and two vessels having overlapping ranges scenario, WLU function with RRU function.

	Vessel #2		
Vessel #1		Threat #1	Threat #3
	Threat #1	0.5050, 0.0000	**0.5050, 0.1750**
	Threat #3	0.1750, 0.5050	0.0000, 0.1750

As one can see from Table 11, again selecting threat #1 is a strictly dominant strategy for vessel #1 (0.5050>0.1750 and 0.5050>0). Thus, vessel #2 must always choose threat #3. Therefore, the Nash equilibrium is at [Threat #1, Threat #3] point. This result shows that with wonderful life utility function we reached an alignment for the individual utilities to the global utility.

The last utility design method along with range restricted utility function that we will show in this study is identical interest utility function. This utility design method makes each vessel's utility equal to the global utility. We made experiment with this utility design method as well. We were able to reach the optimum Nash Equilibria by using this method. In fact, as mentioned in [3], the result must yield to the highest global utility, because the Nash equilibrium for the vessels is exactly that point. If we continue with the same example, Table 12 shows the simulation results when we employ identical interest utility function.

TABLE 12 Simulation results of two threats and two vessels having overlapping ranges scenario, IIU function with RRU.

	Vessel #2		
Vessel #1		Threat #1	Threat #3
	Threat #1	0.5050, 0.5050	**0.6800, 0.6800**
	Threat #3	**0.6800, 0.6800**	0.1750, 0.1750

As one can notice from Table 12, there are two Nash equilibria for this example. [Threat #1, Threat #3] and [Threat #3, Threat #1] are the Nash equilibria. As one can see from Table 11, a vessel cannot make a profitable deviation from these points. They are already most profitable strategies for both of them, and both of them makes the vessels aligned with the global utility.

TABLE 13 Results for Different Scenarios.

#	**Scenario**	**Maximum Global Utility**
1	3 threat #1 against 3 identical vessels RRU Function	1.5150
2	2 threat #1 and 1 threat #3 against 3 Identical vessels RRU Function	1.0100
3	3 threat #1 against 3 identical vessels RRU Function with ESU Function	1.5150
4	2 threat #1 and 1 Threat #3 against 3 identical vessels RRU Function with ESU Function	1.0100
5	2 Threat #1 and 1 Threat #3 against 3 identical vessels RRU function with WLU function	**1.185**
6	2 Threat #1 and 1 Threat #3 against 3 identical vessels RRU function with IIU function	**1.185**
7	1 Threat #1 and 2 Threat #3 against 3 identical vessels RRU function and ESU function	0.5050
8	1 Threat #1 and 2 Threat #3 against 3 identical vessels RRU function with WLU function	**0.8550**
9	1 Threat #1 and 2 Threat #3 against 3 identical vessels RRU function with IIU function	**0.8550**
10	3 Threat #1 against 3 identical vessels RRU function with WLU function	1.5150
11	3 Threat #1 against 3 identical vessels RRU function with IIU function	1.5150
12	3 Threat #1 against 5 identical vessels RRU Function with WLU Function	1.5150
13	3 Threat #1 against 5 identical vessels RRU function with IIU function	1.5150
14	3 Threat #1 against 2 identical vessels RRU function with WLU function	**1.0100**
15	3 Threat #1 against a single vessel RRU function with WLU function	0.5050

According to our results, wonderful life utility function is the optimum utility function for our application, which yields to optimum solution.

On the other hand, the identical interest utility function will ultimately be ineffective, as also mentioned in [3]. Every vessel in the game must know exactly what the global utility is. This adds another communication burden to the system. We also present several simulation results for various scenarios in Table 13.

The first scenario shows that if all of the threats are replicas of each other, then the range restricted utility is enough for a proper alignment even though if there are some overlapping areas. However, the effect of employing other utility functions appears when there exists threats having different properties. Scenarios 2, 4, 5 and 6 show that how the maximum global utility changes when RRU is employed alone, ESU with RRU, WLU with RRU, and IIU with RRU. These trials show that utilization of RRU alone and ESU with RRU have some drawbacks and do not approach to the optimum solution. On the other hand, utilization of WLU with RRU and IIU with RRU have yielded the optimum solution.

Scenarios 7, 8, and 9 show the results of ESU, WLU, and IIU with RRU for a different scenario. Again, this one show that ESU has a limited capacity; on the other hand, WLU and IIU led to an optimum solution.

For the scenarios 10 and 12 as well as 11 and 13 we used same utility functions, but we increased the number of vessels. The number of threats remain the same. Note that the utilities for 10 and 12, and 11 and 13 are the same, because even though the defenders are increased, all the threats are already covered with other weapons, so, they have no contribution to the global utility.

For the scenarios 14 and 15, the number of threats is higher than the number of defenders. For these scenarios even though there are threats that remain uncovered, the global utility is limited with the number of vessels and their contribution to the global utility. Therefore, when number of vessels decreases, the global utility decrease as well.

5. Conclusions

Simulation results show that the system will be stable at Nash equilibria including a pure one as long as it uses a utility function which leads to a potential function. We used a dynamic range limitation. It should be actually a natural result of the system, because if the threats are out of defined range, the utility value is usually lower than not firing the ammunition. Defining it in advance gives an advantage about the computational burden. The threats outside of the range are not considered during weapon assignment process and this leads a faster system, which is essential for this type of systems.

We used other potential functions alongside the range limitation and we were able to see which one is the best choice. According to the results, if we use range restricted utility on its own and if two or more vessels have overlapping areas on the range that they cover, it is not possible to align their utility with the global utility. Range limitation was an important aspect for this scenario. Thus, we decided to use other utility functions with it. Equally shared utility function is failed for some situations, especially if one of the targets has a utility value that is too high. Wonderful life utility function and identical interest utility function have the ability to get the system to the optimum point; however, identical interest utility function has additional communication burden. For this reason, the cumulative distribution functions of the weapon and of the threat function to be used with the RRU function should be wonderful life utility function. As one can see from the results section wonderful life utility function is one of the alignment functions that reached the maximum utility value for the global utility. It is also a better choice than the identical interest utility function which is another alignment function that leads the same result with wonderful life utility function, because, with identical interest utility function every vessel must have the information of the each other's utility value and the global utility value. Thus, wonderful life utility function has less computational burden. For these reasons, we point that wonderful life utility function along with range restricted utility function is the best choice according to our simulation results.

As a future work, the game theory based solution can be extended to a truly dynamic system, which is less discussed in the literature.

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