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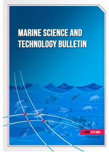

Marine Science and Technology Bulletin

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RESEARCH ARTICLE

Modelling and analysis of pufferfish (*Torquigener albomaculosus*) circular nest on seafloor

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ABSTRACT

This study analyzed the structural features of the circular structures built by the male pufferfish (*Torquigener albomaculosus*) on the seafloor. This impressive circular structure which is built by male pufferfish exists in different depths and sizes on the seafloor. One of the objectives for the male pufferfish constructing these circular structures is to influence the opposite sex. The morphological features of these circular structures built on the sea floor were analyzed mathematically. This analytical study was performed using photographs received from the sea floor as well as the ratios of circular formations discovered in this context. During the image analysis, the histogram equalization method was used to improve the visuals and reveal the patterns of circular structures. The Hough transform method, which is commonly used in the determination of circular structures, was used in the process of precisely determining the dimensions of the obtained circular structures. The circle size ratios obtained from the images and the circular structure dimension ratios obtained from the observation were examined. When the ratio analysis results from 2 images and 6 observation data were examined, it was observed that significant traces of the golden ratio were seen in these nested circular structures. According to this study's analysis of the pufferfish nest, which demonstrated proper development, the percentage difference between the golden ratio and the radius of the circular structures was determined to be as low as 0.185%. Additionally, these circular constructions were recreated in 3D while preserving their proportions, and their hydrostatic pressure characteristics were analyzed depending on their actual depths on the seafloor.

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Introduction

Golden section or ratio (φ) is defined as the magical beauty of mathematics. This ratio is implemented in the area of art, architecture, music, design, etc. and it is frequently encountered as a result of examining the impressive works in these areas. Mankind has created many works by using this impressive ratio or number in nature. Even if this number inspires humans to produce more beautiful products, there is much more impressive work related to this impressive number in nature, which is the main source of this number and this value is also deeply linked with efficiency and environmental usefulness for nature (Meisner, 2018). The golden ratio can generally be defined as the relationship between the components that make up a total structure. This ratio is often defined as length, and sometimes as angular, but it is an impressive and fascinating concept in every aspect. The concept of the golden ratio is shown as the reason why some designs are more impressive than others. Although the concept has been known for a long time, there are many studies on this concept even today. The main reasons for this are because the golden ratio concept is intriguing, and our ability to observe and analyze the environment is increasing day by day as technology advances. There are many examples of traces of the golden ratio in the physical appearance of living organisms. Traces of the golden ratio can be observed in a wide variety of biological phenotypes. Furthermore, research shows that the golden ratio can be found in physiological signals that are central to our life, such as the electrocardiogram (ECG) (Ciucurel et al., 2018).

The trail of the golden ratio can also be found in weather patterns, sea waves and even planets on the solar system (Bartlett, 2019). The golden ratio is found in detailed analyzes of the body structures of living things. The trace of the golden ratio on the body of the monarch butterfly is shown in Figure 1 (Akhtaruzzaman & Shafie, 2011).

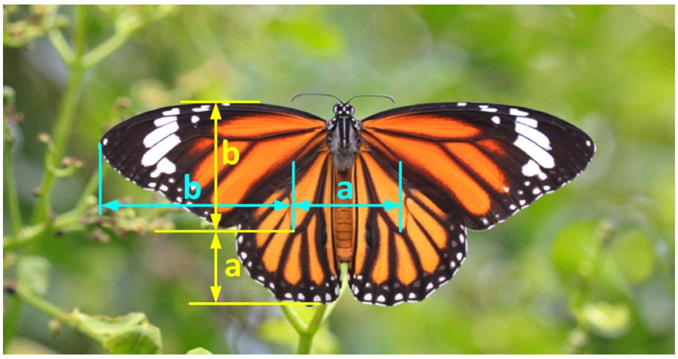

Figure 1. Golden ratio in *Danaus plexippus* wings

In addition, there are various traces of the golden angle, which is the part of the full angle according to the golden ratio in nature. The leaf distribution of *Bromelia humilis* is given in Figure 2 (Lüttge & Souza, 2019).

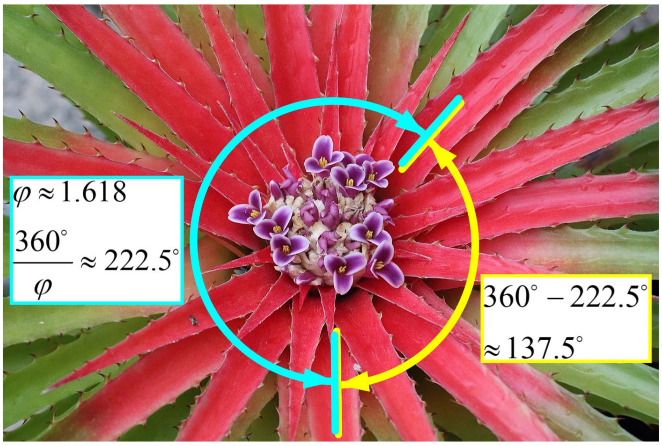

Figure 2. *Bromelia humilis* and golden angle

When the leaf distribution of such leafy plants is examined, there is a relationship between the golden ratio or angle value and the sunshine duration of the leaves in the lower part. With the leaves showing distribution depending on the golden angle, it is possible to capture maximum light in terms of the whole plant. In this way, it is ensured that the plants perform the maximum photosynthetic activity (Strauss et al., 2020). Thanks to the leaf distribution depending on the golden angle, the rate of blocking of the rays of each other is minimized.

In addition to the evidence of the golden ratio that can be seen with the bare eyes, some studies have found traces of the golden angle in internal organs. (Yalta et al., 2016). In a study examining the relationship between the golden ratio and organ morphology, the vertical and transverse dimensions of a healthy human heart structure converge to the golden aspect ratio regardless of ethnic origin. It has been observed that these values significantly deviated from the golden angle in heart-related diseases (Henein et al., 2011). The illustration of the human heart and the golden angle between inlet and outflow tract axes of the right ventricle is given in Figure 3. When the studies on the golden ratio and angle are examined, it has been revealed with impressive studies that these parameters are important not only in terms of efficiency, visuality, art, design, nature etc. but also in the field of health (Akhtaruzzaman & Shafie, 2011). In another study, it was shown that with the golden rhythm depending on the golden ratio, acute gait patterns can be improved in Parkinson's patients (Belluscio et al., 2021).

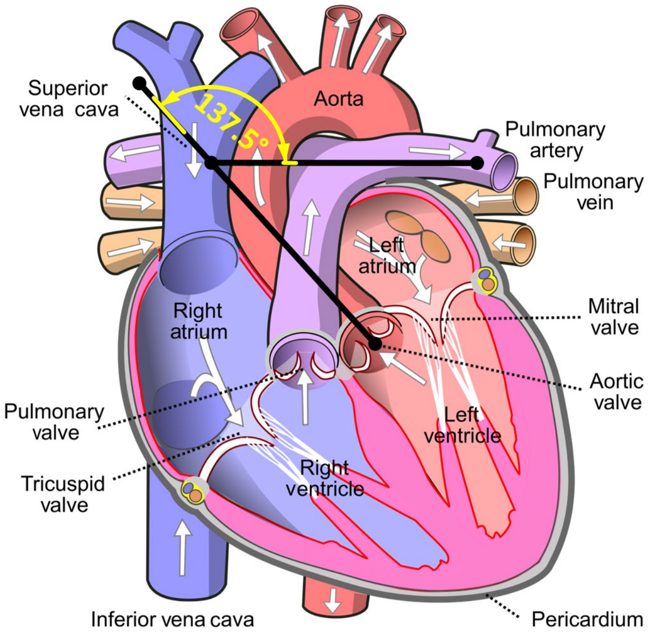

Figure 3. The golden angle between the ascending aorta and pulmonary artery

As can be seen in the examples given, there are very impressive traces of the golden ratio in the structures in nature. In this study, the relationship between the golden ratio and the structural statistical analysis of the circular structures formed by the puffer fish on the seabed was examined. In addition, the modeling outputs of the hydrostatic pressure distribution of this structure, which has traces of golden ratio, are given.

Historical Origins and Development of the Golden Ratio

The golden ratio is the most mysterious and impressive number according to scientists in history. It is believed that the golden ratio concept underlies most events in our daily lives, from why a design or person attracts our interest to the behavior of biological organisms in nature. As a result, the concepts of the golden ratio and its sub-concepts, such as the golden angle and golden rhythm, are extremely exciting and interesting to researchers (Nematollahi et al., 2020).

$$\varphi = \frac{\sqrt{5}+1}{2} = 1.61803\ldots \varphi^2 = \varphi + 1 \quad (1)$$

The illustration of the golden ratio with the line segment is given in Figure 4.

The golden ratio was first mentioned in Euclid's book called *Elements* in 300 BC. He presented a line by dividing the 0.6180399 points as the extreme and mean ratio and this value is called the golden ratio conjugate (Meisner, 2018). The relationship between the golden ratio and golden ratio conjugate (ϕ) is given in Equation 2.

$$\phi = \frac{1}{\varphi} = \frac{\sqrt{5}-1}{2} = 0.61803398\ldots \quad (2)$$

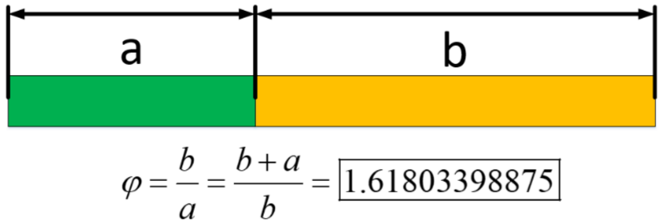

Figure 4. The golden ratio and line segment

Later, Euclid proved the connections of the golden ratio through the pentagram. The illustration of a golden ratio pentagram is given in Figure 5 and the relationship between distance is given in Equation 3 (Livio, 2008).

$$\frac{d}{c} = \frac{c}{b} = \frac{b}{a} = \varphi = 1.61803398875\ldots \quad (3)$$

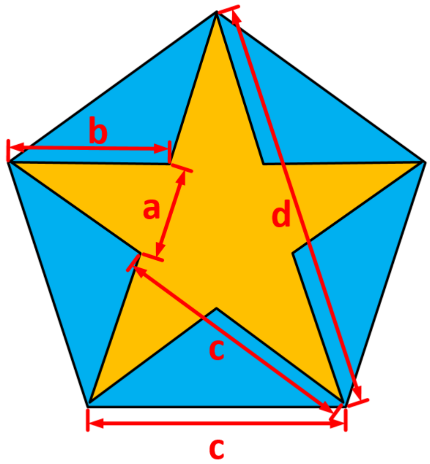

Figure 5. Golden ration and pentagram

The famous mathematician Muhammad ibn Musa al-Khwarizmi divided it into 2 parts of 10 units in length by the golden ratio in his book (The Compendious Book on Calculation by Completion and Balancing). Muhammad ibn Musa al-Khwarizmi's works in this area inspired Leonardo Fibonacci, who invented the Fibonacci series, the well-known mathematical series in history (Aksoy, 2016). The adventure of the golden ratio became even more impressive with the discovery of the Fibonacci series. After that the discovery, the charming relationship between the golden ratio and the Fibonacci series was found (Fletcher, 2006). The relationship between the Fibonacci series and the golden ratio is given in Equation 4. As shown in the equation below, the ratio between the last and previous elements of the Fibonacci series converges to the golden ratio, and the convergence becomes more obvious as the number of elements increases. ($144/89 \approx \varphi$).

Table 1 shows the process of convergence of the Fibonacci series elements to the golden ratio.

$$\underbrace{\mathit{FibonacciSequence} \quad 0,1,1,2,3,5,8,13,21,34,55,89,144,233,377,610,\ldots} \qquad (4)$$

$$\begin{array}{lll} 3 \times 0.618 \approx 2 & 13 \times 0.618 \approx 8 & 55 \times 0.618 \approx 34 \\ 5 \times 0.618 \approx 3 & 21 \times 0.618 \approx 13 & 89 \times 0.618 \approx 55 \\ 8 \times 0.618 \approx 5 & 34 \times 0.618 \approx 21 & 144 \times 0.618 \approx 89 \end{array}$$

Table 1. Golden ratio and Fibonacci series

F_{n+1}	F_n	$\frac{F_{n+1}}{F_n}$
2	1	2
3	2	1.5000
5	3	1.6666
8	5	1.6000
13	8	1.6250
21	13	1.6154
34	21	1.6190
55	34	1.6176
89	55	1.6182

According to studies of the early 1500s, Da Vinci is thought to be the first person to describe the concept of golden ratio with Vitruvian Man as a relationship between golden ratio and human body in literature (Iosa et al., 2018).

In the 17th century, Johannes Kepler discovered that the planets have an elliptical orbit in the solar system and that there are traces of the golden ratio in these orbits (Sugimato, 2021). In literature, the term gold in the golden ratio was first used in the 18th century by the mathematician Martin Ohm in his book named *the Pure Elementary Mathematics* (Meisner, 2018).

Greek letter phi (φ) is used by mathematician Mark Barr at the begging of the 20th century. This number, which has found a place in numerous studies since the day it was discovered and used to open the door to beauty and the spiritual world, is frequently used as the golden ratio due to its unique properties. Although it is not mentioned as much as the golden ratio in the literature, the golden angle which is derived from the golden ratio appears in many different fields such as the arrangement of the leaves of the plants (Strauss et al., 2020). The golden angle is obtained by dividing the 360 degrees of a circle with the golden ratio and as a result of this calculation, circle segments with values of approximately 222.5° and 137.5° degrees are obtained (Lüttge & Souza, 2019). The calculation of the golden angle is given in Equation 5.

$$\frac{b}{a} = \frac{b+a}{b} \approx 1.618 \Rightarrow \frac{360^0}{b} \approx 1.618 \Rightarrow \boxed{b \approx 222.5}^0 \qquad (5)$$
$$a + b = 360^0 \Rightarrow a + 222.5^0 \approx 360^0 \Rightarrow \boxed{a \approx 137.5^0}$$

The illustration of the golden angle which is derived from the golden ratio is given in Figure 6.

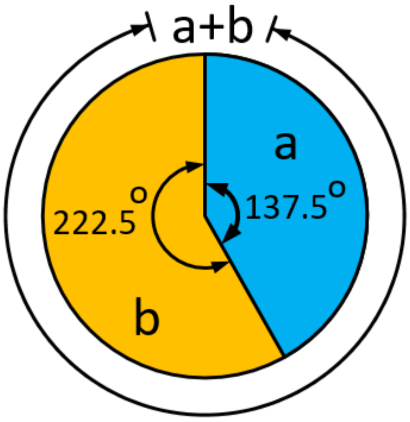

Figure 6. Golden angle

Another geometric shape that is associated with the golden ratio is the Fibonacci (golden) spiral. The golden ratio is the growth factor in the Fibonacci spiral, which is a logarithmic spiral. Every quarter-turn of the golden spiral increases its size by a factor of golden ratio (φ).In nature, the trails of the Fibonacci spiral can be found everywhere, even the universe is full of evidence of the Fibonacci spiral (Herrmann, 2018).

The curves of human ear shape or human bone structure or from the spirals of the nautilus seashell to galaxies contain evidence about the structure of Fibonacci spirals (Akhtaruzzaman & Shafie, 2011).

The illustrations of Fibonacci spiral are given in Figure 7 (Fletcher, 2006).

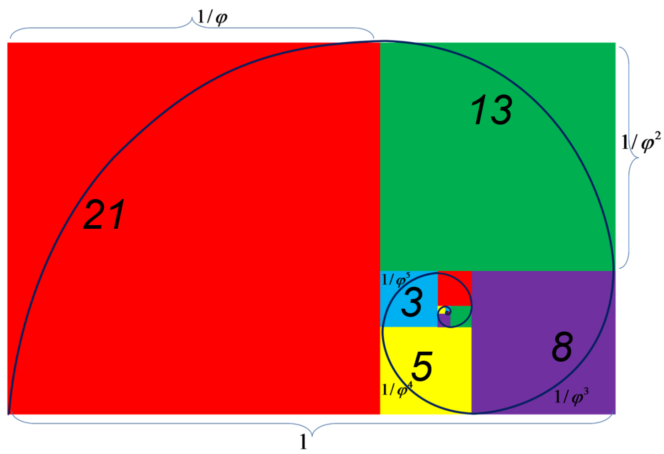

Figure 7. Fibonacci spiral

Samples of examples of the Fibonacci spiral in nature and the human body are given in Figure 8.

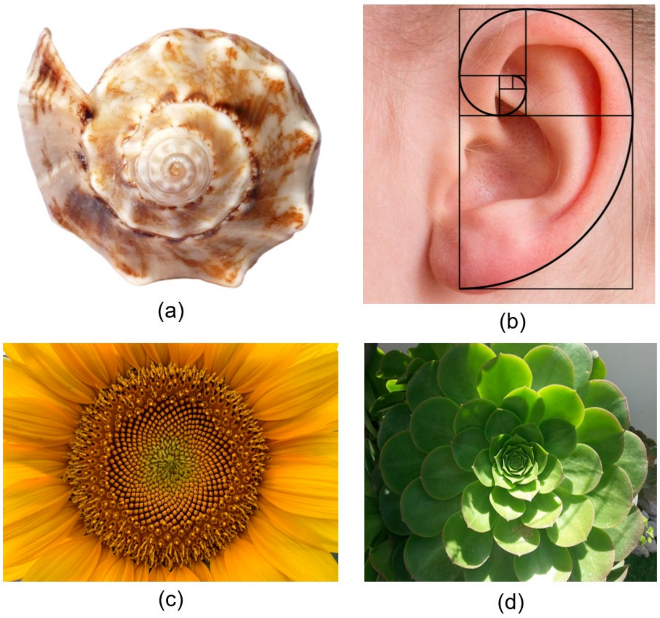

Figure 8. Samples of golden ratio in nature (a) *Turbo marmoratus* (b) Human ear *(Pinna)* (c) *Helianthus annuus* (d) *Aeonium glandulosum*

In addition to the many golden ratio traces found in nature, new structures containing the golden ratio are being discovered every day. Some of these discoveries are on our planet, some in the universe. The shape of a recently discovered fish nest was analyzed according to the golden ratio in this study. The name of this fish is the pufferfish and it has great ability to build a huge structure according to its own size on the seafloor and it is also called white-spotted pufferfish (*Torquigener albomaculosus*) in literature (Matsura, 2015).

Pufferfish (*Torquigener albomaculosus*)

The species of pufferfish named after its behavior, live in the Indian and Pacific oceans and general scientific name is known as "Torquigener" and the name of the fish whose behavior was examined and structured is defined as *Torquigener albomaculosus*. Figure 9 shows an image of male pufferfish. Male pufferfish can reach a maximum length of 12 cm, while females can reach a maximum length of 9.1 cm. (Matsura, 2015). This species is remarkable for its ability to construct huge circular structures on the seafloor and these impressive structures are vital in the decision-making process of female pufferfish during partner selection. (Schaedelin & Taborsky, 2009).

Figure 9. *Torquigener albomaculosus*, (a) lateral view, (b) dorsal view, (c) underwater photographs of *Torquigener albomaculosus* (d) construction stage, size (cm):8.8 SL, 10.9 TL, locality: Ryukyu Islands, Katetsu Cove, Amami-oshima Island, Male, NSMT-P 118118, holotype. Photo by Satoru N. Chiba.

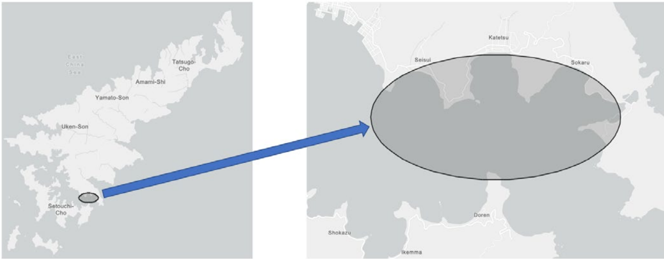

Figure 10. Map of *Torquigener albomaculosus* data collection locations (Ryukyu Islands, Katetsu Cove).

Circular Structures on the Seafloor

In 1995, constructions that are circular were discovered around the island of Amami-Oshima in southern Japan. The emergence of the associated structures is mysterious in the first stage, and it is assumed to be revealed by an organism or natural phenomenon, and these circular shapes are known as mystery circles. For a long time, these round constructions with a diameter of about 2 meters remained a mystery (Matsura, 2015). It was determined that these 2-meter circular structures were built around 2011 by puffer fish, a 12 cm pufferfish species, at depths of 10 to 27 meters. On both sides and with round grooves, related structures of varied shapes are constructed. The objective of these structures developed on the sea floor has been thought to be to impress opposite-sex fish (Kawase et al., 2013).

Underwater photographer Yoji Okata captured comprehensive images of this fish species' construction of circular formations in 2011. In 2012, Kawase and his colleagues recorded the creation of mysterious circular shapes belonging to this fish species and this detailed study formed the basis of our study by providing information about the emergence procedures of circular structures and the dimensions of the circular structure formed on the seafloor (Kawase et al., 2013).

Figure 11 illustrates the circular architecture made by male pufferfish on the seafloor. Inner and outer circles, sand peaks, and a pattern unique to each fish are all part of the circular structure that was built (Mizuuchi, 2018). The circular structure is formed after the separate construction stages of these different parts (Kawase et al., 2017). Females only visit these circular structures at the final stage, when the structure emerges. At the final stage, the male pufferfishes were seen decorating the radially aligned peaks with pieces of shell and coral at the final stage when the circular structure is fully revealed. It has been observed that the frequency of behavior at this stage has been significantly higher than during the other building processes. When a female pufferfish approaches a circular structure, the male mixes the sand particles in the middle of the structure. When the female enters the circular structure, the male advances away from the center zone and rush to her several times (Kawase et al., 2013).

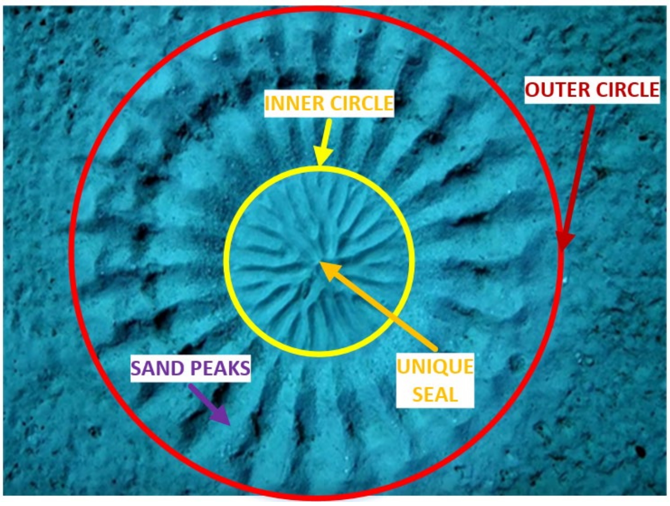

Figure 11. Circular structure of pufferfish and zones

Kawase et al. (2013) examined the developmental processes of 10 male pufferfish's circular structures. Each fish takes 7-9 days to develop the circle structure, and the circular structures formed by each fish vary. The water flow rate in the center was reduced by nearly 25% because of the circular structure's architecture. (Kawase et al., 2013). This is due to the unique seal structure at the center of the circular construction and made of fine sand, can be observed more permanently by the opposite sex. Figure 12 shows and describes circular structures (Catalbas & Gulten, 2022).

As a consequence, on the seafloor, a unique circular structure with a diameter of roughly 2 meters was created. With the influence of discharge following ovulation, this circular shape collapses and flattens (Kawase et al., 2014). The symbols of the puffer nests in the completed process are examined in this study. In Figure 13, the radius of the inner and outer circles is calculated using the pixel-based distance measurement method (Test 1).

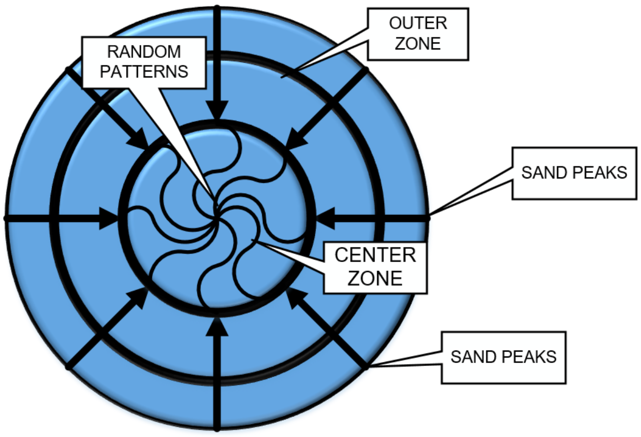

Figure 12. The illustration of the circular structure according to zones

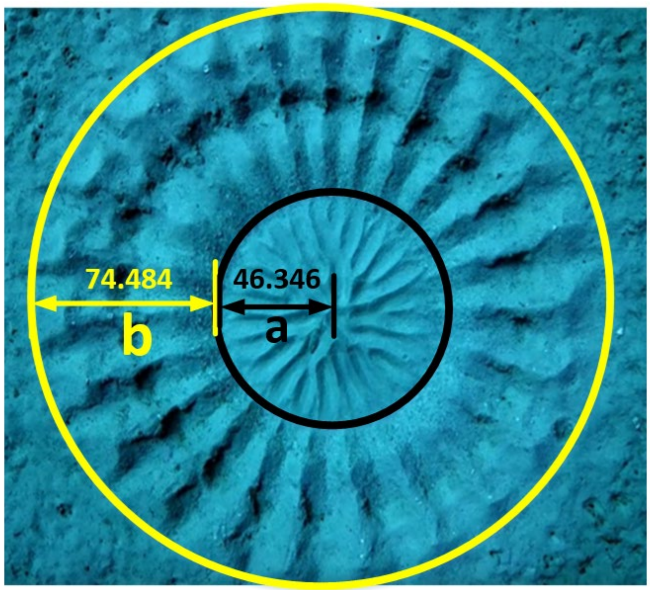

Figure 13. Golden ratio and pufferfish nest (*Test 1*)

When the radius of the circle of the pufferfish circular structure, which has reached the final stage, are examined, it is seen that the determined ratio converges to the golden ratio and it is given in Equation 6.

$$\varphi = 1.6180339 \ldots \qquad (6)$$
$$Ratio(r) = \frac{b}{a} = \frac{74.484}{46.346} = \boxed{1.60712899}$$

In addition, the histogram equalization approach was applied to the images to make the details in the circular structure images more evident, as well as the traces of the inner and outer circle (Dhal et al., 2021). Figure 14 shows the input image of the circular structure and the histogram equalization result according to this input image (Test 2).

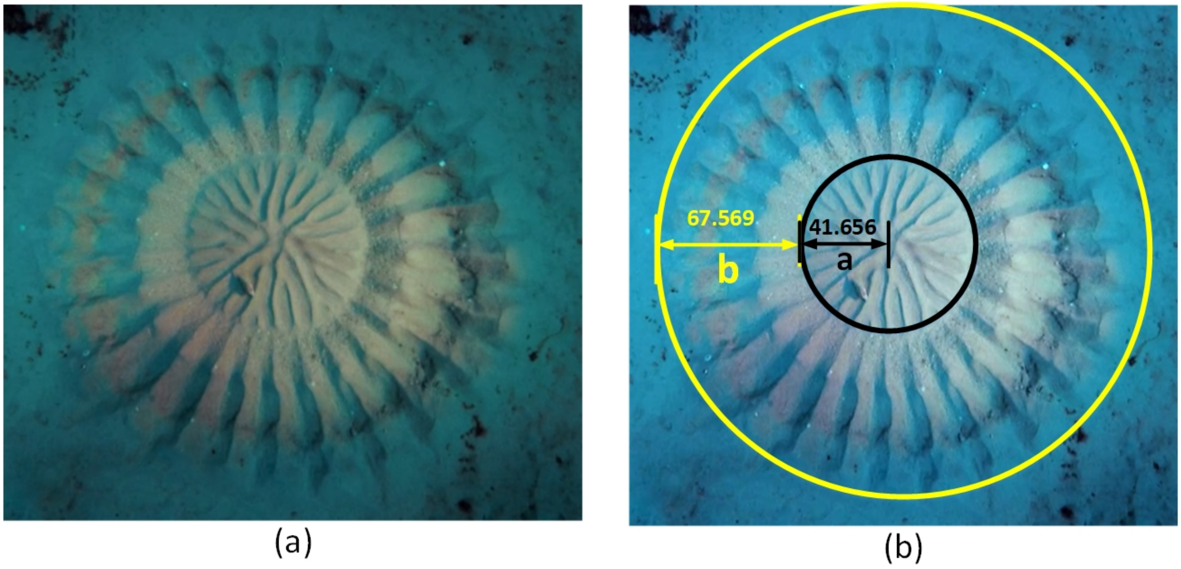

Figure 14. (a) Input image (b) Enhancement image and radius values (*Test 2*)

Table 2. Numerical parameters about pufferfish circular structure

Number of Pufferfish Nest	Number of Peak and Valley Pairs	Radius of Outer Circle (cm) (b+a)	Radius of Inner Circle (cm) (a)	Parameter (cm) (b)	Ratio ($r = \frac{b}{a}$)
S_1	24	82	32	50	**1.5625**
S_1	32	83	35	48	**1.7291**
S_3	24	80	31	49	**1.5806**
S_4	29	79	34	45	*1.3235*
K_1	30	100	50	50	*1*
K_2	27	105	40	65	**1.625**
Test 1	24	120.83	46.346	74.484	**1.6071**
Test2	26	109.225	41.656	67.569	**1.622**

Two circular structure images were analyzed in terms of radius ratio within the scope of this study and the results of these studies are indicated as Tests 1 and 2. Table 2 shows the results of another study in which these circular structures were investigated, as well as the results of the images analyzed in this study (Kawase et al., 2013). Table 2 shows the statistical analysis of the circular structures of the appropriately developed pufferfish nest used in this study. The inner and outer radii of the circular structures in the table are obtained from previous literature and image processing results. The distance of the outer circular structure to the inner circular structure and the distance of the inner circular structure to the center were analyzed. In these processes, distance in cm and pixel values were considered.

Figure 15. 3D model of pufferfish circular structure according to golden ratio

As a result of this analysis, 6 of the 8 radius ratios of circular structures were found to be similar to the golden ratio. It was determined that the average of the radius ratios of these 6 circular structures was 1.621. The average of radius ratios was determined as 1.614 in two test images that were especially improved with the histogram equalization method and whose radius ratios were determined on a pixel-basis measurement method. When these two results are viewed integrally, it is possible to find traces of the golden ratio in the radius ratios of these circular structures' inner and outer circles. Additionally, the special circular structure was realized as a 3D model by using real parameters in the construction processes of circular structures in this work. The 3D model of the circular structure obtained as a result of using parameters similar to the golden ratio is given in Figure 15. The aim of building this 3D model is to investigate the behavior of a circular structure with parameters on the seafloor.

With this 3D model created, the hydrostatic pressure on the seafloor and its effects on the structure have been analyzed in detail and it is given in Figure 16.

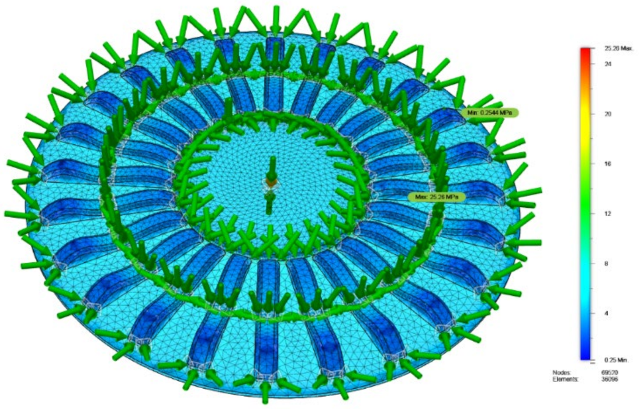

Figure 16. Simulation results of hydrostatic pressure on 3D model pufferfish circular structure

There are various processes involved in the construction of these impressive circular structures. These lengthy construction processes can only be realized if the integrity of the circular structures is preserved at all stages. It should also be noted that the female puffer only visits the circular structure after it has been fully revealed and decorated. As a result, this structure, which was produced on the seafloor and under variable conditions, must be highly resistant to external influences and preserve its integrity. Within the scope of this study, the images of these interesting circular structures were analyzed and reconstructed in a 3D model. When the circular structure was reconstructed as a 3D model depending on these ratios, it was observed that the hydrostatic pressure was uniformly distributed over the structure. As a result, a structure forms that can preserve its structural integrity until the construction processes are finished.

Conclusion

In this work, the morphological analysis of the impressive circular structures created by the male pufferfish to impress the opposite sex has been realized. The process of building these structures, which have a diameter of around 2 meters and are built on the seafloor, is divided into several stages. The process of constructing circular structures takes around 9 days, and the female pufferfish only visits these structures close to the end. The finishing and decoration work is completed by male pufferfish at the last stage. Throughout this process, the circular structures consistency and structural stability must be kept against altering seafloor conditions. When the development process of these structures was examined in detail, it was observed that the architectural ratios were not random. The histogram equalization approach was used in this work to bring out the features in images of two different circular structures, and the radius values of the inner and outer circles of these structures were determined using a pixel-based measuring method. The average value of the ratio between the distances of the circular structures of the pufferfish examined in this study and showing appropriate development is 1.621. The percentage error rate between the values in this calculation and the golden ratio is 0.185% and the absolute error value is 0.003. In the view of these error rates, it can be said that the circular structures of the pufferfish have strong traces of the golden ratio value. The fact that there are such strong traces of the golden ratio in the circular structures of the puffer fish, one of the most impressive structures on the seafloor and recently discovered, may lead to inferences that the traces of the golden ratio in nature may be more than expected. Additionally, the circular structures of pufferfish were reconstructed as a 3D model in this work using real-world characteristics. It has been observed that the 3D-modelled circular structures have an advantageous structure against the distribution of hydrostatic pressure on the seafloor.

Compliance With Ethical Standards

Conflict of Interest

The author declares that there is no conflict of interest.

Ethical Approval

For this type of study, formal consent is not required.

Data Availability Statement

All data generated or analyzed during this study are included in this published article.

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