

Comparison of metaheuristics on multi objective (cost&CO₂) optimization of RC cantilever retaining walls

Betonarme konsol istinat duvarların çok amaçlı(maliyet ve karbondioksit) optimizasyonunda meta-sezgisel yöntemlerin karşılaştırılması

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Abstract

In this study, performance of meta-heuristic methods on optimum design of reinforced concrete (RC) retaining wall has been investigated with respect to minimizing the cost and the CO₂ emission. Biogeography Based Optimization (BBO) and Social Spiders optimization (SSO) methods utilized for investigation. The minimizations of the cost, the CO₂ emission and multi-objective of the cost+CO₂ functions are described as objective functions of the optimization problem. There are thirteen design variables are defined in the optimization problem. Eight of these variables are the cross sectional dimensions of the retaining wall. The other five design variables are the reinforcement detailing of wall members. Flexural and shear strength requirements, minimum and maximum cross section areas of the reinforcement bar, the requirement length for reinforcement details and the factor of safety for failure modes are defined as constraints functions of the optimization problem. The Flexural and shear strength requirements, minimum and maximum limitations of the reinforcement bar areas are adopted from American Concrete Institute (ACI 318-14) design code. In order to test performance of the presented optimization methods literature design examples are used. In addition, efficiency of steel and concrete classes on optimum CO₂ emission and cost have been investigated by using different steel and concrete classes.

Keywords: Biogeography, Social spider, Optimization, Reinforced concrete retaining walls, Sustainable design

Öz

Bu çalışmada, meta-sezgisel optimizasyon yöntemlerinin betonarme konsol istinat duvarlarının minimum maliyet ve CO₂ salınımına göre optimum tasarımı problemindeki performansları araştırılmıştır. Araştırma için, biyo-coğrafya tabanlı ve sosyal örümcek optimizasyon algoritmaları kullanılmıştır. Minimum maliyet fonksiyonu, minimum karbondioksit salınımı fonksiyonu, minimum maliyet ve karbondioksitin salınımını içeren çok amaçlı fonksiyon optimizasyon probleminin amaç fonksiyonları olarak tanımlanmıştır. Optimizasyon probleminde on üç adet tasarım değişkeni tanımlanmıştır. Bunlardan sekiz tanesi istinat duvarının en kesitini oluşturan değişkenlerdir. Diğer beş tanesi ise duvar elemanların donatı detaylandırmasıdır. Eğilme ve kesme kapasite sınırlayıcıları, minimum ve maksimum donatı alanları, donatı detaylandırılmasında gerekli donatı uzunlukları ve göçme modların güvenlik katsayıları optimizasyon probleminin tasarım sınırlayıcıları olarak tanımlanmıştır. Eğilme ve kesme kapasitesi sınırlayıcıları ve donatı alanlarının minimum ve maksimum sınır değerleri Amerikan beton enstitüsü (ACI 318-14) tasarım şartnamesinden alınmıştır. Sunulan optimizasyon yöntemlerinin performanslarını test etmek için literatürdeki tasarım örnekleri kullanılmıştır. Buna ek olarak farklı beton ve çelik malzemeleri kullanılarak malzeme sınıflarının optimum CO₂ salınımı ve maliyeti üzerindeki etkinliği araştırılmıştır.

Anahtar kelimeler: Biyo-coğrafya, Sosyal örümcek, Optimizasyon, Betonarme istinat duvarları, Sürdürülebilir tasarım

1 Introduction

RC cantilever retaining wall is one of the favorite type of retaining structure. Although concrete material seems to be less expensive than steel, it causes more CO₂ emissions to environment and also global warming. Thus, minimizing CO₂ emissions and cost should be considered in the optimum design of the RWs. However, reaching these objectives is difficult as discrete design variables and nonlinear functions are included in the optimization problem. Stochastic search techniques are great tools for the solution of the optimization problem. Ant colony optimization, hunting, particle swarm, firefly and bat algorithms are popular stochastic search techniques that have been mostly used in structural optimization problems for 10 years.

The BBO method [1] is the recent stochastic search technique which mimics the theory of island biogeography. The theory consist two main behaviors. These are speciation (the determine performance of new animals), the extinction of animals and the migration of animals between islands. Despite being the recent optimization algorithm, the BBO has been used

in many optimization problems such as: economic dispatch solution [2], power flow problem [3], cognitive radio systems [4], security audit trail analysis [5], satellite remote sensing images [6], AC transmission system devices [7], approach for segmentation of human head [8], profit maximization of a generation company [9], flexible job shop scheduling problem [10], mirrored traveling tournament problem [11]. However, there is no major study about application of BBO for optimum design of RC structures. Hence, this study is an original study as it includes application of BBO in the optimizations of the RC structures.

A new stochastic search algorithm and an innovative approach called Social Spider Optimization (SSO) technique has been developed in 2013 by adopting movement and mating behaviors of spider colony [12]. Despite being the new technique, the SSO algorithm has been applied on many fields such as: dispatch of thermal power unit [13], design of plug-in electric vehicle [14], wind tribune systems [15], feed forward neural networks learning [16], optical flow methods parameters [17], field weakening control of a DC motor [18] and energy theft detection systems[19]. However, any article

about an application of the SSO algorithm for RW design problems has been found in the literature. Hence, this study is the first study which uses the SSO algorithm to the RW design.

There are numerous studies on optimum design of RC retaining walls: [20]-[27]. However, few researches [28] took into account the environmental effects of retaining wall design. In structural engineering, the environmental aspects have been considered in the recent years [28]-[36]. It is concluded from these studies that RC structure designs having economically low-CO₂ emission can be obtained even from complex design problems.

Outline of the remainder of the study is described as follows. Section 2 briefly describes mathematical modeling of the optimization problem. The BBO and SSO algorithms for the RC Cantilever Retaining Walls are described in the Sections 3 and 4. Parametric study of the optimization algorithm is given in the Section 5. Details of the design examples and their results are given in Section 6. Conclusions of the study are provided in Section 7.

2 Mathematical model

Optimum design of RC retaining wall problems are defined as the selection of dimensions of retaining wall, number and diameter of reinforcements such that safety, stability and stress limitations specified by the concrete building code are satisfied. It is also necessary to consider the economic and environmental aspects in this selection. In this study, three objective functions are defined. The first is the minimization of the cost of the retaining wall which is expressed in Equation (1).

$$f_{cost}(\mathbf{X}) = C_s W_{st} + C_c V_c \quad (1)$$

Where, \mathbf{X} is the vector which contains the sequence numbers of design variables, C_s is the unit cost of steel, C_c is the unit cost of concrete, W_{st} is the weight of steel per unit length of the wall, and V_c is the volume of concrete per unit length of the wall.

The second objective function is minimization of the CO₂ emission of the retaining wall which is expressed in Equation (2).

$$f_{CO_2}(\mathbf{X}) = \sum_{i=1}^{N_{material}} A_i \rho_i E_{CO_2,i} \quad (2)$$

Where A_i , E_{CO_2} and ρ are the cross sectional area, CO₂ emission and density of the structural materials respectively. $N_{material}$ is the number of materials defined in the structure design problem. CO₂ emissions of the structural materials are adopted from literature studies [33],[36] which are shown in Table 1.

In the study, weighted aggregate of the cost and the CO₂ functions of the RC retaining walls are also considered as the objective function. Mathematical formulation of the objective function is described in Equation (3).

$$f_{aggr} = \zeta_{cost} f_{cost} + \zeta_{CO_2} f_{CO_2} \quad (3)$$

Where ζ_{cost} and ζ_{CO_2} are non-negative weights which are taken as 1 in this study [31].

Thirteen design variables are defined in this mathematical model. Eight of these variables describe geometry of the RC cantilever retaining wall and the other five design variables are the reinforcement detailing of wall members (see Figures 1 and 2). Upper and lower limits of cross section dimensions of the retaining wall are illustrated in Table 2.

Reinforcement design variables are considered as discrete design variables which are defined as $n\phi d$ (n : number of bars, d : diameter of bars): the number and diameter of the first stem reinforcement which extends to toe (R1), the number and diameter of the second stem reinforcement which extends to toe (R2), the number and diameter of additional toe reinforcement together (R3), the number and diameter of heel reinforcement together (R4), and the number and diameter of key reinforcement together (R5). Pool of reinforcement design variables are shown in Table 3.

Table 1: Unit CO₂ emissions and unit price of the structural materials.

Material	Strength	Unit Price	CO ₂ emission
Concrete	24 MPa	59.76 \$/m ³	304.75 CO ₂ /m ³
	27 MPa	62.50 \$/m ³	324.76 CO ₂ /m ³
	30 MPa	65.65 \$/m ³	344.54 CO ₂ /m ³
Steel	400 MPa	0.742 \$/kg	0.3857 CO ₂ /kg
	500 MPa	0.770 \$/kg	0.3962 CO ₂ /kg

Table 2: Lower and upper limits of cross sectional design variables [20],[24],[39],[40].

Design Variables	Lower Bound	Upper Bound
X1	0.40H	0.80H
X2	0.10H	0.60H
X3	0.20 m	0.50 m
X4	0.20 m	0.40 m
X5	0.20 m	0.3H
X6	0.5H	0.8H
X7	0.20 m	0.40 m
X8	0.20 m	0.90 m

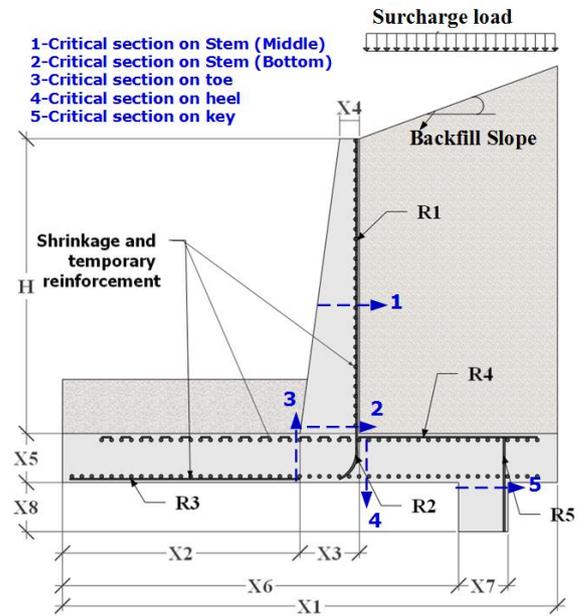


Figure 1: Design variables of the retaining wall.

There are thirty one constraint functions defined in the optimization. First three of them can be grouped into stability constraint functions which are described as: overturning, sliding and bearing capacity constraint functions in (4)-(6). The fourth constraint function is defined from the "no tension" condition (see Equation (7)).

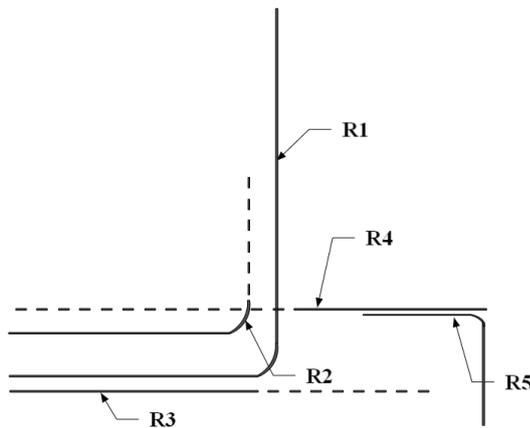


Figure 2: Reinforcement description.

Table 3: Reinforcement design variables.

#	Value	#	Value
1	No Bar	34	3φ30
2	1φ10	35	3φ32
3	1φ12	.	.
4	1φ14	.	.
.	.	78	7φ30
.	.	79	8φ10
.	.	.	.
13	2φ10	100	9φ30
14	2φ12	101	10φ10
.	.	.	.
.	.	109	10φ26
23	2φ30	110	10φ28
24	3φ10	111	10φ30

$$g_1(\mathbf{X}) = \frac{FS_{\text{overturning}}}{FS_{\text{provided for over}}} - 1 \geq 0 \quad (4)$$

$$g_2(\mathbf{X}) = \frac{FS_{\text{sliding}}}{FS_{\text{provided for slide}}} - 1 \geq 0 \quad (5)$$

$$g_3(\mathbf{X}) = \frac{FS_{\text{bearing}}}{FS_{\text{provided for bear}}} - 1 \geq 0 \quad (6)$$

$$g_4(\mathbf{X}) = q_{\min} \geq 0 \quad (7)$$

The 5th-14th constraint functions, defined in the optimization problem, are capacity constraint functions which are formulated in Equation (8) and Equation (9),

$$g_{5-9}(\mathbf{X}) = \left(\frac{M_d}{M_u} \right)_{\text{Critical section}} - 1 \leq 0 \quad (8)$$

$$g_{10-14}(\mathbf{X}) = \left(\frac{V_d}{V_u} \right)_{\text{Critical section}} - 1 \leq 0 \quad (9)$$

where M_u is the ultimate resistance moment, M_d is the design moment, V_u is the ultimate shear capacity, V_d is the design shear force and *Critical section* refers to either the stem, toe, heel, and key of the retaining wall (see Figure 1). Reinforcement arrangement constraint functions are also considered in this optimization problem which is described as:

$$g_{15-19}(\mathbf{X}) = \left(\frac{A_s^{\min}}{A_s} \right)_{\text{Critical section}} - 1 \leq 0 \quad (10)$$

$$g_{20-24}(\mathbf{X}) = \left(\frac{A_s}{A_s^{\max}} \right)_{\text{Critical section}} - 1 \leq 0$$

where A_s^{\min} and A_s^{\max} are the minimum and maximum reinforcement areas defined in code [37]. The last five constraint function groups are the geometric limitation functions of size, reinforcement bars and clear cover limitation of the retaining wall which are formulated in (11)-(16).

$$g_{25}(\mathbf{X}) = \frac{X_2 + X_3}{X_1} - 1 \leq 0 \quad (11)$$

$$g_{26}(\mathbf{X}) = \frac{X_6 + X_7}{X_1} - 1 \leq 0 \quad (12)$$

$$g_{27}(\mathbf{X}) = \frac{L_{db}}{X_5 - (2.c)} - 1 \leq 0 \quad (13)$$

or

$$g_{28}(\mathbf{X}) = \frac{l_{db}}{X_1 - X_2 - C} - 1 \leq 0 \quad (14)$$

and

$$g_{29}(\mathbf{X}) = \frac{12d_b}{X_5 - C} - 1 \leq 0$$

$$g_{30}(\mathbf{X}) = \frac{S_{\min}}{S_{\text{net}}} - 1 \leq 0 \quad (15)$$

$$g_{31}(\mathbf{X}) = \frac{S_{\text{net}}}{Sp_{\max}} - 1 \geq 0 \quad (16)$$

where L_{db} is the minimum development length; L_{dh} is the minimum hook development length; d_b is the diameter of the hooked bar; S_{net} is the clear spacing, and Sp_{\max} is the maximum clear spacing (See Figure 3).

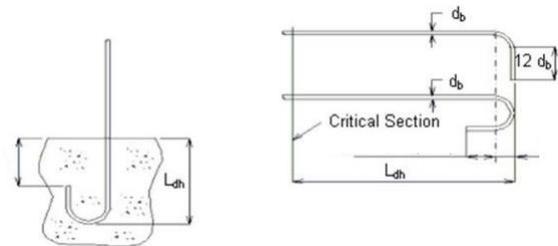


Figure 3: Reinforcement details.

3 Bio-geography based optimization algorithm

BBO algorithm is firstly introduced by D. Simon in 2008 [1], [38] by adopting the theory of island biogeography. The BBO algorithm describes the extinction and migrations of species between islands. The island is defined as an isolated area for species. Two main indexes are related to the extinction and migrations of species between islands. These are habitat suitability index (HIS) and suitability index variables (SIV). HIS parameter describes life quality of habitats in Islands. SIV index characterize habitability which can be considered as independent variables of the habitat. If the habitats have high HIS index, the islands provide good life standards to the species and a large number of them live in the habitats. These habitats have a low species immigration rate because they are already nearly saturated. This assumption is used in the BBO algorithm for carrying out migration. Relationship between number of species and rate of emigration and immigration is illustrated in Figure 4 [1].

In the Figure 4, λ is the immigration rate; μ is the emigration rate; I is the maximum immigration rate; E is the maximum emigration rate; S_0 is the equilibrium number of species and S_{\max} is the maximum number of species.

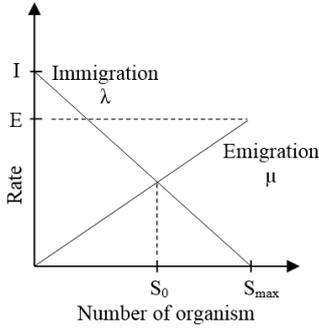


Figure 4: Species model of a single habitat.

In the BBO algorithm, the new candidate design is generated by modifying the independent design variable of the old design which is related to the immigration rate of the design variable. If an independent variable is to be modified, then the emigrating candidate design is chosen by using the roulette wheel selection method that is related to the emigration probability (see Equation (17)).

$$P(x_j) = \frac{\mu_j}{\sum_{i=1}^N \mu_i}, \quad j = 1, \dots, N \quad (17)$$

Where N is the size of population. One more factor is related to the generation of the new design called Mutation. This factor is used to increase the number of species in the islands. Mutation probability of each design is described as follows:

$$m(s) = m_{max} \left(\frac{1 - P_s}{P_{max}} \right) \quad (18)$$

Where m_{max} maximum mutation probability defined by user P_s probability of species and P_{max} is the maximum probability species.

The optimization method based on BBO algorithm tries to find the optimum geometry and reinforcement details of the RC Cantilever Retaining Walls. The steps of the optimization method are described as follows:

Step1: In first step, number of population size (N) RC cantilever retaining wall designs are generated randomly using Equation (19). Then, the designs are evaluated and penalized values of their objective functions are calculated using Equation (20).

$$x_i = x_{li} + r(x_{ui} - x_{li}), i = 1, \dots, N \quad (19)$$

$$f_p = f \cdot (1 + C)^\varepsilon \quad (20)$$

Where f is value of the objective functions described in Equations (1-3), C is the summation of constraint violations calculated using constraint functions stated by Equations (4-16), ε is penalty coefficient which is taken as 2 and r is a random number between $\{0, 1\}$. In general form, constraint violations are calculated as:

$$C_i = \begin{cases} 0 & g_i(\mathbf{X}) \leq 0 \\ g_i(\mathbf{X}) & g_i(\mathbf{X}) > 0 \end{cases} \quad i = 1, 2, \dots, NC \quad (21)$$

Where, $g_i(x)$ is the i^{th} constraint function and NC is the number of constraint functions defined in the optimum design problem.

Step2: Firstly the elite designs which have the lowest penalized objective function values are determined in this step. Then,

immigration and emigration rates of the designs are calculated as follows:

$$\mu_j = \frac{N + 1 - j}{N + 1}, \lambda_j = 1 - \mu_j, j = 1, 2, \dots, N \quad (22)$$

Step3: The immigration and the emigrations parts are performed in the step. In the immigration part, the RC cantilever retaining wall designs are updated with respect to their immigration rates by modifying the independent design variables. A change criterion for the independent design variable is described as:

$$r < \lambda_j^k, j = 1, 2, \dots, N, k = 1, 2, \dots, NDV \quad (23)$$

Where NDV is number of design variables defined in the optimization problem. In emigration part, the designs are modified by using the roulette wheel selection method that is related to the emigration probability described in the Equation (17).

Step4: The designs are mutated in this step. Mutation probabilities of the designs are calculated using Equation (18). If mutation is performed, new RC cantilever retaining wall design is generated randomly.

The steps 2 to 4 are repeated until a pre-assigned maximum number of iterations are completed.

4 Social spider optimization algorithm

Social spider optimization (SSO) algorithm is one of the newest meta-heuristic search algorithm mimics the behaviors of a spider colony. In the spider colony, male and female spiders perform different tasks called movement and mating. In the movement stage, each spider moves to new position which is related to vibrations of its and other colony members. The vibrations of the spiders depend on the gender, distance between the spiders and their weights. In the mating stage, the each male spider having higher weight (dominant males) finds the suitable female spiders in its range and generates a new spider.

The main steps of the SSO algorithm for the optimum design of the optimization problem are described as follows;

Step 1: Initial parameters of the SSO algorithm, which are the number of female spiders (N_f) and the number of male spiders (N_m), are determined in this step using equations (24) and (25) respectively.

$$N_f = \text{round}((0.9 - 0.25 \cdot r)N_s) \quad (24)$$

$$N_m = N_s - N_f \quad (25)$$

Where, round is a function which rounds to the value of the nearest integer.

Step 2: Initial retaining wall designs, assigned to the female (f_{ij}) and the male (m_{ij}) spiders, are generated randomly using equations (26) and (27). Then, penalized objective function values of the designs (f_p) are calculated using equation (20).

$$f_{ij} = x_{li} + r(x_{ui} - x_{li}) \quad i = 1, \dots, N_f, j = 1, \dots, NDV \quad (26)$$

$$m_{kj} = x_{li} + r(x_{ui} - x_{li}), k=1, \dots, N_m, j=1, \dots, NDV \quad (27)$$

Step 3: After the evaluation process, the spider having the lowest objective function value and called the best spider (S_b) and the spider having the highest objective function value and

called the worst spider (S_w) are determined. Then, the weights of the spiders are calculated as follows:

$$w_i = \frac{f_{high} - f_i}{f_{high} - f_{low}} \quad i = 1, \dots, N_s \quad (28)$$

Where, f_{high} , f_{low} and f_i are objective function values of the worst spider, the best spider and the i^{th} spider respectively.

Step 4: In this step, all spiders move to new positions (generate new designs). In the colony, the female and the male spiders use different movement strategies given as follows:

$$f_{ij}^{k+1} = \begin{cases} f_{ij}^k + \alpha \cdot vibc_i(x_{c,j} - f_{ij}^k) + vibb_i(x_{b,j} - f_{ij}^k) + \delta(r-0.5) \leftarrow PF \\ f_{ij}^k + \alpha \cdot vibc_i(x_{c,j} - f_{ij}^k) + vibb_i(x_{b,j} - f_{ij}^k) + \delta(r-0.5) \leftarrow 1-PF \end{cases} \quad (29)$$

$i=1, \dots, N_f, \quad j=1, \dots, n$

$$m_{ij}^{k+1} = \begin{cases} (m_{ij}^k + \alpha \cdot vibf_i(x_{f,j} - m_{ij}^k) + \delta(rand-0.5)) & \text{if } w_{n_r+i} > w_m \\ m_{ij}^k + \alpha \cdot \left(\frac{\sum_{h=1}^{N_m} m_{h,j}^k \cdot w_{n_r+h}}{\sum_{h=1}^{N_m} w_{n_r+h}} - m_{ij}^k \right) & \text{if } w_{n_r+i} \leq w_m \end{cases} \quad (30)$$

$i=1, \dots, N_m, \quad j=1, \dots, n$

Where, α , β and δ rand are the random numbers between {0, 1}; $x_{c,j}$ and $x_{b,j}$ are the j^{th} design variable of the nearest and the best spider; $vibc_i$ is the vibration between the i^{th} spider and the nearest spider to the i^{th} spider calculated using equation (31); $vibb_i$ is the vibration between the i^{th} spider and the best spider calculated using equation (32); $vibf_i$ is the vibration between the i^{th} spider and the nearest female spider to the i^{th} spider calculated using equation (33); w_{med} is the weight of the median spider; k is the iteration number; PF is the female movement parameter between {0, 1}.

$$vibc_i = 0 \quad \text{if } (w_i \geq w_c) \\ vibc_i = w_c \cdot e^{-\sum_{j=1}^n (x_{c,j} - x_{i,j})^2} \quad \text{if } (w_i < w_c) \quad (31)$$

$$vibb_i = w_b \cdot e^{-\sum_{j=1}^n (x_{b,j} - x_{i,j})^2} \quad (32)$$

$$vibf_i = w_f \cdot e^{-\sum_{j=1}^n (x_{f,j} - x_{i,j})^2} \quad (33)$$

Where $x_{f,j}$ is the j^{th} design variable of the nearest female spider; w_c , w_b and w_f are the weights of the nearest spider, the best spider and the nearest female spider respectively. After the movement, the new designs are evaluated, their penalized costs are calculated using equation (20) and the colony is updated.

Step 5: In this step, the mating is performed by the dominant male spiders and the female spiders within the range of the dominant spiders. The dominant male spiders are determined by selecting male spiders whose weights are heavier than weight of the median spider. The female spiders in the range of the dominant male spiders are determined using following conditions:

$$\text{if } \sum_{j=1}^n (x_{m,j} - x_{f,j})^2 \leq \frac{\sum_{j=1}^n (x_{u,j} - x_{l,j})^2}{2 \cdot NDV} \quad (34)$$

$$m=1, \dots, N_{Dm}, \quad f=1, \dots, N_f$$

Where, $x_{m,j}$ is the j^{th} design variable of the m^{th} dominant spider; $x_{f,j}$ is the j^{th} design variable of the female spider; and N_{Dm} is the number of dominant male spiders. If there are no female spiders in the range of the dominant male spiders, mating operation is not performed for the dominant male spider. After determination of female spiders, the new design is generated.

Then, the new design is evaluated, its penalized cost is calculated using equation (20). If cost of the new design is less than the worst design in the colony, the worst design is replaced with the new design and the colony is updated.

Step 6: The termination criteria, which is the reaching maximum iteration number, is checked. If the termination criteria are satisfied, the algorithm is stopped. Otherwise, steps 3 to 6 are repeated.

5 Parametric study of the optimization algorithms

The selection of the search parameters values is considerably vital on the performance of the optimization algorithms. Thus, parametric study is demanded to find suitable values of the search parameters. The cost optimization the 3.5 m height cantilever RW is selected for the parametric study. Detail of the structure is described in section 6.1. The RW is optimized by using the optimization algorithms with different values of the search parameters: $N=25, 50$ and 100 , mutation probability= $0.1\%, 0.5\% 1\%$ and 5% and number of elite population= $0.04 \cdot N, 0.1 \cdot N$ for the BBO algorithm; $N_s=25, 50$ and $100, PF=0.3, 0.4, 0.5, 0.6, 0.7$ and 0.8 for the SSO algorithm. In each test, the example is optimized 50 times using different seed values. The average and best optimum costs obtained from these tests are illustrated in Tables 4 and 5. According to the tables, the most convenient search parameters are bolded in the tables.

Table 4: Minimum cost values for the BBO algorithm with respect to different internal parameters.

Elite D.		Mutation Probability					
		N	0.1%	0.5%	1%	5%	10%
0.04*N	Best	25	116.1	115.2	116.3	114.3	114.7
		50	114.4	114.4	114.6	114.1	113.7
		100	113.9	114.3	113.7	113.9	113.8
	Mean	25	120.6	120.2	121.1	119.5	118.8
		50	118.9	118.7	119.1	117.4	116.6
		100	117.1	116.8	116.3	116.6	116.5
0.1*N	Best	25	115.5	115.8	114.8	116.3	113.9
		50	114.5	114.7	114.6	114.4	113.8
		100	115.3	114.2	113.9	114.2	113.8
	Mean	25	121.6	122.1	121	120.8	118.3
		50	118.1	119.8	118.2	118.2	117.1
		100	117.9	117.3	116.8	117.1	116.8

Table 5: Minimum cost values for the SSO algorithm with respect to different internal parameters.

PF	N_s						
	0.3	0.4	0.5	0.6	0.7	0.8	
Best	25	117.8	117	119.1	117.1	118.2	114.6
	50	119.2	116.3	119.1	119.8	119.8	118.1
	100	123.9	117.1	120.3	119.8	118.5	118.9
Mean	25	122.2	117	120.9	117.9	119.4	116.8
	50	123.3	118.8	120.7	120.8	122.1	119.4
	100	124.4	122	122	124.9	121.4	124.1

6 Design examples

6.1 Case 1: Comparison of metaheuristics on cost optimization of retaining walls

In this case, two design examples (3.5 m and 5.2 m Height Retaining Walls) are solved using presented algorithms which

are previously used in literature [24],[39],[40]. Only cost optimizations are performed in these design examples and obtained results are compared to literature results (Harmony Search (HS) [24], Classical Firefly Algorithm (CFFA) and Adaptive Firefly Algorithm (AFFA) [39]). The input data of the examples are shown in Table 6. The search parameters of the literature algorithms are illustrated in Table 7.

Table 6: Search parameters of literature algorithms.

Algorithm	Parameter	Ex.1.	Ex.2
HS	HMS	15	20
	HMCR	0.9	0.75
	PAR	0.3	0.4
	NFF	80	100
CFFA	B ₀	0.5	0.5
	ACoeff	10	10
	RP	0.4	0.4
	NFF	80	100
AFFA	B ₀	0.5	0.5
	ACoeff	10	10
	RPmin	0.1	0.1
	RPmax	0.8	0.8

Shrinkage and temporary reinforcement area are computed as 0.002% of the cross-sectional area of the retaining wall and the length of these bars is taken as 100 cm (per meter). The design examples are optimized 50 times using different seed values. After 50 runs, the average costs and corresponding standard deviations on optimized costs are illustrated in Table 8. Design variables and cost details of the best optimum design and literature results are illustrated in Tables 8-11. The search histories of the best optimum designs of each algorithm are shown in Figures 5 and 6.

In First example, the minimum cost is obtained as \$113.67 by utilizing the BBO algorithm. This value is 3.79% less than the cost of HS's optimum design, 0.69% less than the cost of AFFA's optimum design, 0.82% less than the cost of SSO's optimum design and 15.1% less than cost of CFFA's optimum design. In the second example, the best design obtained using the SSO algorithm (\$171.89). It is also remarked that all obtained designs satisfy design limitations described in section 2.

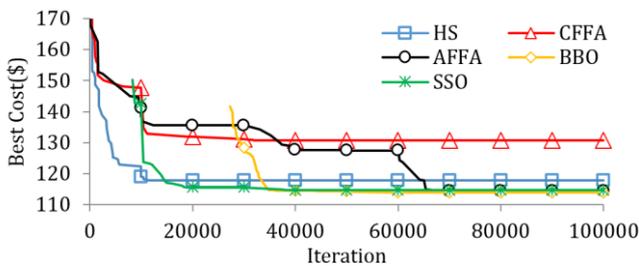


Figure 5: Search histories of the best design for the ex. 1*.

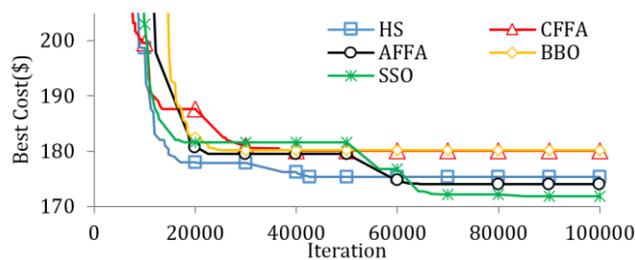


Figure 6: Search histories of the best design for the ex.2*.

*:Search history data of other algorithms are taken by authors of the literature.

Table 7: Statistical results of examples.

		Best	Average	St. Deviation
Ex.1	BBO	113.93	118.70	3.71
	SSO	114.6	117.00	1.92
Ex.2	BBO	180.16	198.40	13.21
	SSO	171.89	182.16	10.16

Table 8: Cost details of the optimum designs for the ex. 1.

Cost Details	HS[24]	BBO	SSO	CFFA[39]	AFFA[39]
Vol.conc.(m ³)	2.03	1.89	1.94	2.17	1.93
Weight _{st} .(kg)	92.20	95.21	77.47	110.05	93.24
Cost _{conc} (\$)	81.10	75.58	92.82	86.78	77.17
Cost _{st} (\$)	36.88	39.09	37.13	44.02	37.29
Cost _{Total} (\$)	117.98	113.67	114.60	130.80	114.46

Table 9: Optimum values of design variables for the ex.1.

Des. Var.	HS[24]	BBO	SSO	CFFA[39]	AFFA[39]
X ₁	2.70	2.61	2.63	3.26	2.91
X ₂	1.60	1.42	1.48	1.47	1.46
X ₃	0.35	0.32	0.34	0.46	0.46
X ₄	0.20	0.20	0.20	0.20	0.20
X ₅	0.35	0.31	0.32	0.35	0.34
X ₆	1.90	2.40	2.40	2.80	2.61
X ₇	0.20	0.20	0.20	0.20	0.20
X ₈	0.60	0.84	0.76	0.20	0.64
R ₁	10φ10	3φ18	10φ10	2φ26	5φ16
R ₂	7φ10	9φ12	8φ10	8φ16	8φ16
R ₃	1φ10	-	1φ10	-	-
R ₄	10φ12	6φ14	9φ12	9φ14	9φ12
R ₅	7φ10	7φ10	7φ10	7φ10	7φ10

Table 10: Cost details of the optimum designs for the ex.2.

	HS[24]	BBO	SSO	CFFA[39]	AFFA[39]
Vol.conc.(m ³)	2.84	2.87	2.27	2.89	2.85
Weight _{st} .(kg)	154.62	163	202.48	160.72	150.27
Cost _{conc} (\$)	113.50	114.84	90.90	115.75	113.93
Cost _{st} (\$)	61.85	65.32	80.99	64.29	60.10
Cost _{Total} (\$)	175.35	180.16	171.89	180.04	174.03

Table 11: Optimum values of design variables for the ex. 2.

Des. Var.	HS[24]	BBO	SSO	CFFA[39]	AFFA[39]
X ₁	2.85	3.25	3.05	3.26	2.91
X ₂	1.40	1.44	1.56	1.47	1.46
X ₃	0.45	0.45	0.21	0.46	0.46
X ₄	0.20	0.20	0.20	0.20	0.20
X ₅	0.35	0.34	0.36	0.35	0.34
X ₆	2.65	2.83	2.71	2.88	2.61
X ₇	0.20	0.20	0.20	0.20	0.20
X ₈	0.75	0.31	0.60	0.20	0.64
R ₁	5φ16	5φ16	9φ16	2φ26	5φ16
R ₂	8φ16	9φ16	6φ20	8φ16	9φ16
R ₃	-	-	-	-	-
R ₄	9φ12	10φ14	8φ16	9φ14	7φ14
R ₅	7φ10	7φ10	5φ12	7φ10	7φ10

6.2 Case 2: Efficiency of steel and concrete classes on optimum CO₂ emission and cost

In this case, a new retaining wall having 4.0m height are optimized using the BBO and the SSO algorithms by considering

minimizing the cost, minimizing the CO₂ emission and minimizing both the cost and the CO₂ emission objective functions. In each optimization test, the example problem is optimized using different concrete and steel material types which are described in Table 1. The unit weight of concrete and steel are taken as 2300 kgf/m³ and 7850 kgf/m³ respectively. Other parameters are same as the first example of the case 1. Optimum cost, CO₂ emission and multi-objective function values of all optimum solutions of the presented algorithms are illustrated in Tables 13 and 14. According to the tables that optimum costs which are obtained by considering different objectives vary from 0.29% (for C27, S400 materials) to 3% (for C30, S500 materials) for the BBO algorithm; from 1.52% (for C24, S400 materials) to 3.93% (for C30, S400 materials) for the SSO algorithm. These differences are not significant. However, the differences between maximum and minimum values of optimum CO₂ emission are 3.44% (for C27, S400 materials) and 8.23% (for C30, S400 materials) for the BBO algorithm; 3.71% (for C27, S400 materials) and 24.4% (for C30, S400 materials) for the BBO algorithm. These are considerably high. In addition there is not supremacy of any optimization can be concluded in the tables.

Table 12: The objective function values of the BBO algorithm for case 2.

Objective		Material					
		C24 S400	C27 S400	C30 S400	C24 S500	C27 S500	C30 S500
Cost		241.7	264.1	269.4	241.2	277.1	281.8
Cost	CO ₂	804.4	815.3	888.1	781.7	792.2	866.0
Multi		1046.1	1079.4	1157.5	1022.9	1069.3	1147.8
Cost		244.6	271.7	274.0	241.9	279.5	282.6
CO ₂	CO ₂	743.0	752.1	853.3	754.7	764.1	807.3
Multi		987.6	1023.8	1127.3	996.6	1043.6	1089.9
Cost		241.8	269.0	269.7	241.8	277.2	281.9
Multi	CO ₂	744.2	753.6	857.3	760.1	765.9	810.1
Multi		986.0	1022.6	1127.0	1001.9	1043.1	1092.0

Table 13: The objective function values of the SSO algorithm for case 2.

Objective		Material					
		C24 S400	C27 S400	C30 S400	C24 S500	C27 S500	C30 S500
Cost		227.8	255.2	261.4	229.4	241.7	259.7
Cost	CO ₂	750.6	807.8	871.6	761.9	800.3	879.5
Multi		978.4	1063.0	1133.0	991.4	1042.0	1139.2
Cost		231.5	261.7	267.4	235.2	252.1	264.6
CO ₂	CO ₂	714.2	735.2	831.2	732.1	741.2	832.2
Multi		945.7	996.9	1098.7	967.3	993.3	1096.8
Cost		229.5	257.1	263.4	231.2	247.3	260.0
Multi	CO ₂	715.9	738.7	835.3	735.5	741.7	836.1
Multi		945.4	995.8	1098.7	966.6	989.0	1096.2

Distribution of optimum values with respect to different materials in case of the minimizing cost and the minimizing CO₂ objectives are plotted in Figures 7, 8 respectively. It is concluded from the figures that the lowest cost value is obtained using the C24 and S400 materials. The second lowest cost is obtained using the C24 and S500 materials which is close to the lowest cost value of the C24 and S400 materials. Moreover, the lowest CO₂ emission is obtained using the C24 and S400 materials. In addition, the optimum cost and CO₂ values increase when concrete strength is increased. On the contrary, any relationship between the steel class and the optimum cost values cannot be defined.

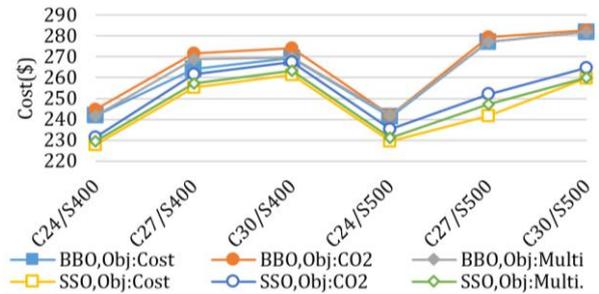


Figure 7: Variations of optimum cost values with respect to different materials.

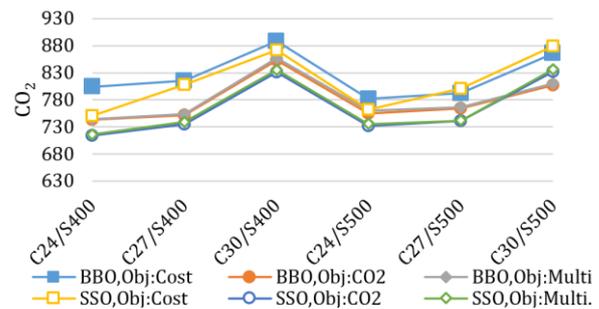


Figure 8: Variations of optimum CO₂ values with respect to different materials

Concrete and steel costs/CO₂ emissions of the optimum designs are illustrated in Figures 9-14 respectively. Five inferences are concluded from these figures. The first one is: concrete cost rate is generally higher than steel cost rate. The second one is: steel cost rate generally increases in higher steel classes. The third one is: steel cost rate for minimize cost and multi objective functions is higher than steel cost rate for minimize cost objective function. The fourth one is: the concrete CO₂ emission rate for minimize cost and multi objective functions is lower than the concrete CO₂ emission rate for minimize cost objective function. The fifth one is: the CO₂ emission of concrete much higher than CO₂ emission of steel.

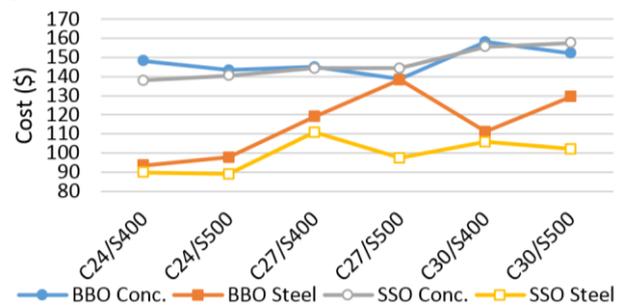


Figure 9: Concrete and steel costs of optimum designs (Objective function: Minimize cost).

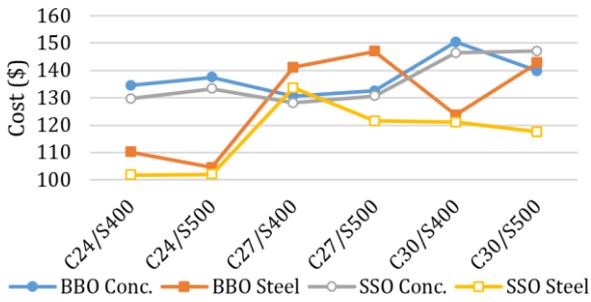


Figure 10: Concrete and steel costs of optimum designs (Objective function: Minimize CO₂ emission).

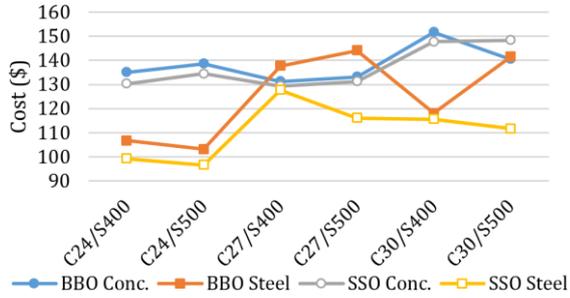


Figure 11: Concrete and steel costs of optimum designs (Objective function: Multi objective).

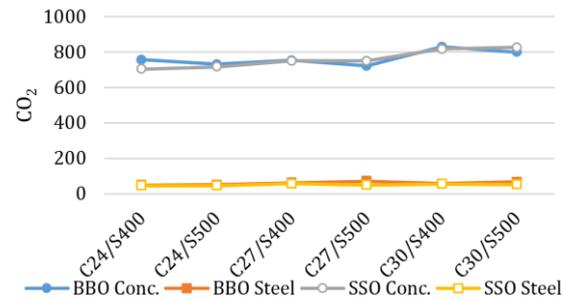


Figure 12: Concrete and steel CO₂ emissions of optimum designs. (Objective function: Minimize cost).

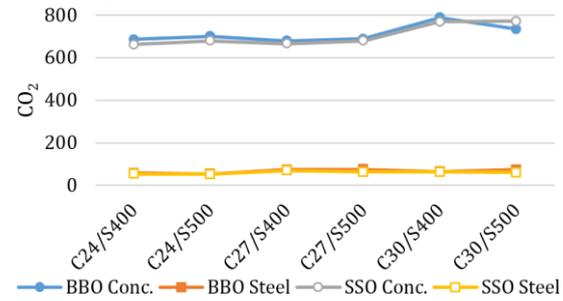


Figure 13: Concrete and steel CO₂ emissions of optimum designs. (Objective function: Minimize CO₂ emission).

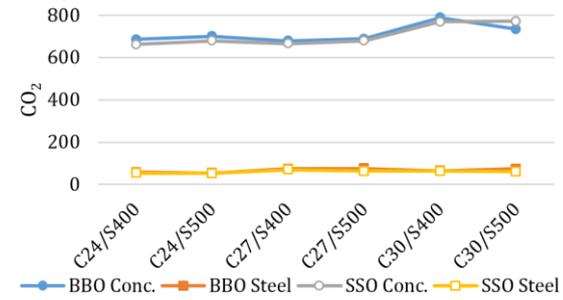


Figure 14: Concrete and steel CO₂ emissions of optimum designs. (Objective function: Multi objective).

7 Conclusion

In this study, the BBO and the SSO algorithms are proposed and utilized to calculate the cost and the CO₂ emission for the RC retaining wall design through investigation of two main subjects. First is the comparison performance of the meta-heuristic search algorithms for retaining wall design problems and the second is the optimization of the retaining wall under environmental considerations.

In first case, the 3.5 m and 5.2 m height retaining walls which are previously used in the literature, are optimized by considering minimizing the cost objective functions and taking unique material properties. The obtained results are compared to literature studies. In the first example, the BBO algorithm shows the best performance and the SSO algorithm has third best performance. Whereas, the SSO algorithm shows the best performance and the BBO algorithm has fifth best performance. Therefore, the supremacy between the SSO and BBO algorithms cannot be defined. However, it can be concluded that, the presented algorithms are powerful and efficient in finding the optimum solution for optimum cost design of RW problems.

In the second case, the new example (4.0 m height retaining wall) is optimized using the presented algorithms by considering minimizing the cost, minimizing the CO₂ and minimizing the weighted aggregate of the cost and the CO₂ objective functions. Three concrete material types (C24, C27 and C30) and two steel material types (S400 and S500) are used. In total, the design example is optimized thirty six times by taking different objectives and different materials. According to obtained results from these runs, six main outcomes are obtained. The first outcome is that the optimization of the retaining wall considering the minimizing CO₂ emission objective function does not have a material influence on the optimum cost of the retaining wall. Therefore, the minimizing CO₂ emission objective function can be used in the cost optimization problem. The second outcome is that when lower material classes (especially concrete class) are used, better optimum cost values and optimum CO₂ emissions are obtained. In summary, if lower class materials are used, lower cost and CO₂ emissions are obtained. The third outcome is: usage of higher strength concrete increases steel material usage. The fourth outcome: concrete material cost constitute majority of total cost and amount of concrete has huge percentage in total CO₂ emission. The fifth outcome: steel cost rate increases when the minimizing CO₂ emission and multi-objective functions are used. The sixth outcome: CO₂ emission rate of concrete is decreases when the minimizing CO₂ emission and multi-objective functions are used.

8 List of symbols

- A : Cross sectional area of the structural material,
- $ACoeff$: Absorption coefficient,
- A_s^{min} : Minimum reinforcement area,
- A_s^{max} : Maximum reinforcement area,
- B_0 : Attractiveness at original location,
- C : Summation of constraint violations,
- C_s : Unit cost of steel,
- d_b : Diameter of the hooked bar,

C_c :	Unit cost of concrete,	R2:	Second vertical steel reinforcement in the stem,
E_{CO_2} :	CO ₂ emission of the structural material,	R3:	Horizontal steel reinforcement in the toe,
f :	Value of the objective function,	R4:	Horizontal steel reinforcement in the heel,
f_{ij} :	Female spider,	R5:	Vertical steel reinforcement of key,
f_{high} :	Objective function of worst spider,	S_b :	Best spider,
f_{low} :	Objective function of best spider,	S_{max} :	Maximum number of species,
f_p :	Penalized value of the objective function,	S_{net} :	Clear spacing,
g_i :	i th constraints function,	Sp_{max} :	Maximum clear spacing,
H :	Height of stem,	S_w :	Worst spider,
HMS :	Harmony memory size,	S_0 :	Equilibrium number of species,
$HMCR$:	Harmony memory considering rate,	V_c :	Volume of concrete per unit length of the Wall,
I :	Maximum immigration rate,	V_d :	Design shear force,
k :	Iteration number,	$vibc_i$:	Vibration between the i th spider and the nearest spider,
L_{db} :	Minimum development length,	$vibb_i$:	Vibration between the i th spider and the best spider,
L_{dh} :	Minimum hook development length,	$vibf_i$:	Vibration between the i th spider and the nearest female spider,
M_d :	Design moment,	V_u :	Ultimate shear capacity,
m_{ij} :	Male spider,	w_b :	Weight of the best spider,
m_{max} :	Maximum mutation probability,	w_c :	Weight of the nearest spider,
M_u :	Ultimate resistance moment,	w_f :	Weight of the nearest female spider,
N :	Size of population,	w_{med} :	Weight of the median spider,
NC :	Number of constraint functions,	W_{st} :	Weight of steel per unit length of the Wall,
N_{Dm} :	Number of dominant male spiders,	X :	Vector of design variables,
NDV :	Number of design variables,	x_b :	Design variable of the best spider,
N_f :	Number of female spiders,	x_c :	Design variable of the nearest spider,
NFF :	Number of firefly,	x_f :	Design variable of female spider,
N_m :	Number of male spiders,	x_l :	Lower boundary of design variable,
$N_{material}$:	Number of materials,	x_m :	Design variable of male spider,
N_s :	Number of spiders,	x_u :	Upper boundary of design variable,
PAR :	Pitch adjusting rate,	X1:	Width of the base,
PF :	Female movement parameter between {0, 1},	X2:	Toe projection,
P_{max} :	Maximum number of species,	X3:	Thickness at the bottom of the stem,
r :	Random number between {0, 1},	X4:	Thickness at the top of the stem,
$round$:	Function which rounds to the value of the nearest integer,	X5:	Thickness of base slab,
RP :	Randomness parameter,	X6:	Distance from toe to the front of the base shear key,
RP_{min} :	Minimum randomness parameter,	X7:	Width of the key,
RP_{max} :	Maximum randomness parameter,	X8:	Depth of the key,
R1:	First vertical steel reinforcement in the stem,	α :	Random number between {0, 1},

β :	Random number between {0, 1},
δ :	Random number between {0, 1},
ε :	Penalty coefficient,
ζ_{cost} :	Non-negative weight coefficient of cost,
ζ_{CO_2} :	Non-negative weight coefficient of CO ₂ ,
λ :	Immigration rate,
μ :	Emigration rate,
ρ :	density of the structural material.

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