SERVO CONTROL APPLICATION OF PREDICTIVE ALGORITHM TO THE CHEESE WHEY DRINK PRODUCTION

Semin ALTUNTAŞ1, Suna ERTUNÇ2, Hale HAPOĞLU3, Mustafa ALPBAZ4

ABSTRACT : Cheese whey drink was produced from the pasteurized mixture which contains cheese whey, glucose, grape juice and milk. The fermentation of this mixture with kefir yeast in a batch bioreactor was investigated. The pH was monitored during the cheese whey drink production without pH control. The system model for generalised predictive control (GPC) algorithm is obtained by using experimental data. The GPC was applied to the bioprocess theoretically in the face of a step change in pH set point (i.e., servo control case). It is noted that GPC with the tuning parameters, \( N_2=2 \), \( N_u=1 \), \( \lambda=0.007 \) provides satisfactory servo control result.

KEY WORDS : Predictive control, Cheese whey drink production, Servo control, Generalised predictive control, Kefir yeast.

PEYNİRALTı SUYı İÇKİSİ ÜRETİMINE ÖNGÖRMELİ ALGORİTMANIN SERVO KONTROL UYGULAMASI

ÖZET: Peyniraltı suyu (PAS) ile bir tür içkisi; PAS, glikoz, üzüm suyu ve sütten oluşan pastörize karışımdan üretilmiştir. Kesikli bir biyoreaktörde kefir mayası ile bu karışımdan fermentasyonu araştırılmıştır. Peynir altı suyu içkisi üretimi sırasında pH değerleri kaydedilmiştir. Genelleştirilmiş öngörümlü kontrol (GPC) algoritması için sistem modeli deneySEL veriYEN yararlanarak elde edilmiştir. pH set noktasına basamak etki verilmesi durumunda (yani servo kontrol durumunda) GPC teorik olarak biyoreaktöre uygulanmıştır. \( N_2=2 \), \( N_u=1 \), \( \lambda=0.007 \) ayar parametreleri ile GPC iyi servo kontrol sonucu sağlanmıştır.

ANAHTAR KELİMEler : Öngörümlü kontrol, Peyniraltı suyu içkisi üretimi, Servo kontrol, Genelleştirilmiş öngörümlü kontrol, Kefir mayası.
I. INTRODUCTION

Cheese whey is a by-product of the cheese making process. Waste of this represents a significant loss of resources and causes serious pollution problems. There are several published work dealt with biotechnological utilization of cheese whey [1-8].

It was noted that kefir was highly resistant to contamination under actual industrial conditions and no serious problems in handling of raw materials and equipment were observed [9]. Kourkoutas et al. proposed a low-alcohol content drink by using continuous whey fermentation using kefir yeast [10]. Paraskevopoulou et al. prepared a kefir-type drink by fermentation with kefir granules from cheese whey containing fructose, black raisin extract and milk [11].

Temperature and pH directly affect the microbial growth; as widely reported in the literature, optimal values of both temperature and pH exist and any deviation from these values may indeed result in a significant affect of food quality.

Generalized Predictive Control (GPC), a popular member of the Model Predictive Control (MPC) strategies, is very effective in the face of many plants that exhibit control difficulties. GPC was introduced by Clarke et al. to overcome the control related many problems [12]. GPC inherent mathematical complexities for the derivation of the control law; moreover its tuning procedures are mainly empirical. Li-Juan et al. proposed a practical GPC algorithm based on online least squares support vector machines which deal with nonlinear systems effectively [13]. Their experiments of GPC on pH neutralizing process showed the effectiveness and practicality of their proposed algorithm. Sato applied a GPC-based PID controller to a weigh feeder [14]. His experimental results showed that a weigh feeder is well controlled using the enhanced GPC-based PID controller. Neshasteriz et al. proposed a modified GPC strategy for the plants described by SOPDT models [15]. They reported that their proposed scheme was less computationally demanding than the conventional GPC methods. They recommended the algorithm for practitioners in industry. Pekel et al. discussed the GPC control strategy and tested by controlling a SISO pH textile treatment system [16]. From their experimental results, it was noted that suggested control system showed good performance for the pH process with nonlinearities and model-plant mismatch.

In this work, cheese whey drink was produced with kefir yeast by controlling pH value of bioreactor medium. This pH control has critical role in quality assurance. The parameterized model was built to closely approximate the pH system. The long range predictive control was applied.
II. GENERALIZED PREDICTIVE CONTROL

Clarke et al. introduced Generalized Predictive Control (GPC) to overcome the control related problems concerning non-minimum phase zeros, open loop unstable or poorly damped poles, unknown or variable dead times and unknown plant orders [12].

GPC is designed on the basis of minimization of the following performance function:

\[ J(u_t) = \mathbb{E} \left\{ \sum_{j=N_1}^{N_2} \left( y_{t+j} - r_{t+j} \right)^2 + \lambda \sum_{j=1}^{N_u} \left( \Delta u_{t+j-1} \right)^2 \right\} \]  \hspace{1cm} (1)

where \( N_1, N_2, N_u \) and \( \lambda \) are minimum predictive horizon, maximum predictive horizon, control horizon and a weighting factor for the variation in control input, respectively. The expectation \( \mathbb{E} \) is used in Equation (1) to denote that the control values selected are estimated from data obtained up to and including time \( t \) and that a stochastic disturbance model has been assumed.

To solve the problem suggested by the minimization of Equation (1), a set of \( j \) step ahead predictions of the output \( y_{t+j} \) have to be calculated for \( j=N_1, \ldots, N_2 \) based upon information known at time \( t \). In order to calculate the \( j \) step ahead prediction, it is necessary to solve the Diophantine equation. Partitioning the system as follows:

\[ \begin{align*}
\frac{C}{\Delta z} e_{t+j} &= E e_{t+j} + z^{-j} F \frac{\Delta z}{\Delta} e_{t+j} \\
\end{align*} \]  \hspace{1cm} (2)

Where \( E e_{t+j} \) presents future data \( F(\Delta z)e_t \) presents past and present data. Since \( E \) embodies all the unknown data at time \( t \):

\[ E = 1 + e_t z^{-1} + \ldots + e_{t+j-1} z^{j-1} \]  \hspace{1cm} (3)

Thus the desired identity is:

\[ C = E A \Delta + z^{-j} F \]  \hspace{1cm} (4)

A model of the plant is expressed in terms of the following form:

\[ A \Delta y_t = B \Delta u_{t+k} + C e_t \]  \hspace{1cm} (5)

Substituting Equation (4) in Equation (5) and putting \( t=t+j \):

\[ y_{t+j} = \frac{B}{A} u_{t+j-1} + E e_{t+j} + z^{-j} \frac{F}{\Delta z} e_t \]  \hspace{1cm} (6)

Substituting \( e_t \) from Equation (5) gives:

\[ y_{t+j} = \frac{B}{A} u_{t+j-1} + E e_{t+j} + \frac{z^{-j} B}{C} y_t + \frac{F}{AC} u_{t+j-1} \]  \hspace{1cm} (7)

where

\[ \frac{z^{-j} B}{AC} = \frac{z^{-j} B}{AC} \left( C - (E A \Delta) \right) = \frac{z^{-j} B}{C} u_{t+j} \]  \hspace{1cm} (8)

and

\[ y_{t+j} = \frac{B}{C} y_t + \frac{z^{-j} B}{C} \Delta u_{t+j-1} + E e_{t+j} \]  \hspace{1cm} (9)
The last term of Equation (9) includes data which are independent of signals measurable at time t. It is then clear that the minimum variance prediction of \( y(t) \) (given data known at time t) is attained by making the last term zero. Hence, from Equation (9):

\[
y_{t|j} = \frac{E}{C} y_t + \frac{EB}{C} \Delta u_{t-j+1}
\]  

(10)

Where \( y_{t|j} \) is a function of known signal values at time t and also of future inputs which have yet to be calculated. A second Diophantine equation is applied now to separate past and future control values, viz:

\[
\frac{EB}{C} \Delta u_{t-j+1} = G \Delta u_{t-j+1} + \frac{F}{C} \Delta u_{t-j+1}
\]  

(11)

Where \( G \Delta u_{t-j+1} \) represents future data and \( (I/C) \Delta u_{t-j+1} \) signifies past and present data. Since G embodies all the unknown data at time t,

\[
G = 1 + g_1 z^{-1} + \ldots + g_j z^{-j+1}
\]  

(12)

Thus the desired identity is:

\[
\frac{EB}{C} = G + z^{-1} F
\]  

(13)

Combining this with Equation (10) gives:

\[
\hat{y}_{t|j} = \frac{G \Delta u_{t-j+1}}{C} + \frac{F}{C} y_t
\]  

(14)

and

\[
\hat{y}_{t+j|t} = \frac{F \Delta u_{t-j+1}}{C} + \frac{F}{C} y_t
\]  

(15)

where \( \hat{y}_{t+j|t} \) represents the ‘free response’ prediction of \( y(t) \) assuming that future control increments after time t-1 are zero.

The future incremental control vector \( \mathbf{u} \) is given by:

\[
\mathbf{u} = (G^T G + \lambda I)^{-1} G^T (\mathbf{r} - \mathbf{f})
\]  

(16)

where the reference signal \( \mathbf{r} \) is given as:

\[
\mathbf{r} = [r_{t+1|t}, r_{t+2|t}, \ldots, r_{t+N_e|t}]^T
\]  

(17)

the vector of free response predictions:

\[
\mathbf{f} = [\hat{y}_{t+1|t}, \hat{y}_{t+2|t}, \ldots, \hat{y}_{t+N_e|t}]^T
\]  

(18)

\( \mathbf{g} \) is the matrix of the impulse response parameters \( g_i \) of the plant model \( B/\Delta \), viz:
Equation (16) gives the future control increments for time t to time \( t+N-1 \) as a strategy built upon data attainable at time t. The procedure employed for closing the loop and implementing the GPC is to apply only the first element of \( \Delta u \), i.e. \( \Delta u_0 \). After this the solution to the optimal control problem is calculated again for the next step.

**III. EXPERIMENTAL SETUP**

A 1 L bioreactor with a cooling jacket as shown in Figure 1 was used. The pH was measured with a pH meter and was recorded on-line every 1 second by a computerized data acquisition system. The system model parameters were calculated by using experimental input-output data obtained from the bioreactor. The prbs signal was given to 0.5 M sodium bicarbonate (NaHCO\(_3\)) flow rate, the system dynamic response data was obtained from the on-line pH monitor.

In the experimental work, the bioreactor was first charged with pasteurized mixture which contains 350 ml cheese whey, 7 g glucose, 70 ml grape juice and 237 ml milk, and immediately inoculum (7 ml kefir yeast) was added. The temperature level was kept at 25\(^\circ\)C by utilizing on-off control. No pH control was provided. pH was monitored throughout the fermentation process.

**Figure 1. Experimental setup**
IV. RESULTS AND DISCUSSIONS

An experimental work is realized to monitore the pH change during the cheese whey drink production with kefir yeast. The temperature level of the bioprocess is kept at 25°C with on-off control. During the bioprocess, the pH decreases from the initial value of 6.9 to 3.9 within 8 hours (Figure 2). It is noted that the pH value should not decrease under a certain value to avoid coagulation. This decrease in pH necessitates control application.

![Figure 2. Experimental pH change during the cheese whey drink production without pH control.](image)

This bioprocess is simulated with a Controlled AutoRegressive Integrated Moving Average (CARIMA) model for control purposes. The pH and sodium bicarbonate flow rate are selected as output and manipulated variable, respectively. A pseudo-random binary sequence (prbs) is utilised as a forcing function in order to determine the dynamics of the process to be controlled. Model parameters are calculated by using the recursive UDU^T algorithm [17]. The following bioreactor model is obtained by using experimental data.

\[
\Delta y(t) = -0.00004976u(t-1) - 0.0001325y(t-1) + \Delta u(t-1)
\]

where \( y \) presents pH value of bioreactor mixture, \( u \) is sodium bicarbonate flow rate.

Equation (20) is used to find closed loop response of pH by using GPC algorithm in the face of setpoint step change. In the closed loop, the setpoint step change given from 5.83 to 6.09. The GPC algorithm is written in MATLAB and apply to the bioprocess theoretically. The GPC algorithm provides a substantial array of tuning knobs which are the minimum costing horizon \( N_1 \), the maximum costing horizon \( N_2 \), the control costing horizon \( N_u \) and the control weighting \( \lambda \). \( N_1 \) is generally employed as a design parameter and should be set at the ratio of dead-time to sampling time. In this work, \( N_1 \) is set at 1. The other tuning parameters are varied to obtain the best control performance. To compare the performance of the GPC
algorithm, the integral square of the error (ISE) and the integral of absolute value of error (IAE) are computed for the closed-loop process output, where:

\[
ISE = \sum_{t=0}^{n} [y(t) - r(t)]^2
\]

(21)

\[
IAE = \sum_{t=0}^{n} |y(t) - r(t)|
\]

(22)

Table 1-3 list the ISE and IAE criteria values for the GPC applications with various values of tuning parameters. The smallest ISE and IAE values are chosen in the Table 1-3 to show the changes of controlled and controlling values in the closed loop. All the time variations of the output pH and base flow rate are shown in Figure 3-5. Related figures numbers are shown in Tables 1-3. They show the best ISE and IAE values for proper control action. There is little to choose between the each controlled variable responses in Figures 3-5. It is shown that the control weighting \( \lambda \) is the most effective tuning parameter for the cases studied.

**Table 1. ISE and IAE values obtained for the GPC algorithm with \( N_u=2 \) and \( N_v=1 \). Where (*) presents the best control result.**

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>IAE</th>
<th>ISE</th>
<th>Figure No</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>0.015</td>
<td>0.00023</td>
<td></td>
</tr>
<tr>
<td>0.006</td>
<td>0.0096</td>
<td>0.00092</td>
<td></td>
</tr>
<tr>
<td>0.0065</td>
<td>0.0047</td>
<td>0.00022</td>
<td></td>
</tr>
<tr>
<td>(*)0.007</td>
<td>0.00028</td>
<td>0.00000079</td>
<td>Figure 3</td>
</tr>
<tr>
<td>0.0075</td>
<td>0.0049</td>
<td>0.000024</td>
<td></td>
</tr>
<tr>
<td>0.008</td>
<td>0.0087</td>
<td>0.000075</td>
<td></td>
</tr>
<tr>
<td>0.009</td>
<td>0.014</td>
<td>0.00018</td>
<td></td>
</tr>
</tbody>
</table>
Figure 3. The control of the pH under the effect of a step increase in setpoint from 5.83 to 6.09. The GPC tuning knobs are $N_I=2$, $N_u=1$, $\lambda=0.007$ (a) the close loop pH change with time, (b) the close loop base flow rate change with time.

Table 2. ISE and IAE values obtained for the GPC algorithm with $N_I=4$ and $N_u=1$. Where (*) presents the best control result.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>IAE</th>
<th>ISE</th>
<th>Figure No</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>0.0048</td>
<td>0.000023</td>
<td></td>
</tr>
<tr>
<td>0.004</td>
<td>0.01</td>
<td>0.0001</td>
<td></td>
</tr>
<tr>
<td>0.0045</td>
<td>0.0052</td>
<td>0.000027</td>
<td></td>
</tr>
<tr>
<td>(*)0.005</td>
<td>0.0014</td>
<td>0.00000002</td>
<td>Figure 4</td>
</tr>
<tr>
<td>0.0055</td>
<td>0.0045</td>
<td>0.00002</td>
<td></td>
</tr>
<tr>
<td>0.006</td>
<td>0.0084</td>
<td>0.00007</td>
<td></td>
</tr>
<tr>
<td>0.008</td>
<td>0.015</td>
<td>0.00023</td>
<td></td>
</tr>
</tbody>
</table>
Figure 4. The control of the pH under the effect of a step increase in setpoint from 5.83 to 6.09. The GPC tuning knobs are $N_2=4, N_u=1, \lambda=0.005$ (a) the close loop pH change with time, (b) the close loop base flow rate change with time.
Table 3. ISE and IAE values obtained for the GPC algorithm with $N_z=4$ and $N_u=2$. Where (*) presents the best control result.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>IAE</th>
<th>ISE</th>
<th>Figure No</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.004</td>
<td>0.012</td>
<td>0.00015</td>
<td></td>
</tr>
<tr>
<td>0.005</td>
<td>0.0023</td>
<td>0.0000053</td>
<td></td>
</tr>
<tr>
<td>0.0051</td>
<td>0.0013</td>
<td>0.0000016</td>
<td></td>
</tr>
<tr>
<td>(*)0.0052</td>
<td>0.00024</td>
<td>0.00000058</td>
<td>Figure 5</td>
</tr>
<tr>
<td>0.0053</td>
<td>0.00076</td>
<td>0.00000058</td>
<td></td>
</tr>
<tr>
<td>0.006</td>
<td>0.007</td>
<td>0.000049</td>
<td></td>
</tr>
<tr>
<td>0.007</td>
<td>0.013</td>
<td>0.00017</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5. The control of the pH under the effect of a step increase in setpoint from 5.83 to 6.09. The GPC tuning knobs are $N_z=4$, $N_u=2$, $\lambda=0.0052$ (a) the close loop pH change with time, (b) the close loop base flow rate change with time.

V. CONCLUSION

Experimental work is realised when the bioreactor is operated without pH control. It is noted that the pH value of the bioreactor mixture must be controlled to prevent the pH value from decreasing under a certain value which causes coagulation.

It can be said that the theoretical basis underlying GPC control is now well established. It is time for such control algorithm to be applied in the cheese whey drink production with kefir yeast. The performance of the consequent control strategy is examined as a trade-off between model following performance and fluctuations in the controlled variable. It is shown that GPC algorithm with $N_z=2$, $N_u=1$, $\lambda=0.007$
provides good control for the cases studied. The control weighting $\lambda$ is the most effective tuning parameter.

**VI. LIST OF SYMBOLS**

A \quad \text{monic polynomial in the z-domain representing the poles of the discrete-time system}

B \quad \text{polynomial in the z-domain representing the zeros of the discrete-time system}

C \quad \text{monic polynomial in z-domain representing the zeros of the process noise}

E \quad \text{polynomial in the z-domain}

e \quad \text{white noise}

F \quad \text{polynomial in the z-domain}

G \quad \text{polynomial in the z-domain}

j \quad \text{objective function for the control algorithm}

k \quad \text{integer number of sampling time steps}

u(t) \quad \text{input variable at time } t

y(t) \quad \text{output variable at time } t

z, z^{-1} \quad \text{forward and backward shift operators}

\mathbb{E} \quad \text{mathematical expectation}

\Delta t \quad \text{sampling time}

\lambda \quad \text{control weighting}

\Delta \quad \text{first difference operator}

**VII. REFERENCES**


