Determining Expected Value and Variance of Demand for Safety Stock Level under Random Parameters

Serap Akcan [1]*, Tahir Khaniyev [2,3], Ali Kokangül[4], Meral Güldeş[4]

[1] Department of Industrial Engineering, University of Aksaray, 68100, Aksaray, Turkey, serapakcan@aksaray.edu.tr
[2]Department of Industrial Engineering, TOBB University of Economics and Technology, 06560, Ankara, Turkey
[3]Azerbaijan National Academy of Sciences, Institute of Cybernetics, Baku, Azerbaijan;
[4]Department of Industrial Engineering, University of Cukurova, 01330, Adana, Turkey
*Corresponding Author

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Abstract

The patients demand is vital in healthcare area. Thus, to determine safety stock level is also significant. But, determining safety stock level in inventory systems based on stochastic lead-time is a much smaller section of literature especially in healthcare area. Therefore, this study aimed to contribute to the literature in this area. This study is investigated for calculating the expected value and variance of demand for determining safety stock level under random parameters (the number of arrivals, transferal rates between levels and length of stays). The lead-time required when supplying medical material in the hospital is also random. A case study is presented using with the real data obtained from Neonatal Intensive Care Unit which has a complex inventory system because of the random parameters and transferal probabilities between levels.

Keywords — Expected Value, Healthcare Systems, Neonatal Intensive Care Unit, Random Parameters, Safety Stock Level, Variance

1 Introduction

There have been many studies on inventory control systems. In most papers, lead-time is assumed to be constant, but this is not the case in the real world. Delays in lead-time can occur in unexpected situations, such as carrying problems, second-class products, production system problems, and supplier problems. Very few studies have considered stochastic lead-time [1,2].

In inventory systems, stochastic lead-time causes either shortages or redundant stock, which should be avoided. For this reason, determining the safety stock is crucial. Few studies have determined safety stock in inventory systems based on stochastic lead-time, with most having focused production. Kelle and Silver [3] proposed safety stock reduction by order splitting when lead-time is random. Lambrecht and Vandaele [4] presented a general approximation for a single-product lot-sizing model with queuing delays. The lead-time probability distribution was lognormal.

Hung and Chang [5] determined safety stock levels for production planning in uncertain manufacturing. Alstrom [6] proposed a model for minimizing the total cost with service levels and safety stock based on order size. Persona et al. [7] presented four different analytical models to calculate the safety stock level for subassemblies and components. Persona et al. [8] modeled cost-based analytical models for optimizing safety stock levels of modular subassemblies and manufacturing components. Louly and Dolgui [1] determined safety stock levels for assembly systems with random component procurement lead-times. Desmet et al. [9] presented an approximation model for retailer replenishment lead-times in two echelon distribution systems. They discussed its imp-

lementation for safety stock optimization. The model assumed normality of demand and nominal lead-times.

Very few relevant studies have been conducted in the healthcare area, where satisfying patient demand is vital. Furthermore, to the best of our knowledge, no studies provide any estimation for the expected value and variance of the total demand based on stochastic lead time. Thus, in this paper, the complex inventory system of a research hospital neonatal intensive care unit (NICU) is investigated. Equations derived for the expected value and variance of the total demand based on stochastic lead-time are presented for determining the safety stock level.

2 The Proposed Procedure to Determine Safety Stock Level

Inventory management is important for the profitability of enterprises, since they spend 45-65% of their incomes on inventory [10]. Also, within the healthcare area, the management of hospital inventory is fundamental to productivity. Around 20-45% of the hospital budget is spent on inventory [11]. Inventory management is critical in hospitals.

A research hospital consists of n units, as shown in Figure 1.1. The number of arrivals, transferal rates between units, and length of stays (LOS) are random in the research hospital. Inpatients can be transferred from one unit to another. Inpatients are either discharged from the same unit to which they were accepted for treatment or are transferred to another unit.

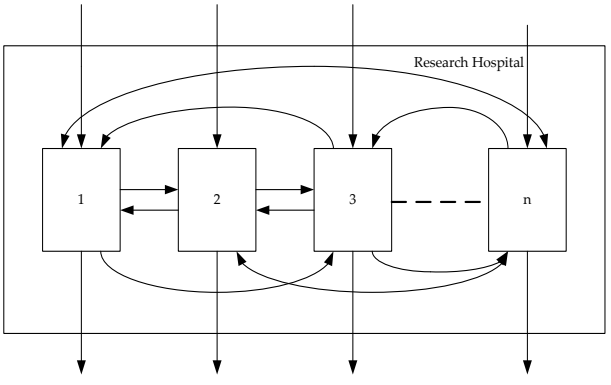

Figure 1.1. A research hospital consist of n unit

Determining the reorder point level for any inventory is more difficult under random parameters (random arrivals, random length of stay and random lead-time). Calculating the safety stock level is crucial for determining the confidence interval of the reorder point level. Therefore, a procedure was developed to calculate the safety stock level under random parameters. The procedure consists of the following steps:

Step1: Calculating ergodic distribution for units.
Step2: Calculating patients' LOS for each unit.
Step3: Calculating safety stock level.

2.1 Calculating ergodic distribution for units

The investigated healthcare system consists of n ($n = 2,3,\dots$) units. It is assumed that the transferal rate between units is known, and units generate an ergodic Markov chain. For this reason, it is important to calculate the ergodic distribution for this system. Because these systems run for a long time, it is more feasible to use the ergodic distribution instead of finite-dimensional distributions. Therefore, we discuss the ergodicity of a Markov chain below.

$X(t)$ corresponds to a patient occupying a particular unit at time t. Let $L(X)$ be the state-space of $X(t)$. $L(X) = \{1,2,\dots,n\}$, and let $X(t)$ be a Markov chain. Because patients can transfer from one unit to another, the system indicated symbolically in Figure 1.1 forms an n-case Markov chain. The one-step transition probability matrix for this chain is:

$$A \equiv ((a_{ij}))_{n\times n}\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}$$

where $a_{ij} \geq 0$, and $\sum_{i=1}^{n} a_{ij} = 1$ $(i, j = 1,2,3,\dots,n)$.

Proposition [12]: The ergodic distribution ($\vec{\pi}_{\infty}^{T}$) of chain $X(t)$ is:

$$\vec{\pi}_{\infty}^{T} A = \vec{\pi}_{\infty}^{T} \quad (2.1)$$

where the vector $\vec{\pi}_{\infty}^{T} = (\varkappa_1, \varkappa_2, \dots, \varkappa_n)$ is the ergodic distribution of chain $X(t)$, and

$$\varkappa_i \equiv \lim_{t\to\infty} P\{X(t) = i\}, i = 1, \dots, n$$

Also, according to the characteristics of an ergodic distribution

$$\varkappa_1, \varkappa_2 \dots, \varkappa_n > 0;\ \varkappa_1 + \varkappa_2 + \dots + \varkappa_n = 1 \quad (2.2)$$

These results were used in the case study.

2.2 Calculating LOS of patient for each unit

For the case study, the next step is calculating the expected value and variance of patients' LOS. Material usage is proportional to the number of patients and depends on the LOS. The ergodic distribution will be used to calculate the expected value and variance of patients' LOS in unit i.

$\hat{\lambda}_{i1}$: the average number of accepted patients in unit i per unit of time.
$\hat{\lambda}_{i0}$: the average number of rejected patients in unit i per unit of time.
$\hat{\lambda}_i$: the average number of arrivals patients in unit i per unit of time.

$$\hat{\lambda}_i = \hat{\lambda}_{i1} + \hat{\lambda}_{i0}\,,\ i = 1,2,\dots,n.$$

Λ: the average number of accepted patients for all unit per unit of time.

$$\Lambda\colon \hat{\lambda}_{11} + \hat{\lambda}_{21} + \cdots + \hat{\lambda}_{n1}$$

$\bar{\lambda}_{i1}$: the average number of accepted patients in unit i per unit of time according to ergodic distribution

$$\bar{\lambda}_{i1} = \Lambda\varkappa_i,\ i = 1,2,\dots,n.$$

T_i: LOS for a patient in unit i

The data for the case study revealed that LOS has a lognormal distribution. For this reason, we will assume that LOS of T_i has a lognormal distribution with parameters (μ_i, σ_i^2).In this case, the expected value and variance of T_i can be calculated as follows:

$$E(T_i) = \exp\left(\mu_i + \frac{\sigma_i^2}{2}\right) \tag{2.3}$$

$$Var(T_i) = \exp(2\mu_i + \sigma_i^2)\,(\exp(\sigma_i^2) - 1),\quad i = 1,\dots,n \tag{2.4}$$

2.3 Calculating safety stock level

The safety stock level is used to calculate the reorder point level safely. $N(t)$ shows the number of registered patients at time t. Generally, it is assumed that process $N(t)$ is a Poisson process. In line with this assumption, the expected value, variance, and standard deviation of $N(t)$ are as follows:

$$E(N(t)) = \Lambda t; Var\big(N(t)\big) = \Lambda t;\ \sigma_{N(t)} = \sqrt{\Lambda t}$$

Using the 3σ rule, we obtain the following inequalities for $N(t)$:

$$E\big(N(t)\big) - 3\sigma_{N(t)} \le N(t) \le E\big(N(t)\big) + 3\sigma_{N(t)}$$

$$\Lambda t - 3\sqrt{\Lambda t} \le N(t) \le \Lambda t + 3\sqrt{\Lambda t}$$

Now, we find the total amount of required material (TRM(t)) for random lead-time τ. Let $\bar{m}_i$ be the average amount of required material in unit i per unit of time. Thus,

$$TRM(\tau) = \sum_{i=1}^{n}\sum_{r=1}^{N_i(\tau)} T_{ir}\bar{m}_i = \sum_{i=1}^{n}\bar{m}_i \sum_{r=1}^{N_i(\tau)} T_{ir} \tag{2.5}$$

$N_i(\tau)$ indicates the number of registered patients during the lead-time τ in unit i. T_{ir} indicates LOS for the patient r in unit i.

It is assumed that lead-time has a uniform distribution with interval $[c, d]$.Under this assumption, the expected value for the total amount of TRM can be calculated as:

$$E\big(TRM(\tau)\big) = \int_c^d E\big(TRM(u)\big) f_\tau(u)\,du = \frac{1}{d-c}\int_c^d E\big(TRM(u)\big)\,du \tag{2.6}$$

where;

$$E\big(TRM(u)\big) = E\left\{\sum_{i=1}^{n}\bar{m}_i \sum_{r=1}^{N_i(u)} T_{ir}\right\} = \sum_{i=1}^{n}\bar{m}_i\, E\left(\sum_{r=1}^{N_i(u)} T_{ir}\right) \tag{2.7}$$

According to the Wald identity [13], the following equation can be written:

$$E\left(\sum_{r=1}^{N_i(u)} T_{ir}\right) = E\big(N_i(u)\big)E(T_i) \tag{2.8}$$

$E(T_i)$ indicates the expected LOS for a patient. Equation (2.9) is obtained from Equation (2.7).

$$E\big(TRM(u)\big) = \sum_{i=1}^{n}\bar{m}_i\, E\big(N_i(u)\big)E(T_i) \tag{2.9}$$

Integrating both sides of Equation (2.9) with respect to parameter u gives Equation (2.10):

$$E\big(TRM(\tau)\big) = \frac{1}{d-c}\sum_{i=1}^{n}\bar{m}_i\, E(T_i)\int_c^d E\big(N_i(u)\big)du = \frac{1}{d-c}\sum_{i=1}^{n}\bar{m}_i\, E(T_i)\int_c^d \tilde{\lambda}_i u\,du$$

$$E\big(TRM(\tau)\big) = \frac{d^2-c^2}{2(d-c)}\sum_{i=1}^{n}\bar{m}_i\,\tilde{\lambda}_i E(T_i) = \frac{c+d}{2}\sum_{i=1}^{n}\tilde{\lambda}_i\bar{m}_i\, E(T_i) \tag{2.10}$$

Where $\tilde{\lambda}_i = \Lambda\varkappa_i$ and $\varkappa_i$ indicate the ergodic probabilities for $i = 1,2,\dots,n$. It is known that LOS for a patient

in unit i has a lognormal distribution with parameters(μ_i, σ_i^2). So accotrding to Equation (2.3)

$$E(T_i) = exp\left(\mu_i + \frac{1}{2}\sigma_i^2\right)$$

Using this equation in Equation (2.10), the following is obtained:

$$E\big(TRM(\tau)\big) = \frac{c+d}{2}\Lambda \sum_{i=1}^{n} \bar{m}_i \varkappa_i exp\left(\mu_i + \frac{1}{2}\sigma_i^2\right) \quad (2.11)$$

Equation (2.11) indicates the average total amount of required material during the lead time.

Remark: Because $TRM(\tau)$ is a random variable, measuring it using only one expected value is incorrect. For this reason, the variance of $TRM(\tau)$ should be calculated with intervals constructed using the 3σ rule for $TRM(\tau)$.

The total amount of required material during leadtime τ is calculated according to Equation (2.5):

$$TRM(\tau) = \sum_{i=1}^{n} \bar{m}_i \sum_{r=1}^{N_i(\tau)} T_{ir}$$

According to the Borovkov identity [14], the following equation can be written:

$$Var\left(\sum_{r=1}^{N_i(\tau)} T_{ir}\right) = E\big(N_i(\tau)\big)\, Var(T_i) + \big(E(T_i)\big)^2 Var\big(N_i(\tau)\big) \quad (2.12)$$

Now, the characteristic in Equation (2.12) is calculated.

$$E\big(N_i(\tau)\big) = \int_c^d E\big(N_i(u)\big) f_\tau(u)du = \frac{1}{d-c}\int_c^d E\big(N_i(u)\big)du = \frac{1}{d-c}\int_c^d \tilde{\lambda}_i u du \frac{d^2-c^2}{2(d-c)}\tilde{\lambda}_i$$

$$E\big(N_i(\tau)\big) = \frac{c+d}{2}\tilde{\lambda}_i = \frac{c+d}{2}\Lambda\varkappa_i \quad (2.13)$$

The probability density function of random variable τ is indicated above with $f_\tau(u)$.For each $u \in [c,d]$;

$$f_\tau(u) = \frac{1}{d-c}$$

The following equation can also be obtained:

$$Var\big(N_i(\tau)\big) = \int_c^d Var\big(N_i(u)\big) f_\tau(u)du = \frac{1}{d-c}\int_c^d Var\big(N_i(u)\big)du = \frac{1}{d-c}\int_c^d \tilde{\lambda}_i u du$$

$$Var\big(N_i(\tau)\big) = \frac{c+d}{2}\bar{\lambda}_i = \frac{c+d}{2}\Lambda\varkappa_i \quad (2.14)$$

Also, we showed the following in Equations (2.3) and (2.4):

$$E(T_i) = \exp\left(\mu_i + \frac{\sigma_i^2}{2}\right)$$

$$Var(T_i) = \exp(2\mu_i + \sigma_i^2)\,(\exp(\sigma_i^2) - 1)$$

Using these to calculate Equation (2.12) yields the following equation:

$$Var\left(\sum_{r=1}^{N_i(\tau)} T_{ir}\right) = \frac{c+d}{2}\bar{\lambda}_i \{\exp(2\mu_i + \sigma_i^2)\exp(\sigma_i^2) - 1 + \exp(2\mu_i + \sigma_i^2)\}$$

$$Var\left(\sum_{r=1}^{N_i(\tau)} T_{ir} = \frac{c+d}{2}\bar{\lambda}_i \exp(2\mu_i + 2\sigma_i^2)\right)$$

$$Var\left(\sum_{r=1}^{N_i(\tau)} T_{ir}\right) = \frac{c+d}{2}\Lambda\varkappa_i \exp(2(\mu_i + \sigma_i^2)),\quad i = 1,2,..,n \quad (2.15)$$

As shown in Equation (2.15), note that because LOS for each unit has a lognormal distribution, the variance of total LOS can be too large. This means that when the real values of LOS are compared with the expected values of LOS, the deviations can be significantly large. In this situation, calculating the variance is vital to avoid very large error.

Considering Equation (2.5) and Equation (2.15), the variance of the total material amount during lead time τ can be calculated as follows:

$$Var\big(TRM(\tau)\big) = Var\left(\sum_{i=1}^{n} \bar{m}_i \sum_{r=1}^{N_i(\tau)} T_{ir}\right)$$

$$Var\big(TRM(\tau)\big) = \sum_{i=1}^{n} \big(\bar{m}_i{}^2\big) Var\left(\sum_{r=1}^{N_i(\tau)} T_{ir}\right) = \sum_{i=1}^{n} \big(\bar{m}_i{}^2\big)\frac{c+d}{2}\Lambda\varkappa_i \exp(2(\mu_i + \sigma_i^2))$$

$$Var\big(TRM(\tau)\big) = \frac{c+d}{2}\Lambda \sum_{i=1}^{n} \big(\bar{m}_i{}^2\big)\varkappa_i \exp(2(\mu_i + \sigma_i^2)) \quad (2.16)$$

Using Equation (2.16), the standard deviation of the total material amount during lead-time τ can be calculated as follows:

$$\sigma_{TRM(\tau)} = \sqrt{Var\big(TRM(\tau)\big)} \qquad (2.17)$$

To determine the safety stock level, the $3s$ rule is applied, and the following interval is obtained:

$$E\big(TRM(\tau)\big) - 3\sigma_{TRM(\tau)} \le TRM(\tau) \le E\big(TRM(\tau)\big) + 3\sigma_{TRM(\tau)}$$

where $E\big(TRM(\tau)\big)$ is defined in Equation (2.11) and $\sigma_{TRM(\tau)}$ is defined in Equation (2.17) .Now we apply these results obtained for $TRM(\tau)$ to the case study.

3. Case Study

3.1 Patient Flow and Material Purchasing System in NICU

The NICU of a university hospital was considered for the case study. The NICU is capable of invasive and non-invasive monitoring of patients' cardio respiratory systems and can provide oxygen and age-appropriate thermoregulation. In addition, heart rate, respiratory rate, blood pressure, and oxygen saturation are monitored continuously.

The unit accepts both inpatients and outpatients and categorizes their illnesses according to the degree of severity. There are three care units where patients may stabilize or worsen and subsequently be re-categorized into one of the other units. The admitted patients are placed in one of three units: Unit-1 patients are non-critical, Unit-2 patients are relatively critical, and Unit-3 patients are highly critical.

When patients arrive at the unit, they are accepted if there is an unoccupied bed; otherwise, they are rejected or transferred elsewhere. Inpatients that are transferred from Unit 1to another have higher priority than outpatients. Upon transfer, if there is no unoccupied bed available, the transferred patient stays in the same unit until a bed becomes available. Table 3.1 [15] shows the rate of patients staying for treatment and then being discharged and the transferal rates between units.

Each patient's requirements for medical supplies vary daily according to their condition. Hence, the amount required for each inventory item depends on the number of patients in the NICU. The daily requirement depends on random parameters such as the patient arrival rate, LOS and the probability of patient transfer. To estimate these random parameters, data from 3330 patients treated in the NICU were collected for five years. The patient arrival rates were calculated as the sum of the accepted patient arrivals and rejected patient arrivals for each unit. The accepted and rejected patient arrivals show a Poisson distribution, and LOS shows a lognormal distribution [16].

To determine the material requirement for any unit in the NICU, medical supply use within the NICU was monitored for one year. Data were collected on approximately 600 types of medical supplies. These materials were classified according to the ABC Analysis method. A filter-pump-set (FPS) is an example from the A group of medical equipment. The minimum leadtime for FPS is 21 days, and the maximum lead-time is 60 days.

3.2 Calculating ergodic distributions for each unit in the NICU

Considering the transferal rate between units in the NICU Table 3.1, Equation (2.1) can be written as follows:

Table 3.1. Patients' staying and then discharging rates and transferal rates between units

Unit of patients' located	Options	Probability
	Staying and then discharging	0.80
Unit-1	Unit-2	0.12
	Unit-3	0.08
	Staying and then discharging	0.65
Unit-2	Unit-1	0.25
	Unit-3	0.10
	Staying and then discharging	0.15
Unit-3	Unit-1	0.55
	Unit-2	0.30

$$(\varkappa_1, \varkappa_2, \varkappa_3)\begin{pmatrix} 0.80 & 0.12 & 0.08 \\ 0.25 & 0.65 & 0.10 \\ 0.55 & 0.30 & 0.15 \end{pmatrix} = (\varkappa_1, \varkappa_2, \varkappa_3)$$

The ergodic distributions $\vec{\pi}_{\infty}^{T}$=(0.617; 0.291; 0.092).

According to the ergodic distribution, after a long time, 61.7% of the patients in the NICU had been in Unit1, 29.1% were in Unit2, and 9.2% were in Unit3.

3.3 Calculating patient LOS for each unit in the NICU

From the data of patients treated in the NICU, the average accepted and rejected patient arrivals for each unit are calculated according to Poisson distributions. These averages are; $\hat{\lambda}_{11} = 0.25$, $\hat{\lambda}_{21} = 0.76$, $\hat{\lambda}_{31} = 0.63$, $\hat{\lambda}_{10} = 0.87$, $\hat{\lambda}_{20} = 0.98$ and $\hat{\lambda}_{30} = 0.96$. Thus, the average number of patient arrivals at each unit per unit of time is calculated as follows:

$$\hat{\lambda}_1 = \hat{\lambda}_{11} + \hat{\lambda}_{10} = 1.12$$
$$\hat{\lambda}_2 = \hat{\lambda}_{21} + \hat{\lambda}_{20} = 1.74$$
$$\hat{\lambda}_3 = \hat{\lambda}_{31} + \hat{\lambda}_{30} = 1.59$$

The accepted patient number to the NICU per unit of time is calculated as follows:

$$\Lambda = \hat{\lambda}_{11} + \hat{\lambda}_{21} + \hat{\lambda}_{31} = 0.25 + 0.76 + 0.63 = 1.64$$

$\hat{\lambda}_{i1}$ indicates the average number of accepted patients in unit i per unit of time according to the ergodic distribution. The following calculations were obtained:

$$\bar{\lambda}_{11} = \Lambda\varkappa_1 = (1.64)(0.617) = 1.012$$
$$\bar{\lambda}_{21} = \Lambda\varkappa_2 = (1.64)(0.292) = 0.477$$
$$\bar{\lambda}_{31} = \Lambda\varkappa_3 = (1.64)(0.092) = 0.151$$

The averages and standard deviations of LOS for each unit are calculated according to the lognormal distribution from patient data. The respective averages and standard deviations were as follows: $\hat{\mu}_1 = 1.67$, $\hat{\sigma}_1 = 0.99$, $\hat{\mu}_2 = 2.16$, $\hat{\sigma}_2 = 0.69$, $\hat{\mu}_3 = 1.55$, $\hat{\sigma}_3 = 1.14$.

$$\mathrm{E}(\mathrm{T_i}) = \exp\left(\mu_i + \frac{\sigma_i^2}{2}\right)$$
$$\mathrm{Var}(\mathrm{T_i}) = \exp(2\mu_i + \sigma_i^2)\,(\exp\,(\sigma_i^2) - 1)$$

Thus the following are obtained: $\mathrm{E}(\mathrm{T_1}) = 8.671$ days; $(\mathrm{T_2}) = 11$ days; $\mathrm{E}(\mathrm{T_3}) = 9$ days; $\mathrm{Var}(\mathrm{T_1}) = 125.114$ and $\sigma_{T1} = 11.2$; $\mathrm{Var}(\mathrm{T_2}) = 73.8$ and $\sigma_{T2} = 8.59$; $\mathrm{Var}(\mathrm{T_3}) = 217.21$ and $\sigma_{T3} = 14.74$. Then the expected value of total LOS per unit time, $\tilde{\lambda}_{i1}\mathrm{E}(\mathrm{T_i})$ is calculated as follows.

Unit 1: $\tilde{\lambda}_{11}\mathrm{E}(\mathrm{T_1}) = (1.012)(8.671) = 8.775$ days
Unit 2: $\tilde{\lambda}_{21}\mathrm{E}(\mathrm{T_2}) = (0.477)(11) = 5.247$ days
Unit 3: $\tilde{\lambda}_{31}\mathrm{E}(\mathrm{T_3}) = (0.151)(9) = 1.363$ days

3.4. Calculating safety stock level in the NICU

Using Equation (2.11), the expected value of the total material amount in the NICU is calculated as follows:

$$\mathrm{E}\big(\mathrm{TRM}(\tau)\big) = \frac{\mathrm{c}+\mathrm{d}}{2}\Lambda\sum_{\mathrm{i}=1}^{3}\bar{\mathrm{m}}_{\mathrm{i}}\,\varkappa_{\mathrm{i}}\exp\left(\mu_{\mathrm{i}} + \frac{1}{2}\sigma_{\mathrm{i}}^2\right)$$

where $\Lambda = 1.64$, $\mathrm{c} = 21$ days, $\mathrm{d} = 60$ days, $\bar{\mathrm{m}}_1 = 1$, $\bar{\mathrm{m}}_2 = 1$, $\bar{\mathrm{m}}_3 = 2.5$, $\varkappa_1 = 0.617$, $\varkappa_2 = 0.291$, $\varkappa_3 = 0.092$,$\hat{\mu}_1 = 1.67$, $\hat{\sigma}_1 = 0.99$, $\hat{\mu}_2 = 2.16$, $\hat{\sigma}_2 = 0.69$, $\hat{\mu}_3 = 1.55, \hat{\sigma}_3 = 1.14$.

The expected value of the total material amount is $E\big(TRM(\tau)\big) = 1710.367$. Using Equation (2.16), the variance of the total material amount in the NICU is:

$$Var\big(TRM(\tau)\big) = \frac{c+d}{2}\Lambda\sum_{i=1}^{3}\big(\bar{m}_i{}^2\big)\varkappa_i\exp(2(\mu_i + \sigma_i^2))$$

Therefore $Var\big(TRM(\tau)\big) = 23390.88$ and $\sigma_{TTM} = 152.94$

For the reorder point level, the following interval is found using the 3σ rule.

$$E\big(TRM(\tau)\big) - 3\sigma_{TRM(\tau)} \le TRM(\tau) \le E\big(TRM(\tau)\big) + 3\sigma_{TRM(\tau)}$$
$$1251.55 \le TRM(\tau) \le 2169.19$$

This result shows that when the amount of material in stock is as low as 2169.19, a new lot should be ordered. Therefore, the hospital can avoid storage issues with a high degree of probability.

4 Conclusions

Few studies have calculated safety stock levels with stochastic leadtime in inventory control systems, especially in healthcare, where inventory levels must be planned carefully to provide better patient care. To determine the safety stock level while considering stochastic leadtime, the expected value and standard deviation of demand during the stochastic leadtime were calculated. Both the safety stock level and confidence interval for the reorder point level were computed using this method and the 3σ rule. A case study was presented with real data obtained from a neonatal intensive care unit.

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