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Research Article

Geometric path planning for parallel parking on double side parked narrow streets

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Abstract: The aim of the paper is to propose collision-free path planning algorithm for autonomous parallel parking based on Ackermann steering geometry. A geometrical path planning algorithm has been employed for generating a path that is based on two straight lines and two identical circles. The algorithm is taking into account that the left side of the street is taken as another parking slot occupied by full of cars. The ego's kinematics model appropriate for the low-velocity parking scenario and the maximum allowable steering angle for the narrow streets is used in the path planning stage by considering mechanical and environmental limits. According to the steering angle, required length of the parking space is obtained. The algorithm can adapt itself according to the lateral position of the ego in case it is parallel to other parked cars. The steering angle to be used and the required parking space length according to the lateral position of the vehicle in the environmental model are given with graphics.

Keywords: Autonomous parallel parking, path planning, Ackermann steering geometry, driver assistance systems

Çift taraflı park edilmiş dar sokaklarda paralel park için geometrik yol planlaması

Özet: Bu yayında, Ackermann yönlendirme geometrisini temel alan, otonom paralel park için çarpışmayı önleyici yol planlayıcı algoritma önerilmektedir. İki düz çizgi ve iki özdeş daireye dayalı bir yol oluşturmak için geometrik yol planlama algoritması kullanılmaktadır. Algoritma, yolun sol tarafının park etmiş araçlarla dolu olduğunu hesaba katmaktadır. Ego araca ait, düşük hızlı park senaryosuna ve izin verilen en geniş direksiyon açısına uygun kinematik model, mekanik ve çevresel sınırlar dikkate alınarak yol planlama aşamasında kullanılmaktadır. Gerekli park yeri uzunluğu, direksiyon açısına göre elde edilmektedir. Algoritma, diğer park etmiş arabalara paralel olması durumunda ego aracın yanal konumuna göre kendini uyarlayabilir. Ego aracın çevre modelindeki yanal konumuna göre kullanılacak direksiyon açısı ve gerekli park yeri uzunluğu grafikler ile ifade edilmektedir.

Anahtar Kelimeler: Otonom paralel park, yol planlama, Ackermann yönlendirme geometrisi, sürücü asistan sistemleri

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1. Introduction

According to the study by INRIX, a provider of transportation analytics, the average driver in the United States spends 17 hours per year searching for parking, costing them over $345 in wasted time, fuel, and emissions. Another study by the National Association of City Transportation Officials (NACTO) shows that nearly 30% of traffic in city centers is caused by drivers searching for parking, with an average search time of 7 minutes per driver. The study also found that the lack of parking availability and high demand often results in illegal parking, which can lead to traffic congestion and safety hazards.

A parallel parking system that uses fuzzy logic is described by (Tavakoli et al, 2011). Problems that caused by the increase in human society and lack of parking spaces are accented. Due to low speed, lateral slip is neglected. Vehicle motion is explained by using kinematic model. Two arcs are obtained to perform required lateral displacement. At the end of the maneuver, the vehicle allocates itself in the middle of parking space. There is no consideration about the left side usage of the road. In brief, left side is completely empty.

The most common approach for parallel parking is well known as the geometrical path planning approach. Rubio et al. (2018) has proposed path tracking for parallel parking by promoting Fuzzy PD+I logic control on a 1:10 scaled non-holonomic mobile robot. Real-time Robot Operating System (ROS) controlled mobile robot is equipped, with a camera and laser-type sensor detecting free park spaces and obstacles. Also, in order to neglect the dynamic disturbance such as slippage, they considered their mobile robot's kinematic modelling in low mass and low-speed range. Error multiplied by PD+I constants is the input of Fuzzy logic for producing the steering angle output. The path consists of two identical arcs and a straight line. The method of creating the arcs is described by (Sungwoo et al, 2011). The last maneuver that is based on straight line path brings the vehicle in the middle of the parking space. The steering angle is selected as 30 degrees by considering the physical limitations of the scaled prototype vehicle.

A method that enhancing Bezier curve by neural network to perform parallel parking is mentioned by (Yu et al, 2021). Flowchart of the maneuver is given. The vehicle is represented with a rectangle that encapsulates the extremities points. As stated in this study, the vehicle body is composed of arc surfaces from top view. However, in order to simplify the modelling, these surfaces are neglected. The drawback is that the driving area seems wider than the real driving area. Ackermann steering model is used in this study. Before the backward maneuvers, the ego vehicle is moving in parallel to stationary vehicles. The parking path is constituted with two arcs and a straight line that connects the ending point of the first arc and starting point of the second arc. Simulations are performed on Matlab. An intelligent Arduino minicar is used for real-world experiments. Flowchart of the autonomous parallel parking process is described.

Instead of the trajectory geometrical path planning regarding parallel parking, study proposed by Jiyuan et al. (2019) approaches desired guidance control actions by taking collide probability into account. In this control technique, three collision actions have been considered according to human experiences in order to create a collision-free path and derive a dynamic differential equation with constraints for position control theory. With this theory, many different parking maneuvers are dynamically characterized if the available parking slot is sufficiently large. Parking maneuvers are performed with a constant speed.

Parallel parking in a single maneuver and several reversed trials are the described by (Metin and Sezer, 2021). The study is based on minimizing the lateral distance constraint in the geometric approach for parallel parking is proposed. According to Ackermann's formulas, kinematic model of the parking one maneuver method (POM) and Parking in several reversed trials method (PSRT) is derived in order to find a free-collision path. Parking maneuver is considered at low speed so kinematic equations supply motion modelling accurately. On the POM, by giving the vehicle its maximum steering angle, the smallest turning radius can be achieved. Designing a path to exit a parked car from its location, then employing the opposite of that path to execute the parking operation is the main Park in Rescuing Parallel Trials (PRT) strategy. Thus, this system has become usable in both narrow and wide parking areas for passenger cars or buses.

Autonomous parallel parking of front wheel steering vehicles is mentioned by Zhang et al (2020) by accenting trajectory planning method and trajectory tracking method. Vehicle is modelled kinematically.

Trajectory planning is discussed as path planning and longitudinal velocity planning. Firstly, a collision free path is obtained with circular arcs and a straight line by considering the kinematic constraint of the vehicle. After that, a continuous curvature path is generated from the previous path by using ß-spline curve. Longitudinal velocity is obtained according to the constraints of the driving and brake systems. Proposed method is designed for one or multiple parking maneuvers.

An automatic parking system is proposed by (Zhang et al, 2022). The parking path is planned according to kinematic model of the vehicle and Model Predictive Controller (MPC) is used for path tracking. Schematic diagram of autonomous parallel parking is given. Parking maneuver constraints are described from the geometrical point of view. A quintic polynomial is used for path smoothing. Simulations are performed on Matlab and CarSim. To verify the performance of controllers, a ROS smart car is tested in laboratory environment.

An automatic parallel parking study is mentioned by (Wang et al, 2017). The ego vehicle can start the maneuver from an arbitrary angle. Two tangential circular arcs that have different radii are used for obtaining the path. Vehicle is modelled kinematically, and Ackermann steering is used due to low speed of the maneuver. Size of the parking space is analyzed in order to consider the collision avoidance conditions. The maneuver is performed on a barrier-free environment. The proposed algorithm is tried for various initial positions and orientations.

Effect of sensor errors on the parallel parking performance of a parking assist system is described by (Garg et al, 2016). In addition, path planning of parallel parking is investigated. Proposed path planning algorithm aims to park the vehicle in one trial to an empty parking space. The path consists of two circular arcs that have same radius in addition to have the same angle at their respective centers. Various control schemes are implemented to lateral controller to track the trajectory. Simulations are performed on IPG CarMaker and Simulink. Sensor errors are modelled detailed in this study.

An autonomous parallel parking study is proposed by (Hatipoglu and Kutluay, 2021). Ackermann steering geometry is used due to low speed of the maneuver. Proposed algorithm can park the vehicle within the lateral limits by creating two identical arcs with minimum radius. The main idea is retrieving the vehicle from the parking spot and then using the same path inversely to park the vehicle. Parameters that effect the steps of the parking maneuver are shown with derived equations. There is no constraint about the left side usage of the road in this study.

In this study, a path planning algorithm for autonomous parallel parking is described. The algorithm handles the maneuver in four steps, two longitudinal and two lateral. Two identical arcs are constituted to place the ego vehicle into the parking space. Proposed algorithm calculates the turning radius according to environment model by taken into consideration of the left side usage of the road in addition to vehicle kinematics which makes the algorithm more convenient for autonomous parallel parking applications in narrow streets. Also, length of the parking space is not constant and calculated for different scenarios. Steering angles and required length of the parking space are obtained for different scenarios.

2. Kinematics and path planning

Motion of the ego vehicle is modelled by using Ackermann steering geometry in this study. Vehicle which has only front wheels for steering are considered. Path planning is based on the perceived environment and geometrical equations. The path is divided in four steps. The first step brings the vehicle to required longitudinal position. Then, right forward and left forward maneuvers are executed to place the ego vehicle into the parking step. As a last step, the ego vehicle performs a straight forward maneuver to place itself in the middle of the parking space longitudinally.

2.1. Vehicle model

Vehicle is assumed to be a light commercial vehicle and the vehicle model is given as Fig.1.

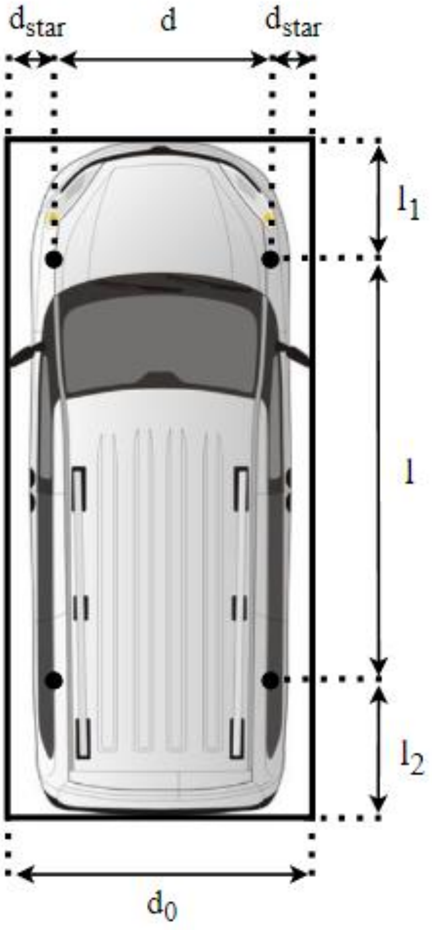

Figure 1. Vehicle model

The distance between the body of the vehicle wheels and the boundary of the vehicle body can be neglected since it is very small. However, the rear-view mirrors of the vehicle take up much more space than the distance between the wheel body and the vehicle body boundary. Therefore, it would not be right to neglect them.

To encapsulate the extremities of the vehicle, rectangular model is used. This approach is also described in (Rubio et al, 2018). However, for the development to be done in a safer range, the vehicle is represented by four moving points representing the ends of the vehicle and each wheel is represented by a point. In order to make the representation more understandable these endpoints are combined, and the vehicle is represented as a rectangle. Variables and parameters are given in Table 1.

Table 1. Variables and Parameters

Variables & Parameters	Notation	Value
Trackwidth	d	1530 mm
Width of the vehicle (side mirrors included)	d_0	2180 mm
Distance between side mirror and center of the wheel	d_{star}	325 mm
Wheelbase	L	3105 mm
Front Overhang	l_1	911 mm
Rear Overhang	l_2, k_3	740 mm
Instantaneous Center of Rotation	ICR	Variable
Turning Radius	R	Variable
Minimum Longitudinal Distance to Exit the Parking Space without Collision	k_1	Variable
Sum of Front Overhang and Wheelbase	k_2	4116 mm
Rear Factor of Safety	k_4	200 mm
Frontal Factor of Safety	k_5	500 mm
Steering Angle	δ_i (inner), δ_o (outer)	[-35 35] in degrees
Lateral Distance between the Right Side of the Ego Vehicle and Left Side of Stationary Cars that are Parked on the Right Side of the Road	X_c	[0.2-1 m] in meters
Lateral Distance between the Left Side of the Ego Vehicle and Right Side of Stationary Cars that are Parked on the Left Side on the Road	Lateral distance	[1.62-0.82 m] in meters
Lateral fos	$lateral_{fos}$	0.1 meters
X Coordinate of the Right Side of Stationary Cars that are Parked on the Left Side of the Road	$X_{st\ car\ right}$	-4
Center Coordinates of the First Circle	(x_A, y_A)	Variable
Center Coordinates of the Second Circle	(x_C, y_C)	Variable

2.2. Ackermann steering

Ackermann steering is a steering mechanism used in vehicles, especially in automobiles, that allows the front wheels to rotate at different angles while the vehicle is turning. Ackermann steering geometry is widely used in modern cars and provides more precise and stable handling in turns. This is achieved by making the inner wheel turn sharper than the outer wheel during a turn, so that the front wheels make two distinct circles with a common center point as can be seen in Figure 2.

The parking maneuver can be considered as a low-speed maneuver. For this reason, the parking maneuver is suitable for modelling with Ackermann.

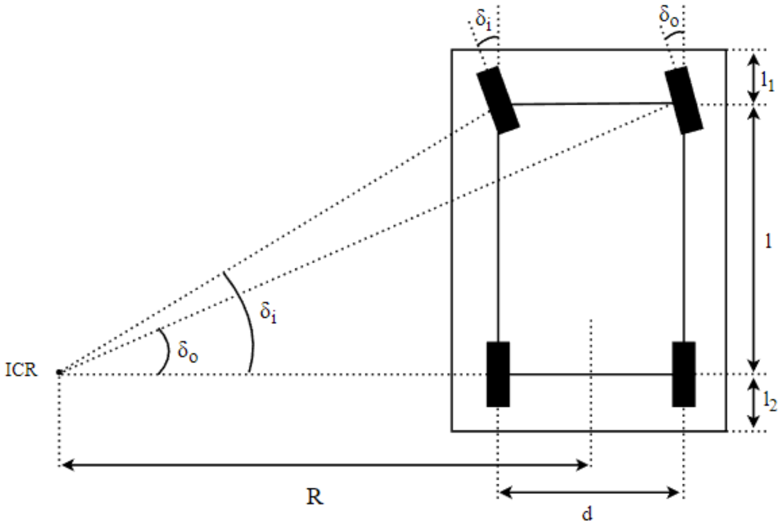

Figure 2. Ackermann steering

The inner front wheel steering angle is δ_i and the outer front wheel steering angle is δ_o.The maximum steering angle (δ_i) of the inner front wheel is assumed to be 35 degrees due to mechanical considerations of the ego vehicle, while the steering angle (δ_o) of the outer front wheel is calculated using geometrical relations. Related formula is given as Equation 1.

$$R = \frac{l}{\tan(\delta_i)} + \frac{d}{2} = \frac{l}{\tan(\delta_o)} - \frac{d}{2} \tag{1}$$

Steering angle, instantaneous center of rotation (ICR), track width and wheelbase values can be used for trajectory calculations of all wheels. The instantaneous center of rotation is fixed as long as the steering angle and the velocity are constant. After these calculations, the instantaneous center of rotation can be taken as a reference for the position of each wheel.

2.3. Path planning

A two-dimensional environment model which consists of six stationary cars, a parking space and an ego vehicle is constituted to simulate a frequently encountered scenario in Matlab and is given in Figure 3. Real-world scenario is also given in Figure 4. Stationary cars are parallel to pavement perfectly and are aligned among themselves. The ego vehicle starts the straight forward maneuver in parallel to the stationary cars.

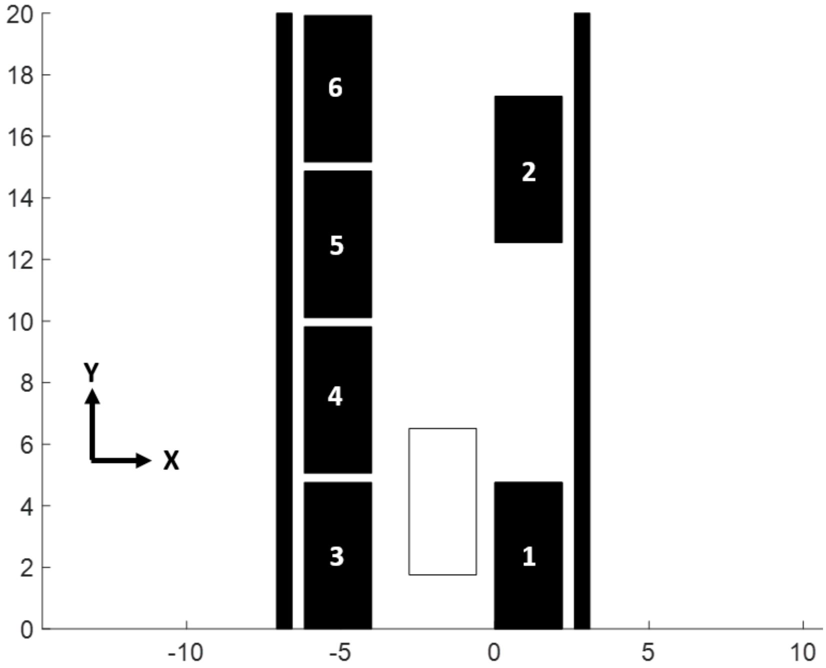

Figure 3. 2-D environment model

Figure 4. Real-world scenario

In order to park to the shortest parking space, maximum value of the steering angle should be used as proposed in (Hatipoglu and Kutluay, 2021). However, left side usage of the road increases as the steering angle increases. In our study, steering angle of the ego vehicle is calculated by considering the left side of the road. It is assumed that the lateral distance between the left border of the ego vehicle and right border of the stationary car is known. Then, this distance is taken as a limit with a factor of safety and is used for calculating the steering angle. The distance is shown in Figure 5. After that, turning radius of the vehicle and required length of the parking space are obtained.

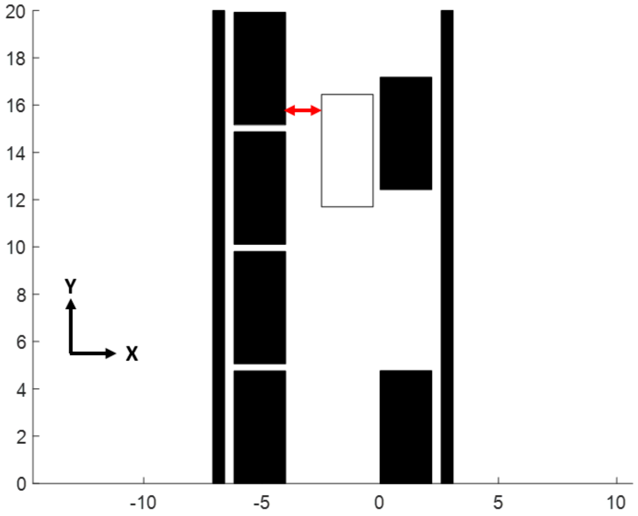

Figure 5. Lateral distance

Length of the parking space is not constant and is changing according to the lateral position of the ego vehicle in the environment model. The schematics of the parking space are given as Figure 6. On the left side, geometrical representation of the parking space that is constituted by Sungwoo et al. (2011) is shown. P_1 and P_2 are locations of the right front edge of the ego vehicle at different instants. P_3 represents the projectile of the rear axle center to the left side of the ego vehicle at the end of backward maneuvers. Then, P_4 shows the center coordinates of the maneuver center which is (x_A,y_A). As described in Rubio et al. (2018), vehicle is modelled as a rectangle by ignoring the smooth curved borders of the body. This approach makes the modelling easier. However, the driving corridor is wider than the real vehicle. By considering Sungwoo et al. (2011) and Rubio et al. (2018), it can be said that the ego vehicle does not collide to stationary car during the backward maneuver. Even so, an additional factor of safety distance is added to the length of the parking space in order to obtain safer parking maneuvers. Parking space with frontal factor of safety distance and related maneuver is given in Figure 7. Also, formula that is used for calculating the length of the parking space is given as Equation 2.

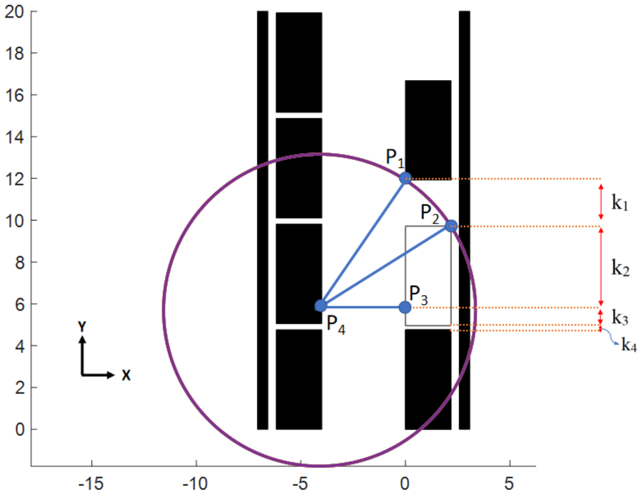

Figure 6. Parking space without frontal safety distance

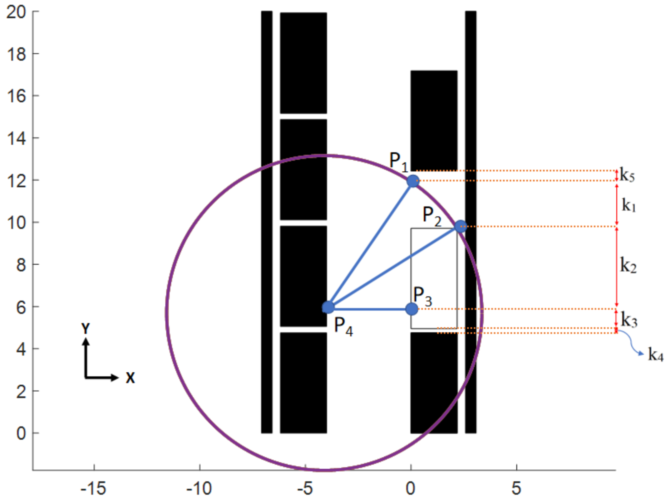

Figure 7. Parking space with frontal safety distance

$$Length\ of\ the\ parking\ space = \sqrt{|P_1P_4|^2 - |P_4P_3|^2} + k_4 + k_5 \tag{2}$$

After these steps, maneuver circles can be constituted. Firstly, the circle that is used for left backward maneuver must be obtained. The circle is given in Figure 8. The center coordinates of this circle can be obtained by using the location of the rear axle center of the ego vehicle at the end of backward maneuvers and turning radius. It is assumed that position of the rear axle center is known during the maneuver.

Turning radius is obtained according to the lateral position of the ego vehicle in the environment model. So, a representative turning radius(R) is added to Equation 3-4. Calculation of the turning radius will be explained detailed in the fourth section. The center coordinates and the radius of the circle are given as Equation 3-4.

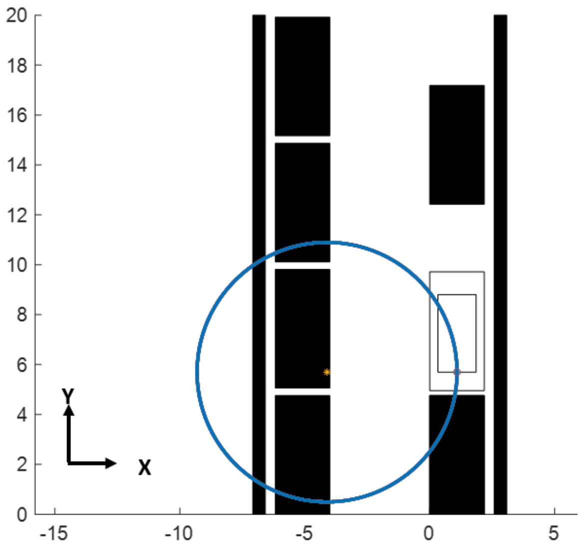

Figure 8. Left backward maneuver circle

$$Center_{X_A} = X_{RAC} - R \quad (3)$$

$$Center_{Y_A} = Y_{RAC} \quad (4)$$

Then, circle that is using for right backward maneuver can be obtained. Center coordinates of this circle is a function of the lateral position and dimensions of the vehicle, position of the first circle center and calculated turning radius. The circle is given in Figure 9. Required change in orientation angle can be calculated by using the center coordinates of the circles. Related formula is given as Equation 7.

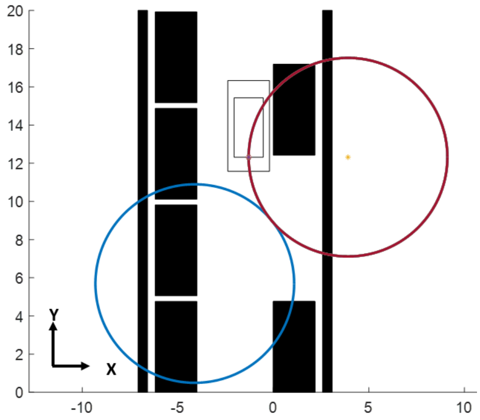

Figure 9. Right backward maneuver circle

$$Center_{X_C} = X_{RAC} + R \quad (5)$$

$$Center_{Y_C} = Y_{RAC} \quad (6)$$

$$Change\ in\ orientation\ angle\ (in\ radian) = \frac{(y_C - y_A)}{(x_C - x_A)} \quad (7)$$

Lastly, the vehicle performs a straight forward maneuver to place itself in the middle of the parking space longitudinally. End of the last maneuver is given in Figure 10. This approach is also mentioned by (Rubio et al, 2018).

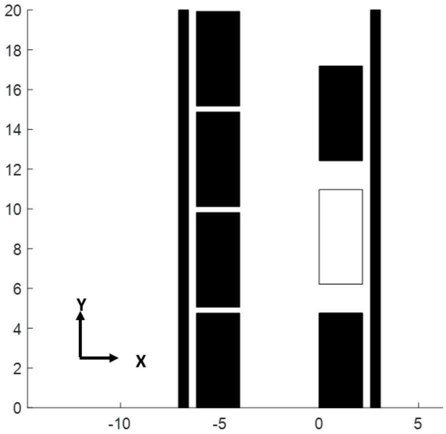

Figure 10. End of the maneuver

3. Scenario and simulation

The proposed scenario is based on a daily parking maneuver in narrow streets. Real-world case is given in Figure 4. Initially, the ego vehicle is in parallel to the stationary cars as given in Figure 3. It starts to go straight until the starting location of backward maneuvers. After that, right backward maneuver is performed. Since the required change in orientation angle has been calculated before the execution of the parking maneuver, the ego vehicle is assumed to be stopped when the change in orientation angle reaches the pre-calculated value. Then, left backward maneuver is performed until the ego vehicle is parallel to the pavement. After that, the vehicle performs a straight forward maneuver to place itself in the middle of the parking space. Under existing assumptions, the described algorithm can park the ego vehicle within lateral limits that are caused by the vehicle kinematics and dimensions of the environment model. Scenarios are given in Figure 11-15.

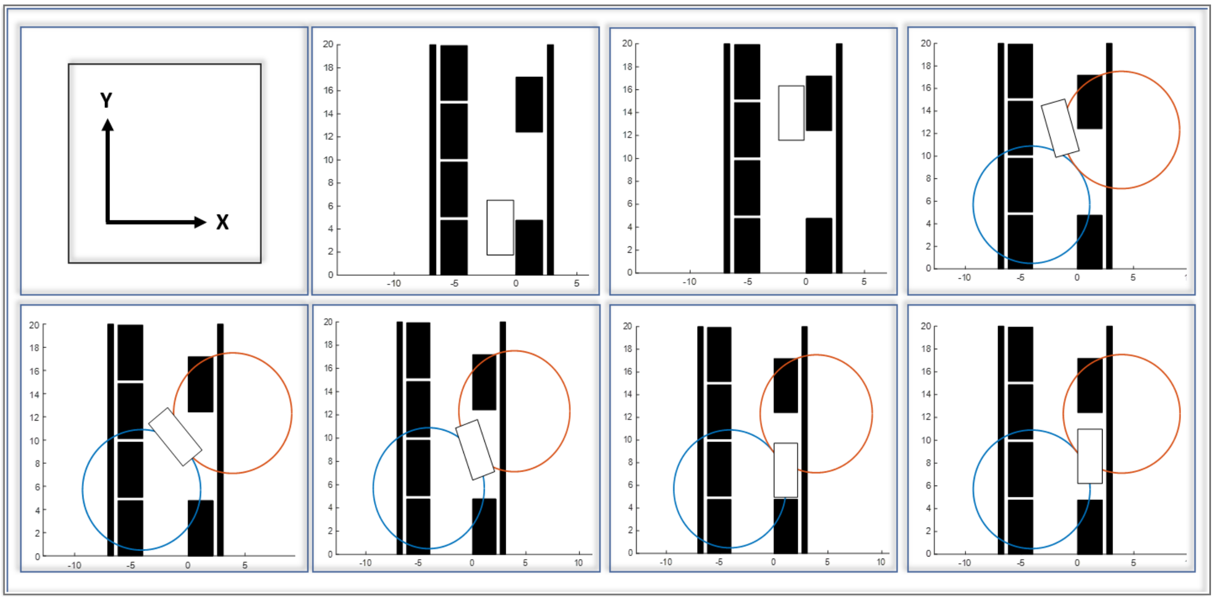

Figure 11. Scenario – 1 at which lateral distance is equal to 1.62 meters and X_c is equal to 0.2 meters

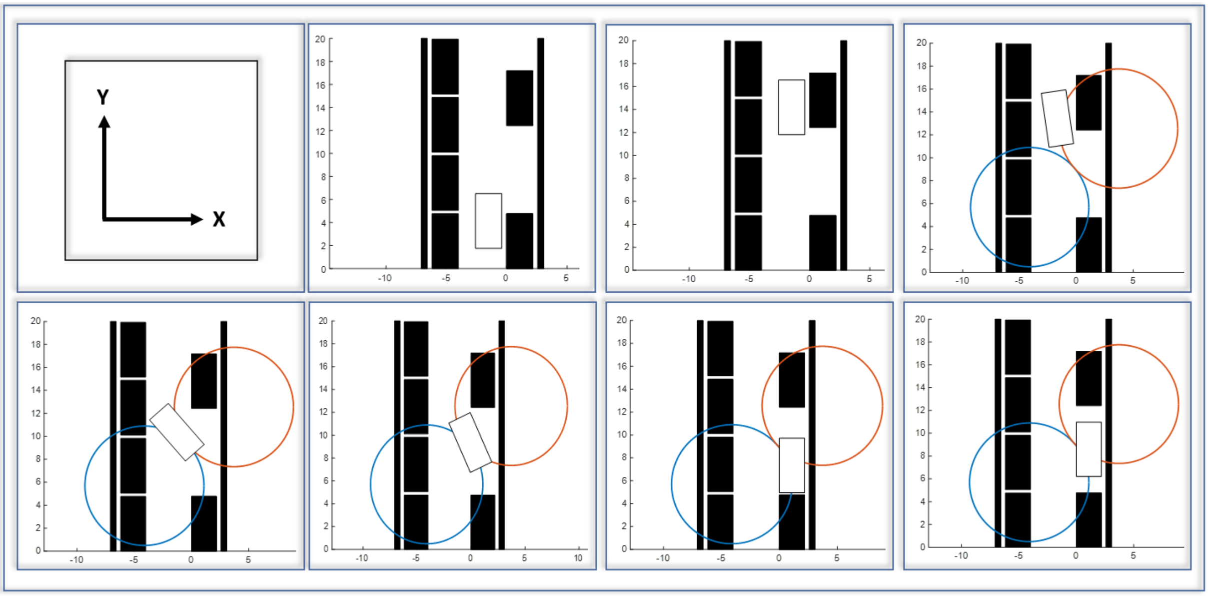

Figure 12. Scenario – 2 at which lateral distance is equal to 1.42 meters and X_c is equal to 0.4 meters

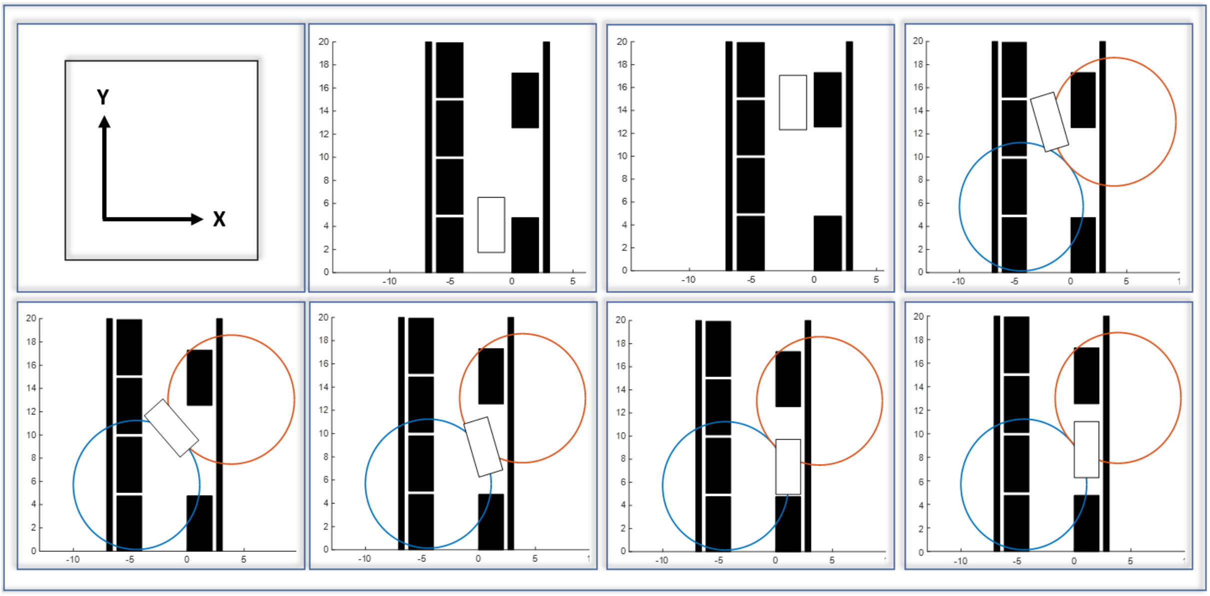

Figure 13. Scenario – 3 at which lateral distance is equal to 1.22 meters and X_c is equal to 0.6 meters

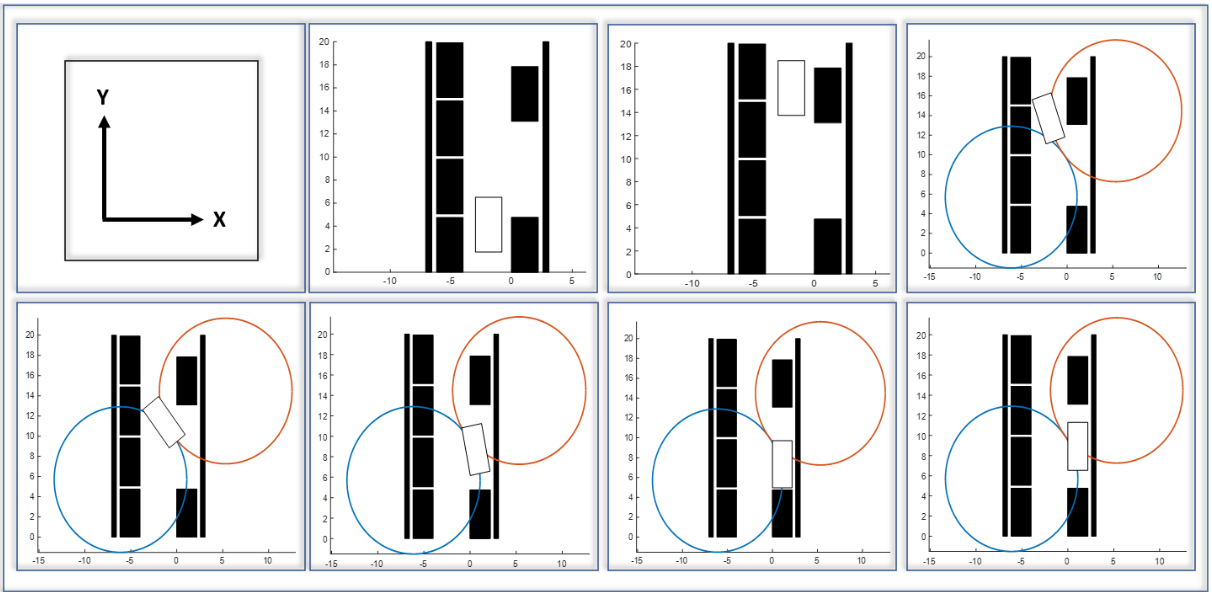

Figure 14. Scenario – 4 at which lateral distance is equal to 1.02 meters and X_c is equal to 0.8 meters

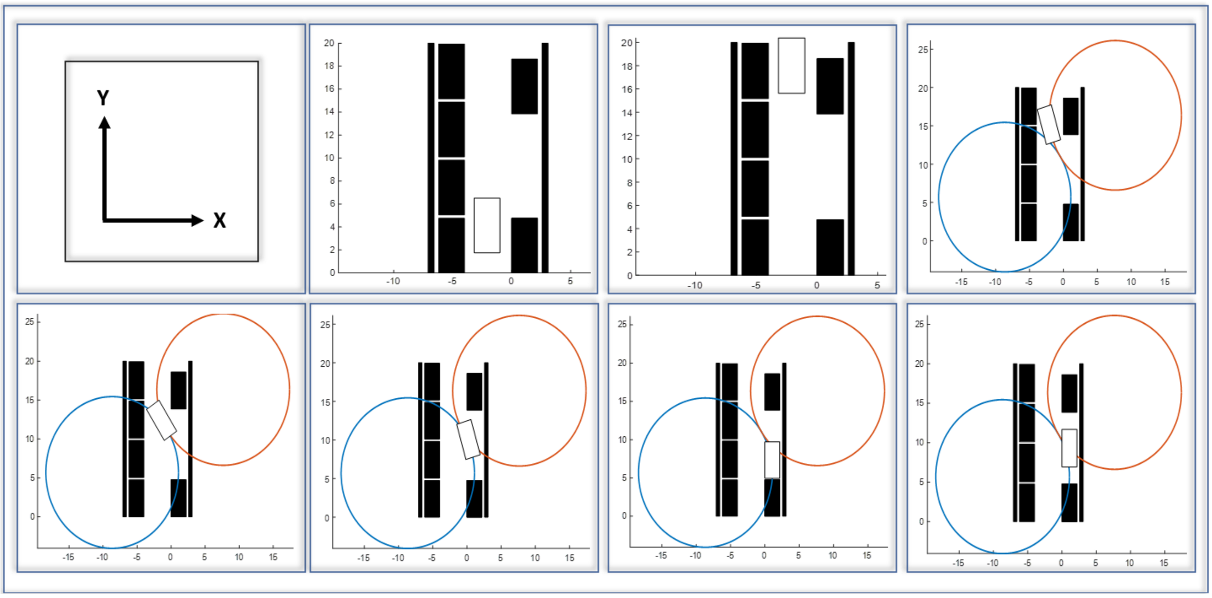

Figure 15. Scenario – 5 at which lateral distance is equal to 0.82 meters and Xc is equal to1 meters

4. Steering angle calculation formula

The steering angle can be selected as large as possible to park the vehicle in the shortest parking space. However, large steering angles require more use of the left side of the road. When the road is narrow, using of the left side of the road must be checked to evaluate the feasibility of the parking maneuver. Proposed formula calculates a turning radius according to the distance on the left. Then, required length of the parking space is obtained. Figure 16 shows the representative starting instant of the backward maneuvers. Maximum lateral displacement of the left front edge of the vehicle is calculated with this geometry. Related points are given in Table 2.

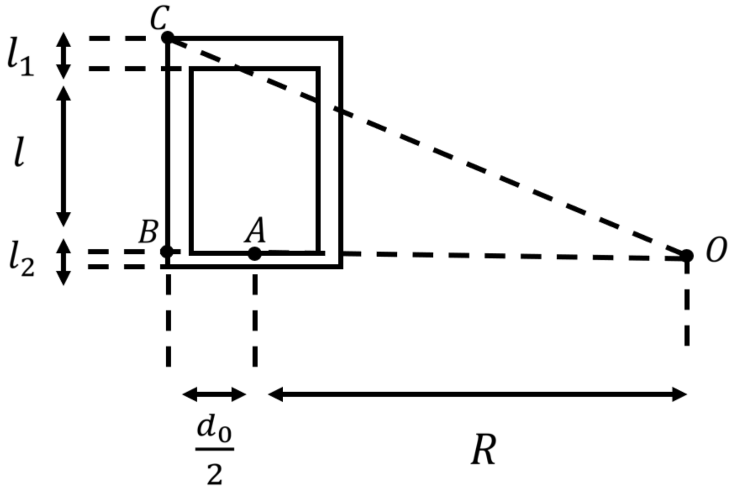

Figure 16. Starting instant of backward maneuvers

Table 2. Related points for steering angle calculation

Point	Explanation
O	Instantaneous center of rotation
A	Rear axle center
B	Projectile of $\overrightarrow{OA}$ on the left side of the vehicle
C	Left front edge of the ego vehicle

As a first step, length of the $|OC|$ must be calculated. Since it is the hypotenuse of the ΔOBC as can be seen in Figure 17., Pythagoras Theorem can be used and given as Equation 8.

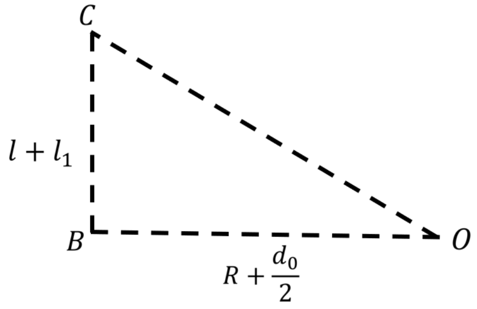

Figure 17. ΔOBC

$$|OC| = \sqrt{\left(R + \frac{d_0}{2}\right)^2 + (l + l_1)^2} \tag{8}$$

After this step, a circle that is used for obtaining the lowest x coordinate of point *C* can be created. Center of this circle is point *O* and radius of the circle is equal to $|OC|$. *C*’ shows the position of point

C when the x coordinate is minimum. The circle is given in Figure 18. At that instant, x coordinate of point *C*’ can be stated with Equation 9.

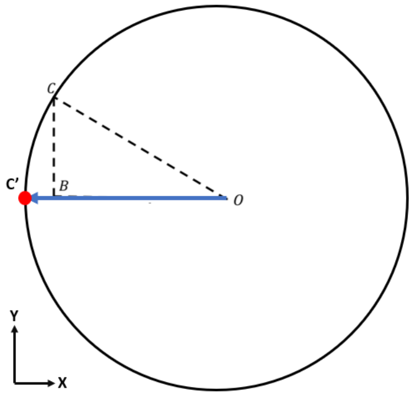

Figure 18. Circle that is used for calculating lowest x coordinate of C

$$\min(X_{C'}) = X_A + R - |OC| = X_A + R - \sqrt{\left(R + \frac{d_0}{2}\right)^2 + (l + l_1)^2} \tag{9}$$

So, the lateral displacement of point C can be stated with Equation 10.

$$lateral\ displacement\ of\ point\ C = R + \frac{d_0}{2} - \sqrt{\left(R + \frac{d_0}{2}\right)^2 + (l + l_1)^2} \tag{10}$$

The main idea is obtaining the turning radius by considering the lateral distance which is given in Figure 5, with a factor of safety distance and equalize the lateral displacement of point C to this distance. Then, the minimum turning radius that the vehicle can perform a non-collision maneuver can be obtained. Related formula is given as Equation 11. This formula is handled as a function and solved by Matlab fsolve command.

$$Turning\ radius\ = X_B + R - \sqrt{\left(R + \frac{d_0}{2}\right)^2 + (l + l_1)^2} - X_{st\ car\ right} - lateral_{fos} \tag{11}$$

In addition to this formula, a comparison section is needed to set the steering angle. For the given vehicle, maximum lateral displacement of point C is calculated as 1.17 meters. This distance is handled with lateral factor of safety distance and compared to lateral distance that is given in Figure 5.

If the difference between initial lateral distance and lateral factor of safety is bigger than the 1.17, then the steering angle is set to maximum which is equal to 35 degrees. If it is less than 1.17, then steering angle is obtained from the turning radius that is calculated by using Equation 11 by applied Ackermann steering relations.

5. Results

The relation between lateral difference and steering angles are given in Figure 19. When the lateral difference is bigger than 1.27 which is equal to the sum of maximum lateral displacement of point C and lateral factor of safety, the steering angle is set to maximum. When this difference is less than 1.27, steering angles are calculated by using Equation 11. As can be seen in Figure 19, the vehicle uses smaller steering angles to prevent left-side crashes during the parking maneuver.

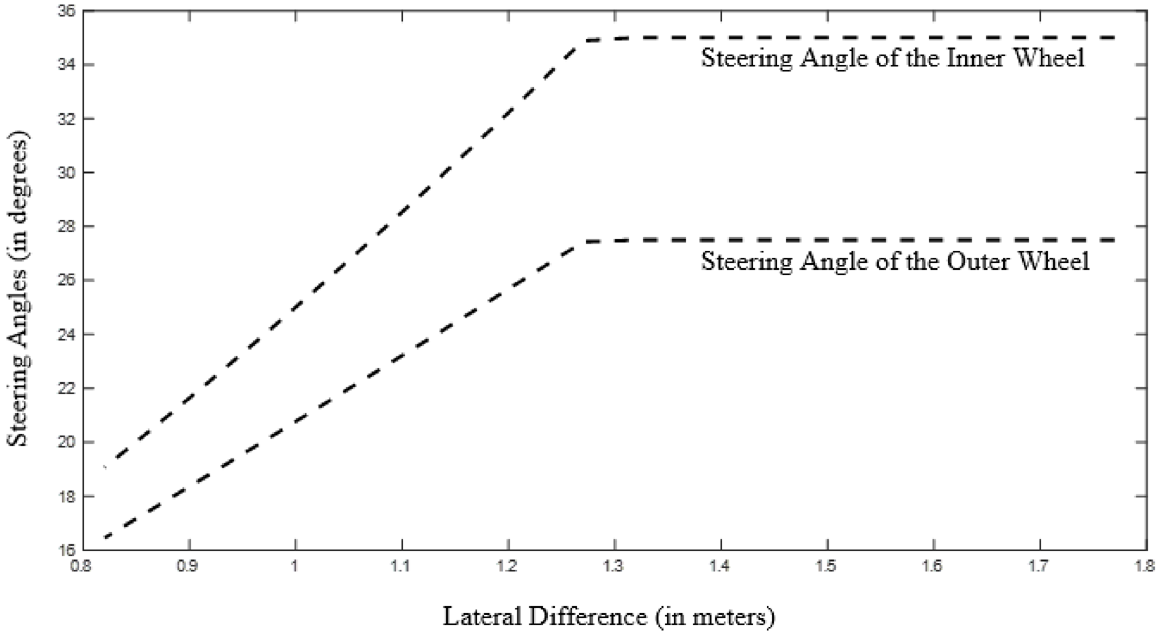

Figure 19. Lateral difference versus inner and outer steering angles

Also, the relation between required length of the parking space and lateral distance is given in Figure 20. When the ego vehicle approaches to the left side of the road, steering angles become smaller. Thus, the turning radius of the vehicle increases and the vehicle needs to be placed in longer parking spaces.

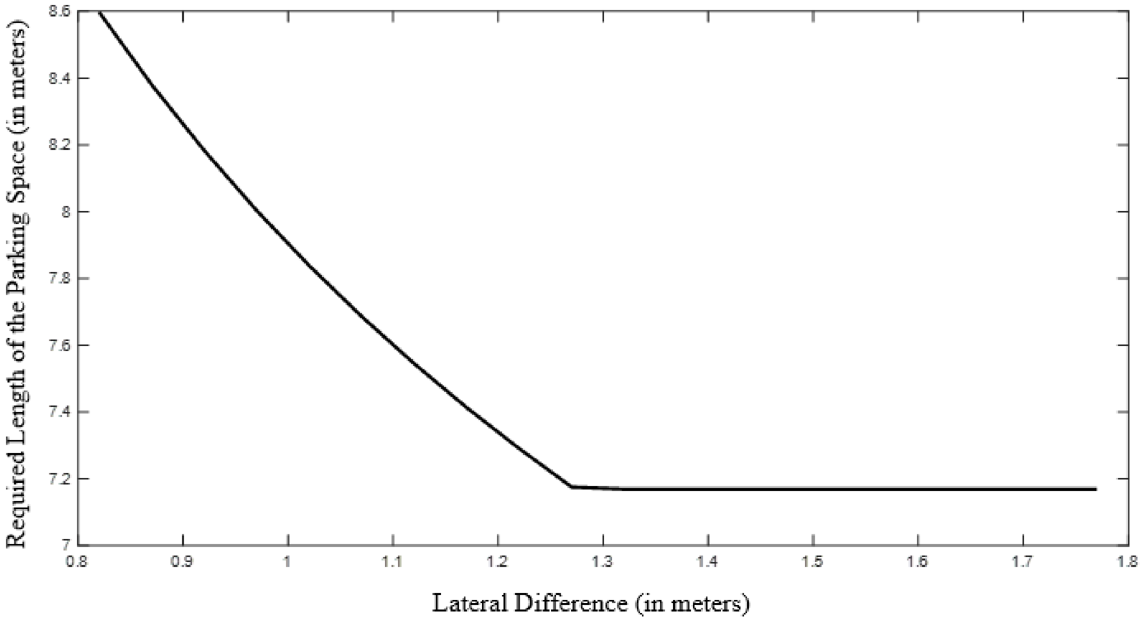

Figure 20. Lateral difference versus required length of the parking space

Finally, the maneuvers that are shown in Figure 11 – 15 are summarized in Table 3. As it can be seen on the Table 3, left side usage of the road increases as the steering angles increase. Therefore, especially in narrow streets, usage of the maximum steering angle is not feasible in order to perform collision-free parking maneuver by considering the left side of the road. In addition, the vehicle needs longer parking spaces due to non-maximum steering angles.

Table 3. Summary of Scenarios

Scenario	**Left side usage of the road (In meters)**	**Lateral distance (In meters)**	**X_c (In meters)**	**δ_{inner} (In degrees)**	**δ_{outer} (In degrees)**	**Parking Space Length (In meters)**
1	1.17	1.62	0.2	35.00	27.50	7.17
2	1.17	1.42	0.4	35.00	27.50	7.17
3	1.12	1.22	0.6	32.98	26.18	7.29
4	0.92	1.02	0.8	25.70	21.26	7.83
5	0.72	0.82	1	19.06	16.45	8.59

6. Conclusion

An autonomous parallel parking algorithm that is based on Ackermann steering geometry is described. The algorithm is independent of the lateral position of the ego vehicle within the limits and dependent of parallelism between the ego vehicle and stationary cars. Calculation of steering angle according to environment model in addition to vehicle constraints is a contribution of this study. The algorithm calculates the minimum turning radius that the vehicle can perform a non-collision parking maneuver with it. Also, required length of the parking space can be calculated and feasibility of the parking maneuver can be evaluated by considering the left side usage of the road. When the ego vehicle approaches to left side of the road, steering angle becomes smaller and required length of the parking space increases. Within the context of the proposed scenarios, it is shown that required length of the parking space can be increased up to 19.8%, however left side lateral displacement of the vehicle can be reduced up to 38.5%. Additionally, the ego vehicle sets the corresponding steering angle when the velocity equals to zero which means that the vehicle takes maximum lateral displacement with minimum longitudinal displacement. So, the smallest driving corridors are seen at proposed scenarios and these corridors can be taken as a limit for control studies. As a future work, multiple maneuver parallel parking will be investigated.

Researchers' Contribution Rate Statement

Emrecan Hatipoglu has contribute to code the autonomous geometrical parking algorithm. Mert Kadir Assoy spent effort on determining the conceptual and design processes of the study and management. Mesut Kaya has contributed to creation of the vehicle model and defining the required parameters. Mert Ezim and Mete Oguz have spent effort on literature search. Emir Kutluay supervised Emrecan Hatipoglu and evaluated the feasibility of the algorithm.

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There is no conflict of interests.

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