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Research Article

Investigation of Seventh Grade Students' Transition Skills among Representations

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Abstract

This descriptive case study was aimed at revealing the level of students' transition skills among representations (verbal, numeric, algebraic, graphic) at the completion of 7th grade. The participants included 133 students attending 7th grade at a state school located in Ankara in Turkey. Students' skills in transitioning between representations were tested using the "Test of Skills in Transitioning between Representations (TSTBR)", which included 12 open-ended questions and analyzed student answers through a graded rubric developed by the researcher. Descriptive statistics such as frequency and percentage were used to examine students' skills in making transitions between representations. Thus, it was determined in the findings that a majority of students had moderate transition skills. According to the mean scores, the number of students with low representation transition skills was greater than the number of students with high representation transition skills. Also, the students were most successful at transitioning from other representations to algebraic, verbal, numeric, and graphic representations, respectively. The mean scores for each representation transition were examined and possible reasons for the difficulties that students faced were discussed.

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Introduction

Contrary to reading, writing and arithmetic skills, algebra is a branch of mathematics which students believe they will not need outside of school. Due to this belief, students are often uninterested in algebra while in school, even though they use algebraic reasoning in their daily lives much more than they realize (such as calculating the amount of fuel used by a vehicle per kilometer) (Williams & Molina, 1997). Although some effort is taken to base curricula on real life content, some teachers shy away from context based instruction when solving equations and instead focus on developing algebraic skills through exercises (Davis, 2005). Through this approach, which resembles an abstract version of arithmetic, students do tend to be successful when following standard solution procedures (Moseley & Brenner, 1997); however, it is possible that algebra instruction based solely on algebraic representation can limit students in problem situations that require upper level skills (Kieran, 1992). To equip students with basic algebraic skills, algebra instruction should avoid abstraction from the beginning and instead make connections between arithmetic and algebra by using appropriate contexts (Tabach & Friedlander, 2003). A crucial part of this structure is the use of representation types. For example, the representations used in middle school algebra instruction comprise the link between elementary and high school, and are important tools needed at all stages of problem solving (Brenner et al., 1997).

Goldin (2003) defines representation as a "configuration of signs, characters, icons and objects that replaces a certain thing and represents it" (p. 276). Representations can be helping tools such as symbols, tables, graphics and diagrams used in the transmission of mathematical ideas to others and in the expression of relationships (Van de Walle, 2013). The representations used in curricula sometimes vary based on age or style (Bruner, 1966). For example, using different representations aligned with the students' level and content within the instructional process, makes students feel more comfortable. Also, learning contexts where more than a single representation is used provide opportunities for students with different learning styles (Schultz & Waters, 2000). Therefore, due to the power and effectiveness of different representations, students discover links that they would likely not recognize with the use of a single representation and thus reach a solution. When the difficulty of a problem increases, students construct new links over existing ones. In this way, concepts become linked to one another, and an effective instructional process can be developed. Additionally, when problems are expressed through different representations, even problems with simple numerical solutions can become richer learning experiences (Choike, 2000). Students also develop better skills to decide which representations to use when solving problems (Schultz & Waters, 2000). The book entitled, "Principles and Standards for School Mathematics", published by the internationally acclaimed National Council of Teachers of Mathematics [NCTM] (based in the USA) recommends that middle school students make frequent use of representations when developing mathematical ideas and solving problems (NCTM, 2000). Importantly, complex ideas can rarely be represented with a single method. Thus, many topics have multiple aspects that must be depicted through different representations. Sometimes, owing to its unique strengths, one representation can clarify a topic better than another (Kaput, 1992). For instance, in the transition from arithmetic to algebra, tables comprise a rather effective representation style as they enable students to obtain a pattern from numbers. Also, graphs clearly reveal change since they are visual. Thus, generalizations may be made, and problems can be solved through algebraic representations. Often, they are also used in generalizing existing solutions (Friedlander & Tabach, 2001).

The skill to make transformations between representations plays a crucial role in concept development (Lesh, 1979). As a result, a distinguishing feature of higher order mathematical thinking is the ability to make transitions between representations. Making strong and detailed links between representations and transforming them from one to the other improves students' conceptual understanding (Even, 1998). Thus, teachers have started to increasingly use multiple representations with the belief that they facilitate the learning of mathematics (Thompson & Chappell, 2007).

There are a variety of studies in the literature regarding multiple representations (Gürbüz & Şahin, 2015; Panasuk & Beyranevand, 2010; Sert, 2007; Yıldırım & Albayrak, 2016). The number of these type of studies conducted within Turkey on this subject has also risen, especially in recent years (Ayyıldız & Aktaş, 2022). For example, research has been conducted into multiple representations at the middle school level focusing on students' representation use and transformation skills.

For instance, Sert (2007) studied 705 eighth graders' transitioning skills between different representations (graphic, table, equation, verbal statement). The results indicated that students had the most difficulty transitioning from other representations to verbal explanations, whereas they found transitioning from other representations to numerical forms via tables tended to be the easiest.

In other research conducted by Panasuk and Beyranevand (2010) with 443 low-achieving seventh and eighth graders investigated their skills in recognizing problem situations presented through different representations (verbal, diagram, symbolic). It was indicated in the results that students are inadequate in recognizing problems shown with different representations and that students who are good at symbolic solutions had a low conceptual understanding. This result was attributed to the lack of emphasis on multiple representations in the curricula.

Similarly, in a qualitative study, Gürbüz and Şahin (2015) investigated four eighth graders' transitioning skills between representations (verbal, table, equation, graph). In this study the representation transition that students found the most difficult tends to be from other representations to graphical representation, while the easiest transition is determined to be from other representations to table representation.

In another research, Yıldırım and Albayrak (2016) not only examined 93 seventh graders' skills in different representations (table, graph, equation) but also their abilities to identify linear relationships which are presented through differing representations. Thus, it was concluded that students are better at forming numerical tables concerning real life situations than other representations. Additionally, students are found to be better at identifying linear relationships in graphic representations than other forms.

In previous studies, the effects of multiple representation based instruction on middle school students' success has been examined. Some study results have shown that multiple representation based instruction increases secondary students' algebra performance (Akkuş-Çıkla, 2004; Kaya, 2015; Moseley & Brenner, 1997) as well as positively affects skills for transformation between representations (Brenner et al., 1995).

İncikabı (2017) identified the representation types included in middle school mathematics textbooks and also examined the transitions between them. It was indicated in the research that textbooks most commonly include algebraic representations and touch on verbal and model representations but rarely focus on table, graphic and real life representations. In 2018, the Turkish Ministry of Education (MoNE) made changes to the "Secondary School Mathematics Curriculum", so that after the fifth grade, curriculum objectives should include the use of verbal, algebraic, numeric and graphic representations as well as focus on transformation from one to another is important. Therefore, textbooks prepared according to the curriculum are expected to provide opportunities for the use and transformation of these representations.

It is shown in the literature on the other hand, that students' skills of making transitions between certain representations are low (Gürbüz & Şahin, 2015; Sert, 2007; Yıldırım & Albayrak, 2016). Therefore, it remains to be seen whether the secondary school mathematics curriculum changes made by MoNE following 2018 and the renewal of textbooks will positively affect Turkish student skills in the transition between representations. According to the objectives in the middle school mathematics curriculum, students are expected to be able to use verbal, numerical, algebraic and graphical representations by the end of the 7th grade. After the changes made in the mathematics curriculum, there was no study examining students' transition skills between representations at the 7th grade level. In line with this goal, the aim of this study was to examine the level of student skills in transitioning between representations (verbal, numeric, algebraic, graphic) at the completion of seventh grade.

Method

The following section includes information regarding the research model, participant demographics, data collection process, and data analysis procedures of the present study.

Research Design

This was a descriptive case study aimed at revealing student skills in transitioning between representations at the completion of seventh grade. This method was chosen to analyze students' transitioning skills in detail as well as reveal the existing situation. Importantly, case studies focusing on a specific phenomenon, enable the description of the case at the end of the study along with facilitating a better understanding of the phenomenon being studied (Merriam, 2013).

Participants

The study group was selected by using convenience sampling. This method is chosen so that the researcher can have proximity and access to the participants (Patton, 2014). A pilot study of the data collection tool was completed at one public school, while another public school within the same city was chosen for the actual research implementation. Thus, the study group included 133 seventh grade students from one public school located in Ankara city during the 2021-2022 academic year. Permissions from the Ministry of National Education were obtained for the ethics committee and voluntary participation was provided.

Data Collection Tools

The "Test of Skills in Transitioning between Representations (TSTBR)" used in the present study to test students' transitions between representations (verbal, numeric, algebraic, graphic) was designed by the researcher. The test included 12 open-ended questions measuring transition skills from each of the representations to all the others. Prior to the pilot study, 22 open-ended questions were prepared and presented to five academicians and three mathematics teachers. Thus, the expert views were gathered regarding the appropriateness of the questions for grade level, whether the questions targeted representation transitions, and whether they were clearly written. A pilot study was conducted with 108 students at the beginning of the second semester during the 2021-2022 academic year. Also, taking into account the expert views and following the pilot study, the skills test was finalized as a 12-item test to measure transitions from each representation to the others. The duration of the implemented test was 40 minutes.

Data Analysis

The TSTBR, designed to measure students' transitioning skills between representations, was implemented in one sitting in which students were given 40 minutes to complete the test. In this open-ended test, a graded scoring rubric was used to analyze students' answers. These rubrics are used because they enable scoring in line with pre-specified criteria (Mertler, 2000). As a result, the study answers were scored using the graded rubric which had been designed by the researcher. Based on this rubric, student answers were grouped as *"inadequate"*, *"needs development"*, *"good"* and *"very good"*. Next, the scoring was done by assigning 0 points for answers in the "inadequate" category, 1 point for those in the "needs development" category, 2 points for those in the "good" category, and 3 points for those in the" very good" category. Thus, for each item in the test, a student could receive a minimum of 0 points and a maximum of 3 points. The lowest score possible for the entire test was zero, and the highest possible score was 36. The grading rubric is provided in Table 1.

Table 1. Grading rubric used in scoring students' answers

	Scoring Criteria
3 Very Good	-The question was fully understood. -The transition between representations was accurate and error-free. No errors were found in the operations done to answer the question. The right answer was reached. A clear and detailed explanation was given if required. -The solution was detailed and could be considered a sample answer.
2 Good	-The question was largely understood. -Even though the transition between representations was made correctly, the right solution had not been reached due to simple calculation error. -The right answer was reached but the solution was not explained clearly enough.
1 Needs development	-The question was partially understood. -The solution started with the right representation but did not proceed correctly. -Even though the representation used was correct, the solution entails conceptual mistakes or numerous calculation errors. -The correct answer was reached but the required explanation was not provided.
0 Inadequate	-The question was not understood. -The representation used was entirely wrong and did not contribute to the solution. - Expressions such as "I didn't understand", "I don't know" were used, or the data presented in the question were repeated. -There was no operation and no explanation.

For inter-rater reliability, a mathematics teacher, with 10 years of experience, who was pursuing a master's degree in the field, scored the answers independently from the researcher. To identify overlap between the scores obtained from these sources, the Miles and Huberman (1994) inter-rater reliability formula was used, and reliability was found to be 0.93. Since this value exceeded 0.70, it denoted consistent scoring.

Also, to identify students' representation transitioning skill levels based on their test scores, the evaluation criteria in Table 2 were used. In the evaluation, students with 25 points and above were grouped under "High Representation Transition Skills (HRTS)", those with scores between 13 and 24 points were grouped under "Moderate Representation Transition Skills (MRTS)", and those whose scores were 12 and below were grouped under "Low Representation Transition Skills (LRTS)".

Table 2. TSTBR evaluation criteria

Score Intervals	Evaluation
25-36	High Representation Transition Skills
13-24	Moderate Representation Transition Skills
0-12	Low Representation Transition Skills

The TSTBR has one item for measuring each of the student skills for transitioning from any of three representations to the fourth one. To analyze students' representation transition skills in more detail, each representation transition was examined according to four subheadings. In this analysis, the total score a student obtained from transitioning from three representations to a given one was calculated. This sum indicated the general level of transition to a given representation. For example, in the three items which measure transition to a given fourth representation, students who scored between 9-7 points were categorized as having "high" skills, whereas those with 6-4 points were categorized as having "moderate" skills, and those with 3-0 points were classified as having "low" skills.

Findings

Presented first the following section are the descriptive statistics pertaining to students' TSTBR scores. The students' minimum and maximum scores, mean averages, and standard deviations are provided in Table 3. Following this, students' skills in transitioning to each representation (verbal, numeric, algebraic, graphic) from the other representations were examined separately.

Table 3. Descriptive statistics for students' TSTBR scores

	Number of Students	Minimum	Maximum	Mean	Standard Deviation
TSTBR Scores	133	0	36	16.68	8.15

As indicated in Table 3, the lowest score of the students who participated in the test was zero and the highest score was 36. The mean score obtained by these students for the TSTBR test, in which the maximum possible score was 36, was 16.68. The distribution of students according to their ability to make transitions between representations is given in Table 4.

Table 4. Distribution of students according to the level of representation transition skills

	f	%
High Representation Transition Skills	19	14.3
Moderate Representation Transition Skills	71	53.4
Low Representation Transition Skills	43	32.3

It is shown in Table 4 that the majority of students had moderate representation transitioning skills. What needs to be noted though, was that the number of students with low representation transitioning skills was more than that of students with high representation transitioning skills.

The mean scores of students for each item that measured a different representation skill in the TSTBR test are provided in Table 5. In the table, verbal representation was indicated as "V", numeric representation as "N", graphic representation as "G", and algebraic representation as "A". Also, the transition between representations was indicated with a "-" symbol. For example, (V-N) denoted the transition from verbal to numerical representation.

Table 5. Mean scores of students in each item of the TSTBR test

Items	Mean Score
Item 1 (V-A)	1.90
Item 2 (V-N)	1.44
Item 3 (A-V)	2.32
Item 4 (V-G)	1.18
Item 5 (A-N)	1.31
Item 6 (G-A)	1.88
Item 7 (N-G)	1.52
Item 8 (N-A)	1.70
Item 9 (G-V)	1.04
Item 10 (A-G)	0.62
Item 11 (G-N)	1.01
Item 12 (N-V)	0.75

As indicated in Table 5, the item where students were most successful was Item 3, which measured the transitioning skill from algebraic to verbal representation. Students' mean score in this item was 2.32 out of 3. The three subsequent most successful representation transitions (item 1 (V-A), item 6 (G-A), item 8 (N-A)) were from others to algebraic representations. For example, the first four items with the highest student mean scores showed that students were good at algebraic representations. While the item in which students were least successful was the one (item 10) that measured student skills in transitioning from algebraic representation to graphic representation. In this item, students' mean score was only 0.62 out of 3. Thus, it was observed that four (item 10, item 11, item 9, item 4) of the five items with the lowest mean scores involved graphic representation.

Transition Skills to Algebraic Representation

The descriptive statistics depicting students' total scores for the three questions which measure transition skills from verbal, graphic and numeric representation to algebraic representation (item 1, item 6, item 8) are provided in Table 6. The lowest possible score for these three questions was zero, and the highest possible score was nine.

Table 6. Students' total score distribution in the transition to algebraic representation

	9-7 points		6-4 points		3-0 points	
	f	%	f	%	f	%
Transition to Algebraic Representation	49	36.9	45	33.8	39	29.3

As can be seen in Table 6, 36.9% of the students had high transition skills from other representations to algebraic representation, 33.8% had moderate skills, and 29.3% had poor transition skills. Additionally, there were 32 students who achieved a full score on all of the items concerning the transition from the other three representations to the algebraic representation, whereas nine students did not score any points on any of these items. As a result, students' mean score from the items concerning transition to algebraic representation was 5.48.

When students' transitions from other representations to the algebraic representation were ranked based on their mean scores, the most successful transition appears to have been from verbal representation to algebraic representation (item 1) with a mean score of 1.90. This was followed by the transition from graphic to algebraic representation (item 6) with a mean score of 1.88, and the least successful transition to the algebraic representation was from numeric representation (item 8) with a mean score of 1.70.

Furthermore, frequent mistakes by students in the transition to algebraic representation included failing to express the relationship between variables algebraically, failing to form an accurate equation, and failing to accurately transform expressions such as ‘times’, ‘half’, ‘same’, ‘more’, and ‘less’ into algebraic expressions. In Figure 1, examples are displayed of students’ incorrect answers to Item 1: “If twice a number increased by five is the same as that number decreased by seven, then halved, what is the number? Write an equation that represents this (Do not solve the equation)”.

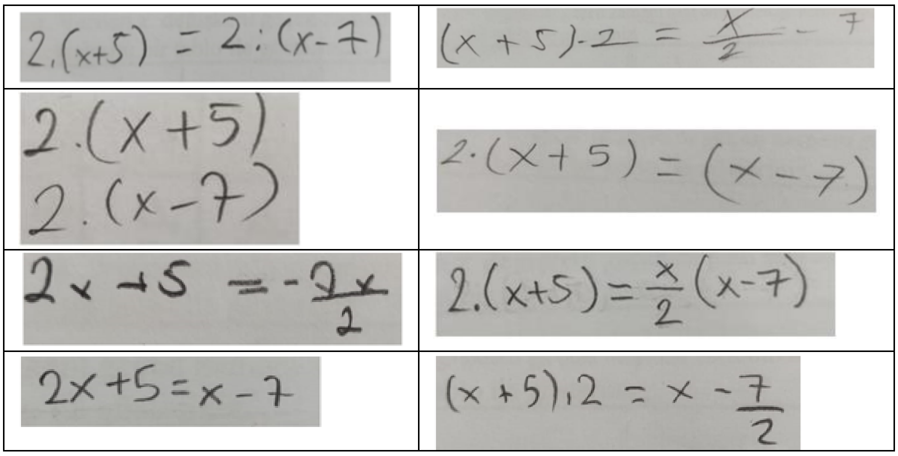


Figure 1. Examples of incorrect answers to item 1

Transition Skills to Verbal Representation

Descriptive statistics representing students’ total scores for the three questions which measured transition skills from algebraic, graphic and numerical representation to verbal representation (items 3, 9, and 12) are provided in Table 7. The lowest possible score for these three questions was zero, and the highest possible score was nine.

Table 7. Students’ total score distribution in the transition to verbal representation

	9-7 points		6-4 points		3-0 points	
	f	%	f	%	f	%
Transition to Verbal Representation	24	18.1	50	37.5	59	44.4

As is shown in Table 7, 18.1% of students had high transition skills from other representations to verbal representation, whereas 37.5% had moderate skills and 44.4% had poor skills. Additionally, there were three students who received a full score on all the items concerning the transition from the other three representations to verbal representation, as well as nine students who did not score any points for any of the items. Next, the mean student score for items concerning the transition to verbal representation was 4.11.

When students' transitions from other representations to the verbal representation were ranked based on their mean scores, the most successful transition seemed to be from algebraic to verbal representation (item 3) a mean score of 2.32, which was followed by the transition from graphic to verbal representation (item 9) with a mean score of 1.04. Finally, the least successful transition to verbal representation was from numeric representation (item 12) with a mean score of 0.75.

In the transition to verbal representations, the students had the most difficulty with the interpretation of numeric and graphic representations. For example, most of the students were not able to verbally explain the relationship between the *nth* term of a sequence and *n* (Item 12). The TSTBR has one item for measuring each of the student skills for transitioning from any of three representations to the fourth one. In a similar question, students were able to algebraically express the *nth* term in the sequence within the table. The fact that students were not able to verbally explain a representation which they were able to express algebraically was worth further consideration. In Figure 2, Item 12 in the TSTBR designed to measure students' transition skills from numerical to verbal representation are provided.

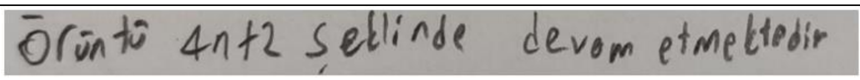
The table below shows the relationship between the number of steps and terms of a number pattern.

Number of steps	Terms
1	6
2	10
3	14
4	18
…	…
n	…

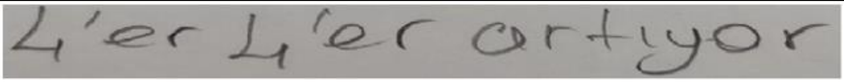
Using the data in the table, verbally express the relationship between the number of steps of the pattern and the terms of the pattern.

Figure 2. Item 12 regarding the transition from numerical representation to verbal representation

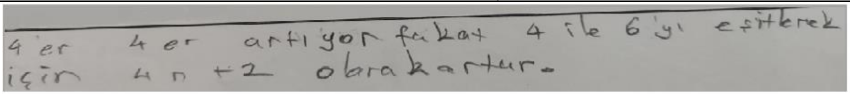
Figure 3 shows examples of incorrect answers for Item 12.

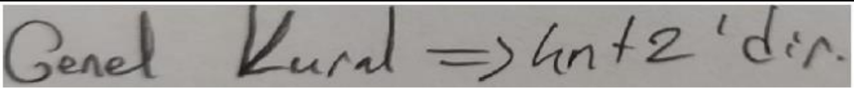


(The sequence continues as 4n+2)

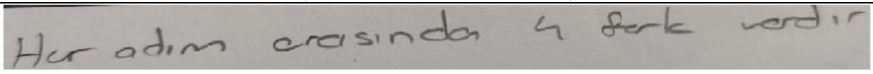


(It increases 4 by 4)


(It increases 4 by 4 but in order to equate 4 and 6, it increases as 4n+2.)



(General rule is =>4n+2)



(There are 4 differences between each step.)

Figure 3. Examples of incorrect answers for item 12

Transition Skills to Numeric Representation

Again, descriptive statistics regarding students' scores, for the total score of the three questions measuring transition skills from verbal, algebraic and graphic representation to numerical representation (items 2, 5, and 11) are provided in Table 8. The lowest possible score for these three items was zero, and the highest possible score was nine.

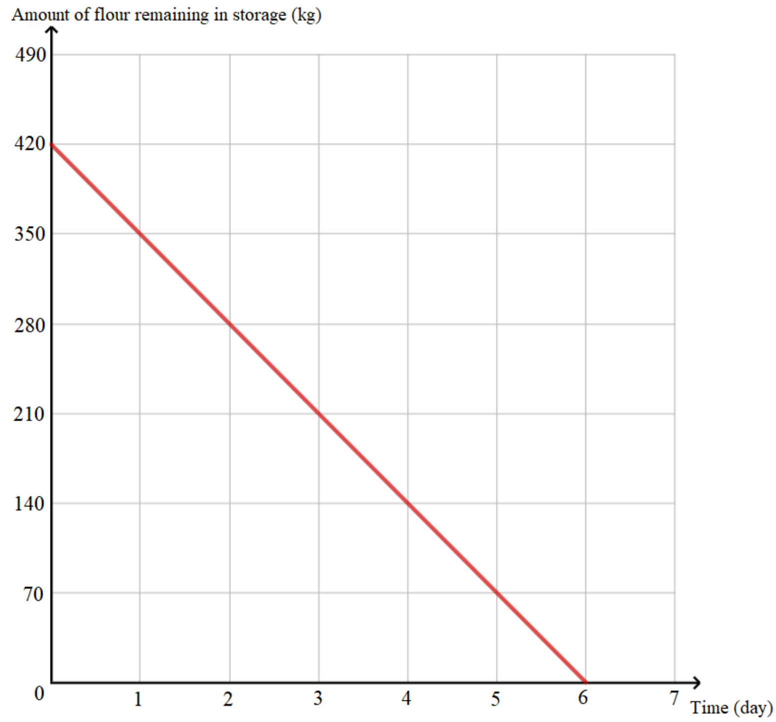
Table 8. Students' total score distribution in the transition to numerical representation

	9-7 points		6-4 points		3-0 points	
	f	%	f	%	f	%
Transition to Numeric Representation	18	13.6	48	36.1	67	50.3

In Table 8 it is indicated that 13.6% of the students had high transition skills from other representations to numeric representation, 36.1% had moderate skills, and 50.3% had poor skills. Furthermore, there were nine students who received a full score on all the items, whereas 22 students did not score any points. The mean score for students regarding the items concerning the transition to numeric representation was 3.76.

Next, when the students' transitions from other representations to numeric representation were ranked based on their mean scores, the most successful transition emerged as the one from verbal to numeric representation (item 2) with a mean score of 1.44. This was followed by the transition from algebraic to numeric representation (item 5) with a mean score of 1.31. Additionally, the least successful transition to numeric representation was from graphic representation (item 11) with a mean score of 1.01.

The most common student mistakes in the transition to numeric representation included starting the data with an incorrect initial value, failing to write the data in tabular format, and drawing a graph in lieu of a table. Other mistakes were not naming the variables in the table and not titling the table. In Figure 4, the TSTBR item (item 11) that measured students' transition skills from graphic representation to numeric representation is presented.



The graph above shows the change in the amount of flour in storage at a patisserie over time. Show the change in the amount of flour in storage per day with a table based on this graph.

Figure 4. Item 11 for the transition from graphic representation to numeric representation

Presented in Figure 5 are examples of incorrect answers for item 11 which required making a numeric table using the information in the graph.

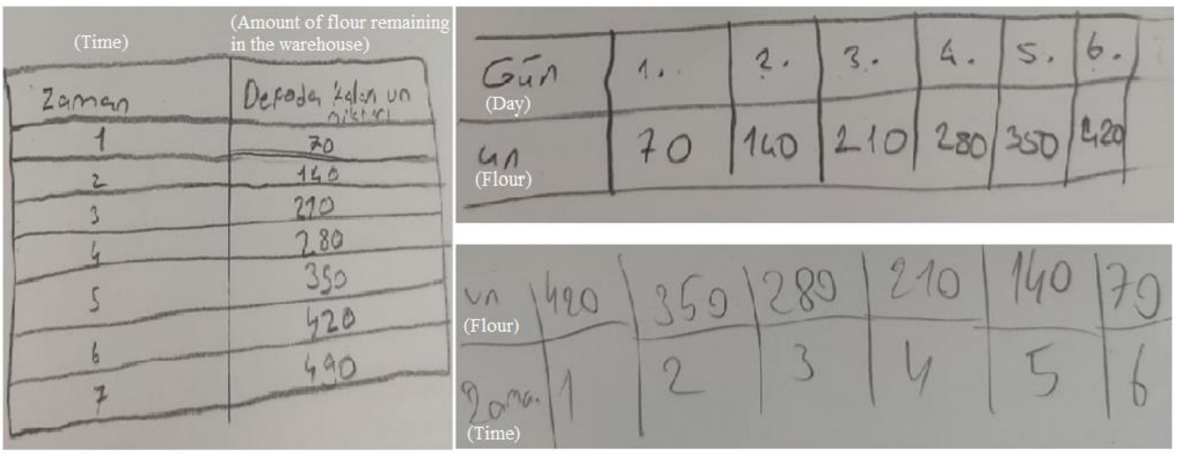


Figure 5. Examples of incorrect answers to item 11

Transition Skills to Graphic Representation

The descriptive statistics depicting students' total scores for the three questions which measure transition skills from verbal, numerical, and algebraic representation to graphic representation (Item 4, 7, and 10) are provided in Table 9. The lowest possible score for these three questions was zero, and the highest possible score was nine.

Table 9. Students total score distribution in the transition to graphic representation

	9-7 points		6-4 points		3-0 points	
	f	%	f	%	f	%
Transition to Graphic Representation	15	11.3	37	27.8	81	60.9

In Table 9, it is shown that 11.3% of the students had high transition skills from other representations to graphic representation, 27.8% had moderate skills, and 60.9% had poor skills. There were four students who received a full score for all items concerning the transition from the other three representations to graphic representation, whereas 21 students did not score any points for any of these items. The mean student scores for the items concerning the transition to numeric representation was 3.32.

When student transitions from other representations to graphic representation were ranked based on the mean scores, the most successful transition seemed to be from numeric to graphic representation (item 7) with a mean score of 1.52, while this was followed by the transition from verbal to graphic representation (item 4) with a mean score of 1.18. The least successful transition to graphic representation was from algebraic representation (item 10) with a mean score of 0.62. As a result, it can be recognized that students' success in the transition from algebraic representation to graphic representation was rather low.

Additionally, the most common mistakes that students made in the transition to graphic representation included placing the numbers incorrectly on the axes, incorrectly identifying the initial point of the graph, failing to scale correctly, and making a numeric table instead of drawing a graph. Also, not naming the axes in the graph as well as not assigning a title to the graph were other common omissions.

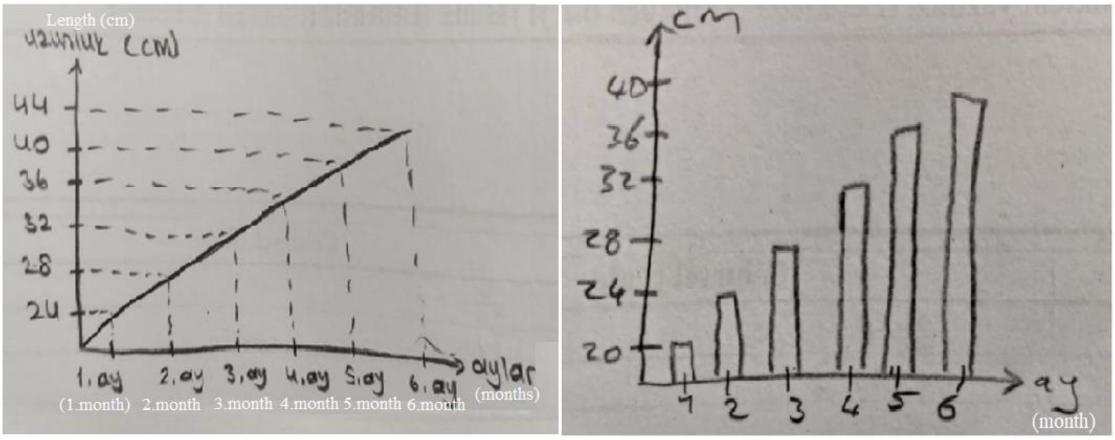


Figure 6. Examples of incorrect answers for the transition to graphic representation

In Figure 6, examples are provided of students' incorrect answers for Item 4: "A 20 cm tall seedling grows by 4 cm each month for six months after being planted. Display the monthly change in the height of the seedling on a graph".

Discussion and Conclusion

The belief that using representations affects mathematics instruction positively as well as the focus placed on representation use in curricula has led to an increase in the number of research studies related to representations. Thus, the present study was aimed at investigating students' representation transition skills (verbal, numeric, algebraic, graphic) at the completion of seventh grade. Furthermore, the aim of this study was to reveal if students were successful or lacking in their representation transition skills. To achieve the study aim, the results of the Test of Skills in Transitioning between Representations (TSTBR) implemented to investigate the research questions were analyzed and discussed.

According to the descriptive statistics of the TSTBR, a great majority of the students investigated had a moderate level of representation transition skills. The mean score obtained by these students for the TSTBR test was 16.68. What stands out from this result was that students with poor representation transition skills outnumbered those with high representation transition skills. Importantly, this may be attributed to the fact that student success was very low in certain representation transitions.

The most successful representation transitions were from other representations to algebraic, verbal, numeric and graphic representations, respectively. Contrary to previous studies at the middle school level which conclude that students are most successful in transition from other representations to numeric representations (Gök & Cansız-Aktaş, 2019;

Gürbüz & Şahin, 2015; Sert, 2007), it was confirmed in the present study as in the findings of Demir and Cansız-Aktaş (2019), that students were most successful in transition from other representations to algebraic representation. Thus, it is posited that students were most successful at transitions from other representations to the algebraic representation due to their instruction being primarily based on algebraic representation. On the other hand, Panasuk and Beyranevand (2011) found in their study with seventh and eighth grade students that successful students primarily prefer to use algebraic representations in the given problem situations. The reason for students' success in using algebraic representations may be the development of their skills in the representations they prefer to use. Furthermore, frequent mistakes by students in the transition to algebraic representation included failing to express the relationship between variables algebraically, failing to form an accurate equation, and failing to accurately transform expressions such as 'times', 'half', 'same', 'more', and 'less' into algebraic expressions.

Next, following algebraic representations, students were most successful at transitions to verbal representations. For example, among the three transitions to verbal representations, students' were most successful at transitioning from algebraic representation to verbal representation. On the other hand, it is worth noting that students were not equally successful at transitions from numeric or graphic representations to verbal representation. This may be due to the fact that the middle school mathematics curriculum includes learning outcomes for algebraic-verbal and verbal-algebraic representation transitions and that students have more practice. Additionally, students were found to have difficulty with verbally expressing themselves when interpreting numeric tables and graphs. Also, Turkish students' tendency to have result-oriented study habits based on the need to prepare for centralized exams, which begin in middle school and continue throughout their academic life, may attribute to this outcome. The possibility that students may not have been provided sufficient opportunities to express problem situations in their own words during class time may have also caused students' verbal skills to be subpar. As mentioned in Gürbüz and Şahin's (2015) study, students' low writing skills also negatively affect verbal representation performance.

Success at the transition from other representations to verbal representation was followed by the success of transition to numeric representation. For example, the most common student mistakes in transition to numeric representation were incorrectly identifying the initial value, not being able to write the data in table format, and/or drawing a graph instead of making a table of values. Other mistakes included not naming the variables in the table and not attaching a title to the table. Importantly, similar mistakes were also present in questions regarding the transition from other representations to graphic representation. Similar to the present study, it has also been determined in several past studies with middle school students, that students are unsuccessful in the transition from other representations to graphic representation (Demir & Cansız Aktaş, 2019; Gök & Cansız Aktaş, 2019; Gürbüz & Şahin, 2015; Hotmanoğlu, 2014). Parallel to the findings of Hotmanoğlu (2014), it was also determined in the present study, that students' most common mistakes in the transition to graphic representation were placing the numbers incorrectly on the axes, incorrectly identifying the initial point of the graph, failing to scale correctly, and making a table of values instead of drawing a graph. Additionally, not naming the axes in the graph and not assigning a title to it were also common mistakes. Despite their knowledge of line graphs, a majority of students preferred to draw bar graphs. Importantly, this may be the result of having worked with bar graphs beginning in the early grades. This can be significant because transitioning from other representations to graphic representation is an important skill for students starting in eighth grade. Thus, if eighth grade students can only utilize graphical representations at an elementary level, then they may face challenges in their future learning.

Results from this study regarding students' success at representation transition, parallel the findings of Incikabı (2017), that textbooks mostly emphasize algebraic, verbal, and model representations but give very little attention to numeric, graphic and real life representations. To duplicate the success levels of algebraic and verbal representations on numeric tables and graphic representations, more time needs to be spent in curriculum development regarding the formation and interpretation of numeric tables and graphic representations.

Suggestions

Data from the present study was limited to seventh graders registered in a public school in Ankara in Turkey. Further studies with more participants from different socio-economic environments and different settlements may contribute to a better understanding of representation skills. Future studies could also be carried out at different grade levels to determine how and if the results vary. After identifying students' representation levels, their development may also be tracked through multiple representation based forms of instruction. Other than the representations used in the present study, representations suited for different topics and at different grade levels should also be included. Furthermore, comparative analysis can be conducted to determine if there is a correlation between the success in mathematics related areas of students in centralized exams and their ability in transitioning between representations. Finally, in future studies within Turkey regarding this research topic, it would be extremely worthwhile to analyze textbooks from different grade levels published following the Turkish Ministry of Education's mathematics curriculum change that occurred in 2018, due to the benefits in understanding to what extent various representations and the transitions between representations were incorporated.

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