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More On Complementary Soft Binary Piecewise Intersection And Union Operations

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Abstract: In order to deal with uncertainty, Molodtsov propoed soft set theory as a mathematical tool in 1999. Since that time, numerous forms of soft set operations have been defined and employed. By establishing the soft binary piecewise operations' distributions over complementary soft binary piecewise intersection and union operations, this study aims to contribute to the literature on soft set theory by giving inspiration to researchers as regards examining and obtaining some algebraic structures using these new soft set operations.

Key words: Soft sets, soft set operations, conditional complements.

Tümleyenli Esnek İkili Parçalı Kesişim ve Birleşim İşlemleri Üzerine Daha Fazla Bilgi

Öz: Molodtsov, belirsizlikle başa çıkmak için 1999'da matematiksel bir araç olarak esnek küme teorisini yarattı. O zamandan bu zamana, çok sayıda esnek küme işlemi tanımlanmış ve kullanılmıştır. Bu çalışma, esnek ikili parçalı işlemlerin, tümleyenli esnek ikili parçalı kesişim ve birleşim işlemleri üzerindeki dağılımlarını ortaya koyarak bu yeni esnek küme işlemlerini kullanarak bazı cebirsel yapıların incelenmesi ve elde edilmesi için araştırmacılara ilham vererek esnek küme teorisi literatürüne katkıda bulunmayı amaçlamaktadır.

Anahtar kelimeler: Esnek kümeler, esnek küme işlemleri, koşullu tümleyenler

1. Introduction

Due to the existence of some types of uncertainty, we are unable to effectively employ traditional ways to address issues in many domains, including engineering, environmental and health sciences, and economics. Three well-known foundational theories that we could use as a mathematical tool to deal with uncertainties are interval mathematics, fuzzy set theory, and probability theory. Molodtsov [1] proposed Soft Set Theory as a mathematical method to deal with these uncertainties, however this method has limits as well because each of these theories has flaws of its own. Since then, this theory has been applied to a variety of fields, including information systems, decision-making [2,3], optimization theory, game theory, operations research, measurement theory, and others [4].

In terms of soft set operations, the first contributions were published in [5,6]. The introduction and analysis of a number of soft set operations (including restricted and extended soft set operations) were followed in [7]. In [8], the fundamental characteristics of soft set operations were covered along with examples of the related to one another. Additionally, the concept of restricted symmetric difference of soft sets were described and explored in [8]. Extended difference and symmetric difference were defined, their properties were examined in [9,10]. When we look at the studies, we can see that restricted soft set operations and extended soft set operations are pr imary categories under which soft set theory's operations fall.

The inclusive complement and exclusive complement of sets, a novel concept in set theory, were proposed and their relationships were investigated in [11]. As a result of the inspiration from this study, certain novel complements of sets were developed in [12]. Additionally, in [13], a number of additional restricted and extended soft set operations were constructed using these complements to soft set theory. Demirci [14], Saralioğlu [15], Akbulut [16] defined a new type of extended operation and in-depth examined their fundamental characteristics by changing the form of extended soft set operations using the complement at the first and second row of the piecewise function of extended soft set operations.

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Çalışıcı and Eren [17] also propesed a new kind of soft difference operations. Yavuz [18] introduced soft binary piecewise operations and examined their fundamental properties extensively. A novel kind of soft binary piecewise operation was also presented by Sezgin and Saralioğlu [19], Sezgin and Demirci [20], Sezgin and Atagün [21], Sezgin and Çağman [22], Sezgin and Aybek [23], Sezgin et al. [24,25] and Sezgin and Yavuz [26] as part of their ongoing studies on soft set operations. They used the complement in the first row of piecewise operations to change the structure of the soft binary operation.

In Sezgin et al. [24,25], complementary soft binary piecewise union and intersection operation were defined, respectively. These novel operations' algebraic characteristics were carefully explored. Particular attention was paid to how these operations distributtes over restricted soft set operations, complementary restricted soft set operations, soft binary piecewise operations, and complementary extended soft set operations. In this study, by establishing the distributions of soft binary piecewise operations over complementary soft binary piecewise union and intersection operations, our purpose is to contribute to the literature on soft set theory by providing researchers with suggestions for studying and extracting some algebraic structures using these new soft set operations.The organization of the paper is as follows: In Introduction Section, literature survey is given with a conclusion paragraph summarizing what is obtained in the paper In Section 2 the main definitions used throughout the paper are given. In Section 3, first of all the distributions of soft binary piecewise operations over complementary soft binary piecewise interection operation, and then distributions of soft binary piecewise operations over complementary soft binary piecewise union operation are handled. This paper is a theoretical study of soft set.

2. Preliminaries

Definition 2.1. [1] Let U be the universal set, E be the parameter set, $P(U)$ be the power set of U and $N \subseteq E$. A pair (K, N) is called a soft set over U where K is a set-valued function such that $K: N \rightarrow P(U)$.

The set of all the soft sets over U is designated by $S_E(U)$, and throughout this paper, all the soft sets are the elements of $S_E(U)$.

Definition 2.2. [7] The relative complement of a soft set (K, N), denoted by $(K, N)^r$, is defined by $(K, N)^r = (K^r, N)$, where $K^r: N \rightarrow P(U)$ is a mapping given by $(K, N)^r = U\backslash K(t)$ for all $t \in N$. From now on, $U\backslash K(t)=[K(t)]'$ will be designated by K'(t) for the sake of designation.

Çağman [11], defined two conditional complements of sets, for the ease of illustration, we show these complements as + and θ, respectively. These complements are defined as following: Let P and C be two subsets of U. C-inclusive complement of P is defined by, P+C= P'∪C and C-exlusive complement of P is defined by PθC= P'∩C. Here, U refers to a universe set, P' refers to the complement of P over U. Sezgin et al. [12], introduced such new three complements as binary operations of sets as following: Then, P*C=P'∪C', PγC= P'∩C, P**λ**C=P∪C' [12]. Aybek [13] conveyed these classical sets to soft sets, and they defined restricted and extended soft set operations and examined their properties.

As a summary for soft set operations, we can categorize all types of soft set operations as following: Let "∇" be used to represent the set operations (i.e., here ∇ can be ∩, ∪,\, Δ, +,θ, *, λ,γ), then restricted operations, extended operations, complementary extended operations, soft binary piecewise operations, complementary soft binary piecewise operations are defined in soft set theory as following:

Definition 2.3. [7, 8,13] Let (Y, D) and (R, J) be soft sets over U. The restricted ∇ operation of (Y, D) and (R, J) is the soft set (S, F), denoted by $(Y, D)\nabla_R(R, J) = (S, F)$, where $F = D \cap J \neq \emptyset$ and $\forall t \in F$, $S(t) = Y(t)\nabla R(t)$

Definition 2.4. [5,7, 9, 10, 13]. Let (Y, D) and (R, J) be soft sets over U. The extended ∇ operation of (Y, D) and (R, J) is the soft set (S, F), denoted by $(Y, D)\nabla_\varepsilon(R, J) = (S, F)$, where $F = D \cup J$ and $\forall t \in F$,

$$S(t) = \begin{cases} Y(t), & t \in D\backslash J, \\ R(t), & t \in J\backslash D, \\ Y(t)\nabla R(t), & t \in D \cap J \end{cases}$$

Definition 2.5. [14-16] Let (Y, D) and (R, J) be soft sets over U. The complementary extended ∇ operation of

(Y, D) and (R, J) is the soft set (S, F), denoted by $(Y, D) \overset{*}{\nabla_{\varepsilon}} (R, J) = (S, F)$, where $F = D \cup J$ and $\forall t \in F$,

$$S(t) = \begin{cases} Y'(t), & t \in D \setminus J, \\ R'(t), & t \in J \setminus D, \\ Y(t) \nabla R(t), & t \in D \cap J \end{cases}$$

Definition 2.6. [17,18] Let (Y, D) and (R, J) be soft sets over U. The soft binary piecewise ∇ operation of (Y, D) and (R, J) is the soft set (S, D), denoted by, $(Y, D) \underset{\nabla}{\sim} (R, J) = (S, D)$, where $\forall t \in D$,

$$S(t) = \begin{cases} Y(t), & t \in D \setminus J \\ Y(t) \nabla R(t), & t \in D \cap J \end{cases}$$

Definition 2.7. [19-26] Let (Y, D) and (R, J) be soft sets over U. The complementary soft binary piecewise ∇ operation of (Y, D) and (R, J) is the soft set (S, D), denoted by, $(Y, D) \underset{\nabla}{\overset{*}{\sim}} (R, J) = (S, D)$, where $\forall t \in D$;

$$S(t) = \begin{cases} Y'(t), & t \in D \setminus J \\ P(t) \nabla R(t), & t \in D \cap J \end{cases}$$

Definition 2.8. [24] Let (Y, D) and (R, J) be soft sets over U. The complementary soft binary piecewise union $(\cup)$ operation of (Y, D) and (R, J) is the soft set (S, D), denoted by, $(Y, D) \underset{\cup}{\overset{*}{\sim}} (R, J) = (S, D)$, where $\forall t \in D$,

$$S(t) = \begin{cases} Y'(t), & t \in D \setminus J \\ Y(t) \cup R(t), & t \in D \cap J \end{cases}$$

Definition 2.9. [25] Let (Y, D) and (R, J) be soft sets over U. The complementary soft binary piecewise intersection $(\cap)$ operation of Let (P, D) and (R, J) is the soft set (S, D), denoted by, $(Y, D) \underset{\cap}{\overset{*}{\sim}} (R, J) = (S, D)$, where $\forall t \in D$,

$$S(t) = \begin{cases} Y'(t), & t \in D \setminus J \\ Y(t) \cap R(t), & t \in D \cap J \end{cases}$$

Example 2.10. [24,25] Let $E=\{e_1, e_2, e_3, e_4\}$ be the parameter set $A=\{e_1, e_3\}$ and $B=\{e_2, e_3, e_4\}$ be the subsets of E and $U=\{h_1, h_2, h_3, h_4, h_5\}$ be the initial universe set. Assume that (F,A) and (G,B) are the soft sets over U defined as following:

$(F,A)=\{(e_1, \{h_2, h_5\}), (e_3, \{h_1, h_2, h_5\})\}$,

$(G,B)=\{(e_2, \{h_1, h_4, h_5\}), (e_3, \{h_2, h_3, h_4\}), (e_4, \{h_3, h_5\})\}$.

Let $(F,A) \underset{\cap}{\overset{*}{\sim}} (G,B)=(H,A)$. Then,

$$H(\omega) = \begin{cases} F'(\omega), & \omega \in A \setminus B \\ F(\omega) \cap G(\omega), & \omega \in A \cap B \end{cases}$$

Since $A=\{e_1, e_3\}$ and $A \setminus B=\{e_1\}$, so $H(e_1) = F'(e_1)=\{h_1, h_3, h_4\}$. And since $A \cap B=\{e_3\}$, so $H(e_3)=F(e_3) \cap G(e_3)=\{h_1, h_2, h_5\} \cap \{h_2, h_3, h_4\}=\{h_2\}$. Thus,

$(F,A) \underset{\cap}{\overset{*}{\sim}} (G,B)=\{(e_1,\{h_1,h_3,h_4\}), (e_3, \{h_2\})\}$.

Let $(F,A) \underset{\cup}{\overset{*}{\sim}} (G,B)=(K,A)$. Then,

$$K(\omega)=\begin{cases} F'(\omega), & \omega\in A\setminus B \\ F(\omega)\cup G(\omega), & \omega\in A\cap B \end{cases}$$

SincE $A=\{e_1, e_3\}$ and $A\setminus B=\{e_1\}$, so $K(e_1)=F'(e_1)=\{h_1,h_3,h_4\}$. And since $A\cap B=\{e_3\}$, so $K(e_3)=F(e_3)\cup G(e_3)=\{h_1,h_2,h_5\}\cup\{h_2,h_3,h_4\}=\{h_1,h_2,h_3,h_4,h_5\}$. Thus,

$(F,A) \underset{\cup}{\overset{*}{\sim}} (G,B)=\{(e_1,\{h_1,h_3,h_4\}), (e_3, \{h_1,h_2,h_3,h_4,h_5\})\}$.

3. Distribution Rules

In this section, distributions of soft binary piecewise operations over complementary soft binary piecewise union and difference operation are examined in detail and many interesting results are obtained.

3.1. Distribution of soft binary piecewise operations over complementary soft binary piecewise intersection (∩) operation:

1) $(F, D) \tilde{\cup}[(S,B) \underset{\cap}{\overset{*}{\sim}} (H,I)]=[(F,D) \underset{\cup}{\sim} (S,B)] \tilde{\cap} [(F,D) \underset{\cup}{\sim}(H,I)]$, where $D\cap B\cap I' =\emptyset$

Proof: Let's first take care of the left hand side of the equality and let $(S,B) \underset{\cap}{\overset{*}{\sim}} (H,I)=(M,B)$, where $\forall\zeta\in B$;

$$M(\zeta)=\begin{cases} S'(\zeta), & \zeta\in B\setminus I \\ S(\zeta)\cap H(\zeta), & \zeta\in B\cap I \end{cases}$$

Let $(F,D) \tilde{\cup}(M,B)=(N,D)$, where $\forall\zeta\in D$;

$$N(\zeta)=\begin{cases} F(\zeta), & \zeta\in D\setminus B \\ F(\zeta)\cup M(\zeta), & \zeta\in D\cap B \end{cases}$$

Thus,

$$N(\zeta)=\begin{cases} F(\zeta), & \zeta\in D\setminus B \\ F(\zeta)\cup S'(\zeta), & \zeta\in D\cap(B\setminus I)= D\cap B\cap I' \\ F(\zeta)\cup [(S(\zeta)\cap H(\zeta)], & \zeta\in D\cap(B\cap I)= D\cap B\cap I \end{cases}$$

Now let's take care of the left hand side of the equality: $[(F,D) \underset{\cup}{\sim} (S,B)] \tilde{\cap} [(F,D) \underset{\cup}{\sim}(H,I)]]$. Let $(F,D) \underset{\cup}{\sim} (S,B)=(V,D)$, where $\forall\zeta\in D$;

$$V(\zeta)=\begin{cases} F(\zeta), & \zeta\in D\setminus B \\ F(\zeta)\cup S(\zeta), & \zeta\in D\cap B \end{cases}$$

Suppose that $(F,D) \tilde{\cup} (H,I)=(W,D)$, where $\forall\zeta\in D$;

$$W(\zeta)=\begin{cases} F(\zeta), & \zeta\in D\backslash I \\ F(\zeta)\cup H(\zeta), & \zeta\in D\cap I \end{cases}$$

Let $(V,D)\ \tilde{\cap}(W,D)=(T,D)$, where $\forall\zeta\in D$;

$$T(\zeta)=\begin{cases} V(\zeta), & \zeta\in D\backslash D=\emptyset \\ V(\zeta)\cap W(\zeta), & \zeta\in D\cap D \end{cases}$$

Thus,

$$T(\zeta)=\begin{cases} F(\zeta)\cap F(\zeta), & \zeta\in(D\backslash B)\cap(D\backslash I)=D\cap B'\cap I' \\ F(\zeta)\cap[(F(\zeta)\cup H(\zeta)], & \zeta\in(D\backslash B)\cap(D\cap I)=D\cap B'\cap I \\ [(F(\zeta)\cup S(\zeta)]\cap F(\zeta), & \zeta\in(D\cap B)\cap(D\backslash I)=D\cap B\cap I' \\ [(F(\zeta)\cup S(\zeta)]\cap[(F(\zeta)\cup H(\zeta)], & \zeta\in(D\cap B)\cap(D\cap I)=D\cap B\cap I \end{cases}$$

Thus,

$$T(\zeta)=\begin{cases} F(\zeta) & \zeta\in(D\backslash B)\cap(D\backslash I)=D\cap B'\cap I' \\ F(\zeta), & \zeta\in(D\backslash B)\cap(D\cap I)=D\cap B'\cap I \\ F(\zeta), & \zeta\in(D\cap B)\cap(D\backslash I)=D\cap B\cap I' \\ [(F(\zeta)\cup S(\zeta)]\cap[(F(\zeta)\cup H(\zeta)], & \zeta\in(D\cap B)\cap(D\cap I)=D\cap B\cap I \end{cases}$$

Here let's take care of $\zeta\in D\backslash B$ in the first equation. Since $D\backslash B= D\cap B'$, if $\zeta\in B'$, then $\zeta\in I\backslash B$ or $\zeta\in(B\cup I)'$. Hence, if $\zeta\in D\backslash B$, $\zeta\in D\cap B'\cap I'$ or $\zeta\in D\cap B'\cap I$. Thus, it is seen that $(N,D)=(T,D)$.

2) $[(F,D)\ \overset{*}{\underset{\cap}{\sim}}\ (S,B)]\ \tilde{\cup}\ (H,I)=[(F,D)\ \tilde{\cup}\ (H,I)]\ \overset{*}{\underset{\cap}{\sim}}\ [(S,B)\ \tilde{\cup}\ (H,I)]$, where $D\cap B'\cap I==\emptyset$.

Proof: Let's first take care of the left hand side of the equality and let $(F,D)\ \overset{*}{\underset{\cap}{\sim}}\ (S,B)=(M,D)$, where $\forall\zeta\in D$;

$$M(\zeta)=\begin{cases} F'(\zeta), & \zeta\in D\backslash B \\ F(\zeta)\cap S(\zeta), & \zeta\in D\cap B \end{cases}$$

Let $(M,D)\ \tilde{\cup}\ (H,I)=(N,D)$, where $\forall\zeta\in D$;

$$N(\zeta)=\begin{cases} M(\zeta), & \zeta\in D\backslash I \\ M(\zeta)\cup H(\zeta), & \zeta\in D\cap I \end{cases}$$

Thus,

$$N(\zeta)=\begin{cases} F'(\zeta), & \zeta\in(D\backslash B)\backslash I=D\cap B'\cap I' \\ F(\zeta)\cap S(\zeta), & \zeta\in(D\cap B)\backslash I=D\cap B\cap I' \\ F'(\zeta)\cup H(\zeta) & \zeta\in(D\backslash B)\cap I=D\cap B'\cap I \\ [F(\zeta)\cap S(\zeta)]\cup H(\zeta) & \zeta\in(D\cap B)\cap I=D\cap B\cap I \end{cases}$$

Now let's take care of the left hand side of the equality: $[(F,D)\ \tilde{\cup}\ (H,I)] \overset{*}{\underset{\cap}{\sim}} [(S,B)\ \tilde{\cup}\ (H,I)]$. Let $(F,D)\ \tilde{\cup}\ (H,I)=(V,D)$, where $\forall\zeta\in D$;

$$V(\zeta)=\begin{cases} F(\zeta), & \zeta\in D\setminus I \\ F(\zeta)\cup H(\zeta), & \zeta\in D\cap I \end{cases}$$

Suppose that $(S,B)\ \tilde{\cup}\ (H,I)=(W,B)$, where $\forall\zeta\in B$;

$$W(\zeta)=\begin{cases} S(\zeta), & \zeta\in B\setminus I \\ S(\zeta)\cup H(\zeta), & \zeta\in B\cap I \end{cases}$$

Let $(V,D) \overset{*}{\underset{\cap}{\sim}} (W,B)=(T,D)$, where $\forall\zeta\in D$;

$$T(\zeta)=\begin{cases} V'(\zeta), & \zeta\in D\setminus B \\ V(\zeta)\cap W(\zeta), & \zeta\in D\cap B \end{cases}$$

Thus,

$$T(\zeta)=\begin{cases} F'(\zeta), & \zeta\in(D\setminus I)\setminus B=D\cap B'\cap I' \\ F'(\zeta)\cap H'(\zeta), & \zeta\in(D\cap I)\setminus B=D\cap B'\cap I \\ F(\zeta)\cap S(\zeta), & \zeta\in(D\setminus I)\cap(B\setminus I)=D\cap B\cap I' \\ F(\zeta)\cap [S(\zeta)\cup H(\zeta)], & \zeta\in(D\setminus I)\cap(B\cap I)=\emptyset \\ [F(\zeta)\cup H(\zeta)]\cap S(\zeta), & \zeta\in(D\cap I)\cap(B\setminus I)=\emptyset \\ [F(\zeta)\cup H(\zeta)]\cap [S(\zeta)\cup H(\zeta)], & \zeta\in(D\cap I)\cap(B\cap I)=D\cap B\cap I \end{cases}$$

It is seen that $(N,D)=(T,D)$.

3) $(F,D)\ \tilde{\setminus}\ [(S,B) \overset{*}{\underset{\cap}{\sim}} (H,I)] = [(F,D)\ \tilde{\setminus}\ (S,B)]\ \tilde{\cup}\ [(F,D)\ \tilde{\setminus}\ (H,I)]$, where $D\cap B\cap I'=\emptyset$

Proof: Let's first take care of the left hand side of the equality and let $(S,B) \overset{*}{\underset{\cap}{\sim}} (H,I)=(M,B)$, where $\forall\zeta\in B$;

$$M(\zeta)=\begin{cases} S'(\zeta), & \zeta\in B\setminus I \\ S(\zeta)\cap H(\zeta), & \zeta\in B\cap I \end{cases}$$

Let $(F,D)\ \tilde{\setminus}\ (M,B)=(N,D)$, where $\forall\zeta\in D$;

$$N(\zeta)=\begin{cases} F(\zeta), & \zeta\in D\setminus B \\ F(\zeta)\cap M'(\zeta), & \zeta\in D\cap B \end{cases}$$

Thus,

$$N(\zeta)=\begin{cases} F(\zeta), & \zeta\in D\setminus B \\ F(\zeta)\cap S(\zeta), & \zeta\in D\cap(B\setminus I)=D\cap B\cap I' \\ F(\zeta)\cap [(S'(\zeta)\cup H'(\zeta)], & \zeta\in D\cap(B\cap I)=D\cap B\cap I \end{cases}$$

Now let's take care of the left hand side of the equality: $[(F,D)\,\tilde{\setminus}\,(S,B)]\ \tilde{\cup}\ [(F,D)\,\tilde{\setminus}(H,I)]$,. Let $(F,D)\,\tilde{\setminus}$ $(S,B)=(V,D)$, where $\forall\zeta\in D$;

$$V(\zeta)=\begin{cases} F(\zeta), & \zeta\in D\setminus B \\ F(\zeta)\cap S'(\zeta), & \zeta\in D\cap B \end{cases}$$

Suppose that $(F,D)\,\tilde{\setminus}(H,I)=(W,D)$, where $\forall\zeta\in D$;

$$W(\zeta)=\begin{cases} F(\zeta), & \zeta\in D\setminus I \\ F(\zeta)\cap H'(\zeta), & \zeta\in D\cap I \end{cases}$$

Let $(V,D)\,\tilde{\cup}(W,D)=(T,D)$, where $\forall\zeta\in D$;

$$T(\zeta)=\begin{cases} V(\zeta), & \zeta\in D\setminus D=\emptyset \\ V(\zeta)\cup W(\zeta), & \zeta\in D\cap D \end{cases}$$

Thus,

$$T(\zeta)=\begin{cases} F(\zeta)\cup F(\zeta), & \zeta\in(D\setminus B)\cap(D\setminus I)=D\cap B'\cap I' \\ F(\zeta)\cup[(F(\zeta)\cap H'(\zeta)], & \zeta\in(D\setminus B)\cap(D\cap I)=D\cap B'\cap I \\ [(F(\zeta)\cap S'(\zeta)]\cup F(\zeta), & \zeta\in(D\cap B)\cap(D\setminus I)=D\cap B\cap I' \\ [(F(\zeta)\cap S'(\zeta)]\cup[(F(\zeta)\cap H'(\zeta)], & \zeta\in(D\cap B)\cap(D\cap I)=D\cap B\cap I \end{cases}$$

Thus,

$$T(\zeta)=\begin{cases} F(\zeta) & \zeta\in(D\setminus B)\cap(D\setminus I)=D\cap B'\cap I' \\ F(\zeta), & \zeta\in(D\setminus B)\cap(D\cap I)=D\cap B'\cap I \\ F(\zeta), & \zeta\in(D\cap B)\cap(D\setminus I)=D\cap B\cap I' \\ [(F(\zeta)\cap S'(\zeta)]\cup[(F(\zeta)\cap H'(\zeta)], & \zeta\in(D\cap B)\cap(D\cap I)=D\cap B\cap I \end{cases}$$

Since $D\setminus B=D\cap B'$, if $\zeta\in B'$, then $\zeta\in I\setminus B$ or $\zeta\in(B\cup I)'$. Hence, if $\zeta\in D\setminus B$, $\zeta\in D\cap B'\cap I'$ or $\zeta\in D\cap B'\cap I$. Thus, it is seen that $(N,D)=(T,D)$.

4) $[(F,D)\ \overset{*}{\underset{\cap}{\sim}}\ (S,B)]\ \tilde{\setminus}\,(H,I)=[(F,D)\,\tilde{\setminus}\,(H,I)]\ \overset{*}{\underset{\cap}{\sim}}\ [(S,B)\,\tilde{\setminus}\,(H,I)]$, where $D\cap B'\cap I=\emptyset$.

Proof: Let's first take care of the left hand side of the equality and let $(F,D)\ \overset{*}{\underset{\cap}{\sim}}\ (S,B)=(M,D)$, where $\forall\zeta\in D$;

$$M(\zeta)=\begin{cases} F'(\zeta), & \zeta\in D\setminus B \\ F(\zeta)\cap S(\zeta), & \zeta\in D\cap B \end{cases}$$

Let $(M,D)\,\tilde{\setminus}\,(H,I)=(N,D)$, where $\forall\zeta\in D$;

$$N(\zeta)=\begin{cases} M(\zeta), & \zeta\in D\backslash I \\ M(\zeta)\cap H'(\zeta), & \zeta\in D\cap I \end{cases}$$

Thus,

$$N(\zeta)=\begin{cases} F'(\zeta), & \zeta\in (D\backslash B)\backslash I=D\cap B'\cap I' \\ F(\zeta)\cap S(\zeta), & \zeta\in (D\cap B)\backslash I=D\cap B\cap I' \\ F'(\zeta)\cap H'(\zeta) & \zeta\in (D\backslash B)\cap I=D\cap B'\cap I \\ [F(\zeta)\cap S(\zeta)]\cap H'(\zeta) & \zeta\in (D\cap B)\cap I=D\cap B\cap I \end{cases}$$

Now let's take care of the left hand side of the equality: $[(F,D)\ \tilde{\backslash}\ (H,I)] \overset{*}{\underset{\cap}{\sim}} [(S,B)\ \tilde{\backslash}\ (H,I)]$. Let $(F,D)\ \tilde{\backslash}$ $(H,I)=(V,D)$, where $\forall\zeta\in D$;

$$V(\zeta)=\begin{cases} F(\zeta), & \zeta\in D\backslash I \\ F(\zeta)\cap H'(\zeta), & \zeta\in D\cap I \end{cases}$$

Suppose that $(S,B)\ \tilde{\backslash}\ (H,I)=(W,B)$, where $\forall\zeta\in B$;

$$W(\zeta)=\begin{cases} S(\zeta), & \zeta\in B\backslash I \\ S(\zeta)\cap H'(\zeta), & \zeta\in B\cap I \end{cases}$$

Let $(V,D) \overset{*}{\underset{\cap}{\sim}} (W,B)=(T,D)$, where $\forall\zeta\in D$;

$$T(\zeta)=\begin{cases} V'(\zeta), & \zeta\in D\backslash B \\ V(\zeta)\cap W(\zeta), & \zeta\in D\cap B \end{cases}$$

Thus,

$$T(\zeta)=\begin{cases} F'(\zeta), & \zeta\in (D\backslash I)\backslash B=D\cap B'\cap I' \\ F'(\zeta)\cup H(\zeta), & \zeta\in (D\cap I)\backslash B=D\cap B'\cap I \\ F(\zeta)\cap S(\zeta), & \zeta\in (D\backslash I)\cap(B\backslash I)=D\cap B\cap I' \\ F(\zeta)\cap [S(\zeta)\cap H'(\zeta)], & \zeta\in (D\backslash I)\cap(B\cap I)=\emptyset \\ [F(\zeta)\cap H'(\zeta)]\cap S(\zeta), & \zeta\in (D\cap I)\cap(B\backslash I)=\emptyset \\ [F(\zeta)\cap H'(\zeta)]\cap [S(\zeta)\cap H'(\zeta)], & \zeta\in (D\cap I)\cap(B\cap I)=D\cap B\cap I \end{cases}$$

It is seen that $(N,D)=(T,D)$.

5) $[(F,D) \overset{*}{\underset{\cap}{\sim}} (S,B)]\ \tilde{\cap}\ (H,I)=[(F,D)\ \tilde{\cap}(H,I)] \overset{*}{\underset{\cap}{\sim}} [(S,B)\ \tilde{\cap}\ (H,I)]$, where $D\cap B'\cap I=\emptyset$.

Proof: Let's first take care of the left hand side of the equality and let $(F,D) \overset{*}{\underset{\cap}{\sim}} (S,B)=(M,D)$, where $\forall\zeta\in D$;

$$M(\zeta)=\begin{cases} F'(\zeta), & \zeta\in D\setminus B \\ F(\zeta)\cap S(\zeta), & \zeta\in D\cap B \end{cases}$$

Let $(M,D)\cap(H,I)=(N,D)$, where $\forall\zeta\in D$;

$$N(\zeta)=\begin{cases} M(\zeta), & \zeta\in D\setminus I \\ M(\zeta)\cap H(\zeta), & \zeta\in D\cap I \end{cases}$$

Thus,

$$N(\zeta)=\begin{cases} F'(\zeta), & \zeta\in(D\setminus B)\setminus I=D\cap B'\cap I' \\ F(\zeta)\cap S(\zeta), & \zeta\in(D\cap B)\setminus I=D\cap B\cap I' \\ F'(\zeta)\cap H(\zeta), & \zeta\in(D\setminus B)\cap I=D\cap B'\cap I \\ [F(\zeta)\cap S(\zeta)]\cap H(\zeta), & \zeta\in(D\cap B)\cap I=D\cap B\cap I \end{cases}$$

Now let's take care of the left hand side of the equality: $[(F,D)\ \tilde{\cap}\ (H,I)]\ \overset{*}{\underset{\cap}{\sim}}\ [(S,B)\ \tilde{\cap}(H,I)]$ .Let $(F,D)\ \tilde{\cap}$ $(H,I)=(V,D)$, where $\forall\zeta\in D$;

$$V(\zeta)=\begin{cases} F(\zeta), & \zeta\in D\setminus I \\ F(\zeta)\cap H(\zeta), & \zeta\in D\cap I \end{cases}$$

Suppose that $(S,B)\ \tilde{\cap}(H,I)=(W,B)$, where $\forall\zeta\in B$;

$$W(\zeta)=\begin{cases} S(\zeta), & \zeta\in B\setminus I \\ S(\zeta)\cap H(\zeta), & \zeta\in B\cap I \end{cases}$$

Let $(V,D)\ \overset{*}{\underset{\cap}{\sim}}\ (W,B)=(T,D)$, where $\forall\zeta\in D$;

$$T(\zeta)=\begin{cases} V'(\zeta), & \zeta\in D\setminus B \\ V(\zeta)\cap W(\zeta), & \zeta\in D\cap B \end{cases}$$

Thus,

$$T(\zeta)=\begin{cases} F'(\zeta), & \zeta\in(D\setminus I)\setminus B=D\cap B'\cap I' \\ F'(\zeta)\cup H'(\zeta), & \zeta\in(D\cap I)\setminus B=D\cap B'\cap I \\ F(\zeta)\cap S(\zeta), & \zeta\in(D\setminus I)\cap(B\setminus I)=D\cap B\cap I' \\ F(\zeta)\cap\ [S(\zeta)\cap H(\zeta)], & \zeta\in(D\setminus I)\cap(B\cap I)=\emptyset \\ [F(\zeta)\cap H(\zeta)]\ \cap S(\zeta), & \zeta\in(D\cap I)\cap(B\setminus I)=\emptyset \\ [F(\zeta)\cap H(\zeta)]\ \cap\ [S(\zeta)\cap H(\zeta)], & \zeta\in(D\cap I)\cap(B\cap I)=D\cap B\cap I \end{cases}$$

It is seen that $(N,D)=(T,D)$.

6) $[(F,D)\overset{*}{\underset{\cap}{\sim}}(S,B)]\ \tilde{\lambda}\ (H,I)=[(F,D)\ \tilde{\lambda}\ (H,I)]\overset{*}{\underset{\cap}{\sim}}[(S,B)\ \tilde{\lambda}\ (H,I)\,)]$ where $D\cap B'\cap I=\emptyset$.

Proof: Let's first take care of the left hand side of the equality and let $(F,D)\overset{*}{\underset{\cap}{\sim}}(S,B)=(M,D)$, where $\forall\zeta\in D$;

$$M(\zeta)=\begin{cases} F'(\zeta), & \zeta\in D\setminus B \\ F(\zeta)\cap S(\zeta), & \zeta\in D\cap B \end{cases}$$

Let $(M,D)\ \tilde{\lambda}\ (H,I)=(N,D)$, where $\forall\zeta\in D$;

$$N(\zeta)=\begin{cases} M(\zeta), & \zeta\in D\setminus I \\ M(\zeta)\cup H'(\zeta), & \zeta\in D\cap I \end{cases}$$

$$N(\zeta)=\begin{cases} F'(\zeta), & \zeta\in(D\setminus B)\setminus I=D\cap B'\cap I' \\ F(\zeta)\cap S(\zeta), & \zeta\in(D\cap B)\setminus I=D\cap B\cap I' \\ F'(\zeta)\cup H'(\zeta) & \zeta\in(D\setminus B)\cap I=D\cap B'\cap I \\ [F(\zeta)\cap S(\zeta)]\cup H'(\zeta) & \zeta\in(D\cap B)\cap I=D\cap B\cap I \end{cases}$$

Now let's take care of the left hand side of the equality: $[(F,D)\ \tilde{\lambda}\ (H,I)]\overset{*}{\underset{\cap}{\sim}}[(S,B)\ \tilde{\lambda}\ (H,I)]$. Let $(F,D)\ \tilde{\lambda}\ (H,I)=(V,D)$, where $\forall\zeta\in D$;

$$V(\zeta)=\begin{cases} F(\zeta), & \zeta\in D\setminus I \\ F(\zeta)\cup H'(\zeta), & \zeta\in D\cap I \end{cases}$$

Suppose that $(S,B)\ \tilde{\lambda}\ (H,I)=(W,B)$, where $\forall\zeta\in B$;

$$W(\zeta)=\begin{cases} S(\zeta), & \zeta\in B\setminus I \\ S(\zeta)\cup H'(\zeta), & \zeta\in B\cap I \end{cases}$$

Let $(V,D)\overset{*}{\underset{\cap}{\sim}}(W,B)=(T,D)$, where $\forall\zeta\in D$;

$$T(\zeta)=\begin{cases} V'(\zeta), & \zeta\in D\setminus B \\ V(\zeta)\cap W(\zeta), & \zeta\in D\cap B \end{cases}$$

$$T(\zeta)=\begin{cases} F'(\zeta), & \zeta\in(D\setminus I)\setminus B=D\cap B'\cap I' \\ F'(\zeta)\cap H(\zeta), & \zeta\in(D\cap I)\setminus B=D\cap B'\cap I \\ F(\zeta)\cap S(\zeta), & \zeta\in(D\setminus I)\cap(B\setminus I)=D\cap B\cap I' \\ F(\zeta)\cap[S(\zeta)\cup H'(\zeta)], & \zeta\in(D\setminus I)\cap(B\cap I)=\emptyset \\ [F(\zeta)\cup H'(\zeta)]\cap S(\zeta), & \zeta\in(D\cap I)\cap(B\setminus I)=\emptyset \\ [F(\zeta)\cup H'(\zeta)]\cap[S(\zeta)\cup H'(\zeta)], & \zeta\in(D\cap I)\cap(B\cap I)=D\cap B\cap I \end{cases}$$

It is seen that (N,D)=(T,D).

3.2. Distribution of soft binary piecewise operations over complementary soft binary piecewise union ($\cup$) operation:

1) $(F, A)\,\tilde{\cap}[(S,B) \overset{*}{\underset{\cup}{\sim}} (H,I)]=[(F,D) \underset{\cap}{\sim} (S,B)]\,\tilde{\cup}\,[(F,D) \underset{\cap}{\sim} (H,I)]$, where $D\cap B\cap I' =\emptyset$

Proof: Let's first take care of the left hand side of the equality and let $(S,B) \overset{*}{\underset{\cup}{\sim}} (H,I)=(M,B)$, where $\forall\zeta\in B$;

$$M(\zeta)=\begin{cases} S'(\zeta), & \zeta\in B\setminus I \\ S(\zeta)\cup H(\zeta), & \zeta\in B\cap I \end{cases}$$

Let $(F,D)\,\tilde{\cap}(M,B)=(N,D)$, where $\forall\zeta\in D$;

$$N(\zeta)=\begin{cases} F(\zeta), & \zeta\in D\setminus B \\ F(\zeta)\cap M(\zeta), & \zeta\in D\cap B \end{cases}$$

Thus,

$$N(\zeta)=\begin{cases} F(\zeta), & \zeta\in D\setminus B \\ F(\zeta)\cap S'(\zeta), & \zeta\in D\cap(B\setminus I)= D\cap B\cap I' \\ F(\zeta)\cap\ [(S(\zeta)\cup H(\zeta)], & \zeta\in D\cap(B\cap I)= D\cap B\cap I \end{cases}$$

Now let's take care of the left hand side of the equality: $[(F,D) \underset{\cap}{\sim} (S,B)]\ \tilde{\cup}\ [(F,D) \underset{\cap}{\sim} (H,I)]$. Let $(F,D) \underset{\cap}{\sim} (S,B)=(V,D)$, where $\forall\zeta\in D$;

$$V(\zeta)=\begin{cases} F(\zeta), & \zeta\in D\setminus B \\ F(\zeta)\cap S(\zeta), & \zeta\in D\cap B \end{cases}$$

Suppose that $(F,D)\,\tilde{\cap}\,(H,I)=(W,D)$, where $\forall\zeta\in D$;

$$W(\zeta)=\begin{cases} F(\zeta), & \zeta\in D\setminus I \\ F(\zeta)\cap H(\zeta), & \zeta\in D\cap I \end{cases}$$

Let $(V,D)\,\tilde{\cup}(W,D)=(T,D)$, where $\forall\zeta\in D$;

$$T(\zeta)=\begin{cases} V(\zeta), & \zeta\in D\setminus D=\emptyset \\ V(\zeta)\cup W(\zeta), & \zeta\in D\cap D=D \end{cases}$$

Thus,

$$T(\zeta)=\begin{cases} F(\zeta)\cup F(\zeta), & \zeta\in(D\setminus B)\cap(D\setminus I)=D\cap B'\cap I' \\ F(\zeta)\cup[(F(\zeta)\cap H(\zeta)], & \zeta\in(D\setminus B)\cap(D\cap I)=D\cap B'\cap I \\ [(F(\zeta)\cap S(\zeta)]\cup F(\zeta), & \zeta\in(D\cap B)\cap(D\setminus I)=D\cap B\cap I' \\ [(F(\zeta)\cap S(\zeta)]\cup[(F(\zeta)\cap H(\zeta)], & \zeta\in(D\cap B)\cap(D\cap I)=D\cap B\cap I \end{cases}$$

Thus,

$$T(\zeta)=\begin{cases} F(\zeta) & \zeta\in(D\backslash B)\cap(D\backslash I)=D\cap B'\cap I' \\ F(\zeta), & \zeta\in(D\backslash B)\cap(D\cap I)=D\cap B'\cap I \\ F(\zeta), & \zeta\in(D\cap B)\cap(D\backslash I)=D\cap B\cap I' \\ [(F(\zeta)\cap S(\zeta)]\cup[(F(\zeta)\cap H(\zeta)], & \zeta\in(D\cap B)\cap(D\cap I)=D\cap B\cap I \end{cases}$$

Here let's take care of $\zeta\in D\backslash B$ in the first equation. Since $D\backslash B= D\cap B'$, if $\zeta\in B'$, then $\zeta\in I\backslash B$ or $\zeta\in(B\cup I)'$. Hence, if $\zeta\in D\backslash B$, $\zeta\in D\cap B'\cap I'$ or $\zeta\in D\cap B'\cap I$. Thus, it is seen that (N,D)=(T,D).

2) $[(F,D)\ \overset{*}{\underset{\cup}{\sim}}\ (S,B)]\ \tilde{\cap}\ (H,I)=\ [(F,D)\ \tilde{\cap}\ (H,I)]\ \overset{*}{\underset{\cup}{\sim}}\ [(S,B)\ \tilde{\cap}\ (H,I)]$, where $D\cap B'\cap I=\emptyset$.

Proof: Let's first take care of the left hand side of the equality and let $(F,D)\ \overset{*}{\underset{\cup}{\sim}}\ (S,B)=(M,D)$, where $\forall\zeta\in D$;

$$M(\zeta)=\begin{cases} F'(\zeta), & \zeta\in D\backslash B \\ F(\zeta)\cup S(\zeta), & \zeta\in D\cap B \end{cases}$$

Let $(M,D)\ \tilde{\cap}\ (H,I)=(N,D)$, where $\forall\zeta\in D$;

$$N(\zeta)=\begin{cases} M(\zeta), & \zeta\in D\backslash I \\ M(\zeta)\cap H(\zeta), & \zeta\in D\cap I \end{cases}$$

Thus,

$$N(\zeta)=\begin{cases} F'(\zeta), & \zeta\in(D\backslash B)\backslash I=D\cap B'\cap I' \\ F(\zeta)\cup S(\zeta), & \zeta\in(D\cap B)\backslash I=D\cap B\cap I' \\ F'(\zeta)\cap H(\zeta) & \zeta\in(D\backslash B)\cap I=D\cap B'\cap I \\ [F(\zeta)\cup S(\zeta)]\cap H(\zeta) & \zeta\in(D\cap B)\cap I=D\cap B\cap I \end{cases}$$

Now let's take care of the left hand side of the equality: $[(F,D)\ \tilde{\cap}\ (H,I)]\ \overset{*}{\underset{\cup}{\sim}}\ [(S,B)\ \tilde{\cap}\ (H,I)]$. Let $(F,D)\ \tilde{\cap}\ (H,I)=(V,D)$, where $\forall\zeta\in D$;

$$V(\zeta)=\begin{cases} F(\zeta), & \zeta\in D\backslash I \\ F(\zeta)\cap H(\zeta), & \zeta\in D\cap I \end{cases}$$

Suppose that $(S,B)\ \tilde{\cap}\ (H,I)=(W,B)$, where $\forall\zeta\in B$;

$$W(\zeta)=\begin{cases} S(\zeta), & \zeta\in B\backslash I \\ S(\zeta)\cap H(\zeta), & \zeta\in B\cap I \end{cases}$$

Let $(V,D)\ \overset{*}{\underset{\cup}{\sim}}\ (W,B)=(T,D)$, where $\forall\zeta\in D$;

$$T(\zeta)=\begin{cases} V'(\zeta), & \zeta\in D\backslash B \\ V(\zeta)\cup W(\zeta), & \zeta\in D\cap B \end{cases}$$

Thus,

$$T(\zeta)=\begin{cases} F'(\zeta), & \zeta\in(D\backslash I)\backslash B=D\cap B'\cap I' \\ F'(\zeta)\cup H'(\zeta), & \zeta\in(D\cap I)\backslash B=D\cap B'\cap I \\ F(\zeta)\cup S(\zeta), & \zeta\in(D\backslash I)\cap(B\backslash I)=D\cap B\cap I' \\ F(\zeta)\cup[S(\zeta)\cap H(\zeta)], & \zeta\in(D\backslash I)\cap(B\cap I)=\emptyset \\ [F(\zeta)\cap H(\zeta)]\cup S(\zeta), & \zeta\in(D\cap I)\cap(B\backslash I)=\emptyset \\ [F(\zeta)\cap H(\zeta)]\cup[S(\zeta)\cap H(\zeta)], & \zeta\in(D\cap I)\cap(B\cap I)=D\cap B\cap I \end{cases}$$

It is seen that (N,D)=(T,D).

3) $(F,D)\,\tilde{\lambda}\,[(S,B)\overset{*}{\underset{\cup}{\sim}}(H,I)] = [(F,D)\,\tilde{\lambda}\,(S,B)]\ \tilde{\cap}\ [(F,D)\,\tilde{\lambda}\,(H,I)]$, where $D\cap B\cap I'=\emptyset$

Proof: Let's first take care of the left hand side of the equality and let $(S,B)\overset{*}{\underset{\cup}{\sim}}(H,I)=(M,B)$, where $\forall\zeta\in B$;

$$M(\zeta)=\begin{cases} S'(\zeta), & \zeta\in B\backslash I \\ S(\zeta)\cup H(\zeta), & \zeta\in B\cap I \end{cases}$$

Let $(F,D)\,\tilde{\lambda}\,(M,B)=(N,D)$, where $\forall\zeta\in D$;

$$N(\zeta)=\begin{cases} F(\zeta), & \zeta\in D\backslash B \\ F(\zeta)\cup M'(\zeta), & \zeta\in D\cap B \end{cases}$$

Thus,

$$N(\zeta)=\begin{cases} F(\zeta), & \zeta\in D\backslash B \\ F(\zeta)\cup S(\zeta), & \zeta\in D\cap(B\backslash I)=D\cap B\cap I' \\ F(\zeta)\cup[(S'(\zeta)\cap H'(\zeta)], & \zeta\in D\cap(B\cap I)=D\cap B\cap I \end{cases}$$

Now let's take care of the left hand side of the equality: $[(F,D)\,\tilde{\lambda}\,(S,B)]\ \tilde{\cap}\ [(F,D)\,\tilde{\lambda}(H,I)]$, Let $(F,D)\,\tilde{\lambda}\,(S,B)=(V,D)$, where $\forall\zeta\in D$;

$$V(\zeta)=\begin{cases} F(\zeta), & \zeta\in D\backslash B \\ F(\zeta)\cup S'(\zeta), & \zeta\in D\cap B \end{cases}$$

Suppose that $(F,D)\,\tilde{\lambda}(H,I)=(W,D)$, where $\forall\zeta\in D$;

$$W(\zeta)=\begin{cases} F(\zeta), & \zeta\in D\backslash I \\ F(\zeta)\cup H'(\zeta), & \zeta\in D\cap I \end{cases}$$

Let $(V,D)\,\tilde{\cap}(W,D)=(T,D)$, where $\forall\zeta\in D$;

$$T(\zeta)=\begin{cases} V(\zeta), & \zeta\in D\backslash D=\emptyset \\ V(\zeta)\cap W(\zeta), & \zeta\in D\cap D=D \end{cases}$$

Thus,

$$T(\zeta)=\begin{cases} F(\zeta) \cap F(\zeta), & \zeta\in(D\backslash B) \cap (D\backslash I)=D\cap B'\cap I' \\ F(\zeta) \cap[(F(\zeta)\cup H'(\zeta)], & \zeta\in(D\backslash B) \cap (D\cap I)=D\cap B'\cap I \\ [(F(\zeta) \cup S'(\zeta)] \cap F(\zeta), & \zeta\in(D\cap B) \cap (D\backslash I)=D\cap B\cap I' \\ [(F(\zeta) \cup S'(\zeta)] \cap [(F(\zeta) \cup H'(\zeta)], & \zeta\in(D\cap B) \cap (D\cap I)=D\cap B\cap I \end{cases}$$

Thus,

$$T(\zeta)=\begin{cases} F(\zeta) & \zeta\in(D\backslash B) \cap (D\backslash I)=D\cap B'\cap I' \\ F(\zeta), & \zeta\in(D\backslash B) \cap (D\cap I)=D\cap B'\cap I \\ F(\zeta), & \zeta\in(D\cap B) \cap (D\backslash I)=D\cap B\cap I' \\ [(F(\zeta) \cup S'(\zeta)] \cap [(F(\zeta) \cup H'(\zeta)], & \zeta\in(D\cap B) \cap (D\cap I)=D\cap B\cap I \end{cases}$$

Since $D\backslash B= D\cap B'$, if $\zeta\in B'$, then $\zeta\in I\backslash B$ or $\zeta\in(B\cup I)'$. Hence, if $\zeta\in D\backslash B$, $\zeta\in D\cap B'\cap I'$ or $\zeta\in D\cap B'\cap I$. Thus, it is seen that $(N,D)=(T,D)$.

4) $[(F,D) \overset{*}{\underset{\cup}{\sim}} (S,B)] \tilde{\lambda} (H,I)= [(F,D) \tilde{\lambda} (H,I)] \overset{*}{\underset{\cup}{\sim}} [(S,B) \tilde{\lambda} (H,I)]$, where $D\cap B'\cap I=\emptyset$.

Proof: Let's first take care of the left hand side of the equality and let $(F,D) \overset{*}{\underset{\cup}{\sim}} (S,B)=(M,D)$, where $\forall\zeta\in D$;

$$M(\zeta)=\begin{cases} F'(\zeta), & \zeta\in D\backslash B \\ F(\zeta) \cup S(\zeta), & \zeta\in D\cap B \end{cases}$$

Let $(M,D) \tilde{\lambda} (H,I)=(N,D)$, where $\forall\zeta\in D$;

$$N(\zeta)=\begin{cases} M(\zeta), & \zeta\in D\backslash I \\ M(\zeta) \cup H'(\zeta), & \zeta\in D\cap I \end{cases}$$

Thus,

$$N(\zeta)=\begin{cases} F'(\zeta), & \zeta\in(D\backslash B)\backslash I=D\cap B'\cap I' \\ F(\zeta) \cup S(\zeta), & \zeta\in(D\cap B)\backslash I=D\cap B\cap I' \\ F'(\zeta) \cup H'(\zeta) & \zeta\in(D\backslash B)\cap I=D\cap B'\cap I \\ [F(\zeta) \cup S(\zeta)] \cup H'(\zeta) & \zeta\in(D\cap B)\cap I=D\cap B\cap I \end{cases}$$

Now let's take care of the left hand side of the equality: $[(F,D) \tilde{\lambda} (H,I)] \overset{*}{\underset{\cup}{\sim}} [(S,B) \tilde{\lambda} (H,I)]$. Let $(F,D) \tilde{\lambda} (H,I)=(V,D)$, where $\forall\zeta\in D$;

$$V(\zeta)=\begin{cases} F(\zeta), & \zeta\in D\backslash I \\ F(\zeta) \cup H'(\zeta), & \zeta\in D\cap I \end{cases}$$

Suppose that $(S,B) \tilde{\lambda} (H,I)=(W,B)$, where $\forall\zeta\in B$;

$$W(\zeta)=\begin{cases} S(\zeta), & \zeta\in B\setminus I \\ S(\zeta)\cup H'(\zeta), & \zeta\in B\cap I \end{cases}$$

Let $(V,D)\overset{*}{\underset{\cup}{\sim}}(W,B)=(T,D)$, where $\forall\zeta\in D$;

$$T(\zeta)=\begin{cases} V'(\zeta), & \zeta\in D\setminus B \\ V(\zeta)\cup W(\zeta), & \zeta\in D\cap B \end{cases}$$

Thus,

$$T(\zeta)=\begin{cases} F'(\zeta), & \zeta\in(D\setminus I)\setminus B=D\cap B'\cap I' \\ F'(\zeta)\cap H(\zeta), & \zeta\in(D\cap I)\setminus B=D\cap B'\cap I \\ F(\zeta)\cup S(\zeta), & \zeta\in(D\setminus I)\cap(B\setminus I)=D\cap B\cap I' \\ F(\zeta)\cup[S(\zeta)\cup H'(\zeta)], & \zeta\in(D\setminus I)\cap(B\cap I)=\emptyset \\ [F(\zeta)\cup H'(\zeta)]\cup S(\zeta), & \zeta\in(D\cap I)\cap(B\setminus I)=\emptyset \\ [F(\zeta)\cup H'(\zeta)]\cup[S(\zeta)\cup H'(\zeta)], & \zeta\in(D\cap I)\cap(B\cap I)=D\cap B\cap I \end{cases}$$

It is seen that $(N,D)=(T,D)$.

5) $[(F,D)\overset{*}{\underset{\cup}{\sim}}(S,B)]\,\tilde{\setminus}\,(H,I)=[(F,D)\,\tilde{\setminus}\,(H,I)]\overset{*}{\underset{\cup}{\sim}}[(S,B)\,\tilde{\setminus}\,(H,I)]$, where $D\cap B'\cap I=\emptyset$.

Proof: Let's first take care of the left hand side of the equality and let $(F,D)\overset{*}{\underset{\cup}{\sim}}(S,B)=(M,D)$, where $\forall\zeta\in D$;

$$M(\zeta)=\begin{cases} F'(\zeta), & \zeta\in D\setminus B \\ F(\zeta)\cup S(\zeta), & \zeta\in D\cap B \end{cases}$$

Let $(M,D)\,\tilde{\setminus}\,(H,I)=(N,D)$, where $\forall\zeta\in D$;

$$N(\zeta)=\begin{cases} M(\zeta), & \zeta\in D\setminus I \\ M(\zeta)\cap H'(\zeta), & \zeta\in D\cap I \end{cases}$$

Thus,

$$N(\zeta)=\begin{cases} F'(\zeta), & \zeta\in(D\setminus B)\setminus I=D\cap B'\cap I' \\ F(\zeta)\cup S(\zeta), & \zeta\in(D\cap B)\setminus I=D\cap B\cap I' \\ F'(\zeta)\cap H'(\zeta) & \zeta\in(D\setminus B)\cap I=D\cap B'\cap I \\ [F(\zeta)\cup S(\zeta)]\cap H'(\zeta) & \zeta\in(D\cap B)\cap I=D\cap B\cap I \end{cases}$$

Now let's take care of the left hand side of the equality: $[(F,D)\,\tilde{\setminus}\,(H,I)]\overset{*}{\underset{\cup}{\sim}}[(S,B)\,\tilde{\setminus}\,(H,I)]$. Let $(F,D)\,\tilde{\setminus}\,(H,I)=(V,D)$, where $\forall\zeta\in D$;

$$V(\zeta)=\begin{cases} F(\zeta), & \zeta\in D\backslash I \\ F(\zeta)\cap H'(\zeta), & \zeta\in D\cap I \end{cases}$$

Suppose that (S,B) $\tilde{\backslash}$(H,I)=(W,B), where $\forall\zeta\in B$;

$$W(\zeta)=\begin{cases} S(\zeta), & \zeta\in B\backslash I \\ S(\zeta)\cap H'(\zeta), & \zeta\in B\cap I \end{cases}$$

Let (V,D) $\underset{\cup}{\overset{*}{\sim}}$ (W,B)=(T,D), where $\forall\zeta\in D$;

$$T(\zeta)=\begin{cases} V'(\zeta), & \zeta\in D\backslash B \\ V(\zeta)\cup W(\zeta), & \zeta\in D\cap B \end{cases}$$

Thus,

$$T(\zeta)=\begin{cases} F'(\zeta), & \zeta\in(D\backslash I)\backslash B=D\cap B'\cap I' \\ F'(\zeta)\cup H(\zeta), & \zeta\in(D\cap I)\backslash B=D\cap B'\cap I \\ F(\zeta)\cup S(\zeta), & \zeta\in(D\backslash I)\cap(B\backslash I)=D\cap B\cap I' \\ F(\zeta)\cup[S(\zeta)\cap H'(\zeta)], & \zeta\in(D\backslash I)\cap(B\cap I)=\emptyset \\ [F(\zeta)\cap H'(\zeta)]\cup S(\zeta), & \zeta\in(D\cap I)\cap(B\backslash I)=\emptyset \\ [F(\zeta)\cap H'(\zeta)]\cup[S(\zeta)\cap H'(\zeta)], & \zeta\in(D\cap I)\cap(B\cap I)=D\cap B\cap I \end{cases}$$

It is seen that (N,D)=(T,D).

6) [(F,D) $\underset{\cup}{\overset{*}{\sim}}$ (S,B)] $\tilde{\cup}$ (H,I)= [(F,D) $\tilde{\cup}$ (H,I)] $\underset{\cup}{\overset{*}{\sim}}$ [(S,B) $\tilde{\cup}$ (H,I))], where $D\cap B'\cap I=\emptyset$.

Proof: Let's first take care of the left hand side of the equality and let (F,D) $\underset{\cup}{\overset{*}{\sim}}$ (S,B)=(M,B), where $\forall\zeta\in D$;

$$M(\zeta)=\begin{cases} F'(\zeta), & \zeta\in D\backslash B \\ F(\zeta)\cup S(\zeta), & \zeta\in D\cap B \end{cases}$$

Let (M,D) $\tilde{\cup}$ (H,I)=(N,D), where $\forall\zeta\in D$;

$$N(\zeta)=\begin{cases} M(\zeta), & \zeta\in D\backslash I \\ M(\zeta)\cup H(\zeta), & \zeta\in D\cap I \end{cases}$$

Thus,

$$N(\zeta)=\begin{cases} F'(\zeta), & \zeta\in(D\backslash B)\backslash I=D\cap B'\cap I' \\ F(\zeta)\cup S(\zeta), & \zeta\in(D\cap B)\backslash I=D\cap B\cap I' \\ F'(\zeta)\cup H(\zeta) & \zeta\in(D\backslash B)\cap I=D\cap B'\cap I \\ [F(\zeta)\cup S(\zeta)]\cup H(\zeta) & \zeta\in(D\cap B)\cap I=D\cap B\cap I \end{cases}$$

Now let's take care of the left hand side of the equality: $[(F,D)\ \tilde{\cup}\ (H,I)]\ \overset{*}{\underset{\cup}{\sim}}\ [(S,B)\ \tilde{\cup}\ (H,I)]$. Let $(F,D)\ \tilde{\cup}\ (H,I)=(V,D)$, where $\forall\zeta\in D$;

$$V(\zeta)=\begin{cases} F(\zeta), & \zeta\in D\backslash I \\ F(\zeta)\cup H(\zeta), & \zeta\in D\cap I \end{cases}$$

Suppose that $(S,B)\ \tilde{\cup}\ (H,I)=(W,B)$, where $\forall\zeta\in B$;

$$W(\zeta)=\begin{cases} S(\zeta), & \zeta\in B\backslash I \\ S(\zeta)\cup H(\zeta), & \zeta\in B\cap I \end{cases}$$

Let $(V,D)\ \overset{*}{\underset{\cup}{\sim}}\ (W,B)=(T,D)$, where $\forall\zeta\in D$;

$$T(\zeta)=\begin{cases} V'(\zeta), & \zeta\in D\backslash B \\ V(\zeta)\cup W(\zeta), & \zeta\in D\cap B \end{cases}$$

Thus,

$$T(\zeta)=\begin{cases} F'(\zeta), & \zeta\in(D\backslash I)\backslash B=D\cap B'\cap I' \\ F'(\zeta)\cap H'(\zeta), & \zeta\in(D\cap I)\backslash B=D\cap B'\cap I \\ F(\zeta)\cup S(\zeta), & \zeta\in(D\backslash I)\cap(B\backslash I)=D\cap B\cap I' \\ F(\zeta)\cup[S(\zeta)\cup H(\zeta)], & \zeta\in(D\backslash I)\cap(B\cap I)=\emptyset \\ [F(\zeta)\cup H(\zeta)]\cup S(\zeta), & \zeta\in(D\cap I)\cap(B\backslash I)=\emptyset \\ [F(\zeta)\cup H(\zeta)]\cup[S(\zeta)\cup H(\zeta)], & \zeta\in(D\cap I)\cap(B\cap I)=D\cap B\cap I \end{cases}$$

It is seen that $(N,D)=(T,D)$.

4. Conclusion

In this paper, we have obtained the distributions of soft binary piecewise operations over complementary soft binary piecewise intersection and union operations with the aim to contribute to the theoretical basis of soft set theory by providing researchers with suggestions for studying and extracting certain algebraic structures using these new soft set operations.The distributions of soft binary piecewise operations over other complementary soft piecewise operations could be the subject of further theoretical research for soft sets in the future. Also, since soft sets are a powerful mathematical tool for detecting uncertain objects, this work will enable researchers to propose some new cryptographic or decision-making methods based on soft sets. Moreover, the study of soft algebraic structures on algebraic properties can be handled again by considereing the operations defined in this article.

Kaynaklar

[1] Molodtsov D. Soft set theory-first results. Comput Math Appl 1999; 37(1), 19-31.

[2] Özlü Ş. Interval Valued q- Rung Orthopair Hesitant Fuzzy Choquet Aggregating Operators in Multi-Criteria Decision Making Problems. Gazi University Journal of Science Part C: Design and Technology 2022a; 10 4). 1006-1025.

[3] Özlü Ş. Interval Valued Bipolar Fuzzy Prioritized Weighted Dombi Averaging Operator Based On Multi-Criteria Decision Making Problems. Gazi University Journal of Science Part C: Design and Technology 2022b; 10 (4). 841-857.

[4] Özlü Ş and Sezgin A., Soft covered ideals in semigroups. Acta Univ. Sapientiae Math 2021; 12 (2). 317-346.

[5] Maji PK, Bismas R & Roy AR. Soft set theory. Comput Math Appl 2003; 45 (1), 555-562.
[6] Pei D, Miao D. From Soft Sets to Information Systems. In: Proceedings of Granular Computing. IEEE 2005; 2, 617-621
[7] Ali MI, Feng F, Liu X, Min WK. Shabir M. On some new operations in soft set theory. Comput Math Appl 2009; 57(9), 1547-1553.
[8] Sezgin A, Atagün AO. On operations of soft sets. Comput Math Appl 2011; 61(5), 1457-1467.
[9] Sezgin A, Shahzad A, Mehmood A. New Operation on Soft Sets: Extended Difference of Soft Sets. J. New Theory 2019; (27), 33-42.
[10] Stojanovic NS. A new operation on soft sets: extended symmetric difference of soft sets. Military Technical Courier 2021; 69(4), 779-791.
[11] Çağman N. Conditional complements of sets and their application to group theory. J New Results Sci 2021; 10 (3), 67-74.
[12] Sezgin A, Çağman N, Aybek F. New conditional complements of sets and their application to group theory. in press in Information Management and Computer Science 2023a.
[13] Aybek F. New restricted and extended soft set operations. MSc, Amasya University, Turkey, 2023, unpublished thesis.
[14] Demirci AM. New type of extended operations of soft set: Complementary extended plus, union and theta operations. MSc, Amasya University, Amasya, Turkey, 2023, unpublished thesis.
[15] Sarıalioğlu M. New type of extended operations of soft set: Complementary extended gamma, intersection and star operations. MSc, Amasya University, Amasya, Turkey, 2023, unpublished thesis.
[16] Akbulut E. New type of extended operations of soft set: Complementary extended lambda and difference operations. MSc, Amasya University, Amasya, Turkey, 2023, unpublished thesis.
[17] Eren ÖF, Çalışıcı H. On some operations of soft sets. The Fourth International Conference on Computational Mathematics and Engineering Sciences; 2019; Antalya, Turkey.
[18] Yavuz E. Soft binary piecewise operations and their properties. MSc, Amasya University, Amasya, Turkey, 2023, unpublished thesis.
[19] Sezgin A, Sarıalioğlu M. New soft set operation: Complementary soft binary piecewise theta operation. in press in Journal of Kadirli Faculty of Applied Sciences 2023b.
[20] Sezgin A, Demirci AM. New soft set operation: Complementary soft binary piecewise star operation. Ikonion Joural of Mathematics, 2023; 5(2), 24-52.
[21] Sezgin A, Atagün AO. New soft set operation: Complementary soft binary piecewise plus operation. in press in Matrix Science Mathematic 2023.
[22] Sezgin A, Çağman N. New soft set operation: Complementary soft binary piecewise difference operation. in press in Osmaniye Korkut Ata University Journal of the Institute of Science and Technology 2023.
[23] Sezgin A, Aybek F. New soft set operation: Complementary soft binary piecewise gamma operation. Matrix Science Mathematic 2023; 7(1), 27-45.
[24] Sezgin A, Aybek F, Bilgili Güngör N. New soft set operation: Complementary soft binary piecewise union operation.in press in Black Sea Journal of Engineering and Science 2023b.
[25] Sezgin A, Aybek F, Atagün AO. New soft set operation: Complementary soft binary piecewise intersection operation. Acta Informatica Malaysia 2023c; 7(1), 38-53.
[26] Sezgin A, Yavuz E. New soft set operation: Complementary soft binary piecewise lamda operation, in press in Sinop University Journal of Science 2023b.