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# Leaky strip aquifer parameter estimation by ratio matching process

## Oran eşleştirme işleyişi ile sızdıran şerit akifer parametre tahmini

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**Abstract**

*In this study, a novel estimation method, called as Ratio Matching Process (RMP), was proposed to determine the hydrologic parameters of a leaky strip aquifer. RMP employs two simple formulations in order to simplify the parameter estimation procedure. The first formula gives the leakage factor from the site data by selecting two different time-drawdown measurements, which are separated by the user-specified distance from the inflection point of the drawdown vs logarithmic time curve. The second formula provides the users to find the dimensionless inflection point from the leakage factor. Thus, the aquifer parameters could be easily acquired with RMP by avoiding the classical curve matching process. The capability of RMP was studied with several hypothetical pumping test scenarios including ideal and noise-contaminated datasets. The results from the implemented cases indicate that RMP can be regarded as a good alternative to the available methods.*

**Keywords:** Aquifer parameters, Groundwater, Inverse problem, Leaky aquifer, Parameter estimation.

**Öz**

*Bu çalışmada, Oran Eşleştirme İşleyişi (OEİ) adı verilen sızdıran şerit akiferinin hidrolojik parametrelerinin belirlenmesi için yeni bir tahmin yöntemi önerilmiştir. OEİ, parametre tahmin sürecini kolaylaştırmak için iki basit formül kullanır. Birinci formül, su düşümü-logaritmik zaman eğrisinin büküm noktasından bir kullanıcı tarafından belirlenen mesafe ile ayrılan iki farklı su düşüm ölçümünden sızdırma faktörünü verir. İkinci formül, kullanıcıların sızdırma faktöründen boyutsuz büküm noktasını bulmasını sağlar. Böylece, akifer parametreleri OEİ ile klasik eğri eşleştirme işleyişinden kaçınarak kolayca elde edilebilir. OEİ'nin kabiliyeti çeşitli ideal ve gürültü ile kirletilmiş hipotetik pompaj testi senaryoları ile çalışılmıştır. İncelenen durumdan elde edilen sonuçlar OEİ'nin hali hazırdaki mevcut yöntemlere iyi bir alternatif olarak değerlendirilebileceğini göstermektedir.*

**Anahtar Kelimeler:** Akifer parametreleri, Yeraltı suları, Ters problem, Sızdıran akifer, Parametre tahmini.

## 1 Introduction

Drains can be constructed for several purposes such as collecting the recharge water or irrigation water from agricultural fields, removing hazardous contaminant from an aquifer, reducing the groundwater levels for urban areas or controlling flood in plains [1]. Several mathematical models described by horizontal flow towards canals or drains in non-leaky or leaky aquifer conditions have been proposed over last few decades. [2] formulated two different drain functions in a non-leaky confined aquifer to describe the groundwater flow subjected to a constant discharge test (CDT) or constant head test (CHT). The type curves for one-dimensional horizontal flow in leaky aquifer case were then constructed by [3]-[5] for both of CDT and CHT. In these studies, the drain functions were given in the integral forms that require numerical integration schemes for the functional evaluation. [6] found an equivalent version of the drain function for the CHT condition in a leaky aquifer, which can be expressed by a linear combination of exponential and complimentary error functions. For the CDT case, a similar solution was also adopted by [7]-[9]. [10] replaced the infinite extent aquifer assumption with finite-width aquifer bounded by no-flow boundary condition to understand the behavior of groundwater flow near the line sink. Employing the double porosity concept, [11] modelled one-dimensional transient flow near a stream under a step rise/fall test for a finite fracture system. [12] implemented a finite-difference based numerical solution to investigate the flow behavior of the model proposed by [11] under non-Darcian flow conditions. [1] studied the two dimensional steady-state flow response of an infinite extent horizontal drain in an unconfined aquifer with changing thickness. [13] studied the storage and skin effect for one dimensional flow in a finite width sink for a leaky strip aquifer.

Apart from the leaky strip aquifer, the extensive literature is available to estimate the aquifer parameters for confined and classical leaky aquifer systems. [14] suggested a derivative based methodology called as Double Inflection Method (DIM). [15] proposed a diagnostic curve approach to identify leaky aquifer parameters with/without aquifer storage. [16] formulated a regression-based analysis for determining aquifer parameters from the drawdown recorded in partial penetrating wells. In addition, some metaheuristic optimization algorithms are strong alternative to find the aquifer parameters from the perspective of inverse problem in groundwater engineering. For instance, [17] used Differential Evolution (DE) to detect the location of pollutant source, [18] used a Particle Swarm Optimization (PSO) for identifying Non-Darcian flow parameters, [19] used a DE based parameter estimation scheme for leaky and non-leaky aquifers. [20] introduced an optimization scheme with water cycle algorithm (WCA) for generalized radial flow.

The determination of aquifer parameters such as transmissivity, storativity and leakage factor using drawdown data obtained from a leaky strip aquifer is as important as the

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forward mathematical models as illustrated above. The aquifer parameters could be typically estimated via graphical curve matching process (i.e. [8] and [10]). However, a specific method associated with determination of aquifer parameters for the horizontal flow towards a drain in leaky aquifer case was very rare in the literature as discussed. This research is therefore motivated to establish a new estimation methodology, referred to as Ratio Match Process (RMP), in order to estimate hydraulic parameters from the leaky strip aquifer in lieu of classical curve matching process. The RMP is derived from a simple logic that links the leakage factor with the ratio of two specific drawdown values separated by a pre-specified log-window from the inflection point of time-drawdown curve. The proposed functional relations are able to estimate the flow parameters in the leaky strip aquifer model. The merits of the RMP are summarized as;

i. The application of the proposed method is straightforward and simple,
ii. The bias in the estimation, which is resulted from the subjectivity of the user, is eliminated since the proposed approach avoids the curve matching process,
iii. The introduced method provides the estimation performance as accurate as the classical ones.

## 2 Material and method

### 2.1 Theory of leaky strip aquifer

The governing equation including the assumptions such as a fully penetrating drain, negligible storage in confining bed, initially constant static water level (SWL), constant confining bed and aquifer thicknesses for one-dimensional horizontal flow in a semi-infinite extent homogenous leaky aquifer, as schematized in Figure 1, was given as [21].

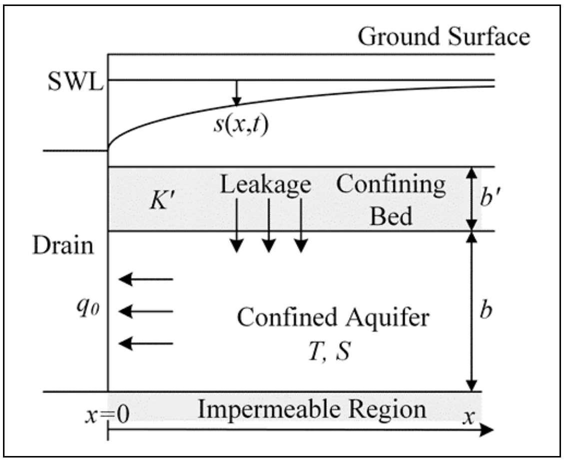


Figure 1. Illustration of 1-D flow to drain in leaky aquifer.

$$\frac{\partial}{\partial x}\left(T\frac{\partial s}{\partial x}\right)-\frac{K'}{b'}s=S\frac{\partial s}{\partial t} \tag{1}$$

Where $s$ is the drawdown in the drain, $t$ is the time, $x$ is the perpendicular distance from the observation point to the drain, $T$ is the transmissivity of the confined aquifer, $S$ is storativity of the confined aquifer, $b'$ is the thickness of confining bed and $K'$ denotes the vertical hydraulic conductivity of confining bed. When Eq. (1) is subjected to initial condition $s(x,0)=0$, and boundary conditions $T\,ds/dx=q_0$ when $x=0$ and $s(\infty,t)=0$, in which $q_0$ is the constant flow rate per unit width of the aquifer at $x=0$, [8] provided the solution of Eq. (1) as;

$$s(x,t)=\frac{q_0 x}{T}F(u,\lambda)_q \tag{2}$$

with drain function $F$ as

$$F(u,\lambda)_q=\frac{\mathrm{e}^{-\lambda}erfc\left(\sqrt{u}-\frac{\lambda}{2\sqrt{u}}\right)-\mathrm{e}^{-\lambda}erfc\left(\sqrt{u}+\frac{\lambda}{2\sqrt{u}}\right)}{2\lambda} \tag{3}$$

Where $erfc$ is the complimentary error function. The dimensionless time $u$ and leakage factor $B$ are

$$u=\frac{x^2S}{4Tt},\ \lambda=\frac{x}{B},\ B=\sqrt{\frac{Tb'}{K'}} \tag{4}$$

It is important to note that, Eq. (1) is the governing equation of groundwater flow to well for classical leaky aquifer system. It was previously solved by [22] with the different initial and boundary conditions than those presented herein.

### 2.2. Proposed ratio matching process (RMP)

The development of RMP starts from determining the inflection point where the first derivative of $F$ vs $\ln u$ curve is maximum [14]. This inflection point serves as a critical point that is unique for each $\lambda$ value. Noting that $\partial F/\partial(\ln u)=u\,\partial F/\partial u$, the inflection point $u_i$ for each $\lambda$ value was then determined by

$$\frac{\partial}{\partial \mathrm{ln}u}\left(\frac{\partial F}{\partial \mathrm{ln}u}\right)=u^2\frac{\partial^2 F}{\partial u^2}+u\frac{\partial F}{\partial u}=0 \tag{5}$$

To find the inflection point of a specific type curve generated with a predefined $\lambda$ value, the following procedure was applied:

i. The first and second derivatives of Eq. (3) with respect to $u$ was computed,
ii. The first and second derivatives were then substituted in Eq. (5). Thus, a non-linear expression with two arguments $u$ and $\lambda$ was obtained,
iii. With a known $\lambda$ value, the obtained expression from previous step becomes a single variable equation and it was solved by Bi-section method in order to find the inflection point for a type curve drawn by $\lambda$ value.

This process was repeated for several $\lambda$ values and each inflection point was noted. Once the inflection point $u_i$ for each $\lambda$ value was determined by solving Eq. (5), after trial of various regression models, a rational fit model was performed well to relate $u_i$ with $\lambda$ as

$$\ln u_i=\frac{124.3\ln\lambda-109}{(\ln\lambda)^2+9.37\ln\lambda+91.06} \tag{6}$$

together with the statistical properties such as the coefficient of determination $R^2=1$, adjusted $R^2=1$, sum squared error ($SSE$)= 0.1356 and root mean squared error ($RMSE$)= 0.0214. It is noted that Eq. (6) was obtained with the $\lambda$ values selected from a range of $0.01\le\lambda\le3.16$ and the dimensionless time values lying on an interval of $-15\le\ln u\le5$. The $\lambda$ and $u$ ranges were selected from the type curve family proposed by [8].

RMP then utilizes two drawdown measurements $(t_1, s_1)$ and $(t_2, s_2)$ on the field curve which are separated by a logarithmic interval Δ from the inflection point $(t_i, s_i)$ as shown in Figure 2(a). In other words, $\Delta = \ln(t_2/t_i) = \ln(t_i/t_1)$. The corresponding dimensionless matching pairs on the type curve then become $(u_1, F_1)$, $(u_2, F_2)$ and $(u_i, F_i)$ respectively. Thus, the drawdowns can be expressed as

$$s_1 = \frac{q_0 x}{T} F_1, \quad F_1 = F(u_1, \lambda)$$
$$s_2 = \frac{q_0 x}{T} F_2, \quad F_2 = F(u_2, \lambda) \tag{7}$$

Also, $u_1$ and $u_2$ can be re-written as

$$u_1 = u_i t_i / t_1 \Rightarrow \ln u_1 = \ln u_i + \Delta$$
$$u_2 = u_i t_i / t_2 \Rightarrow \ln u_2 = \ln u_i - \Delta \tag{8}$$

The drawdown ratio $\phi$ is formulated as

$$\phi = \frac{s_2(t_2)}{s_1(t_1)} = \frac{F_2(u_2, \lambda)}{F_1(u_1, \lambda)} = \frac{F(u_i e^{-\Delta}, \lambda)}{F(u_i e^{\Delta}, \lambda)} \tag{9}$$

Eq. (9) implies that the drawdown ratio is a non-dimensional quantity. The effect of particular pumping test on the drawdown ratio was implicitly characterized by $\lambda$ value and the inflection point $u_i$. The $u_i$ term was already combined with $\lambda$ as given in Eq. (6). Therefore, drawdown ratio, $\phi$, term in Eq. (9) can be regarded as a function of $\lambda$ with the known Δ value. The Δ value is just a user-specified value and may be selected with respect to the duration of pumping test accordingly.

With a prescribed Δ value, the $\phi$ term can be easily related to $\lambda$ by a regression analysis. In other words, the number of variables in Eq. (9) reduces to $\phi$ and $\lambda$. Again, a rational fit equation form was employed to link $\phi$ with $\lambda$ values as

$$\ln \lambda = \frac{p_1 (\ln \phi)^2 + p_2 \ln \phi + p_3}{(\ln \phi)^2 + w_1 \ln \phi + w_2} \tag{10}$$

where $p_1$, $p_2$, $p_3$, $w_1$ and $w_2$ are the model coefficients varying with the user specified Δ. To obtain model coefficients proposed in Eq. (10), the following computations were performed:

i. A Δ value was chosen. (i.e. $\Delta = 0.1$ ),
ii. For a range of $0.01 \le \lambda \le 3.16$ and $-15 \le \ln u \le 5$ values, a number of type curves was generated by Eq. (3),
iii. For each $\lambda$ value, the inflection point was computed by Eq. (5) as stated earlier,
iv. According to the selected Δ value, $u_1$ and $u_2$ values were computed by Eq. (8) and corresponding $F_1$ and $F_2$ values were found to give $\phi$ value in Eq. (9).
v. The $\lambda$ and $\phi$ values were linked by the model proposed in Eq. (10). Note that the form in Eq. (10) was obtained after cumbersome trial and error process,
vi. By a regression analysis, the model coefficients were noted,
vii. Whole process was repeated with different Δ values.

Table 1 summarizes the model coefficients and the statistical performance of the proposed fit equation given in Eq. (10). It is important to note that Eq. (10) is valid for $1.5 < \ln \phi < 10$, which is a broad range for practical applications. Once $\lambda$ was found by Eq. (10) and $u_i$ was determined by Eq. (6), the aquifer parameters were then estimated by

$$\hat{T} = \frac{q_0 x}{s_i} F(\hat{u}_i, \hat{\lambda}), \quad \hat{S} = \frac{4 \hat{T} t_i \hat{u}_i}{x^2} \tag{11}$$

where superscript "^" denotes the estimated parameters. As a summary, the application procedure of RMP is

- Plot $s$ vs $\ln t$ using the available field data,
- Find the inflection point $t_i$ and corresponding drawdown $s_i$ by employing a numerical derivative scheme for $s$ vs $\ln t$ curve,
- Decide Δ , and find $s_1$ and $s_2$ values from the available site data to compute $\phi$ value,
- Estimate $\lambda$ by using Eq. (10) and $u_i$ by using Eq. (6),
- Estimate the aquifer parameters by using Eq. (11).

# 3 Results

## 3.1 Illustrative Example with Homogeneous Data

As an illustrative example, a hypothetical pumping test scenario was constructed with the following inputs: $q_0 = 0.05$ m²/day/m, $\lambda = 0.45$, $x = 25$, $T = 0.5$ m²/day and $S = 0.0001$. The test started from 0.01 day and ended after 1 day with the uniform time increment of 0.05 day in logarithm base 10, which yields to a total of 41 data pairs. In other words, time values were generated with an array of $t = 10^{(-2:0.05:0)}$. Following the RMP as previously outlined, the drawdown at the inflection point was found as $s_i = 2.06$ m that corresponds to $t_i = 0.36$ day. By selecting Δ= 1, the data pairs $(t_1, s_1)$ and $(t_2, s_2)$ were found as (0.13, 0.95) and (0.98, 3.14), respectively as shown in Figure 2(a). The $\lambda$ was estimated as 0.4511 by Eq. (10). The $u_i$ was then found as 0.089 by Eq. (6). The aquifer parameters were then computed as $T = 0.5056$ m²/day and $S = 9.9 \times 10^{-5}$. Using these estimated values, the drawdown values were simulated by Eq. (2) and compared with the observed test data. As shown in Figure 2(b), the proposed RMP is able to achieve a good agreement between the simulated and observed drawdowns on the error metrics such that $RMSE$ was $8.83 \times 10^{-3}$ and $R^2$ was almost 1. Traditionally, the theoretical type curve and the field data curve are superimposed in order to achieve the perfect match. Thus, it enables the practitioners to read $\lambda$ value, matching $t$ and $s$ values from the superimposed curve. The manual curve matching procedure (CMP) was also employed as shown in Figure 2(c).

Table 1. The coefficients and statistical properties of Model Fit in Eq. (10).

| Δ | $p_1$ | $p_2$ | $p_3$ | $w_1$ | $w_2$ | *RMSE* | *SSE* | *Adj-R²* |
|---|---|---|---|---|---|---|---|---|
| 0.1 | 5.0488 | -1.0441 | 0.0335 | 0.2389 | -0.0183 | 0.0008 | 0.0001 | 1.0000 |
| 0.25 | 4.9148 | -2.5471 | 0.2050 | 0.5740 | -0.1104 | 0.0008 | 0.0001 | 1.0000 |
| 0.5 | 4.5134 | -4.7149 | 0.7621 | 1.0159 | -0.3979 | 0.0007 | 0.0001 | 1.0000 |
| 0.75 | 4.0351 | -6.3916 | 1.5417 | 1.3175 | -0.7880 | 0.0006 | 0.0001 | 1.0000 |
| 1 | 3.6057 | -7.7037 | 2.4019 | 1.5882 | -1.2621 | 0.0007 | 0.0001 | 1.0000 |
| 1.25 | 3.2786 | -8.8517 | 3.1907 | 1.9868 | -1.8854 | 0.0010 | 0.0001 | 1.0000 |
| 1.5 | 3.0562 | -10.0110 | 3.7121 | 2.6730 | -2.7851 | 0.0015 | 0.0003 | 1.0000 |
| 2 | 2.8251 | -12.8331 | 2.9073 | 5.3633 | -5.9710 | 0.0024 | 0.0009 | 1.0000 |

By identifying the matching point as $u = 1$ and $F(u, \lambda) = 1$, the corresponding matching pairs on the field curve were read as $t_m = 0.035$ day and $s_m = 2.9$ m when $\lambda = 0.5$. Using these values, $T$ and $S$ were estimated as 0.4310 m$^2$/day and 9.7×10$^{-5}$, respectively. As shown in Figure 2(d), the estimation performance of CMP was assessed by substituting estimated aquifer parameters into Eq. (2). With CMP, *RMSE* was realized as $2.83 \times 10^{-2}$ and $R^2 = 1$.

It is important to note that the estimation performance of CMP may change with the different matching pairs than those presented here. [14] discussed the potential drawbacks of using CMP for interpreting the aquifer parameters of a leaky aquifer. The same situation is still valid for the problem in this study. To address this issue, another CMP application was performed by reading the matching pairs as $\lambda = 0.4$, $s_m = 2.9$ m and $t_m = 0.025$ day. Absolute relative percentage error (ARPE) was computed for each estimated parameter. Table 2 compares the estimation performance of RMP and two possible applications of CMP. The results presented in Table 2 clearly demonstrates that RMP could estimate the aquifer parameter better than the classical curve matching process.

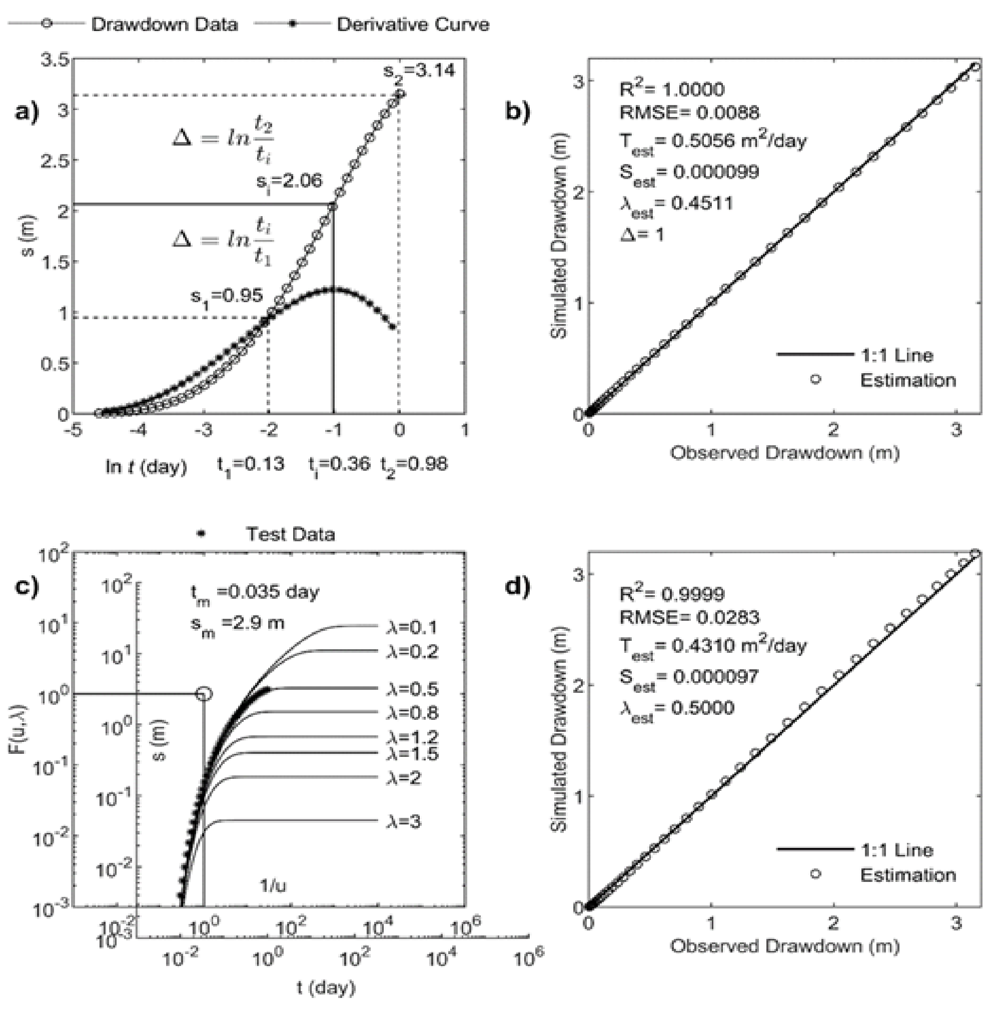


Figure 2(a): Application of RMP. (b): Estimation performance of RMP. (c): Application of CMP. (d): Estimation performance of CMP.

Table 2. Estimation Performance of Implemented methods.

| | RMP | Curve Match 1 | Curve Match 2 |
|---|---|---|---|
| Estimated λ | 0.4511 | 0.5 | 0.4 |
| Estimated T (m$^2$/day) | 0.5056 | 0.4310 | 0.5952 |
| Estimated S | 9.9×10$^{-5}$ | 9.7×10$^{-5}$ | 9.5×10$^{-5}$ |
| ARPE for λ | 0.24% | 11.11% | 11.11% |
| ARPE for T | 1.12% | 13.80% | 19.04% |
| ARPE for S | 1.00% | 3.00% | 5.00% |
| RMSE | 0.0088 | 0.0283 | 0.0401 |
| $R^2$ | 1.0000 | 0.9999 | 0.9996 |

### 3.2 Noise-Augmented data and sensitivity analysis

The RMP was further assessed with a number of numerical benchmark experiments mimicking in-situ aquifer pumping tests. These scenarios were designed to understand to what extent the deviation in the estimates of leakage factor $\lambda$ will affect $T$ , and $S$ predictions, and so to elaborate the overall of performance of RMP. To this end, the model parameters $T$, $S$ and $\lambda$ values were drawn randomly from the uniform distributions for $-2 \leq log_{10}T \leq 2$, $-6 \leq log_{10}S \leq -2$, and $0.2 \leq \lambda \leq 2$, respectively. Remaining parameters to produce the drawdown responses from hypothetical pumping tests were identical to the previous example. Furthermore, the drawdown data for each test case were contaminated with some random noise up to 2% of original drawdown value at each time level in order to mimic the measurement errors occurred in the field conditions. Thus, a total of 100 datasets were generated with randomly selected $T$, $S$ and $\lambda$ from the aforementioned ranges for each parameter.

Another aim of this example is to show the effect of $\Delta$ on the estimation performance. It is important to note that some unrealistic datasets may be synthetized by following the data generation procedure as summarized. For instance, they may contain very small or large drawdown values. Also, the test duration may not be long enough to use a constant pre-specified $\Delta$ value in each estimation. Therefore, RMP was applied with different $\Delta$ values and the coefficients of Eq. (10) given in Table 1 were interpolated accordingly for each test case. If any $\Delta$ value is used to follow the outlined estimation procedure, the coefficient of Eq. (9) can be interpolated by using linear, cubic-spline or other interpolation techniques. In this study, Radial Basis Function Collocation Method (RBFCM) was preferred due to its simplicity and accuracy.

The $\lambda$ estimations shown in Figure 3(a) were scattered around 1:1 line and $R^2$ value was realized as 0.9712. In Figure 3(b), $T$ estimations show a good agreement with the generated $T$ values regarding the error metrics $R^2 = 0.9331$ and *RMSE*=5.6753. The agreement between the estimated and generated $S$ values, as given in Figure 3(c), was realized as $R^2$ of 0.9864. For $\lambda$ estimations, the proposed RMP gives 73 out of 100 realizations less than 10% of ARPE, as shown in Figure 3(d). As shown in Figure 3(e), 94 estimations by RMP have ARPE values less than 25% for $T$ estimations. Finally, RMP was able to perform 84 estimations with ARPE less than 10% as depicted in Figure 3(f). These results also show that the value of $\Delta$, whether it is large or small, does not considerably affect the estimation performance.

## 4 Discussion

The RMP is motivated from the inflection point as a critical point that is unique for each $\lambda$ value. The inflection point could be found from the semi-logarithmic derivative of the available site test data. Therefore, the performance of RMP is closely related to precise determination of the inflection point. A further analysis was also performed to understand the effect of $t_i$ on entire estimation procedure. Suppose $t_i$ was found as 0.3264 day, which corresponds to almost 10% relative error, instead of 0.3614 day as found in the illustrative example, $s_i$ would then be 1.9384 m. By means of $\Delta = 1$ , $(t_1, s_1)$ and $(t_2, s_2)$ pairs would be as (0.12, 0.8553) and (0.89, 3.0545), respectively.

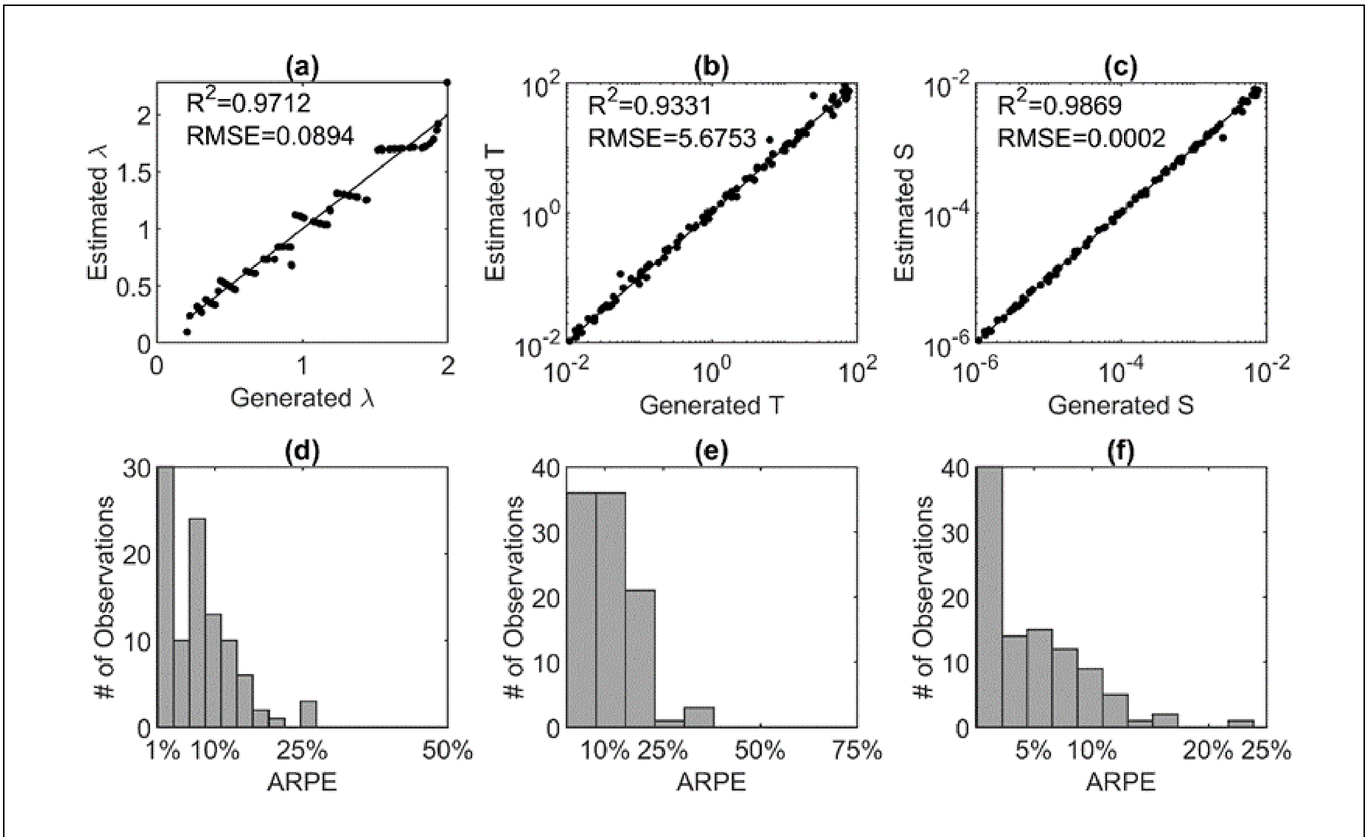


Figure 3. Overall results of 100 realizations for. (a): Estimated $\lambda$ values. (b): Estimated $T$ values. (c): Estimated $S$ values. (d): ARPE distribution for $\lambda$ estimates. (e): ARPE distribution for $T$ estimates. (f): ARPE distribution for $S$ estimates.

Using these values, RMP was able to estimate $\lambda$ as 0.5187 with ARPE of 15.2%, $T$ as 0.4326 m²/day with ARPE of 13.47%, $S$ as $9.68 \times 10^{-5}$ with ARPE of 3.14%. Alternatively, when 5% of error was added to $t_i$, the ARPE values reduced to 7.7% for $\lambda$, 6.6% of $T$ and 1.9% for $S$. The estimation performance increases with the lesser error in $t_i$ as expected. Some numerical techniques could be beneficiary to remedy the determination of $t_i$. The numerical differentiation scheme proposed by [23] is suggested to compute the derivative of the field data. Also, the site dataset is naturally discrete. The Radial Basis Function Collation Method (RBFCM) (i.e. [24]) is recommended to fit the discrete test data. For instance, if $(t_1, s_1)$ , $(t_2, s_2)$ and $(t_i, s_i)$ pairs are not available in the recorded test data, RBFCM is able to produce these values very accurately. The Golden-ratio search method can also be applied to determine the inflection point from maximizing the time derivative of the site data. These improvements were undertaken in this study.

## 5 Conclusions

This study offers simple functional relations to interpret the aquifer parameters for a leaky strip aquifer case. A number of hypothetical test scenarios including smooth and noise-augmented pumping test data was applied to investigate the proposed approach referred to as Ratio Matching Process (RMP). The important advantages and disadvantages associated with the proposed RMP can be summarized as:

i. The proposed RMP is a straight-forward method to estimate the aquifer parameters. From the available site data, the inflection point from $s$ vs $\ln t$ curve is first found. According to user specified $\Delta$ value, the drawdown ratio will be computed. Eq. (10) will be then applied to find $\lambda$, Eq. (6) will be used to find $u_i$. Finally, the aquifer parameters can be estimated by Eq. (11),

ii. The performance of RMP is closely relate to accurate estimation for inflection of time-drawdown data recorded in the field. To this end, the derivative of time-drawdown data is needed. Using derivative is however risky if the site measurements are not smooth, which means data may includes noise as a results of measurement errors or heterogeneity,

iii. As shown in Discussion part, 10% error in the position of inflection point yields absolute relative percentage error (ARPE) of 15.2% in $\lambda$, ARPE of 13.47% in $T$, and ARPE of 3.14% in $S$ for the illustrative example case. These errors reduce with lesser error made in the location of inflection point. For accurate estimation of inflection point, the interpolation of site data is recommended. This also allows the user to compute the derivative for uniform spacing if the site data were not recorded evenly in time,

iv. According to the user specified $\Delta$ value, $(t_1, s_1)$ and $(t_2, s_2)$ data pairs cannot be available in the site data. In this case, an interpolation scheme should be used to estimate these data pairs. Radial Basis Function Collocation Method (RBFCM) is recommended for this purpose,

v. Based on the site data, if the prescribed coefficients of Eq. (10) as shown in Table 1 cannot be used, the coefficients can be interpolated by RBFCM,

vi. According to the results of examples with the synthetic data as implemented in this study, RMP could easily and accurately estimate the aquifer parameters for this aquifer model,

vii. RMP does not suffer from any subjectivity in manual curve matching process, as demonstrated. In addition, RMP is a faster method to retrieve the aquifer parameters. A spread-sheet program is enough to apply the proposed methodology.

The results drawn from the analyses show that RMP is able to achieve a good estimation performance. Also, the logic behind RMP has a potential to be extended for estimating aquifer parameters of classical leaky aquifer whose type curve behavior is very similar to that of leaky strip aquifer presented in this study. As a conclusion, RMP can be regarded as a strong alternative to the traditional curve matching process.

## 6 Author Contributions

A. Ufuk ŞAHİN is responsible for the conceptualization, computations, writing and editing of this study.

## 7 Ethics committee approval and conflict of interest statement

For this paper, it is not necessary to get permission from the ethics committee. The author also states that there is no conflicting interest between authors or with any institution/s.

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