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Capacitated Network Traffic Assignment using Lagrange Neural Networks

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ABSTRACT

The purpose of this paper is to examine the travel time within a network by analyzing the changes in link volume of network traffic assignment issues that are impacted by capacity constraints using a Lagrange neural network methodology. To achieve this objective, the optimization problem associated with network traffic assignment, which includes capacity constraints, is transformed into a Lagrange problem. Following this, the Lagrange function is reduced to a system of differential equations consisting of neural equations using the gradient method. The dynamic system, which includes initial values, is solved through the well-established Runge-Kutta method. Finally, a numerical example is provided to illustrate the solution process and demonstrate the effectiveness of the presented neural network approach.

1. INTRODUCTION

The Traffic Assignment Problem (TAP) is a fundamental topic in transportation analysis and applications. Its objective is to determine the equilibrium flow patterns in a given transportation network while considering the origin-destination demands. In recent years, this problem has received significant attention, leading to the development of various modeling and solution techniques based on Wardrop's two optimization principles: User Equilibrium (UE) and System Optimal (SO). These principles have been extensively researched and analyzed, resulting in a comprehensive body of literature and practical applications [9].

The pioneering research undertaken by Beckmann and colleagues (2) demonstrated the potential for determining traffic flow on a road network through the application of the UE principle. However, the estimation of cost and/ or time functions, selection of appropriate forms, and calibration of parameters present significant challenges in this context. UE solutions may often involve congested links exceeding their designated capacities, thus necessitating the imposition of capacity restrictions on link flows to enhance traffic distribution. While the Frank-Wolfe algorithm has proven effective in addressing the Traffic Assignment Problem (TAP), incorporating capacity constraints complicates the model and poses additional challenges. As such, solving the Capacity-constrained Traffic Assignment Problem (CTAP) becomes a computationally demanding and time-consuming endeavor. To address this intricate problem, researchers have explored various methodologies, such as asymptotic link performance functions or penalty-based approaches, as extensively discussed in the literature by scholars [3-9].

Recent research has been exploring the application of neural networks for solving nonlinear optimization problems. Hopfield network, introduced by Tank and Hopfield [10], was proposed for linear optimization problems and since then, neural networks have been increasingly used in areas such as linear optimization and nonlinear optimization, with their results analyzed on various problems [11-19]. These neural networks are dynamic systems that utilize energy functions, which are a combination of the objective function and constraints of the original problem, similar to the Lagrange function. This particular method has not yet been applied to UE traffic assignment problems with capacity constraints. Therefore, this study aims to analyze a nonlinear network traffic assignment problem with capacity constraints, using the Lagrange neural network method. This will examine the effects of capacity limits on connection flows and the effects of traffic volume changes over time on the travel times of the network. The study begins by introducing the static CTAP model, which is then transformed into a neural dynamic system using the Lagrange function of the static CTAP model to examine the effect of capacity on connection flows and the development of traffic over time. The points obtained from this system satisfy the Karush-Kuhn-Tucker conditions, which is the user optimality condition, and therefore an UE solution is obtained in the static CTAP model.

This research involves utilizing the fourth-order Runge-Kutta method to conduct numerical simulations to solve the neural network model with specific initial conditions. Through observing the changes over time in the system and Lagrange neurons shown on graphs, the impact of traffic formation in connection flows is observed, particularly the volume formed in the network, on user travel time. The findings are then compared to those in previous literature [20] to demonstrate the precision and validity of the proposed solution process in this study. The findings of this paper contribute to the understanding of network traffic behavior and provide a valuable framework for addressing CTAPs. The utilization of neural networks and dynamic modeling techniques offers new perspectives for analyzing and optimizing traffic flow in real-world transportation systems.

The paper's structure is organized as follows: Section 2 details the optimization problem for CTAP. Section 3 outlines the Lagrange neural network utilized to optimize CTAP. Section 4 provides a numerical example, while Section 5 features a discussion. Lastly, Section 6 concludes with remarks concerning the results.

2. PROBLEM FORMULATION

In transportation network analysis, the traffic network can be represented as $G(N,A)$, where N represents the set of nodes and A denotes the set of connections. The sources and destinations of the network are denoted as R and S, respectively. To incorporate user equilibrium traffic assignment with link capacity constraints, a nonlinear programming problem can be formulated:

$$\min z(h) = \sum_a \int_0^{f_a} c_a(x), dx$$

$$s.t. \qquad (1)$$

$$\begin{cases} \sum_k h_k^{rs} = q_{rs} & \forall r \in R, s \in S \\ h_k^{rs} \geq 0 & \forall k \in K_{rs}, r \in R, s \in S \\ f_a = \sum_r \sum_s \sum_k h_k^{rs} \delta_{a,k}^{rs} & \forall a \in A, k \in K_{rs}, r \in R, s \in S \\ f_a \leq C_a & \forall a \in A \end{cases}$$

where z represents the objective function; f_a represents the total flow on link a; c_a represents a separable, piecewise linear link cost function; q_{rs} represents the total traffic demand between r and s; h_k^{rs} represents the flow on chain(route) k between r and s; . K_{rs} represents the set of chains between r and s; $\delta_{a,k}^{rs}$ represents the link-chain incidence matrix; and C_a represents the link capacity on link a.

The problem at hand is non-linear and comprises of both equality and inequality constraints. However, it must be transformed into a problem that solely consists of equality constraints.

$$\min z(h) = \sum_a \int_0^{f_a} c_a(x), dx \qquad (2)$$

$$s.t.$$

$$\begin{cases} q_{rs} - \sum_k h_k^{rs} = 0 & \forall r \in R, s \in S \\ C_a - \left(f_a + (\rho_a)^2\right) = 0 & \forall a \in A \\ h_k^{rs} \geq 0 & \forall k \in K_{rs}, r \in R, s \in S \\ \rho_a \geq 0 & \forall k \in K_{rs}, r \in R, s \in S \end{cases} \qquad (3)$$

where ρ_a represents the slack variables for capasited constraints. In order to maintain simplicity, $\rho_a{}^2$ is chosen as the representation, although other differentiable positive functions of ρ a could be employed, provided they possess the necessary dynamic range. $f_a = \sum_r \sum_s \sum_k h_k^{rs} \delta_{a,k}^{rs} \quad \forall a \in A, k \in K_{rs}, r \in R, s \in S$ is the predefined link flows.

The Lagrange function for this problem can be expressed as follows:

$$\begin{aligned} L\,(h,\rho,\lambda,\nu) &= \sum_a \int_0^{f_a} c_a(x)dx \\ &+ \sum_{r\in R}\sum_{s\in S} \lambda_{rs}\left(q_{rs} - \sum_{k\in K_{rs}} h_k^{rs}\right) \\ &+ \sum_a \nu_a \left(C_a - \left(\sum_{r\in R}\sum_{s\in S}\sum_{k\in K_{rs}} h_k^{rs}\delta_{a,k}^{rs} + (\rho_a)^2\right)\right) \end{aligned} \qquad (4)$$

where L represents the Lagrange function. Additionally, h refers to the chain flow variables, while ρ represents the slack variables for capasited constraints. The Lagrange multiplier associated with the capacity and flow conservation constraints are denoted by λ and ν, respectively. The Lagrange function includes the objective function, flow conservation constraints, flow-variable definition constraints, and capacity constraints, which allows us to determine the best solution by minimizing this function.

Definition 1: A Kuhn-Tucker point is defined as a point $(h^*,\rho^*,\lambda^*,\nu^*)$ that meets specific conditions.

$$\frac{\partial L(h^*,\rho^*,\lambda^*,\nu^*)}{\partial h_k^{rs}} = \frac{\partial z(h^*)}{\partial h_k^{rs}} + \frac{\partial\left(\sum_{r\in R}\sum_{s\in S}\lambda_{rs}^*\left(q_{rs} - \sum_{k\in K_{rs}} h_k^{rs*}\right)\right)}{\partial h_k^{rs}} - \frac{\partial\left(\sum_a \nu_a^*\left(C_a - \left(\sum_{r\in R}\sum_{s\in S}\sum_{k\in K_{rs}} h_k^{rs*}\delta_{a,k}^{rs} + (\rho_a^*)^2\right)\right)\right)}{\partial h_k^{rs}} = 0 \qquad (5)$$

Primal Feasibility:

$$\sum_k h_k^{rs*} = q_{rs} \quad \forall r \in R, s \in S,$$
$$\left(C_a - \left(\sum_r \sum_s \sum_k h_k^{rs*} \delta_{a,k}^{rs} + \rho_a^* \right) \right) \leq 0, \quad \forall a \in A, k \in K_{rs} \quad (6)$$
$$h_k^{rs*} ..0 \quad \forall k \in K_{rs}, r \in R, s \in S, \ \rho_a \geq 0, \forall a \in A$$

Dual Feasibility:

$$\lambda_{rs}^* ..0 \quad \forall r \in \mathrm{R}, s \in \mathrm{S} \text{ and } \nu_p^* ..0 \quad \forall a \in A. \qquad (7)$$

Complementary Slackness:

$$\lambda_{rs}^* \left(q_{rs} - \sum_{k \in K_{rs}} h_k^{rs*} \right) = 0, \quad \forall r \in R, s \in S;$$
$$\nu_a^* \left(C_a - \left(\sum_r \sum_s \sum_k h_k^{rs*} \delta_{a,k}^{rs} + \rho_a^* \right) = \right) = 0, \ \forall a \in A \qquad (8)$$

The presence of certain conditions signifies the activation of the corresponding constraint, which is denoted by positive values of λ_{rs} and ν_a. The gradient of the Lagrange function with respect to variables h_k^{rs}, ρ_a, λ_{rs}, and ν_a can be expressed as follows:

$$\frac{\partial L(h,\rho,\lambda,\nu)}{\partial h_k^{rs}} = \left(c_a(f_a) \delta_{a,k}^{rs} - \sum_{r \in R} \sum_{s \in S} \lambda_{rs} + \sum_a \nu_a \delta_{a,k}^{rs} \right) = 0 \qquad (9)$$

$$\frac{\partial L(h,\rho,\lambda,\nu)}{\partial \rho_a} = 2\nu_a \rho_a = 0 \qquad (10)$$

$$\frac{\partial L(h,\rho,\lambda,\nu)}{\partial \lambda_{rs}} = q_{rs} - \sum_{k \in K_{rs}} h_k^{rs} = 0 \qquad (11)$$

$$\frac{\partial L(h,\rho,\lambda,\nu)}{\partial \nu_a} = C_a - \left(\sum_{r \in R} \sum_{s \in S} \sum_{k \in K_{rs}} h_k^{rs} \delta_{a,k}^{rs} + (\rho_a)^2 \right) = 0 \qquad (12)$$

From equation (9), a generalized route travel cost is obtained as:

$$\lambda_{rs} = \left(c_a(f_a) + \sum_a \nu_a \right) \delta_{a,k}^{rs} \qquad (13)$$

where $c_a(f_a)\delta_{a,k}^{rs}$ denotes the cost on link a.

A capacitated user equilibrium flow can be determined the following conditions:

$$h_k^{rs} > 0 \to \left(c_a(f_a) + \sum_a \nu_a \right) \delta_{a,k}^{rs} = \lambda_{rs} \qquad (14)$$

$$h_k^{rs} = 0 \to \left(c_a(f_a) + \sum_a \nu_a \right) \delta_{a,k}^{rs} \geq \lambda_{rs} \qquad (15)$$

$f_a = \sum_r \sum_s \sum_k h_k^{rs} \delta_{a,k}^{rs} \quad \forall a \in A, k \in K_{rs}, r \in R, s \in S$ is the predefined link flows.

We can also observe from the complementary slackness conditions (8) that

$$\text{if } \nu > 0 \to C_a = f_a \qquad (16)$$

or

$$\text{if } \nu = 0 \to f_a < C_a. \qquad (17)$$

The positive values of Lagrange multipliers, denoted by ν_a, are reasonably associated with delays caused by capacity limitations when link flows become saturated. Empirical observations indicate that the total travel time on a road segment typically comprises two distinct components: the travel time itself and the waiting time at the exit point when it reaches capacity. Lagrange multipliers can be interpreted as the waiting times experienced by a vehicle on the link during equilibrium conditions, reflecting the delay experienced by a vehicle during the equilibrium state.

3. LAGRANGE NEURAL NETWORKS FOR CTAPs

The primary aim of this research is to develop and educate a neural network with the ability to attain a state of equilibrium. This state of equilibrium is indicative of a fixed point of the Lagrange function (4), which implies that the dynamic behavior of the neural network is governed by the gradient of this function. By computing the gradient of the Lagrange function, we can construct the following neural network model to resolve problem (1):

$$\begin{cases} \dfrac{\partial h}{\partial t} = -\nabla_h L\ (h,\rho,\lambda,\nu) \\ \dfrac{\partial \rho}{\partial t} = -\nabla_\rho L\ (h,\rho,\lambda,\nu) \\ \dfrac{\partial \lambda}{\partial t} = \nabla_\lambda L\ (h,\rho,\lambda,\nu) \\ \dfrac{\partial \nu}{\partial t} = \nabla_\nu L\ (h,\rho,\lambda,\nu) \end{cases} \qquad (18)$$

If the network is physically stable, the equilibrium point $(h^*, \rho^*, \lambda^*, \nu^*)$ described by $\frac{\partial h}{\partial t} = 0$, $\frac{\partial \rho}{\partial t} = 0$, $\frac{\partial \lambda}{\partial t} = 0$, and $\frac{\partial \nu}{\partial t} = 0$ at $(h^*, \rho^*, \lambda^*, \nu^*)$ obviously meets (9) and (12) and thus provides a Lagrange solution to traffic assignment problem (1). In component form, we can express as follows:

$$\begin{cases} \frac{\partial h_k^{rs}}{\partial t} = -\left(c_a(f_a)\delta_{a,k}^{rs} - \sum_{r\in R}\sum_{s\in S}\lambda_{rs} + \sum_a \nu_a \delta_{a,k}^{rs} \right) \\ \frac{\partial \rho}{\partial t} = -\left(-2\nu_a \rho_a\right) \\ \frac{\partial \lambda}{\partial t} = q_{rs} - \sum_{k\in K_{rs}} h_k^{rs} \\ \frac{\partial \nu}{\partial t} = C_a - \left(\sum_{r\in R}\sum_{s\in S}\sum_{k\in K_{rs}} h_k^{rs}\delta_{a,k}^{rs} + \left(\rho_a\right)^2 \right) \end{cases} \tag{19}$$

where h and p acquire a physical interpretation as representations of neuronal activity.

The aim is to identify the lowest value of problem (1), thereby giving rise to the creation of the Lagrange dual of problem (4). The objective is to optimize the Lagrange multipliers while concurrently minimizing the decision variables h and p. This indicates a decrease in the system variables as time progresses alongside a corresponding increase in the Lagrange multipliers.

Theorem 1: Consider the stationary point $\left(h^*, \rho^*, \lambda^*, \nu^*\right)$ of problem (4). Assuming that $\nabla_h^2 L(h,\rho,\lambda,\nu) > 0$ and $\left(h^*, \rho^*\right)$ is a regular point of problem (2), it follows that $\left(h^*, \rho^*, \lambda^*, \nu^*\right)$ serves as an asymptotically stable point within the neural network.

Proof: Applying the principles of nonlinear dynamic system theory, we proceed by linearizing equation (18) around the equilibrium point $\left(h^*, \rho^*, \lambda^*, \nu^*\right)$. The local properties of the equilibrium are determined by analyzing the behavior of the linearized system.

To establish the local asymptotic stability of the system, we linearize equation (18) by performing a Taylor series expansion around the equilibrium point $\left(h^*, \rho^*, \lambda^*, \nu^*\right)$. The linearized system represents the behavior of the equilibrium point in the vicinity of $\left(h^*, \rho^*, \lambda^*, \nu^*\right)$.

The linearized system takes the following form:

$$\begin{cases} \frac{\partial h}{\partial t} = -\nabla_h L \mid_{(h^*,\rho^*,\lambda^*,\nu^*)} (h - h^*) \\ \frac{\partial \rho}{\partial t} = -\nabla_\rho L \mid_{(h^*,\rho^*,\lambda^*,\nu^*)} (\rho - \rho^*) \\ \frac{\partial \lambda}{\partial t} = \nabla_\lambda L \mid_{(h^*,\rho^*,\lambda^*,\nu^*)} (\lambda - \lambda^*) \\ \frac{\partial \nu}{\partial t} = \nabla_\nu L \mid_{(h^*,\rho^*,\lambda^*,\nu^*)} (\nu - \nu^*) \end{cases} \tag{20}$$

The local asymptotic stability of the equilibrium point is determined by analyzing eigenvalues of the linearized system in the vicinity of point $\left(h^*, \rho^*, \lambda^*, \nu^*\right)$. Specifically, if all eigenvalues of the linearized system have negative real parts, it indicates that the equilibrium point exhibits local asymptotic stability.

When using neural networks for optimization, our main focus is on achieving global stability. It is essential that the network remains globally stable, meaning it avoids oscillations or chaos regardless of the starting point. This ensures that an optimal solution can always be obtained by initializing the network with any value. Lyapunov's method is a highly effective approach for stability analysis, as it involves finding a suitable Lyapunov function.

Definition 2: Let's define the Lyapunov function using the Euclidean norm of absolute values as follows:

$$\begin{aligned} E(h,\rho,\lambda,\nu) = &\frac{1}{2}\left|\nabla_h L(h,\rho,\lambda,\nu)\right|^2 + \frac{1}{2}\left|\nabla_\rho L(h,\rho,\lambda,\nu)\right|^2 \\ &+ \frac{1}{2}\left|\nabla_\lambda L(h,\rho,\lambda,\nu)\right|^2 + \frac{1}{2}\left|\nabla_\nu L(h,\rho,\lambda,\nu)\right|^2 \end{aligned} \tag{21}$$

where $L(h,\rho,\lambda,\nu)$ represents the Lagrange function of the system.

Proof: To prove the stability of the system, we need to show that Lyapunov function $E(h,\rho,\lambda,\nu)$ satisfies the stability conditions.

1. Positive definiteness:

First, let's prove that $E(h,\rho,\lambda,\nu)$ is positive definite. This means that the function is always greater than zero for any non-zero input. Since $E(h,\rho,\lambda,\nu)$ is defined as the sum of the squares of the absolute values of each term, it is clear that each term is positive or zero. Therefore, $E(h,\rho,\lambda,\nu)$ is a positive definite function.

2. Negativity of the derivative:

To demonstrate the negativity of the derivative, the Lyapunov function $E(h,\rho,\lambda,\nu)$ is the time derivative along the trajectories of the system (within the system of differential equations). We need to show that this time derivative is always negative or zero, and can only be zero at equilibrium points. Let's take the time derivative of $E(h,\rho,\lambda,\nu)$:

$$\frac{dE}{dt}=\frac{\partial E}{\partial h}\frac{dh}{dt}+\frac{\partial E}{\partial \rho}\frac{d\rho}{dt}+\frac{\partial E}{\partial \lambda}\frac{d\lambda}{dt}+\frac{\partial E}{\partial \nu}\frac{d\nu}{dt}$$

By substituting expressions for $\frac{dh}{dt}$, $\frac{d\rho}{dt}$, $\frac{d\lambda}{dt}$, and $\frac{d\nu}{dt}$ from system of equations (18), we obtain:

$$\frac{dE}{dt}=-\left|\nabla_h L(h,\rho,\lambda,\nu)\right|^2-\left|\nabla_\rho L(h,\rho,\lambda,\nu)\right|^2 -\left|\nabla_\lambda L(h,\rho,\lambda,\nu)\right|^2-\left|\nabla_\nu L(h,\rho,\lambda,\nu)\right|^2\leq 0$$

The negative sign in front of each term ensures that the time derivative of $E(h,\rho,\lambda,\nu)$ is always negative or zero, indicating stability. The derivative can only be zero at equilibrium points.

Therefore, since the Lyapunov function $E(h,\rho,\lambda,\nu)$ satisfies the necessary conditions for stability, the system is stable.

4. A NUMERICAL EXAMPLE

Let's examine a traffic system that comprises of three central nodes and ten connections, as described in [20]. Figure 1 represents the travel demands between Origin-Destination (O-D) points. The link costs in the problem are given by piecewise polynomials, as follows:

- For $0\leq f_a<5$, the cost of link a is $\frac{1}{5-f_a}$, where a ranges from 1 to 5. - For $6\leq f_a\leq 10$, the cost of link a is 0, where a ranges from 6 to 10.

It's important to note that only the first five links in the network have flow, while the remaining five links have no flow.

Since link costs are rational expressions, each link capacity must be less than five, and a natural capacity has been added to the problem. The O-D pairs are numbered as follows: $O^{(12)}-D^{(12)}$ represents trips from node 1 to node 2, $O^{(2)}-D^{(2)}$ represents trips from node 1 to node 3, $O^{(3)}-D^{(3)}$ represents trips from node 3 to node 1, and $O^{(4)}-D^{(4)}$ represents trips from node 3 to node 2. Therefore, our notation for $O-D$ travel demand is $q_{12}=3;\ q_{13}=6;\ q_{31}=2;\ q_{32}=5.$

There are four commodities with seven chains, and the chain flows are displayed as matrix form in Figure 2. Each row represents a chain flow from left to right, where the first column indicates the starting point of the chain and the last column indicates the destination point.

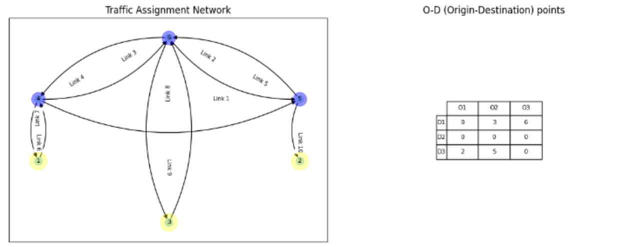

Figure 1. A traffic assignment network with the travel demands between different O-D points.

The conservation equations impose a requirement on the flows within chains, ensuring their adherence.

$$\begin{aligned} &h_1^{(12)}+h_2^{(12)}=3, && h_1^{(13)}+h_2^{(13)}=6,\\ &h_1^{(31)}=2, && h_1^{(32)}+h_2^{(32)}=5,\\ &h_k^{(rs)}..0 \end{aligned} \qquad (22)$$

Additionally, as seen in Figure 2, the link-to-chain flow equations for links 1 to 5 can be expressed by utilizing the link-to-chain flow matrix provided therein.

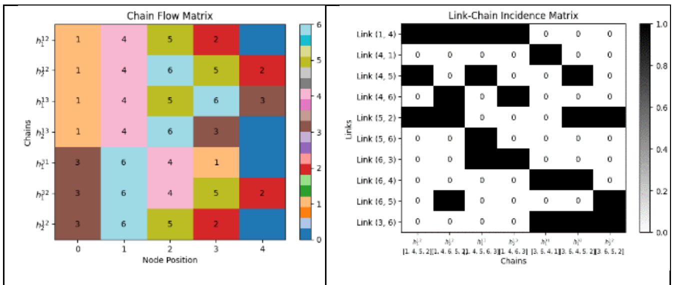

Figure 2. Chain Flow Matrix and Link-Chain Matrix

These equations can be obtained by summing flows of chains that connect relevant nodes, with each column representing the flow of a chain and each row representing the flow of a link between two nodes.

$$\begin{aligned} h_1^{(12)}+h_1^{(13)}+h_1^{(32)} &= f_1,\\ h_1^{(13)} &= f_2,\\ h_2^{(12)}+h_2^{(13)} &= f_3,\\ h_1^{(31)}+h_1^{(32)} &= f_4,\\ h_2^{(12)}+h_2^{(32)} &= f_5. \end{aligned} \qquad (23)$$

Thus, the equivalent problem to problem (1) can be written as follows:

$$\min z(h)=\sum_a \int_0^{f_a}\frac{1}{5-x}dx,\ a=1,2,\ldots,5 \qquad (24)$$

s.t.

Chain Flows Constraints: (25)

$$\begin{cases} h_1^{(12)}+h_2^{(12)}=3, & h_1^{(13)}+h_2^{(13)}=6, \\ h_1^{(31)}=2, & h_1^{(32)}+h_2^{(32)}=5, \\ h_1^{(12)},h_2^{(12)},h_1^{(13)},h_2^{(13)},h_1^{(31)},h_1^{(32)},h_2^{(32)}\geq 0 \end{cases}$$

Capacity Constraints:

$$\begin{cases} h_1^{(12)}+h_1^{(13)}+h_1^{(32)} & \leq 4.999, \\ h_1^{(13)} & \leq 4.999, \\ h_2^{(12)}+h_2^{(13)} & \leq 4.999, \\ h_1^{(31)}+h_1^{(32)} & \leq 4.999, \\ h_2^{(12)}+h_2^{(32)} & \leq 4.999. \end{cases} \quad (26)$$

By introducing additional variables $\rho_a,\ a=1,2,\dots,5$ for the capacity constraints, an equivalent formulation for problems (2) and (3) can be obtained as:

$$\min z(h)=\sum_a \int_0^{f_a} \frac{1}{5-x}dx,\ a=1,2,\dots,5$$

s.t.

Chain Flows Constraints:

$$\begin{cases} h_1^{(12)}+h_2^{(12)}=3, & h_1^{(13)}+h_2^{(13)}=6, \\ h_1^{(31)}=2, & h_1^{(32)}+h_2^{(32)}=5, \\ h_1^{(12)},h_2^{(12)},h_1^{(13)},h_2^{(13)},h_1^{(31)},h_1^{(32)},h_2^{(32)}\geq 0 \end{cases}$$

Capacity Constraints:

$$\begin{cases} h_1^{(12)}+h_1^{(13)}+h_1^{(32)} & +(\rho_1)^2=4.999, \\ h_1^{(13)} & +(\rho_2)^2=4.999, \\ h_2^{(12)}+h_2^{(13)} & +(\rho_3)^2=4.999, \\ h_1^{(31)}+h_1^{(32)} & +(\rho_4)^2=4.999, \\ h_2^{(12)}+h_2^{(32)} & +(\rho_5)^2=4.999. \\ \rho_1,\rho_2,\rho_3,\rho_4,\rho_5\geq 0 \end{cases}$$

As capacity constraints cannot be exact in optimization problems and the capacity constraints in this case are less than 5, they are taken as 4.999. After constructing an equivalent Lagrange function for this problem, the problem reduces to a set of ordinary differential equations, similar to (18), which describe the transient behavior of the neural network. These equations can be solved using the classical fourth-order Runge-Kutta method. All computation and modeling steps were performed using Python 3.11.1 on Jupyter Notebook, executed on a personal computer with the following specifications: CPU: AMD PRO A10-8700B R6, 10 Compute Cores 4C+6G, 1.80 GHz, RAM: 8 GB.

The research findings illustrated in Figures 3 and 4 were obtained through the use of specified initial conditions:

$(1,1,1,1,1,1,1,0.1,0.1,0.1,0.1,$
$0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1)$ for variable h_1^{12}, h_2^{12}, h_1^{13}, h_2^{13}, h_1^{31}, h_1^{32}, h_2^{12}, λ_1, λ_2, λ_3, λ_4, v_1, v_2, v_3, v_4, v_5, ρ_1, ρ_2, ρ_3, ρ_4, ρ_5.

Figure 3 portrays the temporal variations of neurons in the neural network and the corresponding link flows when calculated values are applied to the optimization problem. Additionally, Table I presents the outcomes garnered from solving the neural network. Meanwhile, Figure 4 displays the relationship between link flows and traffic congestion. It is evident that, as traffic congestion increases, there is a noticeable decline in link flows. These outcomes represent the user equilibrium results for the network traffic assignment problem utilizing the Lagrange neural network.

TABLE 1:

USER EQUILIBRIUM RESULTS FOR NETWORK TRAFFIC ASSIGNMENT PROBLEM USING LAGRANGE NEURAL NETWORK.

System Neurons	Lagrange Neurons	Link Flows	Objective Function
$h_1^{12}=3.009537,$	$\lambda_1=0.371703,$	$f_1=4.638500,$	9.086326
$h_2^{12}=1.319719,$	$\lambda_2=2.766433,$	$f_2=1.319719,$	
$h_1^{13}=0.0,$	$\lambda_3=3.038079,$	$f_3=4.680359,$	
$h_2^{13}=4.680359,$	$\lambda_4=3.137963,$	$f_4=2.309238,$	
$h_1^{31}=1.999994,$	$v_1=0.0,$	$f_5=4.690839$	
$h_1^{32}=0.309243,$	$v_2=0.0,$		
$h_2^{12}=4.690839,$	$v_3=0.0,$		
$\rho_1=0.6330851,$	$v_4=0.0,$		
$\rho_2=1.9181470,$	$v_5=0.0,$		
$\rho_3=0.6127112,$			
$\rho_4=1.6400531,$			
$\rho_5=0.6061630$			

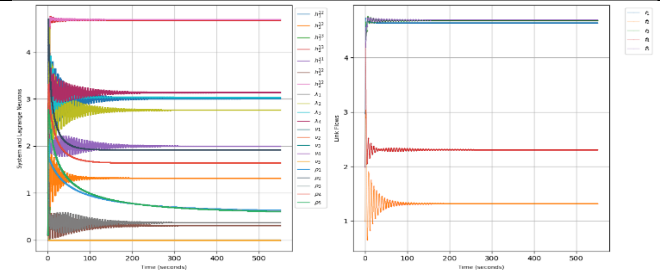

Figure 3. The change in system and Lagrange neurons over time, as well as the variation of link flows over time

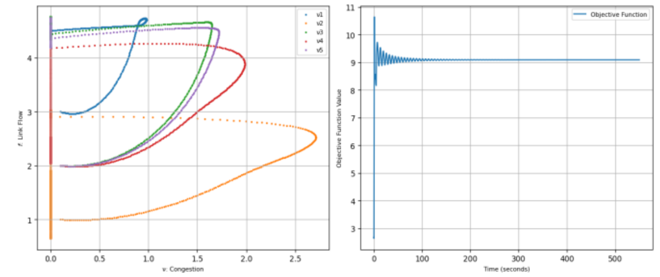

Figure 4. The relationship between link flow and traffic congestion, and tracks the changes in the objective function over time.

5. DISCISSION

The presence of asymptotic stability in a given system indicates that any existing oscillations or disturbances will gradually diminish and converge towards the equilibrium point. This behavior holds true regardless of the initial conditions or perturbations introduced into the system, as they will eventually fade away over an infinite duration, leading to a permanent settlement at the equilibrium point. The convergence of all neurons towards the equilibrium point over time, as demonstrated in Figures 3 and 4, ensures the fulfillment of capacity constraints in the link flows while minimizing the objective function. Furthermore, the accuracy of the obtained results, as presented in Table I, can be validated by comparing them with the outcomes of a previous study employing classical methods, as denoted in literature [20].

6. CONCLUSION

The present study concerns the optimization model of static CTAP and its transformation into a dynamic system to analyze the effects of capacity constraints on connection flows and temporal variations of traffic in the network. Specifically, the gradient of the Lagrange function of the static CTAP optimization problem was transformed into a Lagrangian neural network, which allowed for the derivation of a set of differential equations that capture the dynamic behavior of the network. The numerical solution of the neural network was obtained using the fourth-order Runge-Kutta method, taking into account the initial conditions. By examining the temporal evolution of the system and the Lagrange neurons within the dynamic system, we were able to visualize the changes in traffic volume occurring in the connection flows over time and the traffic occurring on the connections with these changes. We also compared our results with an existing numerical example from the literature and confirmed the accuracy of our proposed Lagrange neural network method.

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