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Research Article

Review of Mathematical Modeling Research: A Descriptive Content Analysis Study

İbrahim ÇETİN [1], Mustafa AYDIN [2], Şerife BİLGİÇ [3]

[1] Necmettin Erbakan University, Department of Mathematics Education, Konya, Turkey, ibrahimcetin44@gmail.com, http://orcid.org/0000-0003-4807-3295

[2] Necmettin Erbakan University, Department of Educational Sciences, Konya Turkey, maydinselcuk@gmail.com, http://orcid.org/0000-0001-8414-0008

[3] Hacettepe University, Department of Special Education, Ankara, Turkey, serifebilgic@hacettepe.edu.tr, http://orcid.org/0000-0001-8476-7134

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Abstract – Recently, mathematical models and modeling practices have become popular in associating mathematics with real-life problems and, hence understanding this relationship. Accordingly, the mathematical modeling skill has been adopted in the standards and by researchers. Understanding the use of mathematical modeling in the learning and teaching processes of mathematics education will contribute to the future of the field. This study aimed to review the trends in mathematical modeling literature using leading research studies. This study reviewed the various types of studies indexed in the Web of Science database between 2000 and 2021 regarding how they addressed modeling. As well as mathematical modeling approaches used, the studies were reviewed in terms of basic characteristics such as publication year, sample, and research method. We evaluated studies using a form developed by the researchers, and the study revealed an increase in the number of studies over the years, and the studies were conducted mostly with pre-service teachers. In addition, we observed that research studies employed mostly small samples to closely monitor the modeling process.

Keywords: mathematical modeling, mathematics, modeling, mathematics education, descriptive content analysis

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Corresponding author: Serife BİLGİC, serifebilgic@hacettepe.edu.tr

Introduction

Mathematical modeling education has attracted ever-increasing interest in the last twenty years. One of the most significant purposes of mathematics education is to cultivate individuals with the ability to apply mathematics in everyday life (Kaiser, 2005) and hence develop students' abilities to solve problems in everyday life (English & Watters, 2005). Besides, mathematical modeling problems develop students' mathematical thinking and problem-solving skills to a greater extent than traditional problems (English & Watters, 2005). The basic components of mathematical modeling are employed within the framework of international research such as PISA, which aims at applying the mathematics taught in schools to everyday life and which centers around mathematics literacy (Organisation for Economic Co-operation and Development [OECD], 2013), has progressively popularized the mathematical modeling approach. The popularization of mathematical modeling in mathematics education has led many countries to include this approach in their curricula from elementary education to higher education (Australia Ministry of Education, 2008; Common Core State Standards for Mathematics, 2011; Ministry of Singapore Education, 2007; National Council of Teachers of Mathematics [NCTM], 2000).

Various pursuits in mathematics education have sought to change the curriculum structure in schools in the mathematical modeling area while inspiring different researchers interested in mathematical modeling. Lack of desired development in students' problem-solving skills (Lesh & Zawojewski, 2007) and recognizing the significance of modeling in problem-solving in terms of coping with real-world situations and a competency-based economy have led to a shift from problem-solving to mathematical modeling in mathematics education (Chan, 2013). Mathematics researchers have turned towards modeling studies because it supports teaching particular concepts in mathematics and developing positive attitudes towards mathematics (Blum & Ferri, 2009). As a result, the number of research studies on mathematical modeling has tended to increase lately, and studies addressing mathematical modeling with respect to different aspects have started to be published in mathematics education (Blum & Ferri, 2009). With advancements in learning approaches and technology, we can come across modeling studies aiming at higher-order cognitive skills supported with technology (Çekmez, 2020; Lingefjärd, 2013; Siller & Greefrath, 2010). Mathematical modeling is also an integral part of the mathematics curriculum in many countries, both at primary and secondary school level, and at the higher education level, where mathematics often functions as a service subject for other disciplines (Durand-Guerrier et al, 2021). Mathematics, especially as a service course, has a critical role in understanding and solving the problems of other disciplines (Çevikbaş et al., 2022). Mathematical modeling, which is employed in various fields such as applied mathematics, Physics, Biology, and Engineering (Damlamian et al., 2013) as well as social fields of application (Ferruzzi &

Almeida, 2013; Laudares & Lachini, 2005), is also used in mathematics education with various aims and approaches (Kaiser et al., 2006; Niss et al., 2007).

Despite the prominent role of mathematical modeling in mathematics education and progressively increasing modeling studies lately, the studies on mathematical modeling are not at the desired level yet (Ferri & Blum, 2013; Stillman et al., 2015). Furthermore, the literature has not reached a consensus on the meaning and use of modeling (Aztekin & Taşpınar-Şener, 2015). However, there is widespread consensus on the critical importance of modeling competencies and the modeling cycle. Despite this, there is no extensive research literature on the specific contributions of short- and long-term mathematical modeling approaches at school and higher education levels to the development of these competencies. This represents a significant research gap in the field and is of critical importance for future scientific investigations (Çevikbaş et al., 2022). Systematizing different approaches and understandings and addressing modeling with a holistic perspective may enable a better understanding of these studies, which have been gaining increasing significance in mathematics education. Therefore, the current study aims at revealing the current state of mathematical modeling studies in mathematics education through descriptive content analysis. Accordingly, research studies in the literature are systematically analyzed in this study, employing the basic concepts related to mathematical modeling, modeling approaches, and modeling types.

Mathematical Modeling

The definitions of mathematical modeling in the literature vary according to the researchers' perspectives and what they attribute to mathematical modeling (Bukova Güzel, 2016). The elaboration of the process and featured characteristics stand out in these definitions (Çavuş Erdem, 2018). Pollak (1979), who is one of those using real-life problems in mathematics educations for the first time, defined mathematical modeling as the interaction of the world outside mathematics with mathematics. Lesh and Doerr (2003) defined mathematical modeling as transforming a real-life problem into a mathematics problem, forming the mathematical models needed to solve the problem, and interpreting the results. They thus emphasized the explanation of real-life with mathematical models. Borromeo-Ferri (2006) considers mathematical modeling a complicated and circular process that includes the transitions between the mathematical world and real life. Galbraith and Clatworthy (1990) figured mathematical modeling as applying mathematics in solving unstructured problems of real-life situations. These definitions suggest that mathematical modeling is a cyclical process in which the real-life and mathematics world are associated. Problems not including needed information for solutions are solved through mathematization with the aid of models, and the results are evaluated and adapted to real life. Individuals need specific competencies to be able to perform this modeling process successfully. Maaß (2006) lists these competencies as the knowledge, skills, ability, willingness to model, and metacognitive skills an individual needs to carry out the mathematical modeling process adequately and in line with the aims. The skill of modeling involves transferring a real-life case to the mathematical world with mathematical operations and hence associating the result with real life; however, modeling competency includes much more than this process. There are several researchers who define this process and various competencies working in the mathematical modeling field in the literature (Berry & Houston, 1995; Blum & Leiβ, 2007; Borromeo-Ferri, 2006; Kapur, 1982; Lesh & Doerr, 2003; Pollak, 1979; Schwarz et al., 2008). The most common and well-known among these is the modeling process introduced by Borromeo-Ferri (2006), and the modeling competencies used in this process are presented in Figure 1.

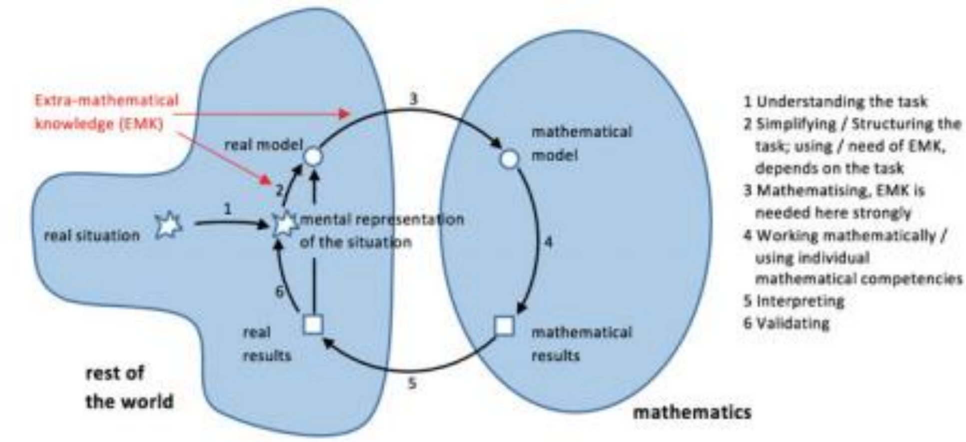

Figure 1 Mathematical Modeling Process (Borromeo-Ferri, 2006)

The mathematical modeling process is illustrated in Figure 1 in a cyclical form involving seven sub-processes. These sub-processes include understanding the task, simplifying and structuring the task (using the extra mathematical knowledge needed for the task), mathematizing, working mathematically (using individual mathematical competencies), interpreting, validating, and presenting. Accordingly, a mathematical modeling cycle starts with a real-life situation or task, and the problem is defined with a non-mathematical language. However, the problem should not be structured, and it should urge students to think. In the second step, students read, imagine, draw and form a table to make sense of the task or problem. Then, students form, examine, associate the needed data, and form hypotheses and assumptions. Thus, they comprehend the associations. At this phase, students perform operations using a mathematical concept and rule to prove the associations, assumptions, and hypotheses with the data at hand. This is the step where the model is math-oriented at the maximum level, and students solve the problem mathematically at the end of this step. The last step is the one where students control the validity of the cycle and decide on its accuracy. If the model formed is mathematically accurate and the process is appropriate, the obtained result is interpreted for real life and then reported. However, if the result is unreasonable and inaccurate, the modeling cycle starts again, and students continue the process until they find an appropriate and accurate result.

The modeling competencies (skills) mentioned above partly share similarities with problem-solving skills. Modeling problems are a particular dimension of problem-solving. According to Galbraith and Catworthy (1990), mathematical modeling is the application of mathematics to unstructured problems in real life. In other words, modeling problems have to do with non-routine real-life problems. Nevertheless, problem-solving involves both routine and non-routine problems. Verbal (routine) problems are not included in mathematical modeling problems. Extra mathematical knowledge regarding ambiguous conditions is needed to solve modeling problems (Schukajlow et al., 2018). Similarly, a number of researchers arguing that routine problems are not adequate to enhance students' problem-solving skills (Blum & Niss, 1991; English & Watters, 2004; Henn, 2007; Lesh & Doerr, 2003) have focused on developing open-ended, non-routine real-life mathematical modeling activities in which there are not fixed instructions to direct students. Research has revealed that mathematical modeling contributes students to gain learning outcomes such as transferring mathematical concepts in real life (Lesh & Doerr, 2003; Lesh & Harel, 2003), having positive attitudes towards mathematics (Blum, 2011; Borromeo Ferri, 2009), and developing metacognitive knowledge and skills (Blum & Ferri, 2009; MaaɃ, 2006) and mathematical reasoning skills (Chamberlin & Moon, 2008; Zawojewski et al., 2003;).

Mathematical Modeling Approaches

Studies on mathematical modeling include different perspectives and theories (Aztekin & Taşpınar-Şener, 2015; Erbaş et al. 2014; Kaiser et al., 2006; Niss et al., 2007). Though these perspectives are not clearly different from each other, it is possible to define the aspects they address. This part includes the opinions of researchers' who discussed various aspects of mathematical modeling.

Kaiser and Sriraman (2006) classified mathematical modeling approaches in six categories, in the most general sense. "Realistic and applied modeling" in this categorization grounds on developing modeling competencies through applications. It aims to develop students' problem-solving and modeling skills. Contextual modeling aims at psychological goals related to the subject. Students are provided with real-life tasks. It is thus assumed that students can attain meaningful learning by experiencing mathematical concepts in appropriate contexts. Educational modeling is one of the most frequently used modeling types. It focuses on structuring learning processes and developing content. Educational modeling can be considered as an intersection of realistic modeling perspective and contextual modeling perspective. This perspective aims to arrange learning environments and processes that are appropriate for mathematical modeling to teach concepts. Socio-critical modeling aims to develop a critical perspective towards the social environment, and it focuses on social and cultural aspects of mathematics. With a socio-critical modeling perspective, mathematics education emphasizes students' gaining critical thinking skills in line with society and social environments. Discussions from simple to complex ones contribute to students' critical thinking. Another perspective is epistemological modeling which is based on a theoretical and philosophical perspective. It gives particular importance to mathematical concepts, the associations between them, and students' interpretations regarding these concepts. According to this perspective, realistic context is of secondary importance in mathematical modeling activities, and each structure that includes mathematics is accepted as a mathematical modeling activity. The last perspective, cognitive modeling, is based on cognitive processes. This perspective is related to analyzing cognitive and metacognitive thinking processes in mathematical modeling (Bukova Güzel, 2016).

The second categorization for mathematical modeling is mathematical modeling as a goal and as a tool. Mathematical modeling as a goal perspective aims at equipping students with modeling skills to solve problems related to real-life situations, and then using and developing these skills while mathematical modeling as a tool perspective targets teaching mathematical concepts in the curricula and using modeling as a tool for this (Galbraith, 2012; Lesh & Doerr, 2003). In addition, mathematical modeling as a goal perspective focuses on modeling competencies and skills, aiming to teach mathematical modeling. On the other hand, in mathematical modeling as a tool, establishing context to teach mathematics in a meaningful way is emphasized. The aim here is to use mathematical modeling to teach mathematics (Aztekin & Taşpınar-Şener, 2015; Kertil et al., 2016). Additionally, mathematical modeling as a goal perspective stresses the process representing forming, developing, and generalizing mathematical structures (Erbaş et al., 2014). In brief, in mathematical modeling as a goal perspective, the focus is modeling competencies, and thus use, and development of modeling skills in daily life problems is emphasized. Therefore, there is a flow from real life to mathematics. In mathematical modeling as a tool perspective, the focus is on teaching a mathematical concept. So, this should be taught in a real-life context, and hence there is a flow from mathematics to real life.

As well as the approaches mentioned above shaping mathematical modeling studies, Berry and Houston (1995) categorized mathematical modeling into four groups: experimental modeling, theoretical modeling, dimensional analysis, and simulation modeling. There are also other modeling approaches that enrich mathematical modeling, such as Realistic Mathematics Education, highlighted by Freudenthal (1991), and Model and Modeling Perspective (MMP), put forth by Lesh and Doerr (2003).

The differentiation observed in modeling studies may be accounted for by the fact that mathematical modeling in classroom environments and instructional processes is not at the desired level across the world (Blum & Borromeo-Ferri, 2009). In the same vein, Aztekin and Taşpınar-Şener (2015) examined the literature and revealed that the studies did not have adequate content and variety, and the studies were mainly carried out with pre-service teachers.

Understanding the modeling processes and cycle can be vital in developing modeling competencies. Therefore, there is a need for systematic review studies that address more variables in this field. It is also known that more research is needed to evaluate measurement tools and approaches to promote mathematical modeling in schools and universities (Schukajlow et al., 2018). Although the studies in modeling literature are limited, carrying out a systematic review of these studies is of critical significance. When looking at the literature, the comprehensive literature review conducted by Kaiser and Brand (2015) can be considered the starting point of systematic review studies on mathematical modeling. However, this review consists of examining modeling studies in the proceedings of conferences such as International Conference on the Teaching of Mathematical Modeling and Applications (ICTMA) and International Congress on Mathematical Education (ICME), rather than reputable databases such as Web of Science (WOS). Çevikbaş et al. (2022) is an important study that compiles research in reputable databases in the field. In this research, mathematical modeling competencies were used as the main keyword and were discussed in terms of different variables. However, in this systematic review, it was determined that mathematical modeling was not discussed as a goal and tool or in terms of mathematical modeling approaches. An examination of review studies on mathematical modeling in the literature suggests that these studies review articles (Albayrak & Çiltaş, 2017; Aztekin & Taşpınar-Şener, 2015) or dissertations (Yenilmez & Yıldız, 2019) published in a particular country. So there are limited and national review studies. The current study has a broader perspective and reviews studies published in other countries and journals indexed in WOS. This study aimed at gathering current studies in the field in a mutual framework, taking into account the themes put forth by studies addressing different aspects of the subject (Aztekin & Taşpınar-Şener, 2015; Erbaş et al., 2014; Kaiser, 1995). This study, which gathers together current literature on mathematical modeling and is expected to guide future research, aimed to review the literature through descriptive content analysis. This study offers a systematic review of the studies regarding mathematical modeling, identified as a result of certain parameters among the studies published in journals indexed in WOS (Social Science Citation Index [SSCI] Journals). The themes created by the researchers are explained in detail in the method section of the study. In line with the purpose of the study, the research questions are as follows:

- How do mathematical modeling studies in the WOS database distribute in terms of characteristics such as publication year, sample level, country, and research method?
- How and with what purposes do the mathematical modeling studies were used in the studies?
- Which mathematical modeling approaches were used commonly in the studies?

Method

Article Selection Process

Researchers define a set of criteria for data collection in systematic review studies. Data quality is of great significance in these research studies (Hwang & Tsai, 2011). Criterion selection strategy is crucial for a better description of the field (Kitchenham et al., 2009). These criteria may include selecting all articles published in a field or limiting the review to indexes such as SSCI, which is thought to have high research quality, and became prominent thanks to indexing articles for a long time (Zhang & Leung, 2014). In the current study, we selected mathematical modeling articles published in journals in the SSCI database. We accessed the SSCI database through the WOS website and performed a search on this page.

By using the advanced search option on the page, the search made with the help of logical expressions [TS= ("math*" AND "education") AND TS= ("modeling" OR "modeling")] has been customized so as not to exclude the studies related to mathematical modeling. The starting point concerning publication year was 2000, and the latest date was 5 January 2021, when the search was performed for the last time. The search resulted in 941 studies. The number of studies decreased to 260 after filtering the search with being indexed in SSCI, open access, and English publication language. In selecting these studies, researchers (experienced in mathematics education) examined the articles elaborately and identified 42 studies that addressed modeling applications in mathematics education. The criteria in this examination were having modeling education content in the study and not including modeling applications in other disciplines (such as physics or biology). The studies on which the researchers could not agree were excluded. Table 1 presents inclusion and exclusion criteria.

Table 1 Article Selection Criteria

Inclusion criteria	Exclusion criteria
• Studies addressing education applications of mathematical modeling. • Articles • Publications in English	• Studies using mathematical models in different disciplines (such as physics or biology) • Editorial studies • Studies focusing on other subjects despite including mathematical modeling

The studies identified in line with inclusion and exclusion criteria are provided in Table 2. Most of the articles were published in high-quality and top-end journals.

Table 2 List of Articles and Journals accessed in WOS Database

Nr	Article	Journal	Publisher	Number of articles	Impact Value	Category
1	Bal & Doğanay (2014); Çiltaş & Işık (2013); Doruk (2012); Eraslan & Kant (2015); Eraslan (2012);); Erbaş et al. (2014); Hidayat et al. (2018);	*Educational Sciences: Theory & Practice*	Codon Publications	7	0.7	Education and education research
2	Jacobs & Durandt (2017); Niss (2017); Shahbari & Peled (2015); Şen Zeytun et al. (2017); Urhan & Dost (2017)	*International Journal of Science and Mathematics Education*	Springer	5	1.578	Mathematics and science education
3	Karalı & Durmuş (2015); Krutikhinaet al. (2018); Tezer & Cumhur (2017); Zapata-Grajales et al. (2018)	*Eurasia Journal of Mathematics, Science and Technology Education*	Modestum	4	0.47	Mathematics, STEM, science and engineering education
4	Paolucci & Wessels (2017)	*Journal of Teacher Education*	Sage	1	3.600	Teacher education and standards
5	Aztekin & Taşpınar-Şener (2015); Şahin & Eraslan (2016)	*Eğitim and Science*	TED	2	0.740	Education and education research

6	Frejd & Bergsten (2016); Hankeln (2020); Shahbari & Tabach, 2020; Wake (2014)	*Educational Studies in Mathematics*	Springer	4	1.500	Mathematics education
7	Dewolf et al. (2015)	*Instructional Science*	Springer	1	1.734	Education and education research
8	Mentzer et al. (2014)	*International Journal of Technology and Design Education*	Springer	1	1.326	Technology and design education
9	Gainsburg (2013)	*Mathematical Thinking and Learning*	Taylor & Francis	1	1.393	Education and education research
10	Kjeldsen & Blomhøj (2013)	*Science & Education*	Springer	1	1.266	Mathematics and science education, education history and research
11	Hickendorff (2013)	*Cognition and Instruction*	Taylor & Francis	1	2.516	Education and education research
12	Doruk & Umay (2011)	*Hacettepe University Journal of Faculty of Education*	Hacettepe University	1	0.18	Education and education research
13	Kim & Kim (2010)	*Asia Pacific Education Review*	Springer	1	0.761	Education and education research
14	Barquero et al. (2018); Chang et al. (2020); Dawn (2018); Frejd & Bergsten (2018); Galleguillos & de Carvalho Borba (2018); Hernandez-Martinez & Vos (2018); Orey & Rosa (2018); Schukajlow et al. (2018); Sevinc & Lesh (2018); Villarreal et al. (2018)	*ZDM – Mathematics Education*	Springer	10	1.256	Mathematics education
15	Cekmez (2020)	*Interactive Learning Environments*	Taylor & Francis	1	1.929	Education and education research
16	Asempapa & Brooks (2020)	*Journal Of Mathematics Teacher Education*	Springer	1	1574	Education and education research

Coding and Data Analysis

The researchers coded all the articles and then analyzed them. To ensure coding reliability, first, randomly selected 15 articles were coded by the researchers independently. The results were close to each other in this coding, and then the researchers continued coding the articles independently. In the next step, the codes were compared, and the researchers evaluated the codes with disagreements together and reached a compromise. After identifying the number of "consensus" and "disagreement" among the codes, reliability was calculated using the reliability formula recommended by Miles and Huberman (1994):

Reliability = Consensus / (Consensus + Disagreement)

Reliability values over 70% indicate reliability (Miles & Huberman, 1994). Reliability was calculated as 92.4% in the current study. The researchers used office programs for coding, and the analysis was recorded using a standard form. The coding procedure was performed in line with research questions. The publication year of the article was determined based on the publication date. The participants were categorized as teachers, pre-service teachers, and students, and the category of students was divided into sub-categories. The samples were also categorized in terms of size. The research designs were grouped as qualitative, quantitative, and mixed designs. In addition, how the modeling was addressed in the studies was also categorized. These categories and their details are provided in the findings section.

Findings and Discussions

Forty-two studies on modeling selected within the scope of the current study were examined in line with the analysis categories. Noteworthy findings in each table are explained below the related tables.

Table 3 Distribution of studies on modeling in terms of sample

Sample	Frequency (f)	Percentage (%)	Articles
Elementary school students	2	4,8	Eraslan & Kant (2015); Şahin & Eraslan (2016)
Lower secondary school students	5	11,9	Doruk (2012); Doruk & Umay (2011); Hickendorff (2013); Shahbari & Peled (2014); Tezer & Cumhur, (2017)
High school students	4	9.5	Chang et al. (2020); Hankeln (2020); Mentzer et al.(2014); Zapata-Grajales et al. (2017)
University students	5	11.9	Dewolf et al. (2013); Gainsburg (2013); Hernandez-Martinez & Vos (2018); Kjeldsen & Blomhøj (2013); Niss (2017)
Pre-service teachers	9	21.4	Bal & Doğanay (2014); Cekmez (2020); English (2017); Çiltaş & Işık (2013); Eraslan (2012); Hidayat er al. (2018); Karalı & Durmuş (2015); Şen Zeytun et al. (2017); Villarreal et al. (2018)
Teachers	5	11.9	Asempapa & Brooks (2020); Barquero et al. (2018); Dawn (2018); Galleguillos & de Carvalho Borba (2018); Sevinc & Lesh (2018)
Others	12	28.6	Aztekin & Taşpınar-Şener (2015); Erbaş et al. (2014); Frejd & Bergsten (2016, 2018); Jacobs & Durandt (2017); Kim & Kim (2010); Krutikhina et al. (2018); Orey & Rosa (2018); Schukajlow et al. (2018); Shahbari & Tabach (2020); Urhan & Dost (2017); Wake (2014);
Total	42	100,0	

It is observed that studies carried out with pre-service teachers are more frequent among studies on modeling in the literature. In addition, "other" studies of literature review, book review, and studies with mixed participants have a similar percentage in the literature. There are few studies carried out with elementary school and high school students.

Table 4 Distribution of studies on modeling in terms of number of participants

Participants	Frequency (f)	Percentage (%)	Articles
Less than 50	24	57.1	Aztekin & Taşpınar-Şener (2015); Bal & Doğanay (2014); Barquero et al. (2018); Cekmez (2020); Çiltaş & Işık (2013); Dewolf et al. (2015); Eraslan (2012); Eraslan & Kant (2015); Frejd & Bergsten (2016, 2018); Gainsburg (2013); Galleguillos & de Carvalho Borba (2018); Hankeln (2020); Hernandez-Martinez & Vos (2018); Karalı & Durmuş (2015); Kjeldsen & Blomhøj (2013); Mentzer et al. (2014); Niss (2017); Sevinc & Lesh (2018); Shahbari & Tabach (2020); Şahin & Eraslan (2016); Şen Zeytun et al. (2017); Villarreal et al. (2018); Zapata-Grajales et al. (2018)
51-99	6	14.3	Dawn (2018); Doruk (2012); Jacobs & Durandt (2017); Kim & Kim (2010); Paolucci & Wessels (2017); Shahbari & Peled (2014)
100+	8	19	Asempapa & Brooks (2020); Chang et al. (2020); Doruk & Umay (2011); Hickendorff (2013); Hidayat et al. (2018); Orey & Rosa (2018); Schukajlow et al. (2018); Tezer & Cumhur (2017)
Other	4	9.5	Erbaş et al. (2014); Krutikhina et al. (2018); Urhan & Dost (2017); Wake (2014)
Total	42	100,0	

More than half of the articles (57.1%) worked with participants that were less than 50. This sample size was followed by participant groups of 100 and over (19%) and between 51 and 99 (14.3%) participants. The ‘other’ category includes studies in which the numbers of participants were not stated, and this group has the least frequency (9.5%).

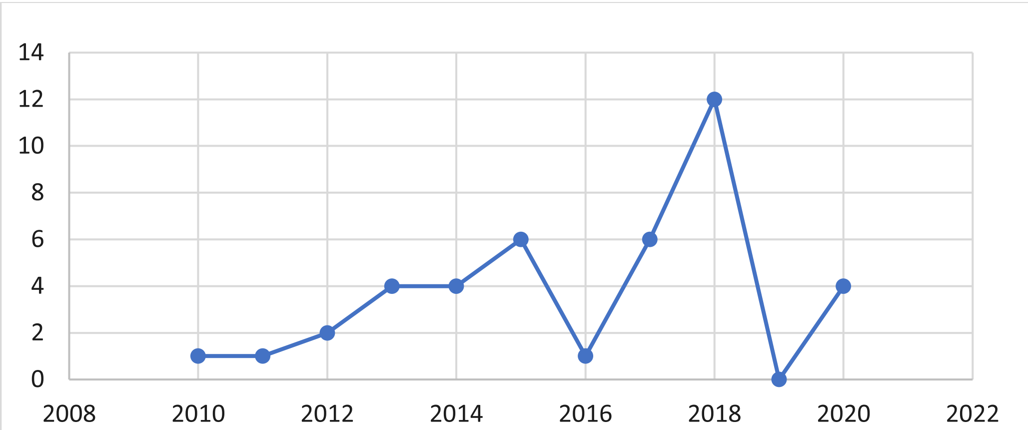

Figure 2 Number of Publications

The number of research studies on mathematical modeling gained pace after 2013. In other words, there was a limited number of studies before 2013, and the number of studies increased rapidly in 2015 and afterwards.

Table 5 Distribution of studies on modeling in terms of research designs

Design	Frequency (f)	Percentage (%)	Articles
Mixed	3	7,1	Frejd & Bergsten (2016); Kim & Kim (2010); Paolucci & Wessels (2017)

Quantitative	9	21,4	Asempapa & Brooks (2020); Chang et al. (2020); Dewolf et al. (2015); Doruk & Umay (2011); Hickendorff (2013); Hidayat et al. (2018); Jacobs & Durandt (2017) Tezer & Cumhur (2017); Wake (2014);
Qualitative	30	71,4	Aztekin & Taşpınar-Şener (2015); Bal & Doğanay (2014); Barquero et al. (2018); Cekmez (2020); Çiltaş & Işık (2013); Dawn (2018); Doruk (2012); Eraslan (2012); Eraslan & Kant (2015); Erbaş et al. (2014); Frejd & Bergsten (2018); Gainsburg (2013); Galleguillos & de Carvalho Borba (2018); Hankeln (2020); Hernandez-Martinez & Vos (2018); Karalı & Durmuş (2015); Kjeldsen & Blomhøj (2013); Krutikhina et al. (2018); Niss (2017); Orey & Rosa (2018); Schukajlow et al. (2018); Shahbari & Peled (2015); Shahbari & Tabach (2020); Sevinc & Lesh (2018); Şahin & Eraslan (2016); Şen Zeytun et al. (2017); Urhan & Dost (2017); Villarreal et al. (2018); Wake (2014); Zapata-Grajales et al. (2018)
Total	42	100,0	

Qualitative research designs predominated the studies on modeling in the literature. In most of these studies, modeling activities were used, and participants' modeling processes were analyzed. On the other hand, quantitative studies constituted 20% of the studies. Studies employing mixed designs were the least. This distribution shows us that the dominant paradigm in mathematical modeling research is the qualitative paradigm.

Table 6 Distribution of studies on modeling in terms of countries

Country	Frequency (f)	Percentage (%)	Articles
The U.S.A.	2	6,9	Fang & Guo (2016); Mentzer et al. (2014)
The U.K.	1	3,4	Wake (2014)
Belgium	1	3,4	Dewolf et al. (2015)
Denmark	2	6,9	Kjeldsen & Blomhøj (2013); Niss (2017)
Indonesia	1	3,4	Hidayat et al. (2018)
South Africa	2	6,9	Jacobs & Durandt (2017); Paolucci & Wessels (2017)
Holland	1	3,4	Hickendorff (2013);
Israel	1	3,4	Shahbari & Peled (2015)
Sweden	1	3,4	Frejd & Bergsten (2016)
California	1	3,4	Gainsburg (2013)
Cyprus	1	3,4	Tezer & Cumhur (2017)
Columbia	1	3,4	Zapata-Grajales et al. (2018)
Korea	1	3,4	Kim & Kim (2010)
Russia	1	3,4	Krutikhina et al. (2018)
Turkey	12	41,4	Aztekin & Taşpınar-Şener (2015); Bal & Doğanay (2014); Çiltaş & Işık (2013); Doruk (2012); Doruk & Umay (2011); Eraslan (2012); Eraslan & Kant (2015); Erbaş et al. (2014); Karalı & Durmuş (2015); Şahin & Eraslan (2016); Şen Zeytun et al. (2017); Urhan & Dost (2017)
Total	42	100,0	

The distribution of the articles in terms of countries demonstrates that the studies conducted in Turkey constituted 41% of all studies in the sample. The studies conducted in the U.S.A., Denmark, and South Africa corresponded to 21% of all studies. Other countries had just one study on mathematical modeling.

Table 7 Distribution of studies on modeling in terms of how modeling is addressed

Examination of Modeling	Frequency (f)	Percentage (%)	Articles
Modeling as a goal	13	31.0	Dawn (2018); Doruk & Umay (2011); Erbaş et al. (2014); Frejd & Bergsten (2016, 2018); Krutikhina et al. (2018); Orey & Rosa (2018); Schukajlow et al. (2018); Sevinc & Lesh (2018); Şahin & Eraslan (2016); Urhan & Dost (2017); Wake (2014); Zapata-Grajales et al. (2018)
Modeling as a tool	29	69.0	Asempapa & Brooks (2020); Aztekin & Taşpınar-Şener (2015); Bal & Doğanay (2014); Barquero et al. (2018); Cekmez (2020); Chang et al. (2020); Çiltaş & Işık (2013); Dewolf et al. (2015); Doruk (2012); Eraslan (2012); Eraslan & Kant (2015); Gainsburg (2013); Galleguillos & de Carvalho Borba (2018); Hankeln (2020); Hernandez-Martinez & Vos (2018); Hickendorff (2013); Hidayat et al. (2018); Jacobs & Durandt (2017); Karalı & Durmuş (2015); Kim & Kim (2010); Kjeldsen & Blomhøj (2013); Mentzer et al. (2014); Niss (2017); Paolucci & Wessels (2017); Shahbari & Peled (2015); Shahbari & Tabach (2020); Şen Zeytun et al. (2017); Tezer & Cumhur (2017); Villarreal et al. (2018)
Total	42	100,0	

The studies were also examined in terms of whether modeling was used as a goal or tool in those studies. In 69% of the studies, modeling was examined as a tool. This demonstrates that the use of modeling is more common in learning-teaching processes. In other words, modeling is used to a greater extent with pedagogical purposes such as conceptual learning and arrangement of learning processes.

Table 8 Distribution of studies on modeling in terms of modeling approaches

Modeling Approach	Frequency (f)	Percentage (%)	Articles
Contextual Modeling	3	7,1	Hickendorff (2013); Paolucci & Wessels (2017); Sevinc & Lesh (2018)
Cognitive Modeling	5	11,9	Eraslan (2012); Eraslan & Kant (2015); Mentzer et al. (2014); Shahbari & Tabach (2020); Şahin & Eraslan (2016)
Educational Modeling	14	33,3	Asempapa & Brooks (2020); Bal & Doğanay (2014); Barquero et al. (2018); Chang et al. (2020); Çiltaş & Işık (2013); Dawn (2018); Doruk (2012); Galleguillos & de Carvalho Borba (2018); Hidayat et al. (2018); Kim & Kim (2010); Orey & Rosa (2018); Şen Zeytun et al. (2017); Tezer & Cumhur (2017); Villarreal et al. (2018)
Epistemological Modeling	9	21,4	Aztekin & Taşpınar-Şener (2015); Erbaş et al. (2014); Frejd & Bergsten (2016, 2018); Kjeldsen & Blomhøj (2013); Krutikhina et al. (2018); Schukajlow et al. (2018); Urhan & Dost (2017); Wake (2014)
Realistic Modeling	11	26,2	Cekmez (2020); Dewolf et al. (2015); Doruk & Umay (2011); Gainsburg (2013); Hankeln (2020); Hernandez-Martinez & Vos (2018); Jacobs & Durandt (2017); Karalı & Durmuş (2015); Niss (2017); Shahbari & Peled (2015); Zapata-Grajales et al. (2018)
Total	42	100,0	

The most frequent modeling approach used in the studies was educational modeling with a percentage of 33%, followed by realistic modeling with 26,2%. 21,4% of the studies used epistemological modeling, 11,9% used cognitive modeling, and 7,1% used contextual modeling.

Conclusions and Suggestions

The studies on mathematical modeling used in mathematics education were reviewed in the current study in terms of publication year, method, participant characteristics, countries, modeling approaches, and intended purpose of modeling use. The study revealed that research studies on mathematical modeling were quite limited at elementary, lower and upper secondary levels, while most of the studies were carried out with pre-service teachers. It is discussed in the literature that mathematical modeling is not adequately covered in elementary mathematics education (Jones et al., 2002), environment-friendly electronic worksheets and modeling activities should be used with student groups at different levels (Rojano, 2015), and, with the technological advancements, opportunities to access mathematical modeling needs to be the primary goal of all mathematics curricula (Amit, 1999; Er-sheng, 1999). Similarly, Çevikbaş et al., (2022) determined the studies on mathematical modeling competencies were mostly conducted with secondary school and high school students, and then with teacher candidates. Research on mathematical modeling is performed with pre-service teachers as opposed to elementary or lower secondary level students due to challenges of bureaucratic procedures (Aztekin & Taşpınar-Şener, 2015). Other reasons for the high number of studies with pre-service teachers may be that they are more accessible for particularly mathematics education faculty, and researchers do their research within the scope of modeling courses they offer to pre-service teachers. Additionally, researchers may believe that mathematical modeling activities with younger age groups and at lower secondary school would not be effective in forming mathematical models at the desired level. Researchers and teachers may find it hard to practice qualified mathematical modeling studies activities with younger age groups because it requires high-level pedagogical mathematics content knowledge (Aztekin & Taşpınar-Şener, 2015). It is vital to support and develop resources for integrating mathematical modeling into early childhood mathematics education (Paolucci & Wessels, 2017). There was only a single study with graduate-level participants, and there were no studies with academic's participants, which is another indicator that mathematical modeling research is performed with a limited sample variety (Albayrak & Çiltaş, 2017). Mathematical modeling studies at the K12 level can be extended and popularized because calculations can now be made with technological elements, and models can be visualized with dynamic software. Thus, the number of research studies in this field will increase. Research at the K12 level has an essential role in identifying the challenges and opportunities students experience in modeling activities, as well as students' mathematical thinking and attitudes towards valuing mathematics. Therefore, an increase of research studies at this level may help us understand mathematical modeling better and allow room for mathematical modeling in education environments at the desired level. Mathematical modeling research from the start of elementary school is important for students to develop their skills of using mathematics in real life (Jones et al., 2002). This argument invites us to research modeling at each schooling level. However, the majority of mathematical modeling research has been carried out with pre-service teachers. This indicates that researchers cannot make good use of schools that are actually areas of application. Due to bureaucratic procedures, researchers may prefer to work with pre-service teachers rather than lower and upper secondary level students. Therefore, schools should make it convenient for universities to research the K12 level. Besides, that mathematical modeling research studies were carried out with mostly pre-service teachers and teachers indicates that the instructional aspect of modeling is featured in the literature. Hence, the learning aspect of modeling should also be addressed with studies examining students' development of modeling skills in real classroom settings at elementary, lower, and upper secondary school levels.

With regard to the sample size of the studies on mathematical modeling, this study revealed that more than half of the studies used samples of less than 50 participants. This result is in parallel with findings on research designs used in mathematical modeling research. Nearly 70% of the studies employed a qualitative design, bringing about fewer numbers of participants. Another reason for lesser participants is the use of the convenience sampling method. Because researchers tend to carry out their research with smaller samples due to time restrictions and ethical procedures. The finding that nearly half of the mathematical modeling studies in Turkey had a sample size of 1-30 (Bayrak & Çiltaş, 2017; Çelik, 2017) lend its support to the current study. Then Çevikbaş et al. (2022) conducted a research and it was determined that the maximum number of samples in mathematical modeling studies was carried out with samples smaller than 50. In addition, this study demonstrated that sample size is directly related to modeling approaches. For instance, a study employing cognitive modeling approach needs to work with fewer participants by its very nature because it addresses challenges students experience in modeling processes (Eraslan, 2012; Shahbari &

Tabach, 2020; Şahin & Eraslan, 2016). However, experimental or correlational survey methods are used in modeling studies employing educational modeling approach, resulting in higher numbers of participants (Hidayat et al., 2018; Kim & Kim, 2010; Tezer & Cumhur, 2017). These findings manifest that sample size is directly affected by the design of the research study, and more importantly, the modeling approach adopted in the study.

The examination of the studies on modeling literature in terms of publication year showed that these studies gained pace after 2003. Similarly, Çevikbaş et al. (2022) found that mathematical modeling research started in 2003. A reason for the increase in the number of studies by year may be related to the fact that several countries (NCTM, 2000; Ministry of Education Singapore, 2007; CCSM, 2011) addressed mathematical modeling as a basic competency in their curricula. Besides, that the aims of mathematics education can be achieved through mathematical modeling has contributed to the increase in research studies in this field (Gürbüz & Doğan, 2019). Another reason urging mathematics educators to research mathematical modeling is anxiety stemming from the inefficiency of traditional methods and problem-solving activities in ensuring the use of students' mathematical knowledge and thinking skills in real life (Mousoulides et al, 2008). The inadequacy of routine problems in developing problem-solving skills (Blum & Niss, 1991; English & Watters, 2004; Henn, 2007; Lesh & Doerr, 2003) may have led researchers to focus on developing open-ended, non-routine, and real-life related mathematical modeling activities (Çekmez, 2020; Hickendorff, 2013). Besides, the results of international comparative examinations such as TIMSS, PISA, and PIRLS, aiming to measure the extent to which students use the knowledge they learned at school in their daily lives, are discussed by people today, and countries have started revising their education system in line with these exams, which has increased the interest in mathematical modeling research the focus of which is real-life problems. There is not an equal distribution in the number of studies in terms of year. There are no studies in some years, while there are a number of studies in some years. However, with the influence of ICTMA conferences, a steady progress has been detected in the number of mathematical modeling studies over time (Çevikbaş et al., 2022). The continuity of research is critical for the quality of mathematical modeling and reaching different samples. Therefore, more elaborate and subsequent research studies are needed.

We also examined the studies on modeling in terms of research design types and revealed that qualitative studies predominated. In some studies within the sample (Eraslan & Kant, 2015; Şahin & Eraslan, 2016; Şen Zeytun et al., 2017), modeling activities were used to figure out participants' modeling processes. Researchers may have preferred qualitative research design because they offer researchers opportunities to thoroughly examine and interpret modeling processes and modeling skills in a contextual problem. That nearly half of the research studies in mathematics education employ qualitative design (Hart et al., 2009) also supports this finding. Additionally, other reasons may be that researchers do not want to go beyond classical methods in mathematical modeling studies, and they prefer qualitative research due to their deficiencies in self-efficacy towards using new methods in application although they may have adequate theoretical and practical knowledge to use those methods (Çelik, 2017). There are few studies addressing modeling with a quantitative perspective. There are quantitative studies that revealed that metacognition affected mathematical modeling positively (Hidayat et al., 2018), students were more successful with the education practiced with mathematical modeling method than 5E model with regard to achievement in mathematics and problem-solving skills (Tezer & Cumhur, 2017), and a scale development study aiming to identify mathematics teachers' attitudes towards mathematical modeling applications (Asempapa & Brooks, 2020). However, these studies have a low percentage among the studies in the literature. On the other hand, Çevikbaş et al. (2022) also found that mostly quantitative studies were carried out, followed by qualitative studies. The emergence of such a result can be interpreted as the fact that the studies examined in the studies are based on different years or the search terms used in the research are different. Therefore, researchers can conduct comparative, causal correlational studies and studies that employ regression and structural equation modeling. Besides, mixed-method studies also have a low percentage among the studies (Frejd & Bergsten, 2016; Kim & Kim, 2010; Paolucci & Wessels, 2017). So, researchers should also carry out research in mixed-method designs.

41% of all research studies on mathematical modeling belonged to Turkey. The studies conducted in the U.S.A., Denmark, and South Africa corresponded to 21%. Other countries had just one study on mathematical modeling. Turkey's high percentage may be because curricula in Turkey clearly include mathematical modeling (Ministry of National Education, 2013; TTKB; 2013), and using mathematical modeling in the instructional processes is advised. The substantial increase in the number of studies on mathematical modeling in Turkey started in 2013 and the following years (Bayrak & Çiltaş, 2017; Çelik, 2017). This suggests that revisions in curricula, achievement or failure in international examinations, and changes in examination systems in countries may have led researchers to study mathematical modeling. On the other hand, curricula emphasized mathematical modeling as a basic competency in some countries such as Singapore (Ministry of Education Singapore, 2007) and Germany (The New German Educational Standards and Curricula, 2006). Yet, the countries are not among the countries included in our sample, indicating that changes in curricula do not affect countries' research trends in the same way. An examination of mathematical modeling studies carried out in Turkey, where most studies are published, shows that despite the increasing number of studies, there are not adequate studies addressing different levels, different components, and different methodologies. Therefore, researchers are advised to integrate mathematical modeling with other mathematical skills such as problem-solving, reasoning, and making associations. However, Çevikbaş et al., (2022) when the geographical regions of the studies were examined, it was determined that these studies were mostly conducted in Europe. While in our research, we almost did not find any research conducted in Germany, an interesting result was found by Çevikbaş et al. (2022) study found that researchers of German origin published more. This can be interpreted as the classification criteria related to mathematical modeling reflecting the research culture of the countries to some extent (Çevikbaş et al., 2022). Therefore, the classification criteria reveal such a result. For example, while mathematical modeling and mathematics education were used as the search terms in our research, the terms modeling competencies and mathematics were used in Çevikbaş et al., (2022)'s research.

Whether modeling was used as a goal or as a tool in modeling studies was also examined within the current study. Modeling was used as a tool in about 70% of the studies, which can be interpreted as that modeling studies paid regard to pedagogical purposes. In other words, studies prioritized forming contexts to teach mathematics in a more meaningful way (Kertil et al., 2016) rather than measuring modeling processes, modeling skills, and competencies. With the increasing use of technology in mathematical modeling activities, studies using modeling as a goal are expected to increase. Very few mathematical modeling studies examined within this study which adopted technology (Orey & Rosa, 2018) used modeling as a goal. The number of studies using modeling as a goal is expected to increase thanks to the increase in environments where students use technology for calculation in mathematical modeling education, they simulate the model in dynamic mathematics and geometry software, and they develop modeling competencies such as forming assumptions for the model, forming models, testing the assumptions and validating. However, in just a single study, pre-service teachers tried to solve a real-life problem in an interactive learning environment using dynamic mathematics software (Çekmez, 2020). Considering the contribution of technology to the development of mathematical modeling skills, mathematical modeling studies supported with technology employing digital materials would enrich the field.

The predominance of mathematical modeling as a tool approach within the research studies in the sample may be related to the fact that concept teaching is still maintaining its traditional place in the field. Particularly the approach of solving real-life problems for reinforcement after concept teaching (Doruk & Umay, 2011) led researchers to use mathematical modeling as a tool. Nearly half of the studies within the scope of the current study were carried out in a classroom setting, which may have also led to this result. Because studies that were not carried out in the classrooms of teachers who do not have adequate knowledge on modeling and modeling competency mostly used modeling as a tool. Teachers still regard mathematical modeling as the use of concrete materials and mathematical models as concrete objects (Gürbüz & Doğan, 2017). Modeling as a tool perspective is more common in studies in which researchers worked with teachers or pre-service teachers (Orey & Rosa, 2018; Dawn, 2018; Sevinç & Lesh, 2018). Considering that teaching modeling skills and strategies will increase in mathematical modeling education, more studies that aim to teach and develop students' mathematical modeling skills (Aztekin & Taşpınar-Şener, 2015) are needed. In addition, modeling as a goal perspective is used more with cognitive modeling approach (Şahin & Eraslan, 2016) and epistemological modeling approach (Frejd & Bergsten, 2016; Krutikhina et al., 2018; Urhan & Dost, 2017), and there were few studies adopting these approaches, which is in line with this finding. Studies adopting mathematical modeling as a goal should increase more because the increase in modeling skills and strategies will contribute to the development of students' modeling skills. Hence, studies towards the development of skills/competencies in the modeling process may contribute to better planning of this process and structuring the process. Further studies using mathematical modeling with approaches such as STEM grounding on interdisciplinary and multidisciplinary studies will help associate mathematics with different disciplines.

Although it was not possible to discriminate between modeling approaches used in the studies clearly, the studies that researchers agreed on were reported. The most challenging part for the researcher was because in most of the studies, how the activities of forming models were implemented was not explained sufficiently (Aztekin & Taşpınar-Şener, 2015). The most frequently used modeling approaches were educational modeling, realistic modeling, and epistemological modeling, respectively. Cognitive modeling and contextual modeling followed these approaches, respectively. That the predominant modeling approach in the studies was educational modeling led us to argue that studies in which learning processes are developed through modeling and particularly concept development is supported this way have gained a place in the modeling literature. In other words, the purposes of mathematics education, its instruction, and learning outputs are prioritized in the studies. In the studies adopting educational modeling approach, pedagogical purposes such as establishing pedagogical strategies (Galleguillos & de Carvalho Borba, 2018), designing and evaluating mathematical modeling activities (Dawn, 2018), and developing competencies of solving mathematical modeling problems (Chang et al., 2020) were featured. That the ratio of realistic modeling approaches that deal with the ability to use mathematics in solving real-life problems is in the second place among the examined studies can be interpreted as the studies on understanding the nature of modeling and improving modeling skills have begun to take place in the literature. That recently popularized issues such as STEM have been addressed in the few studies on the application of mathematics in different fields (engineering, astronomy, or physics) may have to do with the realistic modeling approach. It is noteworthy that cognitive modeling is used in few studies. A reason for this may be that these studies are primarily carried out with pre-service teachers, and they are limited to undergraduate education (Aztekin & Taşpınar-Şener, 2015). Besides, the sample size was mostly minimal in the studies adopting the cognitive modeling approach (Eraslan, 2012; Shahbari & Tabach, 2020; Şahin & Eraslan, 2016). This finding may be due to the fact that the cognitive model is based on the cognitive process, and metacognitive thinking processes are analyzed. For example, the participants of a study in which fourth-grade students' use of modeling activities and the challenges they experience were examined through cognitive and metacognitive thinking were three lower secondary school students (Eraslan & Kant, 2015). Further studies addressing cognitive modeling practices, or cognitive processes and challenges experienced during the modeling process, are needed for modeling studies to gain momentum. Another result of the current study is that limited studies included model forming activities that are considered within the contextual modeling approach (Paolucci & Wessels, 2017; Sevinc & Lesh, 2018). Increased studies on the design of particularly model-forming activities will guide practitioners on how to practice these activities in the classroom. One of the striking results in the study is that there were not any studies based on the socio-critical modeling approach. In particular, it may be suggested that researchers conduct studies on socio-critical modeling, which are based on an approach that can effectively use mathematical modeling to solve problems experienced in the world they live in or in their immediate surroundings.

Compliance with Ethical Standards

Disclosure of potential conflicts of interest

There is no conflict of interests.

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Research involving Human Participants and/or Animals

When reporting studies that involve human participants, authors should include a statement that the studies have been approved by the appropriate institutional and/or national research ethics committee and have been performed in accordance with the ethical standards.

Matematiksel Modelleme Araştırmalarının İncelenmesi: Betimleyici Bir İçerik Analizi Çalışması

Özet:

Matematiğin gerçek yaşamdaki problemlerle ilişkilendirilmesi ve anlamlandırılmasında matematiksel modeller ve modelleme uygulamaları gün geçtikçe daha da benimsenir hale gelmiştir. Bu doğrultuda bir yetkinlik olarak geliştirilen matematiksel modelleme becerisi ise hem standartlar arasında hem de araştırmacılar tarafında yer edinmiştir. Özellikle matematik eğitiminde öğrenme ve öğretme süreçlerinde matematiksel modellemenin kullanımının anlaşılması bu çalışmaların geleceği için katkı sağlayacaktır. Bu araştırmada matematik eğitiminde modelleme çalışmalarına ilişkin eğilimlerin önde gelen çalışmalar üzerinden değerlendirilmesi amaçlanmıştır. Web of Science veri tabanında indekslenen araştırmalarla yürütülen çalışmada 2000-2021 yılları arasındaki farklı türdeki araştırmaların modelleme çalışmalarını hangi yönleriyle ele aldığı incelenmiştir. Bu çalışmalarda kullanılan matematiksel modelleme yaklaşımlarının yanı sıra araştırmalar yıl, örneklem ve kullanılan yöntem gibi temel özellikleri bakımından ele alınmıştır. Araştırmacılar tarafından geliştirilen form üzerinden değerlendirilen çalışmalar yıllara göre bu çalışmalarda beklendiği gibi bir artış meydana geldiğini, öğretmen adayları ile çoğunlukla araştırmaların yürütüldüğünü göstermiştir. Ayrıca araştırmada modelleme sürecinin yakından izlenmesi amacıyla çoğunlukla daha küçük araştırma grupları ile çalışmaların yürütüldüğü görülmüştür. Eğitsel modelleme ve gerçekçi modelleme yaklaşımlarının incelenen araştırmalarda öne çıkan modelleme yaklaşımları olduğu görülmüştür. Araştırma sonuçları, öğrencilerin sorumlu vatandaş olabilmeleri ve toplumsal gelişmelere katılım sağlayabilmeleri için bir yetkinlik olarak görülen modelleme becerisine ilişkin çalışmaların literatürde halen sınırlı bir yer tuttuğunu göstermiştir.

Anahtar kelimeler: matematiksel modelleme, matematik, modelleme, matematik eğitimi, betimsel içerik analizi

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Note. The studies included in the content analysis are marked with asterisks (*)