Bartın University Journal of Faculty of Education
BUEFAD Volume 13, Issue 3
722-734
dergipark.org.tr/buefad
DOI: 10.14686/buefad.1322032

Experiences of Mathematics Teachers of a Test Preparation Center during Professional Development for Incorporating Critical Thinking Skills into Their Practice

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Research Article
Received: 4.7.2023
Revised: 21.1.2024
Accepted: 1.3.2024

Abstract

Higher-order thinking skills are important in conceptual understanding of mathematics, and their development is often emphasized in mathematics curricula. In Turkey, with the new emphasis on higher-order mathematical thinking skills in national high-stakes university entrance exams as well as the curricula, teachers needed to change their instruction to address critical thinking, problem solving, and reasoning both in schools and test-preparation centers. In the Turkish context, teachers in test preparation centers do not usually receive professional development. This study aimed to examine the experiences of 7 mathematics teachers working in a test preparation center during a professional development program for assessment, critical thinking, and higher-order mathematical thinking skills, to understand their perceptions of critical thinking and incorporation of it into instruction. Meetings with teachers were held two days a week, for 12 weeks to work on the assessment and integration of these skills into instruction. Data was collected at the beginning, middle, and end of the implementation, via individual semi-structured interviews and open-ended reflection questions. The findings reflect how participants internalized the relationship between critical thinking and higher-order thinking skills. Results suggest that integrating critical thinking and higher-order thinking skills into an assessment-focused professional development program allowed teachers to experience the critical thinking process and engage in teaching critical thinking, in the socio-cultural context of test preparation centers

Keywords: critical thinking in mathematics education, higher-order thinking skills, mathematics teacher education, professional development.

Bir Sınava Hazırlık Merkezindeki Matematik Öğretmenlerinin Eleştirel Düşünme Becerilerini Uygulamalarına Dahil Etme Konusundaki Mesleki Gelişim Deneyimleri

Öz

Üst düzey düşünme becerileri matematiğin kavramsal olarak anlaşılmasında önemli bir role sahiptir ve bu becerilerin geliştirilmesi matematik müfredatlarında sıklıkla vurgulanmaktadır. Türkiye'de, kritik öneme sahip olan üniversiteye giriş sınavlarında ve müfredatta üst düzey matematiksel düşünme becerilerine yapılan yeni vurguyla birlikte, öğretmenlerin hem okullarda hem de sınava hazırlık merkezlerinde eğitimlerini eleştirel düşünme, problem çözme ve muhakeme becerilerine yer verecek şekilde değiştirmeleri gerekmiştir. Türkiye bağlamında, sınava hazırlık merkezlerinde çalışan öğretmenler genellikle mesleki gelişim programlarına dahil olmamaktadırlar. Bu çalışmanın amacı, bir sınava hazırlık merkezinde çalışan 7 matematik öğretmeninin değerlendirme, eleştirel düşünme ve üst düzey matematiksel düşünme becerilerine yönelik bir mesleki gelişim programı sırasındaki deneyimlerini incelemek, onların eleştirel düşünme ve bunu öğretime dahil etme konusundaki algılarını anlamaktır. Öğretmenlerle 12 hafta boyunca haftada iki gün bu becerilerin değerlendirilmesi ve derslere entegre edilmesi üzerine toplantılar yapılmıştır. Veriler, uygulamanın başında, ortasında ve sonunda bireysel yarı yapılandırılmış görüşmeler ve açık uçlu yansıtma soruları aracılığıyla toplanmıştır. Bulgular, katılımcıların eleştirel düşünme ile üst düzey düşünme becerileri arasındaki ilişkiyi nasıl içselleştirebildiklerini yansıtmaktadır. Sonuçlar, eleştirel düşünme ve üst düzey düşünme becerilerinin değerlendirme odaklı bir mesleki gelişim programına entegre edilmesinin, öğretmenlerin sınav hazırlık merkezlerinin sosyo-kültürel bağlamında, eleştirel düşünme sürecini deneyimlemelerine ve öğretimlerinde de eleştirel düşünmeye yer vermelerine olanak sağladığını göstermektedir.

Anahtar Sözcükler: matematik eğitiminde eleştirel düşünme, üst düzey düşünme becerileri, matematik öğretmen eğitimi, mesleki gelişim

To cite this article in APA Style:

İmamoğlu,Y., Aslan-Tutak,, F. Aşık, G., & Oncar-Ekiz, B. (2024). Experiences of mathematics teachers of a test preparation center during professional development for incorporating critical thinking skills into their practice..*Bartın University Journal of Faculty of Education, 13*(3), 722-734. https://doi.org/10.14686/buefad.1322032

INTRODUCTION

Critical thinking (CT) and problem solving are among the skills related to learning and innovation, which are emphasized in the framework for 21st century learning (Partnership for 21st century learning, 2019). CT has always played an important role in mathematics education too and its development is closely related to higher-order mathematical thinking skills. Along with CT, problem solving, mathematical reasoning and interdisciplinary connections of mathematics with real-life applications are emphasized in the mathematics curricula of many countries. In the Turkish context, even though the importance of the development of such skills is also addressed in the mathematics curricula, until recently this emphasis had not been reflected in the nationwide central high-stakes tests for entering high school or university. Since every individual who intends to attend higher education in Turkey must take the university entrance exam, preparation for these exams is taken very seriously by stakeholders. The current university entrance exam consists of two multiple-choice tests called Basic Proficiency Test (TYT) and Content Proficiency Test (AYT). The aim is to measure verbal and quantitative skills and competencies associated with the application of content knowledge. The latest changes made in curricula and these tests have led to the inclusion of questions embedded in real-life contexts that require reasoning and interpretation rather than just remembering and applying information. With the emphasis on higher-order mathematical thinking skills in national high-stakes university entrance exams, teachers in schools and test preparation centers needed to change their instruction to address these skills. There are various after-school test preparation centers in Turkey that play an important role in the enactment of the curriculum. The teachers in these centers influence the development of students' mathematical skills as much as the classroom teachers but oftentimes they do not receive Professional Development (PD) like school teachers.

There is very limited research on how mathematics teachers of test preparation centers perceive and engage in CT skills. In this regard, this study is carried out in a test-preparation center, affiliated with the local municipality, where students attend lessons to increase their academic achievement. The purpose of this research is to examine the experiences of mathematics teachers in the center participating in a PD program to understand their perception of CT and incorporate CT and higher-order mathematical thinking skills into instruction. The PD was designed to improve the teachers' understanding of higher-order mathematical thinking skills and to enhance their instruction in a way to incorporate CT. Working with a group of high school teachers from a test-preparation center, the focus of the PD was assessment, specifically the construction of mathematical test questions to assess higher-order thinking skills. Participants' experiences during the PD were studied for the following research question:

How did teachers' perception of critical thinking and incorporating critical thinking into instruction change during professional development?

LITERATURE REVIEW

Being able to think critically is an important skill for an individual, and many scholars from different fields have worked on identifying this complex construct. Throughout the past decades, CT had been a focus of interest for philosophical, psychological, and educational traditions of thought (Sternberg, 1986). Various approaches and definitions of CT have emerged and its importance in educational settings have been discussed. According to a study where a panel of experts worked on to define and determine the dimensions of CT, its' role in instruction and how it will be assessed (Facione, 1990), critical thinking is a "purposeful, self-regulatory judgment which results in interpretation, analysis, evaluation, and inference, as well as explanation of the evidential, conceptual, methodological, criteriological, or contextual considerations upon which that judgment is based" (p. 2). Here, CT is considered to contain both cognitive skills and affective dispositions. The cognitive skills involved in CT are "(1) interpretation, (2) analysis, (3) evaluation, (4) inference, (5) explanation and (6) self-regulation" (p. 4). Dispositions that are closely related to these cognitive skills include the following (p. 11):

> Clarity in stating the question or concern, orderliness in working with complexity, diligence in seeking relevant information, reasonableness in selecting and applying criteria, care in focusing attention on the concern at hand, persistence though difficulties are encountered, and precision to the degree permitted by the subject and the circumstance.

Ennis (1996) defined critical thinking as "reasonable reflective thinking focused on deciding what to believe or do" (p. 166), emphasizing "reasonableness, reflection, and the process of making decisions". Ennis (1996) also stated that CT involves some dispositions as well as skills. He identified three broad dispositions as follows (p. 171): Care to "get it right" to the extent possible, represent a position honestly and clearly (theirs as well as others'), and care about the dignity and worth of every person.

Considering the above descriptions and drawing on approaches of CT that were put forward by Glaser (1985) and Halpern (1996), following definition of critical thinking is assumed in this study (Gürkaynak, et.al, 2009, p2):

> Type of thinking that individuals do on purpose and under their own control, where usual approaches and repetition of patterns are prevented; prejudices, assumptions and any information presented is tested, evaluated, judged, and their different aspects, expansions, meanings, and consequences are discussed; ideas are analyzed and evaluated; reasoning, logic and comparison is used; and as a result, certain ideas, theories, or behavior are reached.

CT is closely related to other types of higher-order thinking such as problem solving, decision making and creative thinking (Facione 1990). It differs from these types of thinking, in the sense that it involves judging or assessing the quality of an idea, product or result. Accordingly, critical thinking includes problem solving and it has common processes with creative thinking (Beyer, 1995; Innabi & Sheikh, 2007).

Teaching CT Skills

Halpern (1998) proposed a model for teaching CT which consists of four components: i) dispositional aspect, ii) instruction in CT skills iii) activities designed to facilitate transfer across contexts, and iv) a metacognitive component that is used to direct and assess thinking. For the dispositional aspect, Halpern (1998) suggests keeping in mind that a person may have the ability to think critically but may not have the willingness to use it since it is a process that requires mental effort. Therefore, it is important in instruction to help learners to decide whether a given problem or situation requires using these skills. According to Halpern, the following are among dispositions or attitudes a critical thinker possesses: willingly and persistently engaging in complex tasks, avoiding impulsive activity and acting according to plan, and being flexible and open minded. For the second component, Halpern offered a taxonomy of skills for fostering CT in instruction: verbal reasoning, argument analysis, testing hypothesis, likelihood and uncertainty, and decision making and problem solving. According to her, these five categories of skills are teachable and generalizable so they can be used as a rubric in instruction. For fostering transferring CT across contexts, Halpern suggests using “thoughtful questions” to create meaningful connections between relevant concepts and ads that these questions need to be drawn from the real-world contexts that are frequently encountered. These tasks should include relevant and irrelevant information and require thoughtful analysis and synthesis. In the last component, the importance of meta-cognitive monitoring is emphasized. Some of the questions suggested to be asked are as follows (Halpern 1998, p.454): “How much time and effort are this problem worth?”, “How difficult do you think it will be to solve this problem or reach a conclusion?”, “What CT skills are likely to be useful in solving this problem or analyzing this argument?”, “Are you moving toward a solution?”.

Broadbear (2003) distinguishes CT approach to teaching and learning from educational strategies such as group discussion, lecture, experiments etc., stating that CT should be “infused throughout the educational experience and within these strategies”. Based on the definitions of CT, he proposes that in order to completely address CT, the following should be included in the lessons: ill structured problems, criteria for assessing thinking, student assessment of thinking, and improvement of thinking. Since ill structured problems do not have a single right answer, they enable students to use reasoning and make interpretations and judgements.

Considering that CT requires people to assess the thinking of others as well as of their own, Broadbear (2003) states that a set of criteria should be established in the classroom which can be used to assess thinking. In order to help students to make this assessment, individual feedback on student responses can be used (Snyder & Snyder 2008). Students can be asked to explain the strengths and weaknesses of their thinking and to provide feedback for improvement (Broadbear, 2003). To improve students’ thinking, Broadbear (2003) states that the student work should be revised and resubmitted. After the initial assessment of student work, weak and strong examples can be shared with the students.

Suggestions for teaching CT such as those mentioned above, are general suggestions and not context specific. There has been a debate on whether CT should be taught within a specific subject matter, or separately as a set of general skills. As a result, different approaches to CT instruction have emerged. Ennis (1989) categorized these approaches as follows: general, infusion, immersion, and mixed. In the general instruction approach, CT principles are taught separately from content. The infusion approach also includes explicit CT instruction, but in this case, it is integrated in the content being taught. On the other hand, in the immersed approach, while CT is integrated in the context, it is taught indirectly without explicit explanation to the learners. Finally, in the mixed approach, a separate general instruction for CT skills is given, accompanied with content specific instruction using infusion or immersed approach.

CT in Mathematics Education

As mentioned earlier, CT is closely related to higher-order mathematical skills. The importance of fostering students' higher-order thinking skills is emphasized both in national and international mathematics curricula. Mathematical proficiency is defined as having five interrelated strands: conceptual understanding which requires comprehension of mathematical concepts, operations, and relations; procedural fluency, which is a skill in carrying out procedures flexibly, accurately, efficiently, and appropriately; strategic competence which is the ability to formulate, represent, and solve mathematical problems; adaptive reasoning which is capacity for logical thought, reflection, explanation, and justification; and productive disposition which is seeing mathematics as sensible, useful, and worthwhile in addition to believing in diligence and one's own efficacy (National Research Council, 2001). The following mathematical practices are instrumental in achieving mathematical proficiency: i) Make sense of problems and persevere in solving them ii) reason abstractly and quantitatively, iii) construct viable arguments and critique the reasoning of others iv) model with mathematics v) use appropriate tools strategically vi) attend to precision, vii) look for and make use of structure and viii) look for and express regularity in repeated reasoning (NCTM, 2014). The Common Core Toolkit published by Partnership for 21st century learning (2011) maps out the way mathematical practices emphasized in common core standards are related to skills identified in the framework for 21st century learning. The practices related to CT and problems skills are identified as: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively, modelling with mathematics, and looking for and making use of structure.

As can be seen from these definitions, important practices to attain mathematical proficiency and higher-order mathematical thinking are closely related to CT and problem solving. Since these skills are emphasized in school curricula (e.g. MoNE, 2018), they need to be properly assessed. In a study conducted with prospective mathematics teachers in Turkey (Çelik & Özdemir, 2020), results revealed reasoning, mathematical thinking skills and problem-solving as significant predictors of critical thinking dispositions.

For this study, CT and higher-order mathematical thinking skills were considered while examining the content of the Turkish national high-stake exams. As a result, the most relevant mathematical skills for the needs of teachers at the test prep center were identified as follows: mathematical information literacy, numerical fluency, spatial thinking and mathematical reasoning. These identified skills are considered as the indicators of higher-order mathematical thinking skills. Teachers in the test-preparation centers are expected to both prepare students for the national tests where these skills are measured, and to generate test questions that measure these skills.

Teacher Practices for CT

To be able to teach higher-order thinking skills such as CT, one would expect the teachers to have these skills themselves and consciously aim for incorporating these skills in their practice. In their longitudinal study with teachers, Miri et al. (2007) concluded that persistently teaching for promoting higher-order thinking in a conscious manner is necessary for success. They suggested that PD programs need to be planned so that the teachers have a better understanding of higher-order thinking and can conceptualize CT in a coherent way.

Similarly, Innabi & Sheikh (2007) state that in addition to having a good knowledge of CT, teachers need to know how to teach it and believe that it is possible to teach it to all students. The authors point out the need for programs to train mathematics teachers on how to teach CT to all students, in addition to developing their understanding of critical thinking. Innabi & Sheikh (2007) interviewed mathematics teachers to assess their perception of critical thinking and found that many of them lacked a coherent and comprehensive understanding of critical thinking. Some of the issues they suggest for PD programs are as follows: CT does not necessarily involve "attacking", i.e., dealing with opposite opinions and ideas. People should CT to assess their own acts, thoughts, and feelings. Considering the equity principle, CT in mathematics classes should be the target for all students. Hence the PD programs should consider dealing with diverse students. Using instructional practices that are known to be useful, such as discussion, cooperative learning, self-learning and questioning, usually fall short for teaching CT. Therefore, these practices should be modified to foster CT.

There are few studies conducted in Turkey, focusing on CT skills of mathematics teachers. In one such study, Tunçer and Sapancı (2021) examined the critical thinking dispositions and critical thinking teaching practices of middle-school mathematics teachers. The results showed that there was a significant difference in favor of the group that received in-service training in critical thinking disposition and perceptions of practice.

Another result showed significant positive relationship between mathematics teachers' critical thinking disposition and perceptions of practice, and the metacognition, perseverance and patience and open-mindedness were found to be significant predictors of critical thinking teaching practices.

METHOD

Research Model

The design of this study is based on a qualitative phenomenological approach. According to Ary et al. (2007), this approach is used to “describe and interpret an experience by determining the meaning of the experience as perceived by the people who have participated in it” (p.461). With the phenomenological approach, it is aimed to understand the individual situations and personal experiences of the participating teachers in detail regarding the teaching and learning practice and to best investigate the common points in the behavior of the group (Giorgi & Giorgi, 2003; van Manen, 2023). In this case, teachers’ perceptions of CT and incorporation of CT into instruction during the PD program is the phenomenon being investigated.

Setting and Participants

Increasing demands for higher education have fueled the competition for university entrance, as well as changes in the examination system. The fact that a large part of the exam takers cannot be placed in a program causes students to take the exam a second or third time by having an additional preparation period when they are not enrolled at any high school. Test preparation centers are used to meet the expectations of students who want to improve themselves in this regard. This study is conducted in a test preparation center financed and managed by a local municipality. This educational institution is approved by the Turkish Ministry of National Education and provides free education to students. To provide equal opportunities in education, the institution aims to support the knowledge and skills of students in different disciplines who attend or graduate from school and prepare them for higher education institutions. 21 teachers work in the institution where approximately 700 students participate the educational programs. The test-preparation center offers online and face-to-face courses to students in Science (Physics, Chemistry, Biology), Mathematics, Turkish Literature, Social Sciences (History, Geography). In mathematics lessons, topics in the curricula are reviewed and types of mathematics problems which would prepare students for the national exams are solved. The institution uses textbooks from various publishers as well as the booklets prepared by the teachers. There were eight mathematics teachers working full-time at the institution. The administration of the institution asked all mathematics teachers to attend the PD of this study. This was the first long-term PD for teachers at this institution. One of the teachers could not fully participate in the program activities, so participants of this study were seven.

Demographics information about these seven teachers can be summarized as follows in Table 1. (to ensure anonymity, pseudonyms were used):

Table 1. Demographics of participants

Participant	Age	Teaching Experience (years)	Experience in the institution (years)	Undergraduate Program
Ahmet	29	3	1	Mathematics
Can	29	6	2	Mathematics
Cemre	37	6	1	Mathematics
Defne	34	1	1	Physics
Erdem	30	5	2	Mathematics
Mustafa	26	1	1	Secondary School Mathematics Teaching
Zeki	35	5	5	Mathematical Engineering

As can be seen from Table 1, only one teacher had an undergraduate degree in teaching mathematics. In addition, there is no teacher who has a master's degree in an education-related discipline. This portray can be considered typical for a Turkish test preparation center because undergraduates of mathematics programs cannot be a teacher at a public or private high school without an additional certification. Also there has been a surplus of mathematics teachers. So many young, new graduates of mathematics programs work at different test preparation centers as well as tutoring.

PD Program

A 12-week PD program was implemented, focusing on the development of higher-order thinking skills and teaching skills of teachers. During the program, meetings were held with participating teachers as three-hour sessions, two days a week. The meetings aimed to develop strategies for teachers to integrate CT and higher-order mathematical thinking skills (mathematical information literacy, numerical fluency, spatial thinking and mathematical reasoning) into their lessons and transfer them to students.

The program had three main phases. The first phase started with discussions on higher-order mathematical skills related to the central exams. Then CT was introduced in Phase 2. The last phase focused on discussing mathematics topics, mainly algebra and possible classroom implementations that would develop CT of students. Table 2 depicts these phases with the topic of each and related main activities.

Table 2. Three phases the implementation

Time Period	Content	Activities
Phase 1 (Weeks1-4)	-Introduction of main concepts -Basic principles of Measurement and Evaluation; validity and reliability, item writing, constructing tests -Introduction of higher-order mathematical skills -Writing contextual questions	-Pre-test -Creating a rubric for skills used in question analysis throughout PD. -Analysis of TYT test items (skills) -Item writing exercises (context and skills)
Phase 2 (Weeks 5-7)	-Defining CT -Discussion of CT with higher-order mathematics skills	-Mid-program data collection -Analysis of TYT test items (skills) -Concept mapping on defining CT - Item writing exercises (context and skills)
Phase 3 (Weeks 7-11)	-Discussing mathematical topics (real numbers, function families) with higher-order mathematics skills -Discussing classroom implementations for incorporating CT and higher-order mathematics skills for the topic of discussion	For the math topic of the week: -Analysis of AYT test items - Item writing exercises for each topic (focusing on content)
Week 12	Wrapping up	Post test

As can be seen from Table 2, this PD was designed for participants to first experience CT in their own practice, critically examining math problems, where CT was implicitly addressed. In other words, the immersed approach of teaching CT (Ennis, 1989) was used in the first phase. Since the purpose of this program is not only to develop CT skills of teachers but also to improve their teaching practice, the CT aspect of the program started to be addressed explicitly with Phase 2. So overall, it can be said that the mixed approach (Ennis, 1989) was used in CT instruction during PD. In Phase 2, definition of CT was discussed with teachers along with methods of teaching it. Three main ideas CT were identified: attitude to use problems based on thinking, knowledge of logical questioning, and application skills. Since the main reason for this discussion was to make connection between CT and higher-order mathematical skills, participants were challenged to incorporate "doing mathematics" with the definition of CT. This discussion led the group to consider the relationship between CT and higher-order mathematical skills that had been examined in Phase 1. After this discussion, participants discussed incorporating these skills while working on important algebra topics. For example, the topic of functions was covered in the following manner: first definition of function and big ideas related to the concept were addressed and then the participants discussed how CT can be incorporated in teaching functions. Then they considered ways to assess students' higher-order mathematical skills with questions related to functions.

The worked-sample mathematics problems used in in-service training consist of non-routine problems that require critical and analytical thinking skills. At this stage, problems matching these qualities from university entrance exams (TYT and AYT) and other standardized tests have been selected for study. Subsequently, collaborative work with teachers has been conducted to develop CT skills necessary to give answers to these questions asked in the university entrance exams of the last 5 years. In the second stage, teachers examined all the questions in the university entrance exams of the last five years, and then the analyses made by the teachers were discussed with the researchers. In the final stage, teachers were asked to create non-routine problems that require CT skills, and the problems teachers constructed were examined together with the researchers.

Data Collection and Analysis

In the study, data collection process was carried out in three stages: at the beginning of PD program, at the beginning of Phase 2 (with mid-program questionnaire and interviews), and at the end of PD program. To establish trustworthiness and provide triangulation, different data collection methods were used at each stage (Elo, et al., 2014). In all three stages, participants' written answers were gathered through open ended questions. While focus group interview was carried out in the first stage, individual semi-structured interviews were conducted in the second and last stages. Moreover, one of the researchers kept detailed observation notes during the workshops which were then used as supporting data for analysis. Also, several background variables were collected for each teacher. These included the age of the participants, teaching experience, experience in the institution and the department they graduated from (Creswell & Poth, 2016; Stahl & King, 2020).

Participants' written and interview statements were transcribed and then reviewed by four researchers (first individually and then together) and judged collaboratively with the help of low-inference notes taken. The descriptive codes were applied to the data and categorized to look for patterns. To provide an in-depth description of participants' experiences, the findings were presented by combining both the PD phases and the background variables of the participants. The findings were supported by the quotes captured.

Researchers' role

The research team consists of 4 researchers. Three researchers are academicians with background in mathematics education. The fourth researcher is a graduate student in mathematics education. The three academicians took turns as leaders in different weeks of the implementation, while the other three researchers were sitting among participants, contributing to group discussions and/or taking filed notes. It can be said that during implementation, there was a casual and friendly atmosphere, and the researchers did their best to establish a good rapport with the participants. The three academicians conducted the interviews with the participants in parallel sessions (in different rooms in the institution). Before the interviews, the researchers came together to decide how to approach the participants and who will talk to which participant, in order to make sure that the participants feel as comfortable as possible and share their sincere opinions. At the beginning of the interviews, participants were informed that the contents of the interview will not be shared by anyone, including the administration of the institution.

Research Ethics

Participants were informed about the purposes of the study, and they filled out and signed consent forms to give permission for the data from interviews and open-ended questions, collected as a part of PD, to be used for this study. To ensure anonymity, pseudonyms were used throughout the report. Necessary precautions were taken for data security during all phases of the research.

FINDINGS

To examine participating mathematics teachers' experiences of the PD, the focus group interview, mid-program individual interviews, and end of the program individual interviews were analyzed in addition to participants written answers to pre and post open-ended questions. Since the focus of this research is participants' experiences, the findings section is organized around the following central issues addressed by the participants:

i) Definition of CT in the context of mathematics teaching, ii) implementation of CT in test preparation center mathematics classrooms and iii) experiences on the PD program.

Definition of CT in Mathematics Teaching

The skill of problem solving is one of the basic skills that is considered important in mathematics education, like any educational setting. Mathematics teachers always pay special attention to problem solving, and participants of this study were no exception. At the beginning of the PD program, all the participating teachers stated that they make intentional instructional decisions to develop students' problem-solving skills. During the whole group discussion at the beginning of the program, teachers defined problem solving as a skill to handle issues in life. When specifically asked about mathematical problem solving, they pointed out excelling in problem solving by practicing. Considering that they are working at a test preparation center, the teachers' view of problem solving being limited with solving practice questions/problems in test-books was expected. Instructional practices to improve students' problem-solving skills were also discussed, and one of the teachers, Can, said "I provide why's of the topic, give proofs. For example, proof of square root of 2 [being irrational]". They also acknowledged the complexity of developing problem-solving skills, stating that lots of practice is required and it is connected to other skills such as analytical thinking.

After having teachers' views on problem solving, we asked to define CT. Participants seemed to have very limited understanding of CT at this stage. For example, Erdem defined it as "not to except something as it is but question it". This comment from the participant demonstrates some understanding of CT yet it is very limited. Some participants also stressed questioning. But it's interesting to note that most of the participants defined CT in relation to mathematical problem solving. For example, Can's definition was "A skill to answer different types of questions, eliminating the memorization attitude". So, it can be said that participants had superficial understanding of CT and their discussions were limited to solving math questions/problems.

As stated earlier, in Phase 1 CT was not explicitly addressed. The teachers spent several weeks discussing higher-order mathematical thinking skills and analyzing test items which were intended to measure these skills. It can be said that analyzing the test items this way required them to use their CT skills. So, the teachers had the chance to experience CT, before they were formally introduced to CT. The analysis of the interview data collected at the end of Phase1 revealed that all the participants' answers were still related to solving math problems. While Ahmet provided an unrelated response, answers of other participants were grouped in two perspectives: different aspects, solutions or approaches to the problem (Cemre, Mustafa, Defne) and using reasoning to solve problems (Can, Erdem, Zeki). For example, Cemre explained CT as "being able to use different perspectives, developing interpretation skills, what does the student understand when he/she looks at the problem" while Can defined CT as follows: "It is a type of thinking consisting of cognitive process such as reasoning analysis and evaluation. Thinking differently using basic facts."

After explicitly discussing CT and its relationship with higher-order mathematical thinking skills in Phase 2 and discussing high school algebra concepts with examining how higher-order thinking skills can be developed and assessed in Phase 3, the teachers were asked to define CT again at the end of PD. All the participants' answers were based on solving math problems again, and analyzing test questions, but this time how they approached CT in the context of problem solving differed notably from their initial responses. Six of the participants (Can, Cemre, Defne, Mustafa, Erdem, Zeki) made clear connections between CT and higher-order mathematical skills, especially reasoning. For example, Erdem stated that CT is based on problem solving that brings along abstract and analytical thinking skills while being "automatically loaded" with well-developed mathematical reasoning, procedural fluency, mathematical literacy and spatial reasoning. Some of these participants (Can, Mustafa, Zeki) still described CT as being able to approach solving problems in different ways, but this time they emphasized reasoning supporting CT while talking about exploring different solutions of a problem. Only one participant, Ahmet, limited his definition of CT to solving a problem with different perspectives or aspects.

Findings reported so far portray participants' understanding of CT while they experienced different phases of the PD. First, they experienced CT in their teaching context (analyzing math problems and test questions) without explicit emphasis of CT. As they moved to the second phase, they experienced making sense of CT for mathematics teaching. After the last phase where they examined several algebra concepts with how CT can play a role in teaching those topics, participants' transformed definitions included a web of skills with comprehension of the complex nature of CT. To examine whether this transformation of teachers' CT understanding also transcended into their practice, we asked them about how they would implement CT into their classrooms.

Implementation of CT in Test Preparation Center Mathematics Classroom

To examine participants' understanding of CT from different aspects, we also asked them how they would teach CT in a classroom setting. It should not be forgotten that they have always been test preparation center teachers and even when defining CT, they referred to solving math problems or test questions. During the mid-program interviews, three of the participants (Ahmet, Cemre, Zeki) failed to provide meaningful suggestions for teaching CT. Zeki openly stated that he does not know how while Ahmet mentioned using similar questions to solve in class. Cemre also mentioned using direct instruction to tell students what to do to teach CT. Having given an unrelated response when asked to define CT, Ahmet did not provide a meaningful response to CT instruction that can be expected. While Zeki's definition of CT was based on reasoning, failed to properly discuss teaching CT. This may show that his understanding of CT was not necessarily stemming from CT experiences but from his understanding of the nature of mathematics. Similarly, Cemre defined CT as using different solution methods, but she explained the instruction method directing students. The remaining four participants either stressed using different strategies or approaches to solve math problems (Can, Defne, Erdem) or using sense-making aspect of problem solving to foster CT in a classroom (Can, Erdem, Mustafa). These two types of answers about teaching CT may be expected from participants at this stage because their teaching experience was limited with ration centers and tutoring for the exam.

Participants' responses related to implementation of CT in class at the end of PD were also examined. Ahmet, Cemre, Defne and Zeki addressed using different approaches to teaching mathematics that supports development of students' CT skills. But they emphasized different aspects of teaching CT in a math classroom. Zeki emphasized analyzing learning objectives deeply and examining not just content but also skills, while Ahmet simply suggested using different solutions, saying "I discuss different solutions with students. Even if one person finds the answer, I ask if there is another way for solution". Zeki seems to be transformed notably in incorporating CT with higher-order mathematical skills, when his response for defining CT in the previous part is considered. Yet, his perspective on teaching CT indicates that he relies mostly on mathematical aspects rather than having a rigorous understanding of CT.

Cemre, suggested using the immersed approach like the researchers used in the first phase of the program. It's not clear whether Cemre used such teaching strategy, but she suggested being a model to her students for CT skills. Similarly, Defne, who had suggested using different approaches while solving problems, did not provide details of how it would be possible in the classroom. When we examine answers of these four teachers related to definition and implementation of CT at the end of first Phase 1, it can be said that Ahmet and Zeki transformed from limited understanding of CT to using different perspectives in mathematical problem solving. Even though Cemre and Defne started the PD with some partial understanding of CT with emphasis on using different approaches while solving problems, at the end of the program their perception of CT was still related to using different aspects or solutions. It can be said that their definition and explanation of implementation was more elaborated at the end of program. This is consistent with Zeki's comment, indicating a change in their "point of view and language". In addition to this limited development of four participants in terms of understanding of CT, the other three teachers (Mustafa, Erdem, Can) transformed their understanding to a certain degree.

Mustafa suggested discussing both correct and incorrect answers in the classroom to ensure variability. He also mentioned helping students to "see the big picture". This indicates that he was considering CT as connected to other higher-order thinking skills but without some elaboration of practice. He is a novice teacher so he might not be able to provide examples from his classroom experiences. Whereas more experienced participants Erdem and Can were able to support their ideas with classroom practices. Erdem thinks that it must be conveyed to the student that mathematics develops the way a brain works, and it is not a subject that can be learned by memorization. Can proposed solving a few math problems in class with discussion on main ideas of mathematics rather than solving many problems with superficial understanding.

Overall, when the whole group of participants are considered, it can be said that there were some improvements in their understanding of CT as teachers. While Zeki and Ahmet started with a very limited understanding of CT, they finished the program by considering CT as addressing different problems, solutions and terminology. Cemre and Defne stressed using different problem-solving tasks or strategies at the beginning and at the end of PD, yet their CT understanding improved in a subtle way. This improvement was evident in their answers related to CT implementation in the classroom. These two teachers' responses differed from both Zeki and Ahmet since they mentioned allowing students to experience CT in classroom. The last three participants (Mustafa, Erdem, Can) might have given different answers to definition of CT and implementing CT in classroom but the common element in their answers was approaching CT together with other mathematical thinking skills. All three of them discussed using the connection between CT and higher-order mathematical thinking skills to teach mathematics to students. Having three profiles of participants in terms of change in their understanding of CT, made us examine how participants experienced PD, what the strengths and weaknesses of the program were from their point of view.

Experiences on PD

To examine how participants experienced the PD about assessment with a focus on critical thinking, individual interviews were carried out after Phase 1 (mid-program interviews) and at the end of Phase 3. Participants were asked how the PD program contributed to their teaching as well as strengths and weaknesses of it. Participants' experiences may shed light on the differences in their CT understanding as discussed in the previous section. Participants, Zeki and Ahmet, who considered CT only as having different solutions or different approaches to mathematical problem solving, discussed their PD experiences in a limited manner. Zeki stated that he was not expecting much at the beginning because he did not feel the need for any PD. Yet, he stated at the end of the PD that he realized there were more things for him to learn as a teacher. However, he mostly talked about the limitations of the application that had been discussed during the PD. He mentioned they used predetermined textbooks and materials at the preparation center, which limited them in terms of integrating CT into their classroom practices. Another participant, Ahmet, reported four aspects of his experiences: analyzing questions, identifying student mistakes, revamping his teaching strategies, and improvement in mathematical communication between teachers. He said, “As my awareness of skills increased, the questions I asked myself about my teaching method changed, and accordingly, the strategies I applied for teaching changed”. Even though he mentioned many aspects, when he was asked to elaborate on his experiences, he provided limited answers. For example, he did not provide any examples of classroom practices. So, based on their answers, it can be deduced that these two teachers' experiences were superficial, which might be the explanation for the limited change in their understanding of CT and not being able to implement CT in their classrooms.

Participants Cemre and Defne, on the other hand, were able to transfer CT discussions of PD into their practice. Both stated that they were uncomfortable at the beginning because of not being given of formal definition of CT and higher-order mathematical thinking skills at the beginning of the PD. Cemre stated that “later, all the pieces came together, actually, we all found out what the skills were because we discussed them for weeks”. Defne made a very similar comment and she also stressed realizing how she should have followed a similar approach with her students, guiding them to figure out mathematics. She stated that she felt inexperienced when compared to the other participants and the PD helped her pedagogical skills such as choosing right math problems to promote critical thinking, guiding students step by step:

> Even if a student can't solve the question all the way, I get the student to do it by asking questions on the board. They say, ‘I did it, it wasn't that hard, it's nice’. I don't give the answer, I just lead them using prompting questions, just like you did with us.

It can be deduced that, these two teachers were still not incorporating CT with higher-order mathematical thinking skills and reflecting this incorporation into their practices.

The last three participants, Can, Erdem, Mustafa, differed from other teachers in terms of how they defined CT and how they experienced the PD. Being another novice teacher, Mustafa emphasized the benefit of working on constructing test questions. Even though he was the only participant who is a graduate of a mathematics teaching program, having more preparation of teaching practice, he was new to the settings of a test preparation center. So, the tailor-made PD addressing the purpose and needs of a test preparation center in terms of analyzing and constructing test questions really helped Mustafa. He also stressed not being given the definitions and the facts directly, he was able to internalize these skills. Furthermore, he emphasized reasoning skill: “the most important skill that teachers help students to develop is mathematical reasoning”. Similarly, a little bit more experienced Erdem stated that he was able to understand higher-order thinking skills for constructing test questions. He had never attended a PD before this one and his purpose of attending this PD was only to be able to develop better test questions. He discussed one limitation of the program; spending too much time on some questions when other teachers were discussing in length. He stated that he would prefer to solve more math problems as a whole group. Can had similar motivations as Erdem at the beginning of PD. Can aimed to learn how to construct test questions in addition to some practices to implement in the classroom. He stated that when the group was reflecting on the test questions written by group members, he was learning a great deal about higher-order mathematical skills. According to his observations, the group changed in a way to use the language of the higher-order thinking skills. He gave an example of discourse from the group as “What am I increasing here, procedural fluency or reasoning?” Furthermore, he gave an example of addressing CT in instruction for teaching trigonometric functions. He concluded that it went well but only because the students were high achievers of the test preparation center. He was convinced that the same instruction might not be effective with low achievers as much as with the high achievers

DISCUSSION & CONCLUSION

The PD examined in this study was designed to address a group of teachers who were often neglected. Test-preparation centers are very common in Turkish context and teacher education is not an option for these teachers. Yet these teachers need PD on development of mathematical skills of students. The motivation of participants to attend the PD was to improve themselves in terms of constructing better quality math questions to assess students' higher-order mathematics skills. These skills cannot be considered isolated from other aspects of teaching and their development should be considered together with CT.

Thus, the authors designed a unique 12-week PD program on assessment for test-preparation center teachers. In addition to addressing this neglected group of teachers, there are also two aspects of the program in terms of handling skills: utilizing mixed design of teaching CT and merging higher-order mathematics thinking skills with CT in order to improve teachers' practice. First accepting immersed approach to teach CT then switching to general approach (explicitly introducing CT) (Ennis 1989) for participant teachers was an effective strategy. In a meta-analysis on strategies for teaching CT, Abrami et.al. (2015) concluded that four methods proposed Ennis (1989) for teaching CT "produced significantly positive average effect sizes, but the categories did not differ from one another". Since the target group of this study was teachers, and the aim is to discuss teaching CT as well as focusing on their own CT skills, this using multiple strategies seems appropriate. Similar approach can be used while working with prospective teachers. Another finding of Abrami et.al.'s (2015) meta-analysis is that whole-class and group discussions led by the teacher (dialogue), and exposure of students to authentic or situated problems and examples (authentic instruction) are effective instructional methods for teaching CT. They also report that these two strategies are effective when used together, especially when combined with mentorship. These two methods were also used in this study during the PD. In future implementations, mentoring can also be included in the PD activities proving more opportunities for one-to-one interaction with the participants.

Based on participants' answers, it was clear that most of them were convinced to implement CT in their classrooms. However, it should be noted that the program mostly focused on assessing students' higher-order thinking skills based on what has been emphasized in university entrance tests. As researchers intentionally aimed for developing participants' CT skill, the discussions merged CT with higher-order mathematics skills. The findings of this study reflect how participants were able to internalize the relationship between these two. This program provided a change in language and discussions of the mathematics teachers in the test preparation center. The change in perception and practice for two teachers did not go further than that but all the other teachers further improved their perceptions. Most teachers' initial definitions of CT were limited and some included explanations such as criticizing an opinion and opposing a view. A similar issue was also addressed by Innabi and Sheikh (2007). However, as PD progressed their definitions started to include more aspects of CT. In addition, evidence of CT was observed when they were sharing their experiences about PD in classroom discussions. For example, they were talking about how their 'language' changed and how they started to do things differently while writing questions or talking to their students in the classroom. In addition, in the post test, few teachers implied that CT cannot be attained by every student, and they cannot carry out certain conversations with low level students like they do with high level students. However, as Innabi and Sheikh, (2007) suggested, expectations of CT development should be high for all students.

Based on all the participants' views, it can be said that this PD program met their expectations. According to Matherson and Windle (2017), teachers demand PD learning opportunities that i) are interactive, engaging, and relevant for their students ii) show them a more practical way to deliver content iii) are sustained over time and iv) are teacher driven. Teachers in this study also expected us to show them practical ways that they can immediately use, such as giving the definitions of the terms from the start or providing more concrete examples of classroom practice for teaching higher-order skills. The way PD was structured enabled the teachers to be highly interactive. Most teachers seemed to be engaged in the program activities, however some of the teachers reported that they lost focus in some instances, like when discussion about a test question or a content topic went on too long. In general, it can be said that PD was addressing all five of teacher expectations.

This study focused on experiences and perceptions of participating teachers. Further studies can be conducted that focus on the effectiveness of such a program by evaluating teachers' progress in terms of skills and examining their practices. In addition, similar PD programs can be developed for test preparation centers to accommodate teachers from different disciplines.

Statements of Publication Ethics

Ethics board approval was taken on 13.05.2022 from Boğaziçi University Social and Human Sciences Human Research Ethics Committee (meeting number: 2022/05, approval number: 2022-31).

Researchers' Contribution Rate

Authors	Literature review	Method	Data Collection	Data Analysis	Results	Conclusion
Yeşim İmamoğlu	☒	☒	☒	☒	☒	☒
Fatma Aslan-Tutak	☒	☒	☒	☒	☒	☒
Gürsu Aşık	☒	☒	☒	☒	☒	☒
Beyza Oncar-Ekiz	☒	☒	☒	☒	☐	☐

Conflict of Interest

The authors report there are no competing interests to declare.

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