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A NEW CONSTRAINT HANDLING APPROACH BASED ON ENHANCED QUADRATIC APPROXIMATION: TESTED ON OPTIMAL DESIGN OF MECHANICAL SYSTEMS AND TRUSS STRUCTURES

Mahsa Moloodpoor[a*], Ali Mortazavi[b]

[a*]*Department of Mechanical Engineering, Faculty of Engineering, Ege University, Izmir, Turkey*
*(*corresponding author)*
mahsa.moloodpoor@gmail.com

[b]*Department of Civil Engineering, Faculty of Engineering, Izmir Democracy University, Izmir, Turkey*
ali.mortazavi@idu.edu.tr

Abstract

Optimization refers to the process of identifying the optimal state of a system while ensuring all constraints and requirements are met. In engineering problems, the feasibility of solutions is typically assured by imposing relevant constraints. Since these constraints have different properties, utilizing more systematic and logical methods to handle them has the potential to enhance the search performance of the optimization algorithms. According to this fact, in the current study, a new constraint handling mechanism based on combining the fly-back method, weighted average concept and quadratic approximation approach is developed. The proposed mechanism is then merged with three distinct well-established metaheuristic optimization methods. The effectiveness of the enhanced techniques is evaluated through comparative analysis in solving various mathematical and engineering optimization problems subjected to different constraints. Furthermore, non-parametric statistical tests are conducted to compare the quality of the obtained results. The results show that the developed approach can considerably improve the performance of the search algorithms with regards to accuracy, stability, and computational cost.

Keywords: Metaheuristic algorithm, constrained optimization problems, fly-back method, quadratic approximation, structural and mechanical problems

1. Introduction

Generally, from the aspect of search space boundaries, the optimization problems can be divided into two categories of constrained and unconstrained problems [1]. In contrast with unconstrained problems, some additional boundary conditions should be satisfied to reach a feasible solution in constrained problems. Most engineering optimization problems are in the category of constrained problems [2-17]. On the other hand, the most time-consuming part of these optimization problems is the objective function evaluation (e.g., in structural optimization problems, complex finite element models are required to determine the response of the system). Based on these facts, applying an efficient approach to handle problem's constraints can highly increase the performance of the search algorithm by decreasing the required number of objective function evaluations (OFEs). Also, it can improve the quality of the optimal solution by providing more room for the search algorithm to examine the search space in more detail.

In this regard, there are distinct constrained handling methods developed and integrated with different optimization techniques. For instance, Elaziz et al. (2021) introduced an enhanced version of Harris Hawks Optimizer (HHO) algorithm and utilized the penalty function approach for controlling the constraints [18]. Tsipianitis and Tsompanakis (2020) proposed a modified version of Cuckoo Search (CS) algorithm by implementing a penalty function approach for solving engineering optimization problems [19]. Zhengtong (2019) introduced an improved version of Particle Swarm Optimization (PSO) combined with a dynamic penalty function approach for handling the constraints in solving structural optimization problems [20]. Tejani et al. (2018) investigated simultaneously size, shape and topology optimization of truss structures using metaheuristic techniques and a penalty function approach for handling the stated constraints [21].

Javidi et al. (2019) tested the performance of modified crow search algorithm (CSA) for structural optimization problems. They utilized fly-back approach for handling the corresponding constraints [22]. The main advantage of the Fly-Back (FB) method, compared with different penalty approaches, is that the fly-back rejecting any infeasible solution during the optimization process guarantees the feasibility of the attained solutions. However, the main drawback of this technique is that in the non-convex search spaces, especially in the narrow areas of the domain that are highly restricted the FB might not find any solution due to over rejecting the infeasible solution. In this condition, the algorithm cannot spot a feasible point in the domain and search process cannot be converged. So, in highly constrained problems standard FB approach can cause the process to diverge, or even in some cases any solution may not be found.

To tackle this problem and provide a more flexible approach, in the current study, firstly the quadratic approximation (QA) method is improved by the mean of weighted agent, and then, it is integrated with fly-back approach to provide a more efficient constraint handling method. The new approach called Enhanced Quadratic approximation-based Fly-Back (EQFB) approach not only guarantees the feasibility of the attained solutions by rejecting all unfeasible candidates but also reduces the excessive rejections of standard FB and increases the convergence possibility of the algorithm by giving a second chance to infeasible agents and keeping them inside the feasible region of the search domain. The proposed new strategy is merged with three well-developed optimization metaheuristic algorithms as Teaching and Learning Based Optimization (TLBO) [23], Harris Hawks Optimization (HHO) [24], and Butterfly Optimization Algorithm (BOA) [25]. The performance of these methods using FB and EQFB are tested on handling different mathematical and engineering problems and the attained results are reported and compared in detail.

The rest of the current study is arranged as below. The utilized optimization algorithms are briefly described in Section 2. The proposed constraint-handling module is explained in Section 3. Section 4 is devoted to numerical tests and achieved results are discussed in this section. Finally, the significant points concluded from the current study are summarized in the last Section.

2. Optimization methods

In the current section, initially a brief description of the selected optimization methods is given. These algorithms are Teaching and Learning Based Optimization (TLBO), Butterfly Optimization Algorithm (BOA), and Harris Hawks Optimization (HHO).

2.1. Teaching and Learning Based Optimization (TLBO)

Teaching and Learning Based Optimization (TLBO), introduced by Rao et al. (2011), is inspired by an information transfer in an educational process between the teacher and students. In the method, there is a population in a class and agents are the students. The optimization process utilizing TLBO is based on two main phases as teaching and learning phases. In the teaching phase, the optimal agent acts as the teacher and the remaining agents, serving as students, try to enhance their performance by assimilating knowledge from the best agent. This phase is mathematically expressed as below [23]:

$$\mathbf{X}_i^{new} = \mathbf{X}_i + r.(\mathbf{X}_T - T_F\mathbf{X}_{mean}) \quad (1)$$

where, $\mathbf{X}_T$, $\mathbf{X}_i^{new}$ and $\mathbf{X}_i$ are the best agent's location, the improved location of the *i*th agent and the current position of the *i*th agent, respectively. T_F indicates the teaching factor, which can be selected as 1 or 2. $\mathbf{X}_{mean}$ is the mean vector of the entire agents that can be determined as below [23]:

$$\mathbf{X}_{mean} = \left[m\left(\sum_{j=1}^{np} x_j^1\right), m\left(\sum_{j=1}^{np} x_j^2\right), \ldots, m\left(\sum_{j=1}^{np} x_j^i\right), \ldots, m\left(\sum_{j=1}^{np} x_j^{nd}\right)\right] \quad (2)$$

where, x_j^1 is the first component of the *j*th agent. *np* and *nd* demonstrate the size of the population and number of decision variables, respectively. In addition, *m*(.) is the mean value of the corresponding decision variables.

In the learning phase of the algorithm, the matter is an information interaction between the agents to enhance their location. After a comparison between two distinct agents' objective value the movement occurs. The movement of the agent is toward the agent with a better objective function value. This phase can be mathematically described as below [23]:

$$\mathbf{X}_i^{new} = \mathbf{X}_i + r.\left(\mathbf{X}_i - \mathbf{X}_j\right) \quad \text{if} \quad f(\mathbf{X}_i) < f\left(\mathbf{X}_j\right) \quad (3)$$

$$\mathbf{X}_i^{new} = \mathbf{X}_i + r.\left(\mathbf{X}_j - \mathbf{X}_i\right) \quad \text{if} \quad f(\mathbf{X}_i) > f\left(\mathbf{X}_j\right) \tag{4}$$

where, r is random value which is uniformly distributed in [0,1] interval. Consequently, according to these two phases if $\mathbf{X}_i^{new}$ provides a better position for the agent, it is changed with the old one, otherwise, $\mathbf{X}_i$ is preserved as presented in the following:

$$\mathbf{X}_i = \mathbf{X}_i^{new} \quad \text{if} \quad f(\mathbf{X}_i^{new}) < f(\mathbf{X}_i) \tag{5}$$

$$\mathbf{X}_i = \mathbf{X}_i \quad \text{if} \quad f(\mathbf{X}_i^{new}) \geq f(\mathbf{X}_i) \tag{6}$$

2.2. Harris Hawks Optimizer (HHO)

Harris Hawks Optimizer (HHO) is a non-gradient and swarm-based optimization technique that is inspired by the behavior and chasing style of a kind of bird in the nature named Harris' Hawks. Their surprise pounces for catching prey are mimicked in the algorithm. Three phases are followed in this method; these are *Exploration*, *Transition from exploration to exploitation* and *Exploitation*. Like other swarm-based and nature inspired techniques, each agent in the population is a potential solution of the optimization process. During the search process, the agents change their locations randomly and the average position is attained through the exploration phase of the algorithm. As the prey is escaping from the hunter bird and changes its locations its energy reduces and the algorithm at this point moves to the transition phase. The information attained from the transition phase became a directive for the agent to move toward an improved location. According to the given information about the algorithm, three phases of the HHO is mathematically presented as below [24]:

$Exploration\ phase$

$$X^{t+1} = \begin{cases} X_j^t - r_1\left|X_j^t - 2r_2X^t\right| & q \geq 0.5 \\ \left(X_{prey}^t - X_m^t\right) - r_3\left(LB + r_4(UB - LB)\right) & q < 0.5 \end{cases} \tag{7}$$

$$X_m^t = \frac{1}{N}\sum_{i=1}^{N} X_i^t \tag{8}$$

where, r_n (n=1, 2, 3, 4) and q are random values selected from (0, 1) interval. X^{t+1} and X^t are the next and current location of the predator, respectively. X_{prey}^t is the location vector of the prey. X^t is the current location of the predator. *UB* and *LB* indicate the upper and lower bounds of the variables, respectively. X_j^t is an agent randomly selected from the population. X_m^t is the mean location of the agents in the population and *N* is the population size.

$Transition\ from\ exploration\ to\ exploitation$

$$E = 2E_0\left(1 - \frac{t}{T}\right) \tag{9}$$

where, E indicates the escaping energy of the prey. E_0 is the initial level of the prey's energy, and it is randomly selected from (-1, 1) interval. T is the maximum number of iterations.

$Exploitation\ phase$

for $r \geq 0.5\ \ and\ \ |E| \geq 0.5$

$$X^{t+1} = \Delta X^t - E\left|JX_{prey}^t - X^t\right| \tag{10}$$

$$\Delta X^t = X_{prey}^t - X^t \tag{11}$$

$$J = 2(1 - r_5) \tag{12}$$

for $r \geq 0.5\ \ and\ \ |E| < 0.5$

$$X^{t+1} = X_{prey}^t - E|\Delta X^t| \tag{13}$$

for $r < 0.5\ \ and\ \ |E| \geq 0.5$

$$Y = X_{prey}^t - E\left|JX_{prey}^t - X^t\right| \tag{14}$$

$$Z = Y + S \times LF(D) \tag{15}$$

$$LF(x) = 0.01 \times \frac{u \times \sigma}{|v|^{\frac{1}{\beta}}} \quad , \quad \sigma = \left(\frac{\tau(1+\beta) \times \sin\left(\frac{\pi\beta}{2}\right)}{\tau\left(\frac{1+\beta}{2}\right) \times \beta \times 2^{\left(\frac{\beta-1}{2}\right)}}\right)^{\frac{1}{\beta}} \tag{16}$$

$$X^{t+1} = \begin{cases} Y & if \quad f(Y) < F(X^t) \\ Z & if \quad f(Z) < F(X^t) \end{cases} \tag{17}$$

when $r < 0.5\ \ and\ \ |E| < 0.5$

$$X^{t+1} = \begin{cases} Y & if \quad f(Y) < F(X^t) \\ Z & if \quad f(Z) < F(X^t) \end{cases} \tag{18}$$

$$Y = X_{prey}^t - E\left|JX_{prey}^t - X_m^t\right| \tag{19}$$

$$Z = Y + S \times LF(D) \tag{20}$$

where, r_5 is a random number selected from (0, 1) interval. D is the problem's dimension and S is random vector with a size of $1 \times D$. LF is the levy flight function. The u and v values are randomly selected from (0, 1) interval and β is a constant considered as 1.5.

2.3. Butterfly Optimization Algorithm (BOA)

The Butterfly Optimization Algorithm (BOA) mimics the survival behavior of butterflies to mate and find food sources. This search technique has three main sections as initializing, process iterations and finalizing [25]. In the BOA after the butterflies attract each other by emitting the fragrance, each butterfly moves stochastically toward the best butterfly in the colony. The properties of the search space influence the amount of stimulus intensity for each butterfly. The BOA for global and local searches is defined with different strategies and it is mathematically formulated as below [25]:

Global search

$$x_{i}^{t+1} = x_i^t + (r^2 \times g^* - x_i^t) \times f_i \tag{21}$$

Local search

$$x_{i}^{t+1} = x_i^t + \left(r^2 \times x_j^t - x_k^t\right) \times f_i \tag{22}$$

where, $g^*$is the best agent in the colony. x_j and x_k are randomly selected agents in the colony. t and t+1 designate the current and updated position, respectively. Also, r is a random coefficient uniformly selected from [0,1] interval. f_i presents the fragrance factor of the ith agent and it is also selected from [0, 1] interval.

3. The proposed new constraint handling strategy

This section is devoted to describing the proposed constraint handling mechanism based on combining the fly-back method and quadratic approximation approach. To provide more clarity, firstly the standard Fly-Back method is explained and then the proposed Enhanced Quadratic approximation-based Fly-Back approach is defined.

3.1. Standard Fly-Back (FB) Approach

The standard Fly-Back approach is one of the well-known constraint handling techniques. As presented in Figure 1, in the standard Fly-Back (FB) approach, for holding the agents in the feasible region of the search domain, the violated agents are forced to fly back to its previous feasible location [26]. Although this method has a simple logic, a possible high number of rejections might increase the inefficient iterations and subsequently raise the demanded computational cost. This subject is specially critic in engineering optimization problems with complex and costly objective functions.

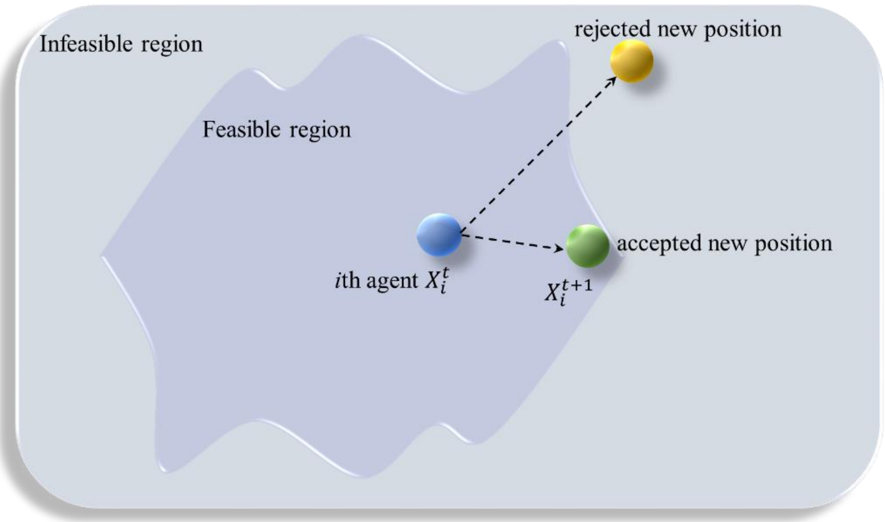


Figure 1. Schematic view of standard Fly-Back

In the current study, to overcome this problem a new approach is introduced. The new quadratic-based constraint-handling approach utilizing the advantages of the weighted agent and quadratic approximation approach puts forward an efficient constraint handling approach. The proposed methodology is described in detail in the following section.

3.2. Enhanced Quadratic Approximation Based Fly-Back (EQFB) approach

In this section, the proposed constraint handling mechanism (EQFB) is described. In this approach, firstly, the constraints are divided to two groups of numeric constraints and characteristic constraints. Numeric constraints are those that do not require any analysis (e.g., bounds of design variables). Characteristic constraints are those that need at least one objective function evaluation to identify the violations [27]. To achieve feasible solutions, it should be checked whether these constraints are violated or not during the optimization process [28, 29]. For further clarification, the schematic view of this expression is presented in Figure 2.

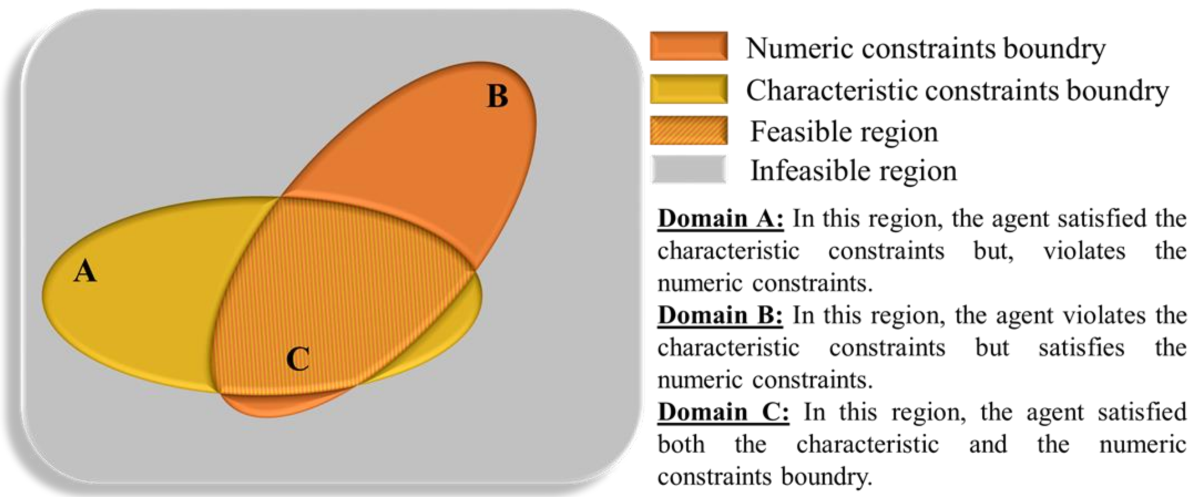


Figure 2. Schematic illustration of numeric and characteristic constraint boundaries

In the original Quadratic Approximation (QA), there are three main primary agents named R_1, R_2 and R_3 generate an infant agent as R^{new}. This generation process is mathematically presented as below [30]:

$$R^{new} = 0.5\,\frac{(R_2^2 - R_3^2)f(R_1) + (R_3^2 - R_1^2)f(R_2) + (R_1^2 - R_2^2)f(R_3)}{(R_2 - R_3)f(R_1) + (R_3 - R_1)f(R_2) + (R_1 - R_2)f(R_3)} \tag{23}$$

The three primary agents are distinct agents ($R_1 \neq R_2 \neq R_3$) randomly selected from the population. $f(.)$ designates the objective function value of the corresponding agent. In this approach, the agents are guided due to a random-based movement, and this causes some shortcomings. For instance, the agents in a better position may be neglected due to the high randomness of the new agent. So, the computational cost of the search process can negatively be affected. Based on this fact, and to decrease the randomness level of standard QA method, one of the three random agents is replaced with a new meaningful agent. The new agent is defined as the weighted average of all available agents in the population and mathematically is expressed as follows [31]:

$$\mathrm{X}^{W} = \sum_{s=1}^{M} \bar{c}_s^{w} \mathbf{X}_s^{P} \tag{24}$$

in which

$$\bar{c}_s^{w} = \left[\hat{c}_s^{w} \Big/ \sum_{s=1}^{M} \hat{c}_s^{w}\right] \tag{25}$$

$$\hat{c}_s^{w} = \frac{\max_{1\le k\le M}\left(f(\mathbf{X}_k^{P})\right) - f(\mathbf{X}_s^{P})}{\max_{1\le k\le M}\left(f(\mathbf{X}_k^{P})\right) - \min_{1\le k\le M}\left(f(\mathbf{X}_k^{P})\right) + \varepsilon}, \quad s = 1,2,\dots,M \tag{26}$$

where, X^{W} represents the weighted agent; M designates the population size; $\hat{c}_s^{w}$ presents the effect coefficient of each agent based on its objective function value ($f(.)$). $\max_{1\le k\le M}\left(f(\mathbf{X}_k^{P})\right)$ and $\min_{1\le k\le M}\left(f(\mathbf{X}_k^{P})\right)$ are the worst and best values of the objective function for all agents, respectively. ε presents a tiny positive quantity (e.g., 0.00001) for preventing any division by zero situation. Consequently, by replacing the R_1 agent with the weighted agent (X^{W}) the enhanced new agent (R^{e-new}) is described as below:

$$R^{e-new} = 0.5\,\frac{(R_2^2 - R_3^2)f(\mathrm{X}^{W}) + (R_3^2 - (\mathrm{X}^{W})^2)f(R_2) + ((\mathrm{X}^{W})^2 - R_2^2)f(R_3)}{(R_2 - R_3)f(\mathrm{X}^{W}) + (R_3 - \mathrm{X}^{W})f(R_2) + (\mathrm{X}^{W} - R_2)f(R_3)} \tag{27}$$

Consequently, as presented in Figure 3 the enhanced new agent is merged with the FB method, and the proposed method is named Enhanced Quadratic Fly-Back (EQFB).

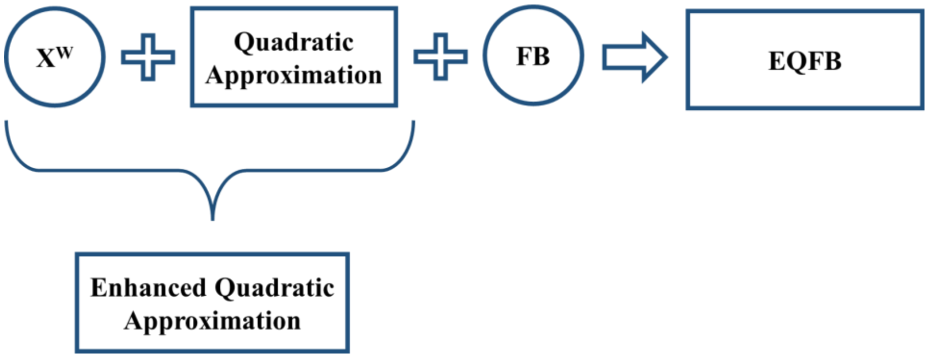


Figure 3. Enhanced Quadratic Approximation based Fly-Back (EQFB)

In contrast with standard FB method that simply reject the violated agents, the introduced Enhanced Quadratic Fly-Back (EQFB) applying the enhanced new agent (R^{e-new}) follows a three-step procedure to evaluate the violated agents. These steps are as follows:

i. The components of the current agent are controlled for any numeric constraint(s) violation.
ii. Changing the violated component(s) in the violated agent (if any) with the equivalent component(s) in the enhanced new agent (R^{e-new}).
iii. Controlling the modified agent for any violation in characteristic constraints, if it is not a violated agent and provides a better solution, it will be replaced with the previous agent and if not, the agent will be rejected.

4. Numerical test

This section is devoted to evaluating the performance of the proposed EQFB technique in handling different constraints of distinct mathematical and engineering problems. In this respect, the FB and EQFB modules are merged with three different methods as TLBO, BOA and HHO. Attained algorithms are utilized for solving distinct constrained optimization problems. Twenty eight mathematical problems are selected from the CEC2017 database [32]. Moreover, five problems such as, a tension/compression spring, a hydro-static thrust bearing, a multiple disc clutch brake, a car side impact and a rolling element bearing design problems are investigated as the mechanical optimization problems. Then, two problems as 72-bar spatial truss and 120-bar dome structures containing dynamic constraints are investigated. The computer system for implementing the algorithms and problem-solving processes is a system equipped with an Intel-i7™ CPU and 12 MB of installed RAM. The results are compared and reported descriptively. For assessing the capability of the proposed methods more evidently, statistical tests are employed on the achieved results.

4.1 Constrained mathematical functions

In the current section, a set of 28 constrained mathematical optimization problems reported in the CEC2017 database [32], with distinct properties, are tested using the combined algorithms. The problems include a wide variety of constraints such as equalities and inequalities. The search range of the problems CF 01-CF 03, CF 08, CF 10-CF 18, CF 20-CF 27 are $[-100,100]^D$, CF 07, CF 19, CF 28 are $[-50,50]^D$, CF 06 is $[-20,20]^D$, CF 04-CF 05, CF 09 are $[-10,10]^D$. *D* designates the dimension of the problems and it is considered as 30 for all functions. The optimization process is run 25 times and the maximum objective function evaluations (OFEs) is considered as 20000**D*. The mathematical optimization problems are defined in the following format [32]:

Minimize: $f(X), \quad X = (x_1, x_2, \ldots, x_n) \; and \; X \in S$ (28)

Subject to: $g_i(X) \leq 0, \quad i = 1, \ldots, p$ (29.1)

$h_i(X) = 0 \quad j = p + 1, \ldots, m$ (29.2)

The equality constraints usually are converted to inequalities using an auxiliary element (δ=0.0001) in the following format [32]:

$$|h_i(X)| - \delta \leq 0, \quad for \quad j = p + 1, \ldots, m \tag{30}$$

The achieved optimal results utilizing different methods are given in Table 1. The accuracy and stability of the selected methods are compared. According to the reported information, the TLBO-EQFB almost provides the most accurate optimal solution among the other combinations. This presents the search capability of the TLBO method and its compatibility with the proposed EQFB module. The obtained standard deviation values reveal that the EQFB module increases the stability of the selected methods in comparison with FB. In addition, based on the Std. values the TLBO-EQFB in most cases exceed the other algorithms in term of stability.

Table 1. Optimal results for CEC2017 constrained test functions

Func.	Value	TLBO-FB	TLBO-EQFB	BOA-FB	BOA-EQFB	HHO-FB	HHO-EQFB
CF 01	Mean	5.37 E-07	**2.95 E-07**	7.53 E-06	7.84 E-06	1.20 E-06	7.56 E-07
	Std.	2.43 E-08	**1.84 E-08**	1.14 E-07	5.81 E-08	4.38 E-08	3.56 E-08
	Rank	2	**1**	5	6	4	3
CF 02	Mean	3.05 E-07	**1.71 E-07**	8.84 E-06	3.44 E-06	2.20 E-06	6.67 E-07
	Std.	9.57 E-09	**9.32 E-09**	9.86 E-07	5.68 E-07	3.13 E-08	1.47 E-08
	Rank	2	**1**	6	5	4	3
CF 03	Mean	5.20 E+05	**2.42 E+05**	9.88 E+05	1.09 E+06	6.43 E+05	8.21 E+05
	Std.	2.88 E-01	**2.15 E-01**	9.00 E+00	8.94 E+00	7.81 E+00	4.59 E+00
	Rank	2	**1**	5	6	3	4
CF 04	Mean	3.60E+02	**3.59 E+02**	5.73 E+02	8.44 E+02	8.59 E+02	4.99 E+02
	Std.	3.15 E-01	**1.64 E-01**	1.21 E+00	6.33 E-01	6.00 E-01	5.76 E-01
	Rank	2	**1**	4	5	6	3
CF 05	Mean	2.75 E+02	**9.17 E+00**	9.51 E+02	3.66 E+02	3.22 E+02	1.75 E+02
	Std.	8.78 E-01	**3.90 E-01**	8.00 E+00	1.99 E+00	4.43 E+00	1.71 E+00
	Rank	3	**1**	6	5	4	2
CF 06	Mean	1.00 E+09	**4.47 E+08**	2.22 E+09	9.17 E+08	3.48 E+09	4.23 E+09
	Std.	3.94 E+01	**2.25 E+01**	7.47 E+02	6.10 E+01	2.97 E+02	5.91 E+01
	Rank	3	**1**	4	2	5	6
CF 07	Mean	2.46 E+06	**6.60 E+05**	5.90 E+06	3.23 E+06	7.87 E+05	7.28 E+05
	Std.	4.89 E+00	**2.61 E+00**	3.36 E+01	8.42 E+00	7.10 E+00	4.29 E+00
	Rank	4	**1**	6	5	3	2
CF 08	Mean	**-4.41 E-03**	-5.11 E-04	6.01E-04	-1.34 E-03	1.33 E-02	-2.90 E-04
	Std.	**4.30 E-06**	4.17 E-06	3.09 E-05	6.10 E-04	4.36 E-06	4.62 E-06
	Rank	**1**	3	5	2	6	4
CF 09	Mean	6.31 E+06	**2.45 E+06**	5.35 E+07	2.26 E+07	7.13 E+06	7.04 E+06
	Std.	2.15 E+01	**4.61 E+00**	8.93 E+01	8.32 E+01	7.59 E+01	1.28 E+01

	Rank	2	**1**	6	5	4	3
CF 10	Mean	-2.15 E-04	-4.69 E-05	3.65 E-04	-1.47 E-04	1.03 E-04	**-5.16 E-04**
	Std.	3.56 E-06	6.75 E-06	3.94 E-05	1.72 E-05	6.32 E-05	**7.98 E-05**
	Rank	2	4	6	3	5	**1**
CF 11	Mean	2.65 E+01	**5.57 E+00**	9.64 E+04	7.03 E+04	8.65 E+01	7.81 E+01
	Std.	3.78 E-01	**2.17 E-02**	6.19 E+01	8.66 E+00	4.08 E-01	3.42 E-01
	Rank	2	**1**	6	5	4	3
CF 12	Mean	3.65 E+03	**2.89 E+03**	2.70 E+07	8.74 E+06	5.49 E+04	4.51 E+04
	Std.	3.56 E+01	**1.11 E+01**	9.04 E+01	6.06 E+01	7.50 E+01	1.49 E+01
	Rank	2	**1**	6	5	4	3
CF 13	Mean	6.32 E+02	**9.40 E+00**	2.83 E+07	3.07 E+05	1.92 E+03	3.89 E+02
	Std.	6.60 E-01	**3.45 E-02**	4.91 E+01	6.39 E+00	4.24 E+00	3.55 E+00
	Rank	3	**1**	6	5	4	2
CF 14	Mean	9.83 E+00	**9.16 E+00**	5.14 E+08	2.43 E+08	7.35 E+02	4.05 E+02
	Std.	6.78 E-02	**6.15 E-02**	7.92 E+02	2.79 E+01	9.01 E+00	6.73 E+00
	Rank	2	**1**	6	5	4	3
CF 15	Mean	8.45 E+02	**6.04 E+01**	2.92 E+06	7.55 E+05	6.34 E+03	1.20 E+03
	Std.	2.48 E-01	**1.70 E-01**	3.02 E+02	8.26 E+01	8.24 E+00	3.88 E+00
	Rank	2	**1**	6	5	4	3

Func.	Value	TLBO-FB	TLBO-EQFB	BOA-FB	BOA-EQFB	HHO-FB	HHO-EQFB
CF 16	Mean	7.72 E+02	**3.70 E+02**	6.53 E+03	6.26 E+03	8.81 E+02	8.66 E+02
	Std.	1.77 E+00	**1.55 E+00**	5.01 E+00	4.29 E+00	3.25 E+00	2.44 E+00
	Rank	2	**1**	6	5	4	3
CF 17	Mean	8.65 E+09	**4.89 E+09**	4.24 E+11	6.96 E+10	7.74 E+09	6.11 E+09
	Std.	2.63 E+00	**2.00 E+00**	3.96 E+01	2.93 E+01	1.39 E+01	1.36 E+01
	Rank	4	**1**	6	5	3	2
CF 18	Mean	5.53 E+16	**7.73 E+15**	1.86 E+18	5.75 E+17	1.74 E+17	3.41 E+16
	Std.	2.50 E+03	**3.97 E+02**	1.08 E+03	5.84 E+03	9.60 E+03	4.42 E+03
	Rank	3	**1**	6	5	4	2
CF 19	Mean	5.96 E+12	**1.16 E+12**	7.36 E+12	6.18 E+12	8.02 E+12	6.05 E+12
	Std.	2.09 E+02	**1.81 E+02**	8.53 E+05	8.15 E+05	5.11 E+05	7.83 E+04
	Rank	2	**1**	5	4	6	3
CF 20	Mean	5.92 E+00	**4.94 E+00**	9.81 E+00	9.04 E+00	6.66 E+00	6.02 E+00
	Std.	9.95 E-03	**8.80 E-03**	3.86 E-02	2.13 E-02	1.54 E-02	1.50 E-02
	Rank	2	**1**	6	5	4	3
CF 21	Mean	7.25 E+01	**9.08 E+00**	9.43 E+07	3.49 E+07	3.89 E+02	1.96 E+02
	Std.	6.66 E-01	**2.76 E-02**	1.67 E+02	9.03 E+01	7.35 E-01	4.05 E-01
	Rank	2	**1**	6	5	4	3
CF 22	Mean	1.01 E+11	**4.82 E+10**	8.49 E+12	4.46 E+12	3.23 E+11	2.35 E+11
	Std.	1.53 E+02	**1.29 E+02**	7.80 E+02	5.10 E+02	5.46 E+02	1.76 E+02
	Rank	2	**1**	6	5	4	3
CF 23	Mean	7.73 E+01	**9.36 E+00**	5.85 E+03	1.96 E+03	2.23 E+02	7.60 E+01
	Std.	8.70 E-03	**4.40 E-04**	9.60 E+01	6.78 E+01	2.32 E+00	1.06 E+00
	Rank	3	**1**	6	5	4	2
CF 24	Mean	9.32 E+01	**2.92 E+01**	5.11 E+02	1.23 E+02	6.45 E+02	9.19 E+01
	Std.	1.89 E-02	**1.51 E-02**	8.69 E+00	6.02 E+00	5.08 E-01	3.12 E-01
	Rank	3	**1**	5	4	6	2
CF 25	Mean	5.18 E+02	**3.06 E+02**	6.92 E+08	2.58 E+08	8.08 E+02	3.41 E+02
	Std.	1.76 E+00	**1.06 E+00**	7.91 E+02	7.34 E+02	5.99 E+00	3.29 E+00
	Rank	3	**1**	6	5	4	2
CF 26	Mean	8.19 E+09	**4.45 E+09**	3.30 E+10	7.19 E+10	4.35 E+10	5.55 E+09
	Std.	9.36 E+01	**4.46 E+01**	9.24 E+02	4.41 E+02	1.72 E+02	1.15 E+02

	Rank	3	**1**	4	6	5	2
CF 27	Mean	2.57 E+17	**7.71 E+16**	8.00 E+19	7.32 E+19	4.75 E+18	3.29 E+18
	Std.	3.12 E+04	**1.09 E+04**	7.22 E+06	6.05 E+06	5.70 E+04	3.20 E+04
	Rank	2	**1**	6	5	4	3
CF 28	Mean	4.31 E+12	**4.30 E+12**	4.34 E+12	4.33 E+12	4.34 E+12	4.33 E+12
	Std.	5.98 E+00	**2.30 E+00**	4.55 E+01	2.18 E+01	8.30 E+00	6.22 E+00
	Rank	2	**1**	5	4	6	3

4.1.1 Non-parametric statistical test (Friedman Rank Test) for constrained mathematical problems

In this section the performance of the proposed algorithms is compared with each other using the Friedman rank test over mean and standard deviation (Std.) values. Attained test results are given in Table 2. Reported results reveal that the performance of the TLBO-EQFB algorithm in terms of accuracy and stability is superior to the other algorithms. In addition, it is observable that the proposed EQFB module increases the performance of all three selected optimization methods.

Table 2. The Friedman rank test for Mean and Std. values for selected constrained functions

Method	Test for optimal Mean value			Test for optimal Std. value		
	Friedman value	Normalized value	Rank	Friedman value	Normalized value	Rank
TLBO-FB	67	0.492537	2	61	0.47541	2
TLBO-EQFB	33	1.000000	1	29	1.000000	1
BOA-FB	156	0.211540	6	161	0.180124	6
BOA-EQFB	132	0.250000	5	135	0.214815	5
HHO-FB	122	0.270492	4	118	0.245763	4
HHO-EQFB	78	0.423080	3	84	0.345238	3

4.2. Constrained engineering problems

In the current section, different mechanical and structural optimization problems with distinct constraints are solved with the considered methods. For handling the constraints, the performance of the proposed EQFB is evaluated. Finally, for investigating the performance of the algorithms a non-parametric statistical test is applied. The problems and their optimal results are given in the following subsections.

4.2.1. Tension/compression spring (T/CS) design problem

In the current problem, it is targeted to minimize the construction cost of a tension/compression spring, which is schematically presented in Figure 4. There are three design variables $\mathbf{X} = [x_1, x_2, x_3]$ for the current problem, where x_1 is the wire diameter (d), x_2 is the mean diameter of spring (D), x_3 is the number of active coils (N) [33].The objective function of the problem, proper constraints and design variables are presented in Table 3.

The achieved optimal solutions for the current problem are reported in Table 4. Based on the attained optimal values and Std. values the TLBO-EQFB outperforms the other selected algorithms from both accuracy and stability aspects. Also, the number of objective function evaluations (OFEs) reveal that the proposed TLBO-EQFB has an acceptable computational cost among other combinations. All these observations indicate that the TLBO-EQFB method, because of its search strategy and constraint handling mechanism puts forward a good performance in comparison with other combinations.

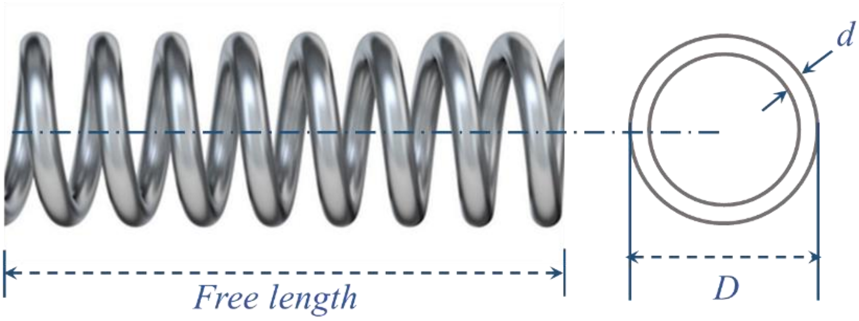


Figure 4. Schematic view of T/CS system

Table 3. Objective function, constraints and design variables of T/CS design problem

Properties	Formulations
Objective function	$f(\mathbf{X}) = (x_3 + 2)x_2x_1^2$
Constraints	$g_1(\mathbf{X}) = 1 - \frac{x_2^3 x_3}{71785x_1^4} \leq 0$ $g_2(\mathbf{X}) = \frac{4x_2^2 - x_1x_2}{125666(x_2x_1^3 - x_1^4)} + \frac{1}{5108x_1^2} \leq 0$ $g_3(\mathbf{X}) = 1 - \frac{140.45x_1}{x_2^2 x_3} \leq 0$ $g_4(\mathbf{X}) = \frac{x_1 + x_2}{1.5} \leq 0$
Design variables	$0.05 \leq x_1 \leq 2$ $0.25 \leq x_2 \leq 1.3$ $2 \leq x_3 \leq 15$

Table 4. The optimal result for T/CS design problem

Algorithms	TLBO-FB	TLBO-EQFB	BOA-FB	BOA-EQFB	HHO-FB	HHO-EQFB
Design Variables						
x_1	0.052041	0.051947	0.054480	0.052816	0.052989	0.052552
x_2	0.365165	0.362953	0.427653	0.383657	0.388826	0.377818
x_3	10.816579	10.932596	8.086149	9.923457	9.628137	10.151806
Optimal Results						
Best	0.012675	0.012666	0.012802	0.012761	0.012695	0.012679
Mean	0.013021	0.129998	0.132579	0.131864	0.131402	0.130663
Std.	1.25E-04	9.81E-05	3.74E-02	2.90E-02	8.54E-03	6.00E-03
OFEs	2140	1620	2860	2500	2440	2100
Constraints						
$g_1(\mathbf{X})$	-0.000325	-0.000002	-0.000079	-0.003221	-0.000073	-0.000001
$g_2(\mathbf{X})$	-0.000174	-0.000004	-0.000004	-0.001658	-0.000046	-0.000037

$g_3(\mathbf{X})$	-4.067563	-4.065918	-4.174091	-4.078530	-4.112751	-4.093337
$g_4(\mathbf{X})$	-0.721863	-0.723400	-0.678578	-0.709018	-0.705457	-0.713087

4.2.2 Hydro-static thrust bearing (HSTB) design problem

Hydro-static thrust bearing design problem which is mentioned by Siddall is a minimization problem [34]. In the current problem, it is required to minimize the power loss in the system during its operation. The system is presented schematically in Figure 5 and aims to withstand a certain load while supplying an axial support. There are four design variables $\mathbf{X} = [x_1, x_2, x_3, x_4]$ for the current problem, where x_1 is the bearing step radius (R), x_2 is the recess radius (R_0), x_3 is the oil viscosity (μ), x_4 is the flow rate (Q) [34]. This problem is subjected to seven constraints such as physical constraints, oil temperature rise, load carrying capacity, oil film thickness and inlet oil pressure. The objective function, proper constraints and design variables of the current problem are listed in Table 5.

Attained outcomes for the HSTB design problem are presented in Table 6. The optimal solutions reveal that the TLBO-FB method finds the most accurate solution after TLBO-EQFB approach. Additionally, the standard deviation values illustrate the higher stability of the TLBO-EQFB in comparison with the other combinations. According to the OFEs, the TLBO-EQFB by eliminating the ineffective iterations more effectively requires lower computational cost than other combine techniques.

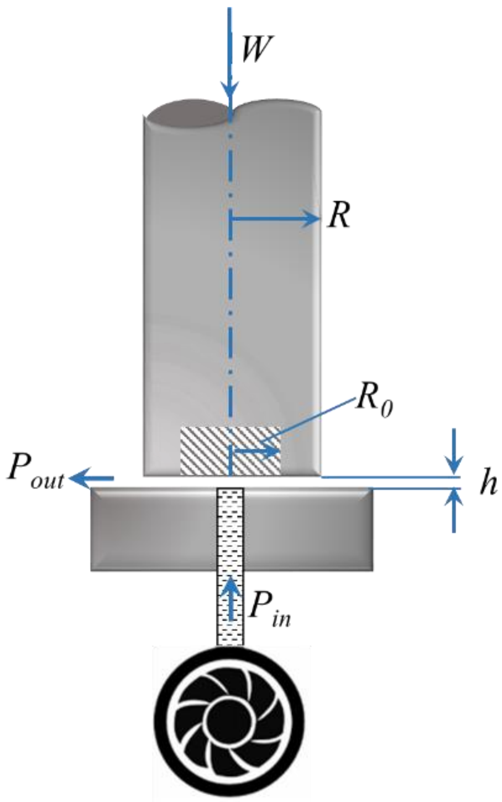


Figure 5. Schematic view of HSTB system

Table 5. Objective function, constraints, and design variables of HSTB design problem

Properties	Formulations
Objective function	$f(\mathbf{X}) = \frac{x_4 P_{in}}{0.7} + E_f$
Constraints	$g_1(\mathbf{X}) = W - 101000 \geq 0$ $g_2(\mathbf{X}) = 1000 - P_{in} \geq 0$ $g_3(\mathbf{X}) = 50 - \Delta\mathrm{T} \geq 0$ $g_4(\mathbf{X}) = h - 0.001 \geq 0$ $g_5(\mathbf{X}) = x_1 - x_2 \geq 0$ $g_6(\mathbf{X}) = 0.001 - \frac{0.0307}{386.4\, P_{in}}\left(\frac{x_4}{2\pi x_1 h}\right) \geq 0$ $g_7(\mathbf{X}) = 5000 - \frac{W}{\pi(x_1^2 - x_2^2)} \geq 0$
Design variables	$1.00 \leq x_1 \leq 16.00$ $1.00 \leq x_2 \leq 16.00$ $1 \times 10^{-6} \leq x_3 \leq 16 \times 10^{-6}$ $1.00 \leq x_4 \leq 16.00$
Other parameters	$W = \frac{\pi P_{in}}{2} \frac{x_1^2 - x_2^2}{\ln \frac{x_1}{x_2}}$ $P_{in} = \frac{6 x_3 x_4}{\pi h^3} \ln \frac{x_1}{x_2}$ $E_f = 143.3076\, x_4 \Delta T$ $\Delta T = 2(10^{P_{out}} - 560)$ $P_{out} = \frac{\log_{10}\log_{10}(8.122 \times 10^6 x_3 + 0.8) - 10.04}{-3.55}$ $h = \left(\frac{1500\pi}{60}\right)^2 \frac{2\pi x_3}{E_f}\left(\frac{x_1^4}{4} - \frac{x_2^4}{4}\right)$

Table 6. The optimal results for HSTB design problem

Algorithms	TLBO-FB	TLBO-EQFB	BOA-FB	BOA-EQFB	HHO-FB	HHO-EQFB
Design Variables						
x_1	5.9567	5.9558	6.7343	6.2874	5.9571	5.9632
x_2	5.3892	5.3890	6.1726	5.7507	5.3873	5.3918
x_3	5.40×10^{-6}	5.36×10^{-6}	5.57×10^{-6}	6.81×10^{-6}	7.25×10^{-6}	5.45×10^{-6}
x_4	2.3016	2.2965	3.1293	4.0707	4.3781	2.3391
Optimal Results						
Best	1632.1306	1625.2757	2029.2110	1901.8800	1852.3102	1640.2114
Mean	1658.2200	1644.4832	2100.5526	1891.8748	1900.6022	1701.4541
Std.	2.61E+01	9.45E+00	6.88E+01	6.51E+01	6.01E+01	5.00E+01
OFEs	8220	6560	11020	9840	9800	7440
Constraints						
$g_1(\mathbf{X})$	34.8165	27.1305	30520.5920	4948.9468	411.2569	52.6320
$g_2(\mathbf{X})$	0.0000	0.0001	204.2231	68.5086	0.0001	0.0001
$g_3(\mathbf{X})$	0.0546	0.1873	2.6812	19.1182	0.0608	0.0592
$g_4(\mathbf{X})$	0.0003	0.0014	0.0005	0.0007	0.0004	0.0003
$g_5(\mathbf{X})$	0.5675	0.5722	0.5617	0.5367	0.5701	0.5831

$g_6(\mathbf{X})$	0.0009	0.0010	0.0009	0.0009	0.0009	0.0009
$g_7(\mathbf{X})$	9.4110	12.5287	4003.0724	3837.2719	9.5611	11.2467

4.2.3 Multiple disc clutch brake (MDCB) design problem

In the current problem, it is desired to minimize the weight of the multiple disc clutch brake, which is schematically presented in Figure 6. There are five design variables $\mathbf{X} = [x_1, x_2, x_3, x_4, x_5]$ for the current problem, where x_1 is the inner radius (r_i), x_2 is the outer radius (r_o), x_3 is the thickness of discs (t), x_4 is actuating force (F), x_5 is the number of friction surfaces (Z). It should be noted that all the design variables are discrete. Considering the geometry and operating requirements, there are eight constraints such as temperature, relative speed of the slip–stick, shear stress, stopping time and physical constraints [35]. The objective function, proper constraints and design variables of the current problem are listed in Table 7.

Optimal results obtained for the MDCB design problem are given in Table 8. Based on the statistical information (i.e., Std. value) reported in this table the TLBO-EQFB has a higher stability than other techniques. Also, the accuracy of the TLBO-EQFB method is observable from the optimal solutions presented in the table. Additionally, the lower computational cost among the techniques belongs to the TLBO-EQFB and HHO-EQFB techniques, respectively.

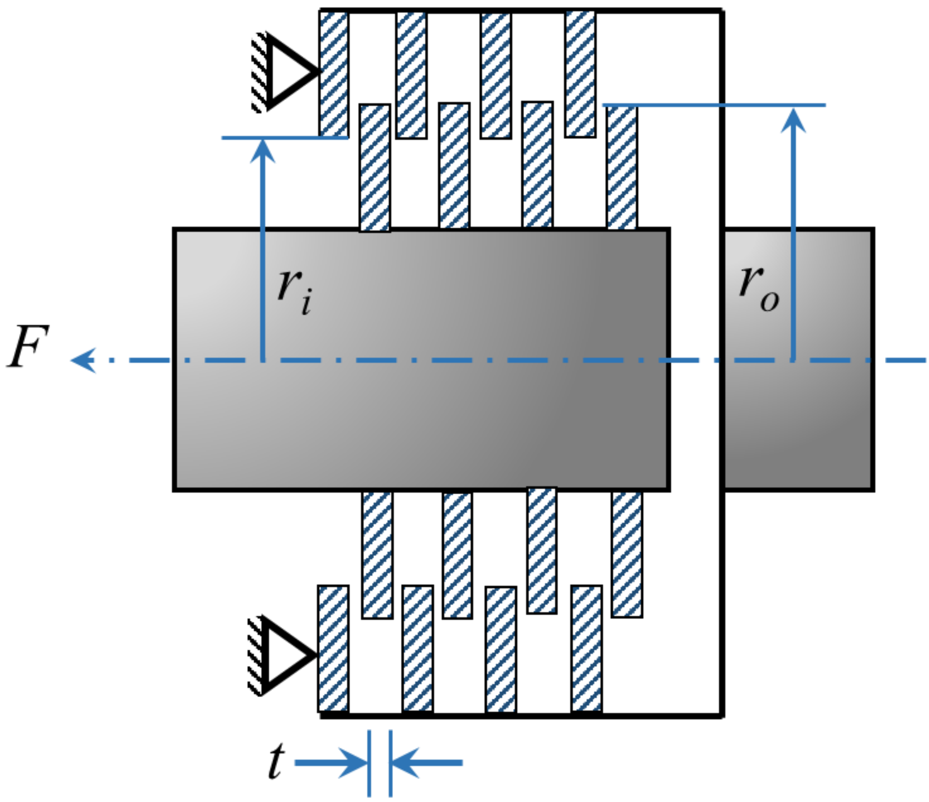


Figure 6. Schematic view of MDCB system

Table 7. Objective function, constraints, and design variables of MDCB design problem

Properties	Formulations
Objective function	$f(\mathbf{X}) = \pi(x_2^2 - x_1^2)(x_5 + 1)0.0000078x_3$
Constraints	$g_1(\mathbf{X}) = x_2 - x_1 - 20 \geq 0$ $g_2(\mathbf{X}) = 30 - (x_5 + 1)(x_3 + 0.5) \geq 0$ $g_3(\mathbf{X}) = 1 - P_{rz} \geq 0$ $g_4(\mathbf{X}) = 10000 - P_{rz}v_{sr} \geq 0$ $g_5(\mathbf{X}) = 10000 - v_{sr} \geq 0$ $g_6(\mathbf{X}) = 15 - T \geq 0$ $g_7(\mathbf{X}) = M_h - 60 \geq 0$ $g_8(\mathbf{X}) = T \geq 0$
Design variables	$x_1 \epsilon [60,61,62, \ldots ,79,80]$ $x_2 \epsilon [90,91,92, \ldots ,109,110]$ $x_3 \epsilon [1.0,1.5,2.0,2.5,3.0]$ $x_4 \epsilon [600,610,620, \ldots ,990,1000]$ $x_5 \epsilon [2,3,4, \ldots ,8,9]$
Other parameters	$M_h = \frac{2}{3}\frac{1}{2}x_4x_5\frac{x_2^3 - x_1^3}{x_2^2 - x_1^2}$ $P_{rz} = \frac{2}{3}\frac{x_4}{\pi(x_2^2 - x_1^2)}$ $v_{sr} = \frac{500\pi(x_2^3 - x_1^3)}{90(x_2^2 - x_1^2)}$ $T = \frac{13750\pi}{30(M_h + 3000)}$

Table 8. The optimal results for MDCB design problem

Algorithms	TLBO-FB	TLBO-EQFB	BOA-FB	BOA-EQFB	HHO-FB	HHO-EQFB
Design Variables						
x_1	70	70	70	76	70	70
x_2	90	90	90	96	90	90
x_3	1.0	1	1.5	1.0	1.0	1.0
x_4	910	600	1000	840	900	810
x_5	3	2	3	3	3	3
Optimal Results						
Best	0.31366	0.23524	0.47048	0.33718	0.31366	0.31366
Mean	0.32051	0.27561	0.49906	0.37600	0.32887	0.32650
Std.	3.44E-02	2.01E-02	8.47E-02	7.90E-02	3.71E-02	3.07E-02
OFEs	4560	4200	5060	4900	4600	4420
Constraints						
$g_1(\mathbf{X})$	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
$g_2(\mathbf{X})$	24.00000	25.50000	22.00000	24.00000	24.00000	24.00000
$g_3(\mathbf{X})$	0.90948	0.94032	0.90053	0.92227	0.91047	0.91943
$g_4(\mathbf{X})$	9809	9874	9790	9824	9811	9830
$g_5(\mathbf{X})$	7894	7894	7894	7738	7894	7894
$g_6(\mathbf{X})$	14.98723	14.97190	14.98835	14.98713	14.98709	14.98570
$g_7(\mathbf{X})$	109708	48190	120565	108788	108502	97646
$g_8(\mathbf{X})$	0.01277	0.02809	0.01165	0.01287	0.01291	0.01429

4.2.4 Car side impact (CSI) design problem

In the current section, the overall weight of a car is desired to be minimized as an engineering optimization problem. In this problem, mixed design variables and constraints are considered. There are seven design variables $\mathbf{X} = [x_1, x_2, x_3, x_4, x_5, x_6, x_7]$ for the current problem. The design variables of a car side impact, which is schematically presented in Figure 7 [36], are B-Pillar reinforcement, thicknesses of B-Pillar inner, cross members, floor side inner, door beltline reinforcement, door beam, roof rail, materials of floor side and floor side inner and barrier height, and hitting position [37]. The objective function, proper constraints and design variables of this problem are given in Table 9.

Optimal results obtained and statistical data information of the optimization process are reported and illustrated in Table 10. According to the optimal outcomes, the TLBO-EQFB obtained the most accurate solution in comparison with other algorithms. This observation shows that the EQFB is well adapted by the TLBO. Also, the standard deviation values reveal that the proposed EQFB mechanism boosts the stability of the algorithms more than the case of using Fly-Back method as a constraint handling method. In addition, the number of objective function evaluations (OFEs) show that an effective constraint handling mechanism highly reduces unnecessary iterations.

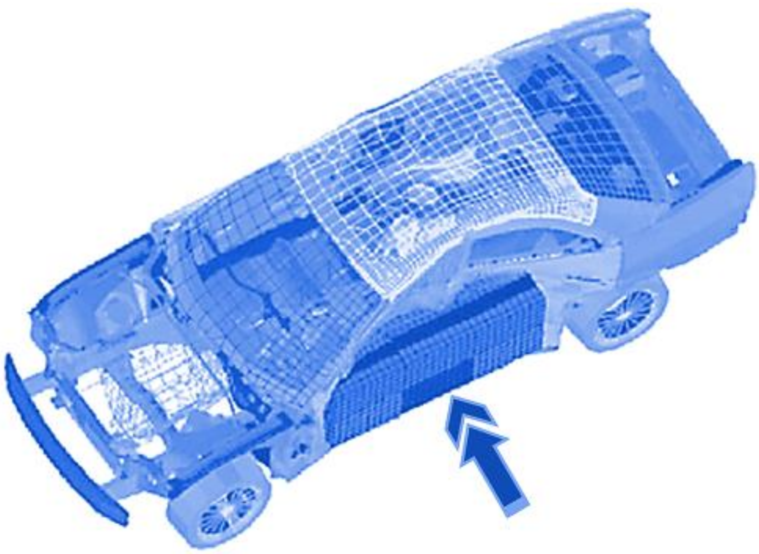


Figure 7. Finite element model of CSI **[36]**

Table 9. Objective function, constraints, and design variables of CSI design problem

Properties	Formulations
Objective function	$f(\mathbf{X}) = 1.98 + 4.90x_1 + 6.67x_2 + 6.98x_3 + 4.01x_4 + 1.78x_5 + 2.73x_7$
Constraints	$g_1(\mathbf{X}) = 1.16 - 0.3717x_2x_4 - 0.00931x_2x_{10} - 0.484x_3x_9 + 0.01343x_6x_{10} - 1 \leq 0$ $g_2(\mathbf{X}) = 0.261 - 0.0159x_1x_2 - 0.188x_1x_8 - 0.019x_2x_7 + 0.0144x_3x_5 + 0.0008757x_5x_{10} + 0.08045x_6x_9 + 0.00139x_8x_{11} + 0.00001575x_{10}x_{11} - 0.32 \leq 0$ $g_3(\mathbf{X}) = 0.214 + 0.00817x_5 - 0.131x_1x_8 - 0.0704x_1x_9 + 0.03099x_2x_6 - 0.018x_2x_7 + 0.0208x_3x_8 + 0.121x_3x_9 - 0.00364x_5x_6 + 0.0007715x_5x_{10} - 0.000535x_6x_{10} + 0.00121x_8x_{11} - 0.32 \leq 0$ $g_4(\mathbf{X}) = 0.074 - 0.061x_2 - 0.163x_3x_8 + 0.001232x_3x_{10} - 0.166x_7x_9 + 0.227x_2^2 - 0.32 \leq 0$ $g_5(\mathbf{X}) = 28.98 + 3.818x_3 - 4.2x_1x_2 + 0.0207x_5x_{10} + 6.63x_6x_9 - 7.7x_7x_8 + 0.32x_9x_{10} - 32 \leq 0$ $g_6(\mathbf{X}) = 33.86 + 2.95x_3 + 0.1792x_{10} - 5.057x_1x_2 - 11.0x_2x_8 - 0.0215x_5x_{10} - 9.98x_7x_8 + 22.0x_8x_9 - 32 \leq 0$ $g_7(\mathbf{X}) = 46.36 - 9.9x_2 - 12.9x_1x_8 + 0.1107x_3x_{10} - 32 \leq 0$ $g_8(\mathbf{X}) = 4.72 - 0.5x_4 - 0.19x_2x_3 - 0.0122x_4x_{10} + 0.009325x_6x_{10} + 0.000191x_{11}^2 - 4 \leq 0$ $g_9(\mathbf{X}) = 10.58 - 0.674x_1x_2 - 1.95x_2x_8 + 0.02054x_3x_{10} - 0.0198x_4x_{10} + 0.028x_6x_{10} - 9.9 \leq 0$ $g_{10}(\mathbf{X}) = 16.45 - 0.489x_3x_7 - 0.843x_5x_6 + 0.0432x_9x_{10} - 0.0556x_9x_{11} - 0.000786x_{11}^2 - 15.7 \leq 0$
Design variables	$0.5 \leq x_1 - x_7 \leq 1.5$ $0.192 \leq x_8 - x_9 \leq 0.345$ $-30 \leq x_{10} - x_{11} \leq 30$

Table 10. The optimal results for CSI design problem

Algorithms	TLBO-FB	TLBO-EQFB	BOA-FB	BOA-EQFB	HHO-FB	HHO-EQFB
Design Variables						
x_1	0.500000	0.500000	0.503762	0.500000	0.501908	0.500143
x_2	1.107071	1.117324	1.104463	1.117704	1.128121	1.117641
x_3	0.500000	0.500000	0.520328	0.500203	0.500570	0.500318
x_4	1.319352	1.301078	1.334846	1.308179	1.286175	1.301739
x_5	0.500013	0.500649	0.509665	0.508999	0.500000	0.500000
x_6	1.499999	1.500000	1.500000	1.499727	1.500000	1.499536
x_7	0.500000	0.500000	0.500000	0.500000	0.500000	0.500265
x_8	0.344999	0.344989	0.337695	0.336619	0.344628	0.345000
x_9	0.344987	0.192000	0.331929	0.344165	0.192000	0.192000
x_{10}	-21.264178	-19.444029	-21.480509	-20.292671	-17.505021	-19.396166
x_{11}	-0.191181	-0.233398	-0.489496	-0.448730	-0.603797	-0.636964
Optimal Results						
Best	22.849788	22.846029	23.072028	22.893319	22.870457	22.853282
Mean	22.867418	22.854449	23.695253	23.001258	23.002963	23.001018
Std.	1.10E-01	0.91E-01	2.21E-01	1.18E-01	1.74E-01	1.30E-01

OFEs	3850	3520	5120	4760	4980	4540
Constraints						
$g_1(\mathbf{X})$	-0.675598	-0.616251	-0.683436	-0.664365	-0.594626	-0.616064
$g_2(\mathbf{X})$	-0.074855	-0.092717	-0.076098	-0.074067	0.092252	-0.092789
$g_3(\mathbf{X})$	-0.064646	-0.068915	-0.064130	-0.064432	-0.069539	-0.069109
$g_4(\mathbf{X})$	-0.105169	-0.086797	-0.106430	-0.099114	-0.080773	-0.086661
$g_5(\mathbf{X})$	-3.900768	-4.272296	-3.877525	-3.779986	-4.160981	-4.270059
$g_6(\mathbf{X})$	-6.350597	-7.269600	-6.354396	-6.174509	-7.015966	-7.264739
$g_7(\mathbf{X})$	-0.002219	-0.002914	-0.005989	-0.000117	-0.009738	-0.004793
$g_8(\mathbf{X})$	-0.000003	-0.000009	-0.007213	-0.000201	-0.000485	-0.000221
$g_9(\mathbf{X})$	-0.993853	-0.963629	-0.986325	-0.965343	-0.929157	-0.962433
$g_{10}(\mathbf{X})$	-0.317787	-0.164148	-0.320862	-0.309093	-0.143674	-0.158846

4.2.5 Rolling element bearing (REB) design problem

As another engineering problem, a rolling element bearing (REB) design is considered, in which dynamic load carrying capacity is desired to be maximized [38]. The schematic illustration of this system is presented in Figure 8. There are ten geometric design variables $\mathbf{X} = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}]$ for the current problem, where x_1 is the pitch diameter (D_m), x_2 is the ball diameter (D_b), x_3 is the number of balls (Z), x_4 and x_5 are the inner and outer raceway curvature coefficients, respectively (f_i, f_o), x_6 and x_7 are the minimum and maximum ball diameter limiters, respectively ($K_{D\,\min}$, $K_{D\,\max}$), x_8 is the parameter for outer ring strength consideration (ε), x_9 is the parameter for mobility condition (e), x_{10} is the bearing width limiter (ζ). It should be noted that Z is a discrete design variable, and other variables are continuous design variables. Also, there are nine proper constraints according to manufacturing conditions and kinematic considerations. Objective function, constraints and design variables of the current problem are given in Table 11.
Optimal results attained for the REB design problem using the selected algorithms are shown in Table 12. According to the optimal solution and Std. values the TLBO-EQFB puts forward a promising performance from both accuracy and stability aspects. In addition, the number of objective function evaluations (OFEs) reveal that the TLBO-EQFB method exceeds the other approaches in terms of computational cost.

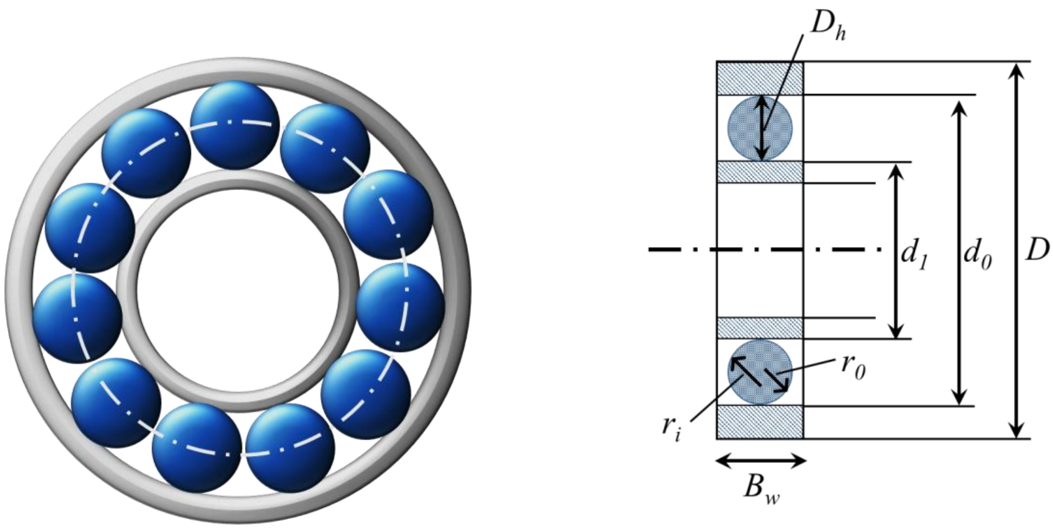


Figure 8. Schematic view of REB system

Table 11. Objective function, constraints, and design variables of REB design problem

Properties	**Formulations**	
Objective function	$f(\mathbf{X}) = \frac{1}{C_d}$ where $C_d = \begin{cases} f_c x_3^{\frac{2}{3}} x_2^{1.8} & if \quad x_2 \leq 25.4mm \\ 3.647 f_c x_3^{\frac{2}{3}} x_2^{1.4} & if \quad x_2 > 25.4mm \end{cases}$	
Constraints	$g_1(\mathbf{X}) = x_3 - \frac{\varphi_o}{2\sin^{-1}\left(\frac{x_2}{x_1}\right)} - 1 \leq 0$ $g_2(\mathbf{X}) = x_6(70) - 2x_2 \leq 0$ $g_3(\mathbf{X}) = 2x_2 - x_7(70) \leq 0$ $g_4(\mathbf{X}) = x_{10}(30) - x_2 \leq 0$ $g_5(\mathbf{X}) = 0.5(250) - x_1 \leq 0$ $g_6(\mathbf{X}) = x_1 - (0.5 + x_9)(250) \leq 0$ $g_7(\mathbf{X}) = x_8 x_2 - 0.5(160 - x_1 - x_2) \leq 0$ $g_8(\mathbf{X}) = 0.515 - x_4 \leq 0$ $g_9(\mathbf{X}) = 0.515 - x_5 \leq 0$	
Design variables	$0.5(250) \leq x_1 \leq 0.6(250)$ $0.15(70) \leq x_2 \leq 0.45(70)$ $4 \leq x_3 \leq 50$ $0.515 \leq x_4 \leq 0.6$ $0.515 \leq x_5 \leq 0.6$	$0.4 \leq x_6 \leq 0.5$ $0.6 \leq x_7 \leq 0.7$ $0.02 \leq x_8 \leq 0.1$ $0.3 \leq x_9 \leq 0.4$ $0.6 \leq x_{10} \leq 0.85$
Other parameters	$f_c = 37.91\left[1+\left\{1.04\left(\frac{1-\gamma}{1+\gamma}\right)^{1.72}\left(\frac{x_4(2x_5-1)}{x_5(2x_4-1)}\right)^{0.41}\right\}^{\frac{10}{3}}\right]^{-0.3} \times \left[\frac{\gamma^{0.3}(1-\gamma)^{1.39}}{(1-\gamma)^{\frac{1}{3}}}\right]\left[\frac{2x_4}{2x_4-1}\right]^{0.41}$ $x = \left[\left\{\frac{(70)}{2}+3\left(\frac{T}{4}\right)\right\}^2 + \left\{\frac{160}{2}-\frac{T}{4}-x_2\right\}^2 - \left\{\frac{90}{2}+\frac{T}{4}\right\}^2\right]$ $y = 2\left\{\frac{(70)}{2}+3\left(\frac{T}{4}\right)\right\}\left\{\frac{160}{2}-\frac{T}{4}-x_2\right\}$ $\varphi_o = 2\pi - \cos^{-1}\left(\frac{x}{y}\right)$ $\gamma = \frac{x_2}{x_1}$, $x_4 = \frac{11.034}{x_2}$, $x_5 = \frac{11.034}{x_2}$, $T = 70 - 2x_2$ $D = 160$, $d = 90$, $B_w = 30$, $r_i = r_o = 11.034$	

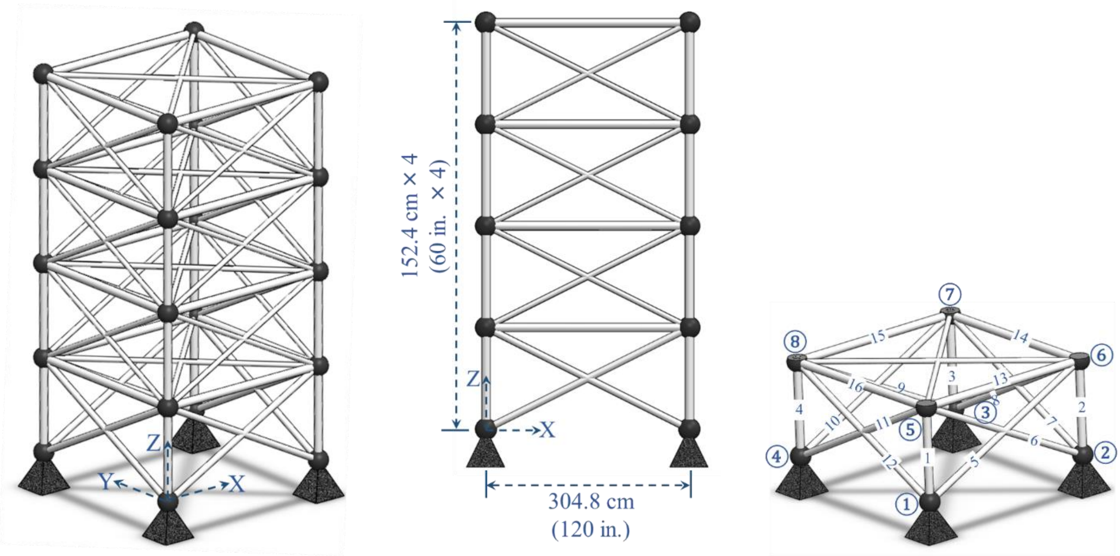
Table 12. The optimal results for REB design problem

Algorithms	TLBO-FB	TLBO-EQFB	BOA-FB	BOA-EQFB	HHO-FB	HHO-EQFB
Design Variables						
x_1	125.7190	125.7200	125.0000	125.7150	125.7211	125.7153
x_2	21.4250	21.4250	21.0000	21.4230	21.4233	21.4233
x_3	11	11	11	11	11	11
x_4	0.515	0.515	0.515	0.515	0.515	0.515
x_5	0.515	0.5115	0.515	0.515	0.515	0.515
x_6	0.4000	0.4212	0.4000	0.4888	0.4015	0.4800
x_7	0.7000	0.6997	0.6000	0.6278	0.6590	0.6278

x_8	0.300000	0.300000	0.300000	0.300149	0.300032	0.300100
x_9	0.1000	0.0953	0.0505	0.0973	0.0400	0.0973
x_{10}	0.6261	0.6455	0.6000	0.6461	0.6000	0.6459
Optimal Results						
Best	1.08924E-05	1.08924E-05	1.13195E-05	1.08943E-05	1.08941E-05	1.08940E-05
Mean	1.16851E-05	1.12909E-05	1.22194E-05	1.22120E-05	1.20443E-05	1.16876E-05
Std.	2.17E-06	1.09E-06	7.22E-06	6.14E-06	4.14E-06	4.00E-06
OFEs	11,520	10,200	31,520	29,600	18,940	16,440
Constraints						
$g_1(\mathbf{X})$	-5.4048	-5.4049	-5.6427	-5.4058	-5.4064	-5.4057
$g_2(\mathbf{X})$	-14.8500	-13.3660	-14.0000	-8.6299	-14.7416	-14.7416
$g_3(\mathbf{X})$	-6.1499	-6.1289	0.0000	-5.3000	-3.2834	-3.2834
$g_4(\mathbf{X})$	-2.6420	-2.0600	-3.0000	-2.0399	-3.4233	-2.0463
$g_5(\mathbf{X})$	-0.7189	-0.7199	0.0000	-1.7175	-0.7211	-0.7153
$g_6(\mathbf{X})$	-24.2810	-23.1049	-12.6250	-1.7175	-9.2789	-23.6097
$g_7(\mathbf{X})$	-0.0005	0.0000	-0.7000	-0.0009	-0.0001	-0.0016
$g_8(\mathbf{X})$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$g_9(\mathbf{X})$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

4.2.6 72-bar spatial truss structure

In the current problem, it is targeted to minimize the weight of a 72-bar spatial truss structure. The schematic view of the system from different points is given in Figure 9. The elements of the truss are listed in sixteen groups according to a symmetric categorization principle [1]. Allowable displacement limits in all directions for all nodes is ±0.25 in. The properties of the utilized material are E=10,000 ksi (module of elasticity) and ρ=0.1 lb/in.^{3} (density). The limit of the compressive and tensile stress for all elements is ±25 ksi. The lower and upper bound of the design variables are considered continuously between 0.1 in.^{2} and 3.0 in.^{2}. For the current problem two independent loading conditions are considered, where the 17, 18, 19, and 20 nodes are exposed to a 5.0 kips load in the negative direction of the z axis and the 17 node is exposed to 5.0 kips in the positive direction of x axis, 5.0 kips in the positive direction of y axis, and 5.0 kips load in the negative direction of the z axis [39].



3D view side view partial view

Figure 9. The 72-bar spatial truss structure

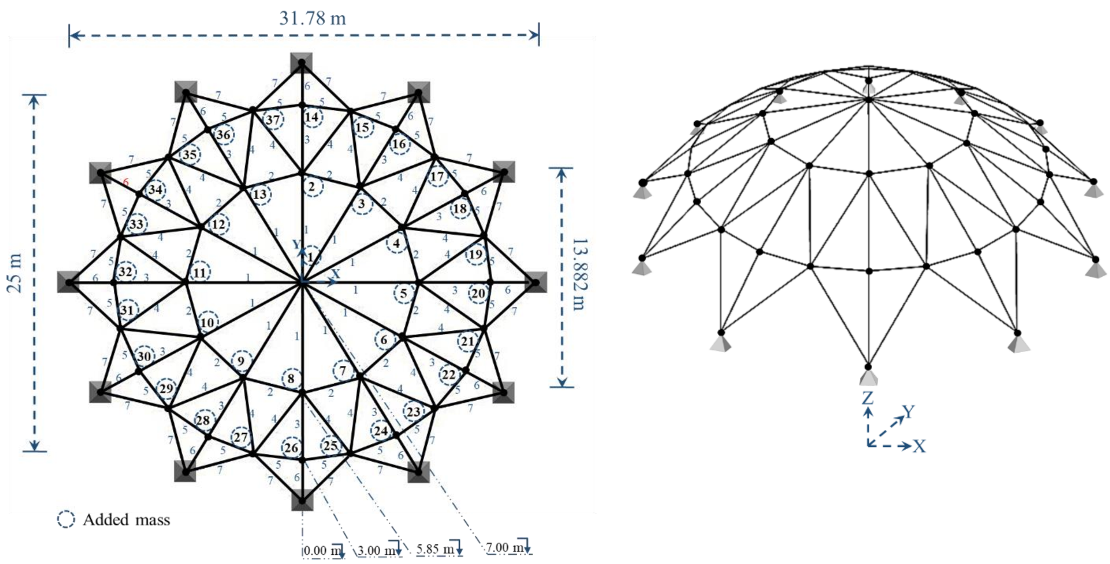
The problem is solved with the selected optimization algorithms and the achieved optimal results are presented in Table 13. According to the reported data, the most accurate optimal solution belongs to the TLBO-EQFB combination. Additionally, according to the Std. values, the stability of the TLBO-EQFB is higher than other optimization algorithms. Objective function values (OFEs) reveal that the EQFB module decreases the computational cost of the optimization process.

Table 13. The optimal result for 72-bar spatial truss system

Algorithms	TLBO-FB	TLBO-EQFB	BOA-FB	BOA-EQFB	HHO-FB	HHO-EQFB
Design Variables (in.^{2})						
A_1- A_4	1.8577	1.8519	1.7430	1.8600	1.8600	1.8364
A_5- A_{12}	0.5059	0.5141	0.5181	0.5209	0.5210	0.5021
A_{13}- A_{16}	0.1000	0.1000	0.1000	0.1000	0.1000	0.1000
A_{17}- A_{18}	0.1000	0.1000	0.1000	0.1000	0.1000	0.1004
A_{19}- A_{22}	1.2476	1.2819	1.3079	1.2710	1.2710	1.2522
A_{23}- A_{30}	0.5269	0.5091	0.5190	0.5090	0.5090	0.5033
A_{31}- A_{34}	0.1000	0.1000	0.1000	0.1000	0.1000	0.1002
A_{35}- A_{36}	0.1012	0.1000	0.1000	0.1000	0.1000	0.1002
A_{37}- A_{40}	0.5209	0.5312	0.5140	0.4849	0.4850	0.5729
A_{41}- A_{48}	0.5172	0.5173	0.5460	0.5010	0.5010	0.5498
A_{49}- A_{52}	0.1004	0.1000	0.1000	0.1000	0.1000	0.1004
A_{53}- A_{54}	0.1005	0.1000	0.1090	0.1000	0.1000	0.1001
A_{55}- A_{58}	0.1565	0.1560	0.1610	0.1680	0.1680	0.1576
A_{59}- A_{66}	0.5507	0.5572	0.5089	0.5839	0.5840	0.5222
A_{67}- A_{70}	0.3922	0.4259	0.4970	0.4330	0.4330	0.4356
A_{71}- A_{72}	0.5922	0.5271	0.5620	0.5200	0.5200	0.5972
Optimal Results						
Best weight (lb)	379.8511	379.7687	381.9100	380.8135	380.6200	380.4417
Mean weight (lb)	381.2522	380.9963	383.5940	382.0971	382.0004	381.9970
Std. (lb)	1.9842	1.1153	3.2506	2.9588	2.3380	2.1778
OFEs	6720	6400	8500	8120	8040	7900

4.2.7 120-bar dome structure with dynamic constraints

In the current problem, the weight of a 120-bar dome structure subjected to multiple natural frequency constraints is aimed to be minimized. The schematic presentation of the system is shown in Figure 10. The properties of the utilized material are E=210 MPa (module of elasticity) and ρ=7971.81 kg/m^3 (density). The elements of the structure are categorized into seven independent groups. For the current problem the nodes 1, 2-13 and 14-37 are exposed to a non-structural mass of 3000 kg, 500 kg, and 100 kg, respectively. Two first natural frequencies $\Phi_1 \geq 9$ Hz and $\Phi_2 \geq 11$ Hz. are the limitations of the current problem. The lower and upper bound of the design variables are considered continuously between 1 cm.^{2} and 129.3 cm.^{2} [30].



Top view 3D view

Figure 10. The 120-bar dome structure

Optimal solutions acquired by the selected approaches are given in Table 14. Based on the presented outcomes and statistical data, the proposed TLBO-EQFB in terms of accuracy and stability is superior to other techniques. Also, the constraints of the problem are not violated, this means that the algorithms are able to handle the dynamic constraints. The EQFB module improves the algorithms’ performance. This observation indicates that the constraint-handling module EQFB is well adopted to all of selected algorithms. The number of objective function evaluations (OFEs) indicate that the unnecessary iterations are reduced, and this decreases the computational cost of the optimization process. Also, the algorithm does not require separate treatment for each type of violation.

Table 14. The optimal result for 120-bar dome structure

Algorithms	TLBO-FB	TLBO-EQFB	BOA-FB	BOA-EQFB	HHO-FB	HHO-EQFB
Design Variables (cm^2)						
A_1	19.5093	19.4867	19.6070	18.9791	20.2631	19.5107
A_2	40.3911	40.4260	41.2901	41.0046	39.2942	40.3368
A_3	10.6066	10.6099	11.1360	10.6124	9.9892	10.6274
A_4	21.1368	21.0910	21.0253	21.8776	20.5630	21.1037
A_5	9.8134	9.8491	10.0601	10.7519	9.6031	9.8450
A_6	11.7798	11.7639	12.7582	12.4286	11.7384	11.7369
A_7	14.8192	14.8556	15.4144	13.7772	15.8771	14.8595
Optimal Results						
Best weight (kg)	8707.2802	8707.2715	8790.4812	8748.2105	8724.9722	8707.2808
Mean weight (kg)	8708. 6005	8708.0009	8793.6752	8751.6634	8727.0025	8709.8960
Std. (kg)	2.1259	1.9763	5.0021	4.1254	3.8996	3.7561
OFEs	4240	4100	6040	5900	5280	5020

4.2.8 Non-parametric statistical tests for constrained engineering problems

To investigate the performance of the selected algorithms in comparison with each other, a non-parametric statistical test is approved. For this aim, the Friedman rank test is implemented. The acquired outcomes are reported in Table 15. Based on the test results the TLBO-EQFB in comparison with other algorithms ranks in a better position from both stability and accuracy features.

Table 15. The Friedman rank test for mean and Std. values for engineering problems

Method	Test for optimal mean value			Test for optimal Std. value		
	Friedman value	Normalized value	Rank	Friedman value	Normalized value	Rank
TLBO-FB	13	0.615384	2	15	0.466667	2
TLBO-EQFB	8	1.000000	1	7	1.000000	1
BOA-FB	42	0.190476	6	42	0.166667	6
BOA-EQFB	33	0.242424	5	33	0.212121	5
HHO-FB	30	0.266666	4	29	0.241379	4
HHO-EQFB	21	0.380952	3	21	0.333333	3

5. Conclusion

In the current study, a new constraint handling mechanism is introduced to incorporate the restrictions of the restricted optimization problems into the search process. For this aim, initially the concept of weighted agent is utilized to reinforce the quadratic approximation search approach. Subsequently, the proposed reinforced strategy is integrated with Fly-Back (FB) mechanism to develop an efficient and capable constraint handling approach so-called Enhanced Quadratic Fly-Back (EQFB) technique. On the one hand, this approach leverages the weighted average to use information collected by entire population more efficiently (This is possible because the weighted agent shares the data collected by all agents). On the other hand, taking advantage of the logic of the quadratic approximation strategy provides more reasonable alternatives for violated agents. These advantages enable the EQFB to enhance the main optimization algorithm's efficiency in scanning the problem's domain. To assess the effect of EQFB method on the search performance of the metaheuristic algorithms, it is combined with three different methods. The selected pilot methods are Teaching and Learning Based Optimization (TLBO), Harris Hawks Optimization (HHO), and Butterfly Optimization Algorithm (BOA). The performance of the proposed combined algorithms is tested on solving constrained mathematical, mechanical and structural optimization problems.

The optimal solutions obtained for all tested problems demonstrate that the EQFB module enhances the precision of the optimization algorithm and boosts its performance. The outcomes of carried out statistical test (i.e., for standard deviation values) demonstrate that the EQFB has an effective role in raising the stability of the optimization process. Furthermore, the number of required objective function evaluations (OFEs) indicates that the EQFB approach substantially decreases the number of ineffective iterations (i.e., iterations without improvement), resulting in a significant reduction in computational cost. This issue specialty is very important in complex engineering optimization problems in which the objective function evaluation is the most time-consuming part of the optimization process. Consequently, it should be noted that

EQFB is a stand-alone module that can be independently integrated with various optimization algorithms. As a future plan, it is targeted to test the performance of the EQFB on solving more complex and large-scale problems in the field of energy.

References

[1] Mortazavi, A., Moloodpoor, M., "Differential evolution method integrated with a fuzzy decision-making mechanism and Virtual Mutant agent: Theory and application", Applied Soft Computing 112 (2021) : 107808.

[2] Rajeev, S., Krishnamoorthy, C.S., "Discrete Optimization of Structures Using Genetic Algorithms", Journal of Structural Engineering 118(5) (1992).

[3] Yadav, A., Kumar, N., "Artificial electric field algorithm for engineering optimization problems", Expert Systems with Applications 149 (2020) : 113308.

[4] Moloodpoor, M., Mortazavi, A., "Simultaneous optimization of fuel type and exterior walls insulation attributes for residential buildings using a swarm intelligence", International Journal of Environmental Science and Technology (2022).

[5] Gandomi, A.H., Alavi, A.H., "An introduction of Krill Herd algorithm for engineering optimization", Journal of Civil Engineering and Management 22(3) (2016) : 302-310.

[6] Hayyolalam, V., Pourhaji Kazem, A.A., "Black Widow Optimization Algorithm: A novel meta-heuristic approach for solving engineering optimization problems", Engineering Applications of Artificial Intelligence 87 (2020) : 103249.

[7] Houssein, E.H., Saad, M.R., Hashim, F.A., Shaban, H., Hassaballah, M., "Lévy flight distribution: A new metaheuristic algorithm for solving engineering optimization problems", Engineering Applications of Artificial Intelligence 94 (2020) : 103731.

[8] Moloodpoor, M., Mortazavi, A., "Thermo-Economic optimization for saving energy in residential bildings using population-based optimization techniques", Journal of Construction Engineering, Management & Innovation 5(1) (2022) : 45-63.

[9] Lee, K.S., Geem, Z.W., "A new meta-heuristic algorithm for continuous engineering optimization: harmony search theory and practice", Computer Methods in Applied Mechanics and Engineering 194(36) (2005) : 3902-33.

[10] Liu, H., Cai, Z., Wang, Y., "Hybridizing particle swarm optimization with differential evolution for constrained numerical and engineering optimization", Applied Soft Computing 10(2) (2010) : 629-640.

[11] Rao, S.S., "Engineering optimization: theory and practice. 4th edition", John Wiley & Sons, Inc., Hoboken, New Jersey (2009).

[12] Moloodpoor, M., Mortazavi, A., Ozbalta, N., "Thermo-economic optimization of double-pipe heat exchanger using a compound swarm intelligence", Heat Tansfer Research 52(6) (2021) : 1-20.

[13] Zamani, H., Nadimi-Shahraki, M.H., Gandomi, A.H., "Starling murmuration optimizer: A novel bio-inspired algorithm for global and engineering optimization", Computer Methods in Applied Mechanics and Engineering 392 (2022) : 114616.

[14] Toğan, V., Mortazavi, A., "Sizing optimization of skeletal structures using teaching-learning based optimization", An International Journal of Optimization and Control: Theories & Applications (IJOCTA) 7(2) (2017) : 130-141.

[15] Mendi, F., Baskal, T., Külekci, M.K., "Application of Genetic Algorithm (GA) for Optimum Design of Module, Shaft Diameter and Bearing for Bevel Gearbox", Materials Testing 54(6) (2012) : 431-436.
[16] Liu, H., Duan, S., Luo, H., "A hybrid engineering algorithm of the seeker algorithm and particle swarm optimization", Materials Testing 64(7) (2022) : 1051-1089.
[17] Moloodpoor, M., Mortazavi, A., Ozbalta, N., "Performance Assessment of Parabolic Trough Collectors under Climatic Conditions of Izmir, Turkey: A Case Study", Heat Transfer Research 55(4) (2024) : 47-76.
[18] Abd Elaziz, M., Yousri, D., Mirjalili, S., "A hybrid Harris hawks-moth-flame optimization algorithm including fractional-order chaos maps and evolutionary population dynamics", Advances in Engineering Software 154 (2021) : 102973.
[19] Tsipianitis, A., Tsompanakis, Y., "Improved Cuckoo Search algorithmic variants for constrained nonlinear optimization", Advances in Engineering Software 149 (2020) : 102865.
[20] Zhengtong, H., Zhengqi, G., Xiaokui, M., Wanglin, C., "Multimaterial layout optimization of truss structures via an improved particle swarm optimization algorithm", Computers & Structures 222 (2019) : 10-24.
[21] Tejani, G.G., Savsani, V.J., Patel, V.K., Savsani, P.V., "Size, shape, and topology optimization of planar and space trusses using mutation-based improved metaheuristics", Journal of Computational Design and Engineering 5(2) (2018) : 198-214.
[22] Javidi, A., Salajegheh, E., Salajegheh, J., "Enhanced crow search algorithm for optimum design of structures", Applied Soft Computing 77 (2019) : 274-289.
[23] Rao, R.V., Savsani, V.J., Vakharia, D.P., "Teaching–learning-based optimization: A novel method for constrained mechanical design optimization problems", Computer-Aided Design 43(3) (2011) : 303-315.
[24] Heidari, A.A., Mirjalili, S., Faris, H., Aljarah, I., Mafarja, M., Chen, H., "Harris hawks optimization: Algorithm and applications", Future Generation Computer Systems 97 (2019) : 849-872.
[25] Arora, S., Singh, S., "Butterfly optimization algorithm: a novel approach for global optimization", Soft Computing 23(3) (2019) : 715-734.
[26] He, S., Prempain, E., Wu, Q.H., "An improved particle swarm optimizer for mechanical design optimization problems", Engineering Optimization 36(5) (2004) : 585-605.
[27] Mortazavi, A., "Bayesian Interactive Search Algorithm: A New Probabilistic Swarm Intelligence Tested on Mathematical and Structural Optimization Problems", Advances in Engineering Software 155 (2021) : 102994.
[28] Mortazavi, A., "Interactive fuzzy Bayesian search algorithm: A new reinforced swarm intelligence tested on engineering and mathematical optimization problems", Expert Systems with Applications 187 (2022) : 115954.
[29] Mortazavi, A., "Comparative assessment of five metaheuristic methods on distinct problems", Dicle University Journal of Engineering 10(3) (2019) : 879-898.
[30] Mortazavi, A., "The Performance Comparison of Three Metaheuristic Algorithms On the Size, Layout and Topology Optimization of Truss Structures", Mugla Journal of Science and Technology 5(2) (2019) : 28-41.

[31] Mortazavi, A., "Size and layout optimization of truss structures with dynamic constraints using the interactive fuzzy search algorithm", Engineering Optimization (2021) : 1-23.

[32] Guohua, W., Mallipeddi, R., Suganthan, P., "Problem Definitions and Evaluation Criteria for the CEC 2017 Competition and Special Session on Constrained Single Objective Real-Parameter Optimization", Nanyang Technological Universityr: Singapore (2016).

[33] Belegundu, A.D., "A study of mathematical programming methods for structural optimization", [Ph.D.]. Ann Arbor: The University of Iowa (1982).

[34] Siddall, J.N., "Optimal Engineering Design. Principles and Applications", CRC Press (1982).

[35] Osyczka, A.. "Evolutionary Algorithms for Single and Multicriteria Design Optimization", Heidelberg: Physica-Verlag (2002).

[36] Youn, B.D., Choi, K.K., Yang, R.J., Gu, L., "Reliability-based design optimization for crashworthiness of vehicle side impact", Structural and Multidisciplinary Optimization 26(3) (2004) : 272-283.

[37] Gandomi, A.H., Yang, X.S., Alavi, A.H., "Cuckoo search algorithm: a metaheuristic approach to solve structural optimization problems", Engineering with Computers 29(1) (2013) : 17-35.

[38] Gupta, S., Tiwari, R., Nair, S.B., "Multi-objective design optimisation of rolling bearings using genetic algorithms", Mechanism and Machine Theory 42(10) (2007) : 1418-1443.

[39] Mortazavi, A., "A fuzzy reinforced Jaya algorithm for solving mathematical and structural optimization problems", Soft Computing 28(3) (2024) : 2181-2206.