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An Investigation for Soliton Solutions of the Extended (2+1)-Dimensional Kadomtsev–Petviashvili Equation

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Genişletilmiş (2+1)-boyutlu Kadomtsev–Petviashvili Denkleminin Soliton Çözümlerinin Araştırılması

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Abstract

This article presents an investigation for soliton solutions of the extended (2+1)-dimensional Kadomtsev–Petviashvili equation which describes wave behavior in shallow water. We utilize the unified Riccati equation expansion method. By employing the powerful method, many soliton solutions are successfully derived, and it is verified by Wolfram Mathematica that the solutions satisfy the main equation. Additionally, Matlab is utilized to generate plots and examine the properties of the obtained solitons. The results reveal that the considered equation exhibits a wide range of soliton solutions, including dark, bright, singular, and periodic solutions. This comprehensive investigation of soliton solutions for the Kadomtsev–Petviashvili equation holds significant relevance in various fields such as oceanography and nonlinear optics, contributing to practical applications.

Anahtar Kelimeler: *Nonlinear Optic; Unified Riccati Equation Expansion Method; Shallow Water Waves; Kadomtsev–Petviashvili Equation.*

Öz

Bu makale, sığ suda dalga davranışını tanımlayan genişletilmiş (2+1) boyutlu Kadomtsev–Petviashvili denkleminin soliton çözümlerinin birleşik Riccati denklemi genişletme yöntemini kullanarak bir araştırmasını sunmaktadır. Söz konusu yöntem kullanılarak, birçok soliton çözümü başarıyla elde edildi ve çözümlerin ana denklemi sağladığı Wolfram Mathematica programı kullanılarak doğrulandı. Grafikler oluşturmak ve elde edilen solitonların özelliklerini incelemek için Matlab programı kullanıldı. Sonuçlar, ele alınan denklemin karanlık, parlak, tekil ve periyodik çözümler dahil olmak üzere çok çeşitli soliton çözümler sergilediğini ortaya koymaktadır. Kadomtsev-Petviashvili denklemi için soliton çözümlerinin bu kapsamlı araştırması, pratik uygulamalara katkıda bulunan oşinografi ve doğrusal olmayan optik gibi çeşitli alanlarda önemli bir öneme sahip olduğu için bu alanlardaki ileri çalışmalara ışık tutacağı öngörülmektedir.

Keywords: *Doğrusal Olmayan Optik; Birleşik Riccati Denklemi Genişletme Yöntemi; Sığ Su Dalgaları; Kadomtsev–Petviashvili Denklemi*

1. Introduction

Nonlinear partial differential equations (NLPDEs) play a crucial role in understanding and predicting the behavior of complex systems in numerous scientific areas such as physics and engineering to chemistry, biology, and economics (Braun, 1983b),), (Cinar et al., 2022), (Debnath, 2012), (Albayrak, P. 2022), (Das, S. E. 2022). These equations provide a mathematical framework to describe various physical phenomena, including fluid dynamics, heat transfer, wave propagation, and electromagnetism, etc. (Weigand 2015), (Cinar *et al*. 2023) (Robinson & Rodrigo 2009), (Davis 2012), (Albayrak 2023), (Ozisik 2022). The ability to accurately model phenomena in nature through NLPDEs and solve these equations is vital for advancing scientific knowledge and facilitating engineering design (Farlow 2012).

In 2022, Wazwaz studied the extended (2+1)-dimensional Kadomtsev–Petviashvili (KP) equation (Wazwaz 2022).

$$\alpha\, z_{xt} - \frac{\alpha^4-6\alpha^2\beta^2+\beta^4}{16} z_{xxxx} - \frac{3(\beta^2-\alpha^2)}{4}(z^2)_{xx} + \kappa\, z_{xx} + \sigma\, z_{xy} + \gamma\, z_{yy} = 0, \quad (1)$$

in which $z = z(x, y, t)$ and $\alpha, \beta, \kappa, \sigma$ and γ are reals such that $\alpha \neq 0$.

The growing fascination with the KP equation has sparked significant attention towards developing and investigating its numerous extensions such as extended, generalized and variable coefficient, etc. In (Ma et al., 2023), the extended (2+1)-dimensional KP equation was solved using conjugate complex and long-wave limit methods. In (Mohanty et al., 2023), authors studied (2+1)-dimensional Kadomtsev–Petviashvili equation with

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variable coefficients by extended generalized $\frac{G'}{G}$-expansion method. In (Li *et al.* 2021), the generalized (2+1)-dimensional KP equation was studied. Ozisik et al. studied soliton solutions of the (2+1)-dimensional KP equation via two different integration technique, namely modified F-expansion and modified generalized Kudryashov methods. In (Ma *et al.* 2023), (2+1)-dimensional KP equation was solved using Painlevé and Lie symmetry analysis.

In this article, the optical solitons of the extended (2+1)-dimensional KP equation are investigated using the unified Riccati equation expansion method (UREEM) (Sirendaoreji 2017), (Zayed *et al.* 2020).

The framework of the subsequent sections is given as follows: In Section 2, a nonlinear ordinary differential equation (NLODE) is obtained by using a wave transformation and then, the unified Riccati Equation expansion method is applied to the NLODE. In Section 3, the results of the paper is given and also a discussion is included.

2. Method

2.1. Wave Transformation

The extended KP equation can be converted into an ordinary differential equation (ODE) utilizing the following transformation:

$$z(x,y,t)=\mathcal{H}(\mu),\ \mu=p\,x+q\,y-r\,t, \qquad (2)$$

where p,q, and r are reals. So, one attains:

$$p^4(\alpha^4-6\alpha^2\beta^2+\beta^4)\mathcal{H}''-16(\kappa p^2+pq\sigma-\alpha pr+\gamma q^2)\,\mathcal{H}-12\,p^2(\alpha^2-\beta^2)\mathcal{H}^2=0, \qquad (3)$$

in which $\mathcal{H}=\mathcal{H}(\mu)$.

2.2. Unified Riccati Equation Expansion

The solutions of the eq. (3) can be supposed in the following form (Sirendaoreji 2017), (Zayed *et al.* 2020):

$$\mathcal{H}(\mu)=A_0+\sum_{i=1}^{N}A_i\psi^i(\mu), \qquad (4)$$

where $A_N\neq 0$ and N represents a balance number. Balancing the terms $\mathcal{H}^2$ and $\mathcal{H}\,\mathcal{H}''$ in Eq. (3), we get $N=2$. So, Eq. (4) is converted into:

$$\mathcal{H}(\mu)=A_0+A_1\,\psi(\mu)+A_2\,\psi(\mu)^2, \qquad (5)$$

where $A_2\neq 0$ and $\psi(\mu)$ represents the solutions of the following equation:

$$\psi'(\mu)=c_0+c_1\psi(\mu)+c_2\psi^2(\mu), \qquad (6)$$

in which Eq. (6) has the following solutions (Sirendaoreji 2017), (Zayed *et al.* 2020):

If $\Delta>0$, then

$$\psi_1(\mu)=-\frac{c_1}{2c_2}-\frac{\sqrt{\Delta}\left(b_1\tanh\left(\frac{\sqrt{\Delta}}{2}\mu\right)+b_2\right)}{2c_2\left(b_1+b_2\tanh\left(\frac{\sqrt{\Delta}}{2}\mu\right)\right)},$$

$$\psi_2(\mu)=-\frac{c_1}{2c_2}-\frac{\sqrt{\Delta}\left(b_1\coth\left(\frac{\sqrt{\Delta}}{2}\mu\right)+b_2\right)}{2c_2\left(b_1+b_2\coth\left(\frac{\sqrt{\Delta}}{2}\mu\right)\right)},$$

If $\Delta>0$, then

$$\psi_3(\mu)=-\frac{c_1}{2c_2}-\frac{1}{2c_2\mu+b_3},$$

If $\Delta<0$, then

$$\psi_4(\mu)=-\frac{c_1}{2c_2}+\frac{\sqrt{-\Delta}\left(b_4\tan\left(\frac{\sqrt{-\Delta}}{2}\mu\right)-b_5\right)}{2c_2\left(b_4+b_5\tan\left(\frac{\sqrt{-\Delta}}{2}\mu\right)\right)},$$

$$\psi_5(\mu)=-\frac{c_1}{2c_2}-\frac{\sqrt{-\Delta}\left(b_4\cot\left(\frac{\sqrt{-\Delta}}{2}\mu\right)-b_5\right)}{2c_2\left(b_4+b_5\cot\left(\frac{\sqrt{-\Delta}}{2}\mu\right)\right)}$$

where $\Delta=c_1^2-4c_0c_2$ and b_1,b_2,b_3,b_4, and b_5 are arbitrary constants. The solutions $\psi_1(\mu)$ and $\psi_2(\mu)$ are non-trivial and nondegenerate if and only if $b_2=\pm b_1, b_1^2+b_2^2\neq 0$. The solutions $\psi_4(\mu)$ and $\psi_5(\mu)$ are non-trivial and nondegenerate if and only if $b_4^2+b_5^2\neq 0$. By collecting all terms with the same power of $\psi(\mu)$ in Eq. (3), and then setting all coefficients to zero, the following system of equations is obtained:

The coefficient of $\psi^0(\mu)$:

$$-c_0p^4Q_1(2A_2c_0+A_1c_1)+12A_0^2p^2Q_3+16\,A_0(\kappa p^2+pq\sigma-\alpha pr+\gamma q^2)=0. \qquad (7)$$

The coefficient of $\psi^1(\mu)$:

$$-6A_2c_0c_1p^4Q_1-A_1(-24A_0p^2Q_3+c_1^2p^4Q_1+2c_0c_2p^4Q_1-16Q_2)=0. \qquad (8)$$

The coefficient of $\psi^2(\mu)$:

$3A_1c_1c_2p^4Q_1 + 4A_2(-6A_0p^2Q_3 + c_1^2p^4Q_1 + 2c_0c_2p^4Q_1 - 4Q_2) - 12A_1^2p^2Q_3 = 0.$ (9)

The coefficient of $\psi^3(\mu)$:

$$p^2\left(5A_2c_1c_2p^2Q_1 + A_1(c_2^2p^2Q_1 - 12A_2Q_3)\right) = 0. \quad (10)$$

The coefficient of $\psi^4(\mu)$:

$$p^2A_2(-2Q_3A_2 + p^2Q_1c_2^2) = 0. \quad (11)$$

where

$$Q_1 = \alpha^4 - 6\alpha^2\beta^2 + \beta^4,$$

$$Q_2 = \kappa p^2 + pq\sigma - \alpha pr + \gamma q^2,$$

$$Q_3 = \alpha^2 - \beta^2. \quad (14)$$

Upon solving the overdetermined system of algebraic equations using a computer algebra system, the sets are obtained:

SET 1.

$$\sigma = \frac{c_1^2p^4Q_1 - 4\left(c_0c_2p^4Q_1 + 4(\kappa p^2 - \alpha pr + \gamma q^2)\right)}{16pq},$$

$$A_0 = \frac{c_0c_2p^2Q_1}{2Q_3},$$

$$A_1 = \frac{c_1c_2p^2Q_1}{2Q_3},$$

$$A_2 = \frac{c_2^2p^2Q_1}{2Q_3}.$$

SET 2.

$$\sigma = -\frac{16(-pr\alpha + q^2\gamma + p^2\kappa) + p^4Q_1c_1^2 - 4p^4Q_1c_0c_2}{16pq},$$

$$A_0 = \frac{(c_1^2 + 2c_0c_2)p^2Q_1}{12Q_3},$$

$$A_1 = \frac{c_1c_2p^2Q_1}{2Q_3},$$

$$A_2 = \frac{c_2^2p^2Q_1}{2Q_3}.$$

By substituting the obtained sets above into Eq. (4) and considering Eq. (2), we can derive the following solutions for the extended (2+1)-dimensional KP equations mentioned in Eq. (1):

$$z_{1,1}(x,y,t) = \frac{c_0c_2p^2Q_1}{2Q_3} + \frac{c_2^2p^2Q_1\left(-\frac{\sqrt{\Delta}\left(b_2+b_1\tanh\left(\frac{\mu}{2}\sqrt{\Delta}\right)\right)}{2c_2\left(b_1+b_2\tanh\left(\frac{\mu}{2}\sqrt{\Delta}\right)\right)} - \frac{c_1}{2c_2}\right)^2}{2Q_3} + \frac{c_1c_2Q_1\left(-\frac{\sqrt{\Delta}\left(b_2+b_1\tanh\left(\frac{\mu}{2}\sqrt{\Delta}\right)\right)}{2c_2\left(b_1+b_2\tanh\left(\frac{\mu}{2}\sqrt{\Delta}\right)\right)} - \frac{c_1}{2c_2}\right)}{2Q_3},$$

$$z_{1,2}(x,y,t) = \frac{c_0c_2p^2Q_1}{2Q_3} + \frac{c_2^2p^2Q_1\left(-\frac{\sqrt{\Delta}\left(b_2+b_1\coth\left(\frac{\mu}{2}\sqrt{\Delta}\right)\right)}{2c_2\left(b_1+b_2\coth\left(\frac{\mu}{2}\sqrt{\Delta}\right)\right)} - \frac{c_1}{2c_2}\right)^2}{2Q_3} + \frac{c_1c_2Q_1\left(-\frac{\sqrt{\Delta}\left(b_2+b_1\coth\left(\frac{\mu}{2}\sqrt{\Delta}\right)\right)}{2c_2\left(b_1+b_2\coth\left(\frac{\mu}{2}\sqrt{\Delta}\right)\right)} - \frac{c_1}{2c_2}\right)}{2Q_3},$$

$$\mathbf{z_{1,3}(x,y,t)} = \frac{\mathbf{c_0c_2p^2Q_1}}{\mathbf{2Q_3}} + \frac{\mathbf{c_2^2p^2Q_1}\left(-\frac{\mathbf{1}}{\mathbf{c_2(px+qy-rt)+b_3}} - \frac{\mathbf{c_1}}{\mathbf{2c_2}}\right)^2}{\mathbf{2Q_3}} + \frac{\mathbf{c_1c_2p^2Q_1}\left(-\frac{\mathbf{1}}{\mathbf{c_2(px+qy-rt)+b_3}} - \frac{\mathbf{c_1}}{\mathbf{2c_2}}\right)}{\mathbf{2Q_3}},$$

$$z_{1,4}(x,y,t) = \frac{c_0c_2p^2Q_1}{2Q_3} + \frac{c_2^2p^2Q_1\left(\frac{\sqrt{-\Delta}\left(b_4\tan\left(\frac{\mu}{2}\sqrt{-\Delta}\right)-b_5\right)}{2c_2\left(b_4+b_5\tan\left(\frac{\mu}{2}\sqrt{-\Delta}\right)\right)} - \frac{c_1}{2c_2}\right)^2}{2Q_3} + \frac{c_1c_2p^2Q_1\left(\frac{\sqrt{-\Delta}\left(b_4\tan\left(\frac{\mu}{2}\sqrt{-\Delta}\right)-b_5\right)}{2c_2\left(b_4+b_5\tan\left(\frac{\mu}{2}\sqrt{-\Delta}\right)\right)} - \frac{c_1}{2c_2}\right)}{2Q_3},$$

$$z_{1,5}(x,y,t) = \frac{c_0c_2p^2Q_1}{2Q_3} + \frac{c_2^2p^2Q_1\left(-\frac{\sqrt{-\Delta}\left(b_4\cot\left(\frac{\mu}{2}\sqrt{-\Delta}\right)-b_5\right)}{2c_2\left(b_4+b_5\cot\left(\frac{\mu}{2}\sqrt{-\Delta}\right)\right)} - \frac{c_1}{2c_2}\right)^2}{2Q_3} + \frac{c_1c_2p^2Q_1\left(-\frac{\sqrt{-\Delta}\left(b_5\cot\left(\frac{\mu}{2}\sqrt{-\Delta}\right)-b_5\right)}{2c_2\left(b_4+b_5\cot\left(\frac{\mu}{2}\sqrt{-\Delta}\right)\right)} - \frac{c_1}{2c_2}\right)}{2Q_3},$$

$$z_{2,1}(x,y,t) = \frac{(c_1^2+2c_0c_2)p^2Q_1}{12Q_3} + \frac{c_2^2p^2Q_1\left(-\frac{\sqrt{\Delta}\left(b_2+b_1\tanh\left(\frac{\mu}{2}\sqrt{\Delta}\right)\right)}{2c_2\left(b_1+b_2\tanh\left(\frac{\mu}{2}\sqrt{\Delta}\right)\right)}-\frac{c_1}{2c_2}\right)^2}{2Q_3} + \frac{c_1c_2p^2Q_1\left(-\frac{\sqrt{\Delta}\left(b_2+b_1\tanh\left(\frac{\mu}{2}\sqrt{\Delta}\right)\right)}{2c_2\left(b_1+b_2\tanh\left(\frac{\mu}{2}\sqrt{\Delta}\right)\right)}-\frac{c_1}{2c_2}\right)}{2Q_3},$$

$$z_{2,2}(x,y,t) = \frac{(c_1^2+2c_0c_2)p^2Q_1}{12Q_3} + \frac{c_2^2p^2Q_1\left(-\frac{\sqrt{\Delta}\left(b_2+b_1\coth\left(\frac{\mu}{2}\sqrt{\Delta}\right)\right)}{2c_2\left(b_1+b_2\coth\left(\frac{\mu}{2}\sqrt{\Delta}\right)\right)}-\frac{c_1}{2c_2}\right)^2}{2Q_3} + \frac{c_1c_2p^2Q_1\left(-\frac{\sqrt{\Delta}\left(b_2+b_1\coth\left(\frac{\mu}{2}\sqrt{\Delta}\right)\right)}{2c_2\left(b_1+b_2\coth\left(\frac{\mu}{2}\sqrt{\Delta}\right)\right)}-\frac{c_1}{2c_2}\right)}{2Q_3},$$

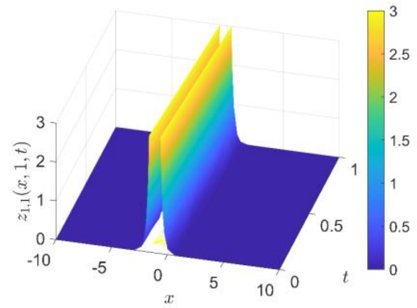
$$\mathbf{z_{2,3}(x,y,t)} = \frac{(c_1^2+2c_0c_2)p^2Q_1}{12Q_3} + \frac{c_2^2p^2Q_1\left(-\frac{1}{c_2(px+qy-rt)+b_3}-\frac{c_1}{2c_2}\right)^2}{2Q_3} + \frac{c_1c_2p^2Q_1\left(-\frac{1}{c_2(px+qy-rt)+b_3}-\frac{c_1}{2c_2}\right)}{2Q_3},$$

$$z_{2,4}(x,y,t) = \frac{(c_1^2+2c_0c_2)p^2Q_1}{12Q_3} + \frac{c_2^2p^2Q_1\left(\frac{\sqrt{-\Delta}\left(b_4\tan\left(\frac{\mu}{2}\sqrt{-\Delta}\right)-b_5\right)}{2c_2\left(b_4+b_5\tan\left(\frac{\mu}{2}\sqrt{-\Delta}\right)\right)}-\frac{c_1}{2c_2}\right)^2}{2Q_3} + \frac{c_1c_2p^2Q_1\left(\frac{\sqrt{-\Delta}\left(b_4\tan\left(\frac{\mu}{2}\sqrt{-\Delta}\right)-b_5\right)}{2c_2\left(b_4+b_5\tan\left(\frac{\mu}{2}\sqrt{-\Delta}\right)\right)}-\frac{c_1}{2c_2}\right)}{2Q_3},$$

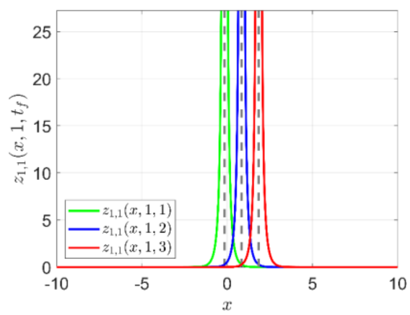
$$z_{2,5}(x,y,t) = \frac{(c_1^2+2c_0c_2)p^2Q_1}{12Q_3} + \frac{c_2^2p^2Q_1\left(-\frac{\sqrt{-\Delta}\left(b_4\cot\left(\frac{\mu}{2}\sqrt{-\Delta}\right)-b_5\right)}{2c_2\left(b_4+b_5\cot\left(\frac{\mu}{2}\sqrt{-\Delta}\right)\right)}-\frac{c_1}{2c_2}\right)^2}{2Q_3} + \frac{c_1c_2p^2Q_1\left(-\frac{\sqrt{-\Delta}\left(b_4\cot\left(\frac{\mu}{2}\sqrt{-\Delta}\right)-b_5\right)}{2c_2\left(b_4+b_5\cot\left(\frac{\mu}{2}\sqrt{-\Delta}\right)\right)}-\frac{c_1}{2c_2}\right)}{2Q_3}.$$

3. Results and Discussion

In this paper, utilizing the UREEM, many soliton solutions of the extended (2+1)-dimensional KP equation were obtained. The diverse graphs of some obtained solutions are demonstrated using two- and three- dimensional plots. The results of this study imply that the considered equation admits dark, bright, singular, and periodic solutions. Figure 1 covers the demonstration o f the solution $z_{1,1}(x,1,t)$ utilizing the parameters $a=1, p=2, q=2, r=2, \beta=2, \gamma=2, \zeta=2, \kappa=2, b_1=1, b_2=3, c_0=1, c_1=3,$ and $c_2=1$. It admits a singular solution. In Fig. (1-a) and Fig. (1-b), 3-dimensional and 2-dimensional demonstrations of the soliton are shown for $t_f=1,2,$ and 3. As can be deduced from Fig. (1-b), the soliton goes to the right on the horizontal axis.



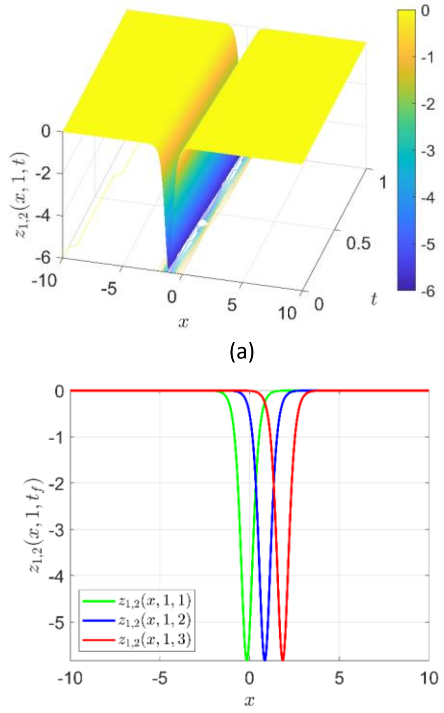
(a)



(b)

Figure 1. Three and two-dimensional plots of $z_{1,1}(x,1,t)$ (singular solution)

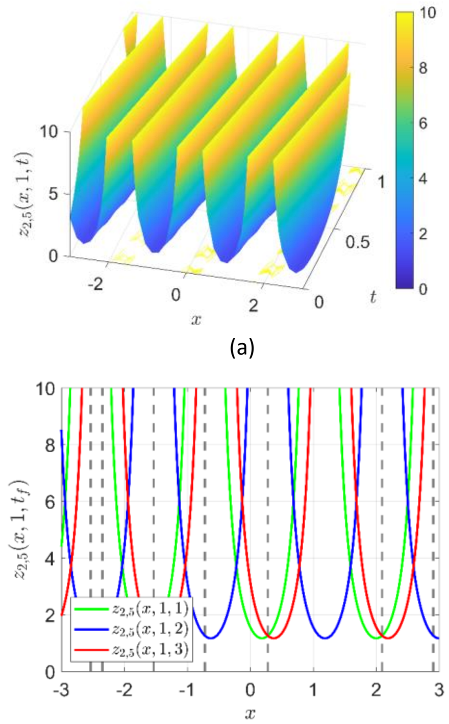
Fig. (2-a) and Fig. (2-b) include some plots of the solution $z_{1,2}(x,1,t)$ for $a=1, p=2, q=2, r=2, \beta=2, \gamma=2, \zeta=2, \kappa=2, b_1=1, b_2=3, c_0=1, c_1=3,$ and $c_2=1$. It demonstrates a dark soliton. The wave goes to the right on the horizontal axis while $t_f=1{,}2,$ and 3.



(a)

(b)

Figure 2. Three and two-dimensional plots of $z_{1,2}(x,1,t)$ (dark soliton)



(a)

(b)

Figure 3. Three and two-dimensional plots of $z_{2,5}(x,1,t)$ (periodic solution)

Finally, we present the 3D and 2D graphs of the periodic solution $z_{2,5}(x,1,t)$ with $a=1, p=2, q=2, r=2, \beta=2, \gamma=2, \zeta=2, \kappa=2, b_4=1, b_5=3, c_0=1, c_1=1,\ and\ c_2=1$, in Fig. (1-a) and Fig. (1-b), respectively.

4. Conclusion

This research deals with the extracting the soliton solutions of the extended (2+1)-dimensional KP equation. Various soliton solutions including kink, bright, and singular solitons were obtained using the UREEM. The plots of the obtained solutions are demonstrated in figures. We anticipate that the obtained outcomes in this study will prove valuable to researchers working in the field of nonlinear wave dynamics and beyond, extending their applicability across various disciplines.

Declaration of Ethical Standards

All authors have complied with the ethical standards.

Credit Authorship Contribution Statement

In this article, all components - conceptualization, methodology, coding and analysis, writing (both original draft and review and editing), visualization- are solely attributed to a single author who is Melih Cinar.

Declaration of Competing Interest

The authors hereby declare that they have no competing interests related to the content of this article.

Data Availability

All data created or analyzed during this study are included in this article.

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