DARBOUX VECTOR FIELD OF THE THIRD ORDER MANNHEIM PARTNER CURVE IN E^3

Şeyda KILIÇOĞLU [1], Süleyman ŞENYURT [2 *]

[1]Baskent University, Primary Mathematics Teaching Program Ankara, Turkey
[2]Ordu University, Faculty of Arts and Sciences Department of Geometry, Ordu, Turkey

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Abstract

In this study we worked on the modified Darboux vector on Mannheim partner curve α_3 which is called the *third order Mannheim partner* of Mannheim curve α. Further we give the offset property of the *third order Mannheim partner* α_3 based on Frenet apparatus of Mannheim curve α.

Mathematics Subject Classification (2010): 53A04, 53A05.

Keywords: Mannheim curves, Frenet apparatus, third order Mannheim curve.

Özet

Bu çalışmada α Mannheim eğrisinin α_3 Mannheim partner eğrisine ait modified Darboux vektörünün Mannheim eğrisinin Frenet aparatlarına bağlı ifadesi verildi. Daha sonra α_3 eğrisinin Mannheim eğrisi olma şartı verildi.

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Keywords: Mannheim eğrileri, Frenet aparatları, modified Darboux vektörü.

**senyutsuleyman@hotmail.com*

1 Introduction and Preliminaries

$\alpha : I \to E^3$ be the $C^2-class$ differentiable unit speed and the quantities $\{T, N, B, \kappa, \tau\}$ are collectively Frenet-Serret apparatus of the curve α. Darboux vector can be expressed as (Gray 1997),

$$D(s) = \tau(s)T(s) + \kappa(s)B(s). \tag{1.1}$$

Let a vector field be

$$\tilde{D}(s) = \frac{\tau}{\kappa}(s)T(s) + B(s) \tag{1.2}$$

along $\alpha(s)$ under the condition that $\kappa(s) \neq 0$ and it is called the modified Darboux vector field of α (Izumiya et al. 2003). A curve is called a Mannheim curve if and only if $\frac{\kappa}{(\kappa^2 + \tau^2)}$ is a nonzero constant, κ is the curvature and τ is the torsion. Recently, a new definition of the associated curves was given by Liu and Wang (Liu & Wang 2008) Mannheim curve was redefined by Liu and Wang as ; if the principal normal vector of first curve and binormal vector of second curve are linearly dependent, then first curve is called Mannheim curve, and the second curve is called Mannheim partner curve, see in (Liu & Wang 2008) . In (Orbay & Kasap 2005) Mannheim offsets of ruled surfaces are defined and characterized. The quantities $\{T_3, N_3, B_3, \kappa_3, \tau_3\}$ are collectively Frenet-Serret apparatus of the curve α_3. We called as α_3 is a *third order Mannheim partner* of the curve α, which has the following parameterizations , *third order Mannheim partner* α_3 can be written as

$$\alpha_3(s) = \alpha + \left(\lambda_1 \sin\theta + \lambda_2 \sin\theta_1 \cos\theta\right)T - \left(\lambda N - \lambda_2 \cos\theta_1\right)N + \left(\lambda_1 \cos\theta - \lambda_2 \sin\theta_1 \sin\theta\right)B \tag{1.3}$$

Also $\left|\lambda + \lambda_1 + \lambda_2\right|$ is the distance between the arclengthed curves α and α_3. Since we have

$$d\left(\alpha(s), \alpha_3(s)\right) = \left\|\alpha_3(s) - \alpha(s)\right\| = \left\|\left(\lambda + \lambda_1 + \lambda_2\right)N(s)\right\| = \left|\lambda + \lambda_1 + \lambda_2\right|.$$

The Frenet apparatus of *third order Mannheim partner* α_3 of a Mannheim curve α, based on the Frenet apparatus of Mannheim curve α are

$$\begin{cases} T_3 = \left(\cos\theta_2 \cos\theta_1 \cos\theta - \sin\theta_2 \sin\theta\right)T - \cos\theta_2 \sin\theta_1 N - \left(\sin\theta_2 \cos\theta + \cos\theta_2 \cos\theta_1 \sin\theta\right)B \\ N_3 = \left(\sin\theta_2 \cos\theta_1 \cos\theta + \cos\theta_2 \sin\theta\right)T - \sin\theta_2 \sin\theta_1 N + \left(\cos\theta_2 \cos\theta - \sin\theta_2 \cos\theta_1 \sin\theta\right)B \\ B_3 = \sin\theta_1 \cos\theta T + \cos\theta_1 N - \sin\theta_1 \sin\theta B \end{cases} \tag{1.4}$$

$$\kappa_3 = \frac{-\dot{\theta}_2}{\cos\theta \cos\theta_1 \cos\theta_2} \quad \text{and} \quad \tau_3 = \frac{\lambda_1 \kappa \dot{\theta}_1}{\lambda_2 \lambda \tau \dot{\theta} \cos\theta_1} \tag{1.5}$$

are the first and second curvatures of the *third order Mannheim partner* α_3, respectively] (Kılıçoğlu & Şenyurt 2017).

2 Darboux vector field of the third order Mannheim partner curve in E^3

Theorem 2.1 *The modified Darboux vector of third order Mannheim partner* α_3 *of a Mannheim curve* α, *based on the Frenet apparatus of Mannheim curve* α *is*

$$\widetilde{D}_3(s) = \left[\frac{-\lambda_1 \kappa \dot{\theta}_1 \cos\theta \cos\theta_2}{\lambda \lambda_2 \tau \dot{\theta} \dot{\theta}_2}\left(\cos\theta_2 \cos\theta_1 \cos\theta - \sin\theta_2 \sin\theta\right) + \sin\theta_1 \cos\theta\right]T$$

$$+\left[\frac{\lambda_1\kappa\dot{\theta}_1\cos\theta\cos\theta_2}{\lambda\lambda_2\tau\dot{\theta}\dot{\theta}_2}\cos\theta_2\sin\theta_1-\cos\theta_1\right]N$$

$$+\left[\frac{\lambda_1\kappa\dot{\theta}_1\cos\theta\cos\theta_2}{\lambda\lambda_2\tau\dot{\theta}\dot{\theta}_2}\left(\sin\theta_2\cos\theta+\cos\theta_2\cos\theta_1\sin\theta\right)+\sin\theta_1\sin\theta\right]B$$

Proof. Since $\tilde{D}_3(s)=\frac{\tau_3}{\kappa_3}T_3(s)+B_3(s)$, and we have the proof as in the following way

$$\tilde{D}_3(s)=\frac{\tau_3}{\kappa_3}T_3(s)+B_3(s),$$

$$=\left[\frac{-\lambda_1\kappa\dot{\theta}_1\cos\theta\cos\theta_2}{\lambda\lambda_2\tau\dot{\theta}\dot{\theta}_2}\left(\cos\theta_2\cos\theta_1\cos\theta-\sin\theta_2\sin\theta\right)+\sin\theta_1\cos\theta\right]T$$

$$-\left[\frac{-\lambda_1\kappa\dot{\theta}_1\cos\theta\cos\theta_2}{\lambda\lambda_2\tau\dot{\theta}\dot{\theta}_2}\cos\theta_2\sin\theta_1+\cos\theta_1\right]N$$

$$-\left[\frac{-\lambda_1\kappa\dot{\theta}_1\cos\theta\cos\theta_2}{\lambda\lambda_2\tau\dot{\theta}\dot{\theta}_2}\left(\sin\theta_2\cos\theta+\cos\theta_2\cos\theta_1\sin\theta\right)-\sin\theta_1\sin\theta\right]B$$

$$\tilde{D}_3(s)=\left[\frac{-\lambda_1\kappa\dot{\theta}_1\cos\theta\cos\theta_2}{\lambda\lambda_2\tau\dot{\theta}\dot{\theta}_2}\left(\cos\theta_2\cos\theta_1\cos\theta-\sin\theta_2\sin\theta\right)+\sin\theta_1\cos\theta\right]T$$

$$+\left[\frac{\lambda_1\kappa\dot{\theta}_1\cos\theta\cos\theta_2}{\lambda\lambda_2\tau\dot{\theta}\dot{\theta}_2}\cos\theta_2\sin\theta_1-\cos\theta_1\right]N$$

$$+\left[\frac{\lambda_1\kappa\dot{\theta}_1\cos\theta\cos\theta_2}{\lambda\lambda_2\tau\dot{\theta}\dot{\theta}_2}\left(\sin\theta_2\cos\theta+\cos\theta_2\cos\theta_1\sin\theta\right)+\sin\theta_1\sin\theta\right]B$$

where we use dot to denote the derivative with respect to the arclength parameter of the curve α.

Theorem 2.2 *The offset property of the third order Mannheim partner* α_3 *based on the frenet apparatus Mannheim curve* α, *can be given if and only if the curvature* κ *and the torsion* τ *of* α *satisfy the following equation*

$$\lambda_3 = \frac{\lambda^2 (\lambda_2)^2 \tau^2 \dot{\theta}^2 \dot{\theta}_2 \cos\theta \cos\theta_1 \cos\theta_2}{\lambda^2 (\lambda_2)^2 \tau^2 \dot{\theta}^2 (\dot{\theta}_2)^2 + (\lambda_1 \kappa \dot{\theta}_1)^2 \cos^2\theta \cos^2\theta_2} = constant.$$

Proof. The offset property of the *third order Mannheim partner* α_3 is

$$\lambda_3 = \frac{-\kappa_3}{\kappa_3^2 + \tau_3^2} = constant.$$

Hence

$$\lambda_3 = \frac{-\kappa_3}{\kappa_3^2 + \tau_3^2} = \frac{-\dfrac{-\dot{\theta}_2}{\cos\theta \cos\theta_1 \cos\theta_2}}{\left(\dfrac{-\dot{\theta}_2}{\cos\theta \cos\theta_1 \cos\theta_2}\right)^2 + \left(\dfrac{\lambda_1 \kappa \dot{\theta}_1}{\lambda \lambda_2 \tau \dot{\theta} \cos\theta_1}\right)^2}$$

$$= \frac{\dfrac{\dot{\theta}_2}{\cos\theta \cos\theta_1 \cos\theta_2}}{\dfrac{\lambda^2 (\lambda_2)^2 \dot{\theta}^2 (\dot{\theta}_2)^2 \tau^2 + (\dot{\theta}_1 \lambda_1 \kappa)^2 \cos^2\theta \cos^2\theta_2}{\lambda^2 (\lambda_2)^2 \dot{\theta}^2 \cos^2\theta \cos^2\theta_1 \cos^2\theta_2 \tau^2}}$$

$$= \frac{\lambda^2 (\lambda_2)^2 \tau^2 \dot{\theta}^2 \dot{\theta}_2 \cos\theta \cos\theta_1 \cos\theta_2}{\lambda^2 (\lambda_2)^2 \tau^2 \dot{\theta}^2 (\dot{\theta}_2)^2 + (\lambda_1 \kappa \dot{\theta}_1)^2 \cos^2\theta \cos^2\theta_2}.$$

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