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The Analysis Of Middle School Students' Mathematical Understanding In Terms Of Different Variables

Ortaokul Öğrencilerinin Matematiksel Anlamalarının Farklı Değişkenler Açısından İncelenmesi

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Abstract

Mathematical understanding which is dynamic, refined but non-linear and which is passed through different levels is a self-renewing process. Determining the factors which affects this process can be seen as the first steps of understanding the mathematical understanding. With this regard, in this study, it is aimed to analyze mathematical understandings of middle school students in terms of different variables (gender, academic achievement etc.). In this research relational screening model was used. The study group is consisted of 466 students who are studying in different grades of a public middle school in Zeytinburnu region of İstanbul. The data was obtained by using 'Demographics Information Form (DIF)' and 'Determining the Mathematical Understanding Levels Scale (DMULS)'. According to research results it was appeared that the mathematical understandings of male students were lower than female students. In addition to that, it was determined that mathematical understandings of middle school students differed significantly according to their grade levels, their academic achievements, whether they received extra help for the mathematics lessons other than the school or not and educational levels of their parents.

Keywords: *Middle school students, mathematical understanding, different variables*

Özet

Kendisini yenileyen bir süreç olan matematiksel anlama, dinamik, seviyeli fakat doğrusal olmayan bir yapıda olup ve farklı aşamalardan geçer. Bu süreci etkileyen faktörlerin belirlenmesi ise, matematiksel anlamayı anlamanın ilk adımları olarak görülebilir. Bu bağlamda araştırmada, ortaokul öğrencilerinin matematiksel anlamalarının farklı değişkenler (cinsiyet, akademik başarı vb.) açısından incelenmesi amaçlanmıştır. Araştırmada ilişkisel tarama modeli kullanılmıştır. Çalışma grubunu, İstanbul ili Zeytinburnu ilçesinde bulunan bir devlet ortaokulunun farklı sınıf seviyelerinde öğrenim görmekte olan 466 öğrenci oluşturmaktadır. Veriler, "Demografik Bilgi Formu (DBF)" ve "Matematiksel Anlama Düzeylerini Belirleme Ölçeği (MADBÖ)" ile elde edilmiştir. Araştırma sonuçlarına göre; erkek öğrencilerin

matematiksel anlamalarının kız öğrencilere göre daha düşük olduğu belirlenmiştir. Ayrıca ortaokul öğrencilerinin matematiksel anlamalarının sınıf seviyelerine, akademik başarılarına, okul dışı matematik dersine yardımcı ders alıp almama durumlarına ve anne-baba eğitim düzeylerine göre anlamlı bir şekilde farklılaştığı ortaya çıkmıştır.

Anahtar Kelimeler: *Ortaokul öğrencileri, matematiksel anlama, farklı değişkenler*

1. Introduction

Although the term of 'understanding' is used in math education literature, the studies carried out for expressing the definition of understanding have taken years. Especially Brownell and Simps (1946) felt the difficult structure of mathematical understanding and expressed that technically, specifying the definitions of 'understanding' and 'perception' overtly was not an easy job. Sierpinska (1990) also referred the difficulty of the subject and according to Sierpinska many writers used understanding with the assumption that it was a well-defined concept. In this sense, they had difficulties as they tried to analyze the meaning of understanding with philosophy (Sierpinska, 1990). On this point, article of Skemp (1976) entitled as 'relational' and 'instrumental' understanding, have become cornerstones on understanding and information subjects in mathematics education. Before Skemp's (1976) study, researchers generally tried to define understanding with information (Sierpinska, 1990).

When the questions 'What is the meaning of understanding?' and 'What do we understand from understanding?' are analyzed; according to Skemp (1976) the term understanding has two different meanings as instrumental understanding and relational understanding. Skemp stated relational understanding as using information by answering the question 'why' meaningfully through relating them with the previous information. For Skemp instrumental understanding provides to reach results in a short time by learning quickly and making the solution of the problem easier. But in relational understanding although structuring information is difficult and time consuming, remembering and the ability to transform information into different forms is easier. Students in this case are able to produce different solution strategies. Deep mathematical understanding must be primarily relational understanding. For this reason, both types of understanding have different advantages according to their own peculiarities (Skemp, 1976).

The study of Skemp separated understanding from information and emphasized the categories of mathematical understanding (Byers & Erlwanger, 1985). Although the researchers separated understanding from information, mathematics educators could not reach a consensus on the definition of 'understanding' since various researchers approached this subject with different perspectives (Schroeder, 1987). For Smith (1996) understanding; is the ability to connect a representation to another representation or situation or to transfer. According to von Glasersfeld who contributed to the development of constructive approach, understanding is structuring new meaningful structures by correlating the information in one's cognitive structure and this process is an ongoing process (von Glasersfeld, 1987). The use of this definition by Glasersfeld helped Pirie and Kieren to structure their theoretical opinions regarding mathematical understanding (Meel, 2003). For Pirie the development of mathemati-

cal understanding; 'is completely a dynamic, refined but non-linear, passing through different levels and a self-renewing process' (Pirie & Kieren, 1994).

A two-dimensional model which called as 'the dynamic model of understanding' was developed by Pirie and Kieren (1994). This model was composed of eight nested layers. The aim of this model is to observe how an individual structure a new subject or concept by passing through. There are eight layers that may occur on any matter for each individual at different levels in the developmental process of understanding (Pirie & Kieren, 1994). These layers are; *1) primitive knowing, 2) image making, 3) image having, 4) property noticing, 5) formalising, 6) observing, 7) structuring* and *8) inventizing*. Learner's development of the understanding is denoted a qualitative change by each layer (Meel, 2003). Each layer covers the previous one. In addition to this each layer is covered by all the subsequent layers. It is necessary for describing the integrated nature of the mathematical understanding (Martin, 2008). In Pirie-Kieren theory, students pass on different levels of understanding layers by structuring new information with their pre-cognitive knowledge (primitive knowing) (Cavey, 2002).

Together with the new perceptions occurred in mathematics education, Smith (2000) expressed the basic purpose of the educational reforms as helping students to learn mathematics by understanding. At this point Amit and Fried (2002) stated that in 1990's teaching mathematics was re-structured in Israel; in order to overcome understanding and learning problems in teaching, students should be more interested in mathematics and they should incorporate mathematics into their daily lives. In this sense, the term of understanding is also used in our country for indicating the purpose of teaching mathematics in middle school teaching mathematics program (OMDÖP, 2013). The point emphasized here is mathematics need to be clear and understandable. The development of understanding is considered as equal in relational context to problem solving and the performance of making algebraic operations (Brownell, 1945; Brownell & Simps, 1946; Polya, 1945; Van Engen 1949; Wertheimer, 1959).

Besides mathematics teachers helps to make mathematics understandable for students but this is not enough either. Many studies on this subject showed that mathematical understanding of prospective teachers that occurred during their university education was not enough for teaching at primary school level (Ball, 1990a, 1990b; Tirosh, 2000; Toluk-Uçar, 2009). Basically there is a necessity to specify the real problem. The important thing for the solution of all problems is to try to explain the mechanism that enables students to understand mathematics and to determine the meaning of students' mathematical understanding. In other words there is a need to understand understanding. Although this is not easy we need to make an effort for that.

When teaching mathematics studies are considered, it is appeared that the real problem especially in cognitive psychology studies is the problem of understand the understanding. For this reason, it is necessary to provide students' mathematical development. In this sense, while there are some studies carried out for defining the process of understanding from different perspectives (Skemp, 1982; Byers & Herscovics, 1977; Davis, 1978; Herscovics & Bergerson, 1988; Pirie & Kieren, 1989) it is seen that there are not studies for determining the factors that affected the mathematical understanding of students. The aim of this study is to analyze mathematical under-

standing of middle school students in terms of various variables (gender, grade levels, etc) to remedy this outstanding gap in literature. In this sense, the research problems are as follows.

1. Is there a difference in mathematical understanding of students according to their gender?
2. Is there a difference in mathematical understanding of students according to their grade levels?
3. Is there a difference in mathematical understanding of students according to their academic achievements?
4. Is there a difference in mathematical understanding of students according to the fact whether they received extra lessons to help mathematics lessons out of school education or not?
5. Is there a difference in mathematical understanding of students according to educational levels of their mothers?
6. Is there a difference in mathematical understanding of students according to educational levels of their fathers?

2. Method

Research Design

The relational screening model was used in this study. Relational screening models are research models which aim to determine the existence of covariance between two or more variants (Karasar, 2002).

Study Group

Purposeful sampling method was used for determining the study group. Purposeful sampling is a probable and non-probability sampling approach (Büyüköztürk, Kılıç-Çakmak, Akgün, Karadeniz, & Demirel, 2011). Patton (1990) mentioned about 14 strategies about the purposeful sampling. In this convenience sampling method was used. In this method, the researcher starts creating the sample starting from the most accessible respondent until he/she reaches the size of the group that he/she needs or studies with the most accessible sample which provides maximum saving (Ravid, 1994). In this regard, the study was carried out in a public school administered by the ministry of education in Zeytinburnu district in the European side of İstanbul. The distribution of the students included in the study according to gender, grade levels, academic achievements, whether to take extra lessons for mathematics lessons or not and educational levels of the parents is seen in Table 1.

Table 1. Independent variables of the study

Independent Variables		f	%
Gender	Female (F)	245	52.58
	Male (M)	221	47.42

Independent Variables		f	%
Grade levels	5th Grade	125	26.82
	6th Grade	122	26.18
	7th Grade	111	23.82
	8th Grade	108	23.18
Mathematics marks of the previous term /academic achievement	1	79	16.95
	2	97	20.82
	3	133	28.54
	4	99	21.24
	5	58	12.45
Whether to receive extra lessons out of school to help mathematic lessons or not	Yes	126	27.04
	No	340	72.96
Mothers' educational levels	Illiterate	74	15.88
	Elementary	181	38.84
	Middle School	140	30.04
	High School	50	10.73
	University	21	4.51
Fathers' educational levels	Illiterate	19	4.08
	Elementary	142	30.47
	Middle School	165	35.41
	High School	108	23.18
	University	32	6.87

When Table 1 is analyzed, 52.58% of the study group of 466 students is composed of female students and 47.42% of the study group is composed of male students. 26.82% of the sample of 466 students are 5th grade students, 26.18% of them 6th grade students, 23.82% of them 7th grade students and 23.18% of them are 8th grade students. When we look at the students' mathematics marks in the previous term, it is seen that 16.95% of the students has 1, 20.82% of the students has 2, 28.54% of them has 3, 21.24% of them has 4 and 12.45% of them has 5. While 27.04% of the students who participated in the study received extra lessons out of school to help their mathematic lessons, 72.96% of them did not receive any extra lessons out of school. According to Table 1, among 466 students, 15.88% of the students' mothers are illiterate. 38.84% of them graduated from elementary schools, 30.04% of them graduated from middle schools, 10.73% of them graduated from high schools 4.51% of them graduated from university. 4.08% of the students' fathers are illiterate. 30.47% of them graduated from elementary schools, 35.41% of them graduated from middle schools, 23.18% of them graduated from high schools 6.87% of them graduated from university.

Data Collection Tools and Data Collection

Demographics Information Form (DIF): In this form, there are questions about the students' gender, their grade levels, whether they received extra lessons out of school to help their maths lessons, educational levels of their parents in order to determine some of the personal or demographic information of the students. The data obtained from DIF were used as the independent variables to compare mathematical understandings of the students.

Determining the Mathematical Understanding Levels Scale (DMULS): This scale used in this study was developed by Kaba and Şengül (2015) for determining the students' mathematical understanding. The scale included in 56 items. The maximum score that one can get from the scale is 280 and the minimum score is 56.

Confirmatory and exploratory factor analyses were conducted for the construct validity and for the reliability of the scale Cronbach Alpha, Spearman-Brown and Guttman Split-Half reliability analysis were conducted and test-retest method was utilized. As a result of the exploratory factor analysis, it was appeared that the scale was composed of a single factor. According to the confirmatory factor analysis results, which were conducted for the validity of DMULS's structure which was composed of 56 items, ratio of the scale was calculated as 2.06. This value shows that the scale is compatible with the real data.

The Cronbach Alpha reliability coefficient of this developed scale was calculated as ".969"; Spearman-Brown reliability coefficient as ".946" and Guttman Split-Half reliability coefficient as ".946". As a result of paired samples t-test which was conducted for testing the invariance of DMULS with respect to time, it can be said that there was not a significant difference between the students' DMULS average scores at 0.05 level after two applications ($t= -1.619$, $p> .05$). In addition to that, it was determined that the correlation between scale scores obtained after two applications was high. In this study, first of all, reliability analyses were conducted. The data obtained in this regard were displayed in Table 2.

Table 2. The results of internal consistency coefficients of DMULS

Reliability	r	p
Cronbach Alpha	.959	$p < .05$
Spearman-Brown	.925	$p < .05$
Guttman Split-Half	.923	$p < .05$

When Table 2 is analyzed, it is seen that Cronbach Alpha internal consistency coefficient of the scale is ".959"; Spearman-Brown internal consistency coefficient is ".925" and Guttman Split-Half internal consistency coefficient is ".923". In this sense, it can be said that the reliability coefficient of the scale is at the desired level. Later on, confirmatory factor analysis was conducted for testing the validity of the scale's factor structure. The results of confirmatory factor analyses were given in Table 3.

Table 3. The results of confirmatory factor analysis of DMULS

Indexes	Value	Model-Data Fit
N	466	
X^2	3012.40	
df	1484	
X^2/df	2.03	*Perfect fit* (Sümer, 2000)
GFI	0.81	*Sufficient fit* (Aydın, 2009)
AGFI	0.80	*Sufficient fit* (Aydın, 2009)

Indexes	Value	Model-Data Fit
PGFI	0.76	*Plain and simple* (Sümer, 2000)
CFI	0.98	*Perfect fit* (Hu & Bentler, 1999)
NFI	0.96	*Perfect fit* (Hu & Bentler, 1999)
NNFI	0.98	*Perfect fit* (Hu & Bentler, 1999)
RMR	0.052	*Fit well* (Brown, 2006)
SRMR	0.045	*Perfect fit* (Kline, 2005)
RMSEA	0.047	*Perfect fit* (Sümer, 2000)

As it can be seen from Table 3, all the fit values of the scale used in this study were at desired level. Data collection tools were applied to the study group between the dates of 14.04.2014/18.04.2014. The students were given 25 minutes for completing the data collection tools. As a result of the application, 478 scales were obtained in total.

Data Analysis

Firstly, all the data (478 DIF and scale) obtained in the study was analyzed one by one by the researchers. As a result of the analysis, it was ascertained that some students marked only one option (for instance; they only marked the option of strongly agree) and some students did not complete the DIF. In this sense, 4 data collection instruments at 5^{th} grade level, 2 at 6^{th} grade level, 5 at 7^{th} grade level and 1 at 8^{th} grade level. 12 data collection tools were not evaluated in total. The necessary analyses for the study were carried out with the data collected from the remaining 466 DIF and scale.

First of all, the data obtained from DIF which was related with gender, grade levels, academic achievements, extra lessons to help mathematics lessons and the educational levels of the parents were organized in frequency (f) and percentage (%) tables.

Later on, it is necessary to decide whether the data will be analyzed by using non-parametric or parametric techniques. In order to make this decision, there are various assumptions. One of these assumptions is the necessity to have a normal distribution or to be close to normal distribution for the data. In this sense, whether the data have normal distribution or not was analyzed. Whether the data were suitable to normal distribution or not was determined by conducting tests which were used about the case of normality. As it is suggested to use Kolmogorov-Smirnov (K-S) test when the group size is bigger than 50 and to use Shapiro-Wilks test when the group size is smaller than 50 (Büyüköztürk, 2012; Büyüköztürk, Çokluk, & Köklü, 2010) in order to make a decision about the normality of the data in this study Kolmogorov-Smirnov (K-S) test (N=466) was used. Having p value bigger than ".05" is interpreted as scores do not show significant (excessive) deviation from normal distribution at this significance level and they are suitable (Büyüköztürk, 2012). It was determined that the data obtained in this study did not have a normal distribution ($p = .000 < .05$). In this sense, it was decided to analyze the data by using non-parametric tests.

In the process coming after making this decision, "Kruskal-Wallis" and "Mann-Whitney U" tests were used for analyzing the data. While Mann-Whitney U test was used for analyzing the data regarding gender and receiving extra lessons out

of school to help mathematics lesson, Kruskal-Wallis test was used for analyzing the grade levels, academic achievement and the educational levels of the parents. The occurrence of the difference, which was observed between the groups according to Kruskal-Wallis test, depending on the significant differences between which groups can be identified with a calculation by hand. Multiple comparison test command related with this is not available in SPSS 15.0 module. In this case, the source of the difference can be analyzed by using Mann-Whitney U test over binary combinations of the groups (Büyüköztürk, 2012). Therefore in this study, the source of the difference occurred between the groups according to Kruskal-Wallis test results was analyzed by using Mann-Whitney U test. In all statistical operations, the significance level was accepted as ".05". SPSS 15.0 software was used for the analysis.

3. Findings

The first problem of the study was determined as *"Is there a difference in mathematical understandings of the students according to their gender?"*. The mathematical understandings of students according to their gender were compared by using Mann-Whitney U test. The results were given in the following Table 4.

Table 4. Mathematical understandings & gender

Gender	N	Mean Rank	Rank Sum	U	p
Female	245	254.02	62235.50	22044.5	.001
Male	221	210.75	46575.50		

When Table 4 is analyzed, it can be said that there is a significant difference between the mathematical understandings of the students according to their gender (U=22044.5; p= .001 < .05). When mean rank is considered, it is understood that mathematical understanding of male students is lower than female students.

The second problem of the study was determined as *"Is there a difference in mathematical understandings of the students according to their grade levels?"*. The mathematical understandings of students were compared by using Kruskal-Wallis test according to grade level variable. The results were given in Table 5.

Table 5. Mathematical understandings & grade levels

Grade Level	N	Mean Rank	df	X^2	p	Significant Difference
5	125	255.64	3	14.14	.003	5-8 6-8 7-8
6	122	243.18				
7	111	237.82				
8	108	192.50				

When Table 5 is analyzed, it can be said that there is a significant difference between the mathematical understandings of the students according to their grade levels (X^2= 14.14; p= .003 < .05). When mean rank is considered, it is seen that 5th grade students have the highest level of mathematical understanding and it is followed by the 6th and 7th grade students. The occurrence of the significant difference appeared

among grade levels, depending on the significant differences between which groups was determined by conducting Mann-Whitney U test over binary combinations of grade levels. According to the results of the analysis, it was understood that mathematical understandings of 5th, 6th and 7th grade students is higher than 8th grade students and the differences are significant.

The third problem of the study was determined as *"Is there a difference in mathematical understandings of the students according to their academic achievements?"*. The mathematical understandings of students were compared by using Kruskal-Wallis test according to academic achievement variable. The results were given in Table 6.

Table 6. Mathematical understandings & academic achievements

Academic Achievement	N	Mean Rank	df	X^2	p	Significant Difference
1	79	128.76				
2	97	189.70				1-2; 1-3; 1-4; 1-5
3	133	237.12	4	112.07	.000	2-3; 2-4; 2-5
4	99	295.98				3-4; 3-5
5	58	334.46				

When Table 6 is analyzed, it is seen that there is a significant difference between the mathematical understandings of the students according to their academic achievements (X^2= 112.07; p= .000 < .05). When mean rank is considered, it is seen that students whose academic achievement score is '5' have the highest level of mathematical understanding and it is followed by the students with marks '4'. The occurrence of the difference appeared among academic achievements, depending on the significant differences between which scores was determined by conducting Mann-Whitney U test over binary combinations of academic achievements. According to the results, it was appeared that there was not a significant difference between students whose maths marks are 4 and 5.

The fourth problem of the study was determined as *"Is there a difference in mathematical understandings of the students according to the fact whether they received extra mathematics lessons out of school to help their mathematics lessons or not?"*. The mathematical understandings of students were compared by using Mann-Whitney U test according to the fact whether they received extra mathematics lessons out of school to help their mathematics lessons or not. The results were given in Table 7.

Table 7. Mathematical understandings & extra mathematics lessons

Extra Maths Lessons out of School	N	Mean Rank	Rank Sum	U	p
Yes	126	255.54	32197.5	18643.5	.032
No	340	225.33	76613.5		

When Table 7 is analyzed, it can be said that there is a significant difference between the mathematical understandings of the students according to the fact whether they received extra mathematic lessons out of school (U=18643.5; p= .032 < .05).

When mean rank is considered, it is understood that mathematical understanding of students who had extra maths lessons is higher than students who did not have extra maths lessons out of school.

The fifth problem of the study was stated as *"Is there a difference in mathematical understandings of the students according to the educational levels of their mothers?"*. The mathematical understandings of students were compared by using Kruskal-Wallis test according to mothers' educational level variable. The results were given in Table 8.

Table 8. Mathematical understandings & mothers' educational levels

Mothers' Educational Levels	N	Mean Rank	df	X^2	p	Significant Difference
1. Illiterate	74	186.17				
2. Elementary	181	220.54				
3. Middle School	140	251.47	4	22.130	.000	1-3; 1-4; 1-5 2-3; 2-4
4. High School	50	285.01				
5. University	21	269.52				

When Table 8 is analyzed, it can be said that there is a significant difference between the mathematical understandings of the students according to their mothers' educational levels (X^2= 22.130; p= .000 < .05). When mean rank is considered, it is seen that students whose mothers are high school graduates have the highest level of mathematical understanding and it is followed by the university and middle school graduates.

The occurrence of the difference appeared according to mothers' educational levels, depending on the significant differences between which educational levels was determined by conducting Mann-Whitney U test over binary combinations of mothers' educational levels. According to the results of the analysis; it was determined that the mathematical understanding of the students whose mother are illiterate is lower than the students whose mothers are middle school, high school or university graduates. Besides, it was also identified that mathematical understanding of the students whose mother are elementary school graduates is lower than the students whose mothers are middle school and high school graduates.

The sixth problem of the study was stated as *"Is there a difference in mathematical understandings of the students according to the educational levels of their fathers?"*. The mathematical understandings of students were compared by using Kruskal-Wallis test according to fathers' educational level variable. The results were given in Table 9.

Table 9. Mathematical understandings & fathers' educational levels

Fathers' Educational Levels	N	Mean Rank	df	X^2	p	Significant Difference
1. Illiterate	19	167.61				
2. Elementary	142	216.37				1-3
3. Middle School	165	243.31	4	10.129	.038	1-4
4. High School	108	242.98				1-5
5. University	32	266.05				

When Table 9 is analyzed, it can be said that there is a significant difference between the mathematical understandings of the students according to their fathers' educational levels (X^2= 10.129; p= .038 < .05). When mean rank is considered, it is seen that students whose fathers are university graduates have the highest level of mathematical understanding and it is followed by the middle school and high school graduates. The occurrence of the difference appeared according to fathers' educational levels, depending on the significant differences between which educational levels was determined by conducting Mann-Whitney U test over binary combinations of fathers' educational levels. According to the results of the analysis; it was determined that the mathematical understanding of the students whose fathers are illiterate is lower than the students whose fathers are middle school, high school and university graduates. It was appeared that there was not any difference between other educational levels.

4. Conclusion, Discussion and Suggestions

The aim of this study is to analyze mathematical understandings of middle school students according to their gender, academic achievements, receiving extra lessons out of schools to help mathematics lessons and educational levels of their parents. As a result of the analysis conducted with respect to this purpose, mathematical understandings of middle school students significantly differed according to their gender. It was appeared that mathematical understandings of female students were higher than male students. Bal (2006) analyzed the difference between operational skills of fifth grade students and mathematical comprehension in maths lessons on the subjects such as natural numbers, four operations on fractions and problems regarding natural numbers and fractions. She identified whether or not this difference was significant in terms of academic achievement, gender and reading comprehension skills. It was appeared that the scores that students obtained from the comprehension and operation test did not create any significant difference according to gender variable. In this sense it can be said that this study is not parallel with Bal's study.

Students' mathematical understanding significantly differed according to grade level variable. It is thought that as the grade level of the students increase mathematical understandings of the students will also increase. However, the students with higher level of mathematical understandings were 5th grade students. It was concluded in the study that as the grade levels increased mathematical understanding levels of the students were decreased. It is seen important to search the reasons of this situation appeared in this study. In this regard, it is suggested to make interviews with students to learn the reasons of this situation in the future studies.

Students' mathematical understanding significantly differed according to their academic achievements. It can be said in this case that students who have higher academic achievements in mathematics lessons also have higher level of mathematical understandings. Bal (2006) concluded in her study that students' scores in comprehension and operation tests did not significantly differ in terms of academic achievement variable. Therefore it was determined that two results did not fit.

Students' mathematical understanding significantly differed according to the fact

whether they received extra lessons out of school to help their mathematics lessons. It was understood that mathematical understandings of students who received extra lessons out of school to help their mathematics lessons were higher. In this case, it is suggested to parents to provide extra lessons to their children out of school to help their mathematics lessons.

Students' mathematical understanding significantly differed according to their mothers' educational levels. It was determined that students whose mothers are high school graduates had the highest level of mathematical understanding. This was followed by the students whose mothers are university graduates. This shows that educational levels of mothers have an effect on increasing the mathematical understandings of the students. Kaya, Bozaslan and Genç (2012) found in their studies that mothers with high level of education had democratic attitudes and their children had high level academic and problem solving achievements. From this perspective the educational levels of mothers are important and it is suggested to focus on this point.

In addition to that students' mathematical understanding significantly differed according to their fathers' educational levels. It was determined that students whose fathers are university graduates had the highest level of mathematical understanding. It was understood that there was not any difference between other levels. This finding can be interpreted as it shows that the educational level of a father is not effective in increasing the mathematical understandings of the students. In this sense, except being an illiterate, it is seen sufficient for a father to have any kind of educational degree. However, it should be remembered that children whose fathers are university graduates have higher level of mathematical understanding.

One of the most important objectives of teaching mathematics is to make students understand mathematics (Hiebert & Carpenter, 1992). However, students expressed in Dursun and Peker's (2003) study about mathematical understanding which has an important place in teaching mathematics that they had difficulties in understanding, comprehending and interpreting the mathematics lesson. In this regard, it is suggested to conduct studies in different subjects and grade levels in order to determine the difficulties that students have during mathematical understanding process.

It is thought that there are many more variables that affect students' mathematical understandings (for instance; attitudes towards mathematics, belief, number of siblings, socio-economic level of the family etc.). Thus, Kastberg (2002) found in his study which he analyzed the mathematical understandings of the students that students' beliefs towards mathematics was effective in understanding the mathematical concepts. In conclusion it is thought that it will be useful to repeat this study by considering the other variables mentioned above. Besides, it is also thought that it is necessary to repeat this study with a broader sample.

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