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PROSPECTIVE MATHEMATICS TEACHERS' MAKING SENSE OF THE DECIMAL REPRESENTATION OF REAL NUMBERS AS RATIONAL NUMBER SEQUENCES THROUGH QUANTITATIVE REASONING

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Abstract: Previous studies have revealed that students have misconceptions on numbers specifically on real numbers (Tall & Schwarzenberger, 1978; Ely, 2010). In order to eliminate the misconceptions, Voskoglou (2013) suggested that teaching should emphasize the use of multiple representations of real numbers and flexible transformations among the representations. In the current study, we conducted classroom teaching experiments (Cobb, 2000) with 19 prospective mathematics teachers in an English-medium university in İstanbul about the decimal representation of real numbers with the emphasis of quantitative reasoning (Thompson, 2011; Karagöz-Akar, 2016). The ongoing and retrospective data analysis was done through line by line analysis of the transcriptions of the video records and the written artifacts. Results showed that thinking through quantities depicted in diagrams, once prospective teachers related long division with multiple representations of rational numbers such as fractions, equivalent fractions and decimals through the mental actions of equal partitioning, grouping and counting, they were able to deduce that all these representations corresponded to the same number and squeezing the decimal representation of both rational and irrational numbers, prospective teachers were able to deduce that real numbers could be represented by the limits of rational number sequences. Results might contribute to the mathematics education field by providing task sequences showing how difficulties regarding the real numbers could be eliminated via focusing on quantities.

***Keywords*:** Real numbers, quantitative reasoning, decimal representation, prospective teachers

Introductıon

Numbers are in the center of mathematics learning and curriculum. Due to the importance of the numbers in mathematics education, researchers focused on students, pre-service and in-service teachers conceptions of numbers. Particularly, Tall and Schwarzenberger (1978) emphasized that students have misconceptions about the relationships between decimal and limit concept and decimal and fractions. Difficulty aroused about real numbers was that students had a confusion when two different decimals represented the same real number (Tall & Schwarzenberger, 1978). Similarly, in an another study (Ely, 2010) about students' conceptions on infinitesimals, an undergraduate calculus student was interviewed. The researcher revealed a similar misconception: the student did not think that 3,99999.... and 4 represents the same number. In addition, Voskoglou and Kosyvas (2012) examined students' difficulties in understanding real numbers via quantitative and qualitative studies. The results of the study showed that students had an incomplete understanding of rational numbers;they preferred to deal with decimal representation more than fractions; and, students having difficulties in rational numbers could not answer the questions about irrational numbers, too. They suggested that while teaching real numbers, the use of multiple representations of real numbers and flexible transformations among them shoucl be emphasized (Voskoglou, 2013, p.41). Similarly, it is proposed that for the teaching of real numbers, quantitative reasoning (Thompson, 2011) could be used (Karagöz-Akar, 2016). Therefore, in the study, we investigated the following research questions: How do prospective mathematics teachers reason while

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developing the decimal representation of real numbers through quantitative reasoning? What meanings of real numbers do prospective mathematics teachers develop during an instructional sequence involving quantitative reasoning?

Method

In this section, we describe participants, data collection and data analysis. The sample of the study (N=19) was the prospective primary and secondary mathematics teachers (from now on called as students) in the same university. The design of the study was classroom teaching experiment (Cobb, 2000) in which "the researchers engage in promoting the development as part of cycle of interaction and reflection" (Simon & Tzur, 1999, p. 253). The teaching sequencewas divided into four phases: Phase I included a review on rational numbers, fractions, equivalent fractions and their relationships. Phase II included students' representing examples of terminating and non-terminating positive rational numbers and their making sense of the relationships among different representations such as long division, division algorithm and diagrams. Phase IIIa-b included students' re-writing and examining the properties of the decimal representations of any rational number as sequences. Phase IV involved examining irrational numbers through decimal representations and students' reaching at an altenative definition of real numbers (Usiskin, Peressini, Marchisotto & Stanley, 2003). Throughout the teaching sessions done by the first author, along with the individually written artifacts, students' discussions and class work were videotaped and recorded. Also, before and after the teaching sessions, to assess students' current knowledge a written assessment was given. After the completion of teaching sessions, the data were transcribed by the first researcher. For the analysis, both ongoing andretrospective analysis (Steffe & Thompson, 2000) was followed:Reading line by line the transcipts of the teaching sessions,the researcher thought of both the individuals' mathematical thinking and class' mathematical thinking as a group (Heinz, 2000). The reason of focusing on individual students' mathematical reasoning was that "the individual students' mathematical activity and the activity of the classroom was related" (Heinz, 2000, pp.59).

Results and Findings

Since the focus of analysis was students' quantitative reasoning throughout the teaching sessions, while doing that individual students' and also the inclassroom's thought processes wererevealed. Mainly the results about Phase II, Phase IIIa and Phase IV are shared in this section. Though we would like to briefly poin to the taken-as-shared meanings from Phase I. At the begining of the instruction, in Phase I, students were asked to define rational numbers, fractions,equivalent fractions and the relationship among them. Mostly students stated rational numbers as the ratio of two integers with the denominator not equal to zero. Students thought that rational numbers differed from fractions in the sense that rational numbers could take negative values. Also, some students knew and explained that fractions represent amounts. That is why all the students agreed upon the idea that fractions could be represented as positive rationals. Some students also stated and all the others agreed that equal fractions represented the same amounts. They were also able to draw diagrams representing both fractions and equivalent fractions. Realizing students' current stage of knowing, we emphasized that they would work on fractions and equivalent fractions as representing amounts and they would be asked to draw diagrams throughtout the whole teaching sessions. Also, during the Phase I, we asked students to show the decimal representation of 7/2 using three representations: long division, division algorithm and diagrams. Then, in Phase II, we asked students to express the decimal representation of 11/3 using three representations again: long division, division algorithm and diagrams. The reason together with working on the example 7/2 was to take their attention to the fractions with both non- repeating and repeating decimals. In the following excerpt we share the classroom discussion on the use of diagrams for finding the decimals of 11/3 with relation to the long division:

R: How did you continue to division?
S1: After the remainder 2, I put comma next to quotient and zero next to remainder.
R: Why did you do like that?
S1: Since I could not divide 2 by 3.
R: What does this mean in terms of diagrams?
S4: Expanding the fraction 2/3 by 10.
S9: Equal-partitioning.
R: So what happens to 2/3?
S1: 20/30.
R: How can you represent 20/30 in the diagram?
Class: We partition each piece into 10.

S1: We are looking for 1/10 ths in 2/3. We know that the little piece is 1/30. Therefore, I can take 3 piece (groups) of 1/30. Then I get 1/10 (showing the first three pieces). Actually I am searching for 3/30 ths in 2/3.

R: So how many 1/10 th do you have in the diagram?

S1: 1 2 3 4 5 6 many 1/10 th. I scanned 6 many 1/10 in the diagram by grouping 1/30 th by 3 and there is remaining portion 2/30.

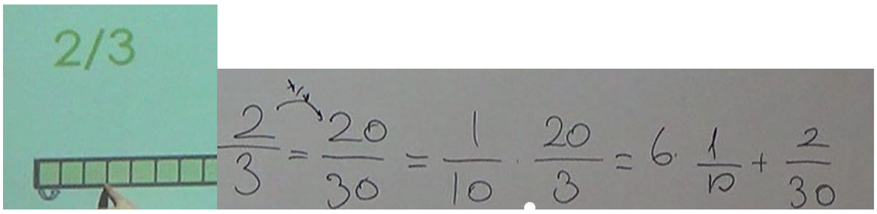

Figure 1. Expression for 2/3 through decimals

As the excerpt shows, the students related putting zero next to remainder and comma next to quotient in the long division through diagrams. They knew that getting equivalent fractions to 2/3 by partitioning each piece into 10 pieces meant putting zero next to the remainder in the long division. They also knew that they ooked for 1/10's after comma in the long division. Thus, in the diagram, they focused on the little piece they obtained through re-partitioning which was 1/30. Through grouping 3 of 1/30's which equaled to 1/10 students came up with the number of 1/10's in 2/3=20/30. As a result they found 6 many 1/10's and a remaining portion and grouped portion as 2/30. This reasoning showed that students worked on the fractional parts (e.g., 2/3) as quantities and engaged in partitioning and grouping. leading to That is, they not only regarded 2/3 as a number but as also a quantity. Therefore, quantitatively, they were able to measure 2/3 by 1/10's, resulting in 6 many of them with the left over part of 2/30. The following student work illustrated how they continued finding further decimals by just zooming in the ungrouped-left over part (2/30, 2/300 etc.) through re-partitioning by 10 and grouping by 3 again:

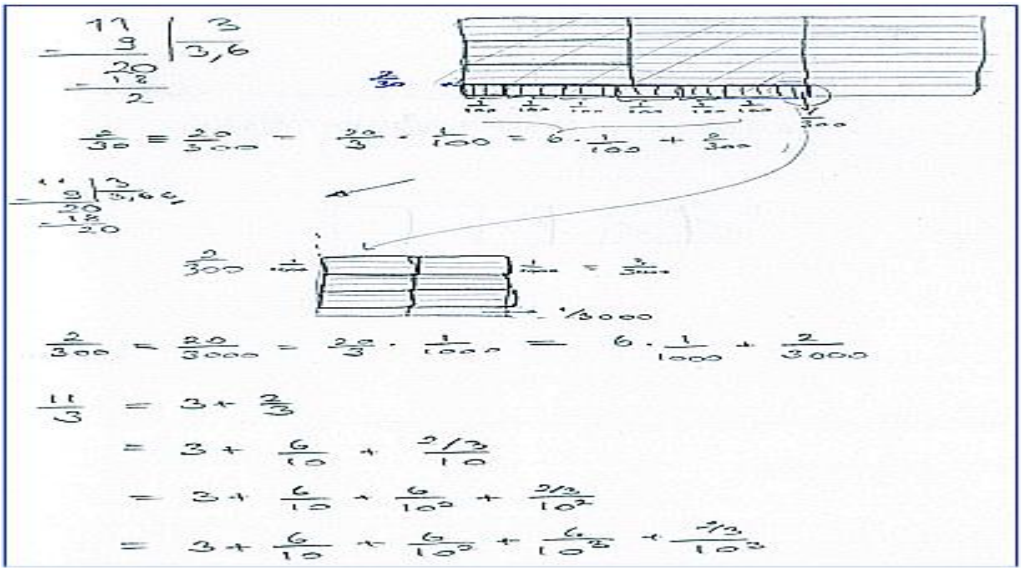

Figure 2. A student's work on the decimal representation of 11/3 through 3 representations

As the data indicated, students were able to think of 2/30 (the left over part) in the diagram as a quantity and re-partition it into 10 more equal pieces getting at 20/300. Then, regrouping 3 of the 1/300 getting at 1/100, they were able to find the number of 1/100 in 20/300. They knew they looked for 1/100s because they did not have any 1/10 any more in the left over part, 20/300 (e.i., 2/30). This way they knew there were 6 many 1/100 in 20/300 with left over 2/300. The Figure 2 also points to their repartitioning 2/300 (i.e., 20/3000) and regrouping 1/300 to come up with the number of 1/1000 in 20/3000 (i.e., 2/300).

Then, writing the decimal expansion of 11/3 as in the figure above (the last line in the figure), students were asked to squezze the decimals in nested intervals. By doing this students focused on the ungrouped parts of the diagram and based on these parts they obtained the intervals as in the figure 3.

After that students were asked to focus on the lower bounds and upper bounds separately and they were asked whether these lower and upper bounds formed sequences or not. Students agreed that the were sequences. Then, they examined the properties of the sequences and obtained that the lower bounds formed increasing bounded rational number sequences and upper bounds formed decreasing bounded rational number sequences. By finding the differences between corresponding terms of the sequences starting from the first setp and ending at the last step as in Figure 3, and finding that the difference in the last step was equal to $1/10^n$ they realized that when n goes to infinity the difference approached zero. Since the number between these sequences was 11/3, they cameup with the result that both sequences represented 11/3.

1.Step:

$3 + 6/10 < 3 + 6/10 + 2/3/10 < 3 + 6/10 + 1/10$

2.Step:

$3 + 6/10 + 6/10^2 < 3 + 6/10 + 6/10^2 + 2/3/10^2 < 3 + 6/10 + 6/10^2 + 1/10^2$

3.Step:

$3 + 6/10 + 6/10^2 + 6/10^3 < 3 + 6/10 + 6/10^2 + 6/10^3 + 2/3/10^3 < 3 + 6/10 + 6/10^2 + 6/10^3 + 1/10^3$

n.Step:

$3 + 6/10 + 6/10^2 + \ldots\ldots + 6/10^n < 3 + 6/10 + 6/10^2 + \ldots\ldots + 6/10^n + 2/3/10^n < 3 + 6/10 + 6/10^2 + \ldots\ldots + 6/10^n + 1/10^n$

Figure 3. Interval for 11/3

Then during Phase III, they generalized their findings to any positive rational number using symbols and notations. Then, in Phase IV, they focused on the irrational number case using the generic example √3. For the irrational number case, students thought of number line and as in the rational number case, they did repartitioning by 10 on the number line for the interval of √3. By taking squares of the bounds they placed 3 in the interval and then they took the square root of 3 for finding the interval of √3. By going this way they obtained nested intervals as (1.7, 1.8) ⊃(1.73, 1.74) ⊃(1.732, 1.733) ⊃.... Again focusing on then lower and upper bounds separately, they concluded that the lower bounds formed increasing bounded rational number sequences and upper bounds formed decreasing bounded rational number sequences. By finding the differences between corresponding terms of the sequences as they did earlier, theyanalyzed that when n goes to infinity the difference at the las step, $1/10^n$,approached zero. Since the number between these sequences was √3., they again concluded that both rational number seqeunces represented √3.

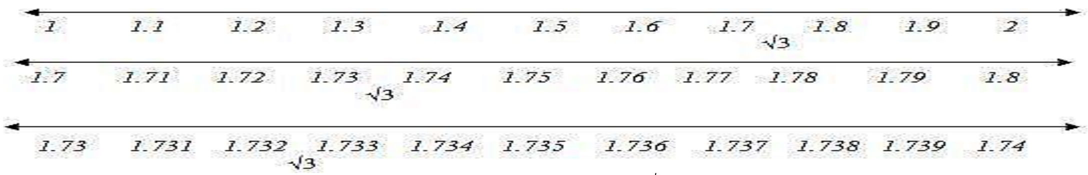

Figure 4. Interval of √3

Finally, by examining both a positive rational number and a positive irrational number, students concluded that positive real numbers can be represented by both increasing and decreasing bounded rational number sequences.

Conclusıon

Results from this study showed that reasoning on quantities depicted in diagrams, once students related long division with multiple representations of rational numbers such as fractions, equivalent fractions and decimals through the actions of equal partitioning, grouping and counting, they were able to deduce that all these representations corresponded to the same number. This was important because research has shown that students had difficulties regarding different representations showing the same number (Voskoglou & Kosyvas 2012). Similarly, squeezing the decimal representation of positive rational numbers, they were able to deduce that rational numbers could be represented by rational number sequences. Also, through partitioning and re-partitioning of the intervals on an example of an irrational number, they realized that irrational numbers also could be squeezed by two rational number sequences. Therefore, they could conclude that not only rational numbers but also irrational numbers could be thought of as the limits of sequences of rational numbers. This was also important because previous research has shown that students had difficulties in reasoning about rational numbers, irrational numbers and hence real numbers (Voskoglou, 2013). In our case, since students were able to deduce an alternative defintion of real number by using rational number sequences multiple representations as fractions diagrams, decimals and rational number sequences, they were able to think that different decimals could correspond to the same real number.

References

Cobb, P. (2000). Conducting teaching experiments in collaboration with teachers. In A. E. Kelly, R. A. Lesh (Eds), *Handbook of Research Design in Mathematics and Science Education,* pp. 307–333. Lawrence Erlbaum Associates.

Ely, R. (2010). Nonstandard student conceptions about infinitesimals. *Journal for Research in Mathematics Education*, *41*(2), 117–146.

Heinz, K. A. (2000). *Conceptions of ratio in a class of pre-service and practicing teachers.* Ph.D. Thesis, The Pennsylvania State University.

Karagoz Akar, G. (2016). Nicel muhakeme ve nicel muhakeme yoluyla kesirler üzerinden gerçek sayıların inşaası. In E. Bingölbali, S. Arslan, & İ. Ö., Zembat (Eds.), *Matematik Eğitimi Teorileri* (pp.117–133). Ankara: Pegem Akademi.

Simon, M. A. and Tzur, R. (1999). Explicating the teacher's perspective from the re- searchers' perspectives: Generating accounts of mathematics teachers' practice. *Journal for Research in Mathematics Education*, *30*(3), 252–264.

Steffe, L. P. and Thompson, P. W. (2000). Teaching experiment methodology: Underlying principles and essential elements. *Handbook of research design in mathematics and science education*, pp. 267–306.

Tall, D., & Schwarzenberger, R. L. (1978). Conflicts in the learning of real numbers and limits. *Mathematics Teaching*, *82*, 44–49.

Thompson, P.W. (2011). Quantitative reasoning and mathematical modeling. In S. Chamberlin, L.L. Hatfield, & S. Belbase (Eds.), *New perspectives and directions for collaborative research in mathematics education: Papers from a planning conference for WISDOM* (pp. 33-57). Laramie: WY: University of Wyoming.

Usiskin, Z., Peressini, A., Marchisotto, E. A. & Stanley, D. (2003). *Mathematics for high school teachers.* Prentice Hall: Upper Saddle River, New Jersey.

Voskoglou, M. and Kosyvas, G.D. (2012). Analyzing students' difficulties in understanding real numbers. *Journal of Research in Mathematics Education*, *1*(3), 301–336.

Voskoglou, M. G. (2013). An application of the APOS/ACE approach in teaching the irrational numbers. *Journal of Mathematical Sciences and Mathematics Education, 8*(1), 30–47.