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# Capacitated Multi Drone Assisted Vehicle Routing Problem

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**Highlights**

• This study differs by considering simultaneous operations of different vehicles.
• A model is proposed for Capacitated Multi Drone Assisted Vehicle Routing Problem.
• Efficient greedy heuristics for the problem is developed.

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**Keywords**

*Drone*
*Unmanned aerial vehicle,*
*Capacitated vehicle routing problem,*
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**Abstract**

This research delves into the dynamic landscape of transportation systems, with a specific focus on the integration of drones and conventional vehicles. The study presents a Mixed Integer Programming (MIP) model for the Capacitated Multi-Drone Assisted Vehicle Routing Problem (mDroneCVRP), aiming to minimize the time of the last vehicle's arrival at the warehouse. It is essential to highlight that the proposed model was effectively solved using the CPLEX algorithm within the GAMS framework, underscoring the sophistication of the solution approach. The integration of multiple drones into the routing process proves to be instrumental in significantly reducing service time, demonstrating the efficacy of synergizing drone and truck operations. As the number of nodes escalates, emphasizing the necessity for heuristic approaches to address larger instances, the study provides valuable insights into the judicious use of drones in synchronized routing operations. Furthermore, the research challenges conventional assumptions by permitting drones to take off from and land on different vehicles, thereby augmenting operational capabilities and adeptly tackling contemporary transportation challenges.

## 1. INTRODUCTION

In the context of rapid technological evolution, transportation systems are witnessing substantial development, especially within the logistics sector. The sector's goal is to forge resilient and cost-effective transportation networks by closely analyzing cost factors and seeking efficiency enhancements. This endeavor is bolstered by the scientific community's efforts to address the inherent challenges of such systems, spurred by an escalating demand for improved logistics solutions.

The exploration of network problems, notably the Traveling Salesman Problem (TSP) initially modeled by Dantzig et al. [1] and identified as an NP-Hard issue, and its subsequent evolution into the Vehicle Routing Problem (VRP) introduced by Dantzig and Ramser [2], exemplifies the ongoing scientific quest for optimizing distribution channels for both cost and time efficiency.

The VRP has been a focal point of study across various domains, showcasing adaptations tailored to the distinct requirements of different systems, influenced by factors such as vehicle types, node configurations, warehouses, and time constraints. The motivation behind these studies is driven by the commercial and organizational need to refine distribution networks to minimize costs and meet delivery standards, crucial for enhancing customer satisfaction and operational success, as seen in both production and marketing spheres.

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Beyond the commercial domain, distribution systems are garnering attention in fields like humanitarian logistics, where the efficiency of delivery—marked by timely and material-specific requirements—is paramount. The scientific literature is rich with vehicle routing studies catering to these areas.

As technology advances, the scope of VRP studies has broadened to include new forms of transportation, evidenced by the categorization of transport modes into air, land, or water, and further into multimodal and intermodal transport. Notably, the advent of unmanned aerial vehicles (UAVs or drones) introduces unique routing challenges and opportunities distinct from classical VRPs, facilitating the integration with other vehicle types and thus transforming the vehicle routing problem landscape.

This study delves into the burgeoning area of new-generation vehicle routing problems, emphasizing the role of cutting-edge technology. It specifically focuses on the Unmanned Aerial Vehicle-assisted Traveling Salesman Problem (TSP-drone), as categorized by Murray and Chu [3], with additional classifications and assumptions outlined by Otto et al. [4]. Unlike prior research, this work introduces a mixed-integer programming model for the Multiple-Drone Assisted Capacitated Vehicle Routing Problem (mDroneCVRP), aiming to minimize the time for the last vehicle to return to the warehouse. This model, adeptly solved using the CPLEX algorithm within the GAMS framework, represents a significant stride in computational methods applied to transportation problems. It innovatively allows drones to take off from and land on different vehicles, offering a flexible approach to minimizing makespan, diverging from the traditional constraints of returning to the launching vehicle.

Following an overview of related studies and the TSP-Drone classifications, the paper delineates the problem definition, assumptions, and the mathematical model devised for mDroneCVRP. The creation of test data for this model and the derived solutions are elaborated upon, showcasing the model's application and effectiveness. An innovative algorithm developed to solve this complex problem is detailed, followed by a presentation of the results obtained, underscoring the model's potential and the efficiency gains from employing drones in vehicle routing. The conclusion discusses the implications of these findings and suggests directions for future research, emphasizing the transformative impact of drone technology on logistics and vehicle routing systems.

## 2. RESEARCH METHODOLOGY

The literature on drone operations is examined in the first stage of the study. In the second stage, the literature gap is identified, with a specific focus on the synchronous operations of drones and vehicles, as emphasized by Otto et al. [4]. Once the problem is determined, a mixed-integer programming model is developed. This process is illustrated in Figure 1.

Moving to the fifth stage, datasets are derived to validate the model. Subsequently, the model undergoes validation through an experimental study. The seventh stage involves the development of a greedy heuristic designed to solve the identified problem. Finally, the results obtained from different stages are compared, and conclusions are drawn to wrap up the study.

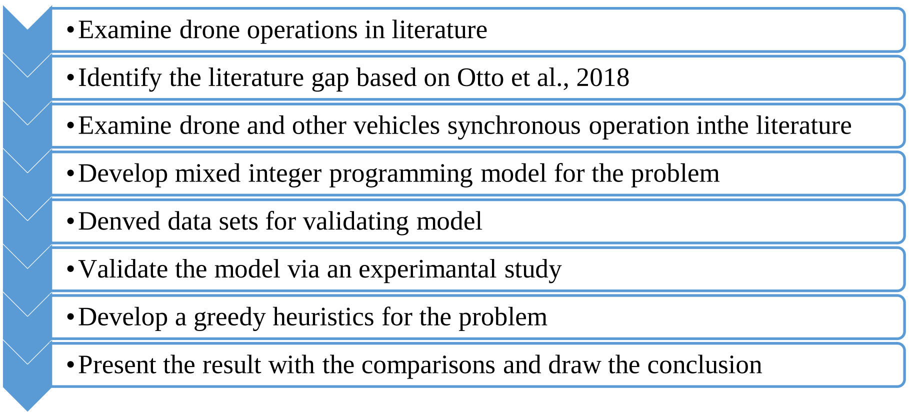


***Figure 1.*** *Research methodology steps*

## 3. LITERATURE REVIEW

The research by Murray and Chu [3] integrates drones with vehicles in the Traveling Salesman Problem, introducing MILP models for the Flying Sidekick TSP (FSTSP) and Parallel Drone Scheduling TSP (PDSTSP), along with a heuristic algorithm. FSTSP plans routes for both trucks and drones to serve customers simultaneously, with drones launching from and returning to the truck. PDSTSP allows trucks and drones to independently serve customers.

Bouman et al. [5] explore all potential truck and drone route combinations, differing from Murray and Chu [3] by considering rendezvous points as potential departure points, and introduce heuristic solutions based on local search and dynamic programming.

Ha et al. [6] focus on cost minimization, proposing local search and grasp heuristic algorithms for problems with 50 and 100 nodes. A subsequent study by Ha et al. [7] introduces a "cluster first-route second" heuristic, optimizing drone paths before truck routing, and a "route first-cluster second" method.

Wang et al. [8] pioneer the investigation of multi-truck and multi-drone systems, conducting theoretical analyses to establish performance benchmarks. Poikonen et al. [9] further this research, defining theoretical limits for expanded scenarios. Ponza [10] applies simulated annealing in TSP-Drone, marking the first use of meta-heuristics in this field.

Ferrandez et al. [11] experiment to find the optimal clusters and speed ratios for trucks and drones, using the K-means algorithm for efficient delivery point clustering. The classification of the studies according to the approach and solution methods is given in Table 1a and Table 1b.

The broader scope of vehicle and drone collaboration research, as summarized by Otto et al. [4], spans various applications from disaster response to enhancing vehicle coverage and connectivity. The potential for ships supporting drone operations highlights the evolving nature of these systems in future logistical and surveillance tasks.

Otto et al. [4] classify research on vehicle and drone operations by their roles within the system, emphasizing that their importance is determined by their assigned task weight. Should their roles be equivalent, they can operate concurrently or take turns leading to optimize performance. The primary objective for vehicles or drones is usually predefined, with their operations imposing certain constraints [4]. Operations can be synchronous or asynchronous, where synchronization depends on various factors like traffic conditions and task times (Mathew et al. [12]; Tokekar et al. [13]). The need for coordination between drones and vehicles leads to novel optimization challenges [4]. Research in this area typically aims to minimize expenses by considering aspects such as the number of drones (Jia and Zhang [14]) or operation duration. Other studies focus on enhancing service quality through different metrics, including the quantity of deliveries (Savuran and Karakaya [15]), the amount of sensor data collected (Tokekar et al. [13]), and the efficiency of drone communication (Wu et al. [16]). Additionally, Viguria and Maza [17] explored how penalizing the distribution of a single task among multiple vehicles could improve the cooperation between drones and vehicles. This body of work collectively examines the division of combined operations based on actual performance drivers and the need for synchronization. Figure 2 describes the separation of combined operations according to actual performance servers and synchronizations.

Vehicles assist drones in scenarios where drones play a crucial role, typically in situations where vehicles do not make deliveries and move slower than drones. Drones are able to cover specific distances over vehicles, with research varying based on objectives, vehicle-drone interaction patterns, and energy limitations. Tokekar et al. [13] explored using speed ratios to facilitate drone-vehicle rendezvous, focusing on transport without delivery. Other studies, like those by Luo et al. [18], Mathew et al. [12], and Garone et al. [19], consider the possibility of drones recharging on vehicles. Commonly, drones deliver packages directly to customers, while vehicles serve as mobile bases, extending the drones' operational range (Ferrandez et al. [11]; Mathew et al. [12]). Garone et al. [19] addressed optimizing sea rescue missions in a continuous space, defining drone destinations within vehicle reach. Mathew et al. [12] tackled the asymmetric Traveling Salesperson Problem (TSP), omitting wait times to minimize delivery costs.

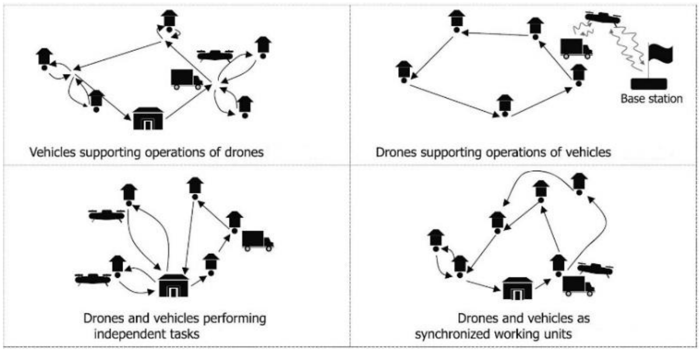


***Figure 2.*** *Combined working patterns of drones and vehicles, (Otto et al. [4])*

In operations with trucks as the main performance driver, drones are used as support vehicles for trucks. Drones can be seen as vehicles supporting mobile warehouses in these operations. Studies have been conducted to minimize the cost of the drone tour (Savuran and Karakaya [15]) or visit the maximum number of customers (Savuran and Karakaya [20]), leaving the carrier with a certain speed and route. Such studies include applications that can contribute significantly to mobile warehouse problems (Otto et al. [4]).

In scenarios where drones and vehicles operate independently and both significantly contribute to performance, no coordination is needed. Murray and Chu [3] outlined a scenario where trucks and drones deliver to individual customers, aiming to minimize the number of drones and optimize service times for both trucks and drones. Here, drones operate without truck support, delivering packages to each customer before returning to the warehouse for the next delivery. Ulmer and Thomas [21] investigated same-day deliveries by either trucks or drones, considering random customer orders to decide on order acceptance.

Synchronization between drones and vehicles becomes essential when they need to meet at specific points. This is evident in the Generalized Service Problem (GSP) where they collaborate closely, as explored by Murray and Chu [3], Agatz et al. [30], and Ha et al. [6]. In such settings, the vehicle departs from the warehouse to serve customers and completes a round trip. The drone, stationed on the vehicle, delivers packages within its reach and returns to the vehicle for battery recharge (Kundu and Matis [45]). The drone's operational range is limited by its maximum flight time, leaving further distances to the vehicle. It must rendezvous with the vehicle at customer locations and can only depart from and return to the vehicle it launched from. Murray and Chu [3] observed that synchronization significantly impacts costs due to the waiting times incurred by both the vehicle and drone during their meet-ups.

The Vehicle Routing Problem (VRP) with drones has been expanded by Wang et al. [8] and Poikonen et al. [9] to include analyses of cost and efficiency, showing that combined truck and drone delivery can significantly reduce delivery times when they share a network and move at similar speeds. Poikonen et al. [9] also explored the maximum efficiency gains under energy constraints and varying distances. Carlsson and Song [23] and Campbell et al. [22] used a continuous approach to predict delivery costs and times, assuming customer locations are spread over a plane, with different metrics for vehicle and drone distances. This approach demonstrated the cost benefits of integrating drones, especially when customer locations are randomly set. Daknama and Kraus [24] tested various metaheuristics for drone and vehicle delivery, allowing drones to land on any safely parked vehicle.

***Table 1a.*** *Classification of Studies of Until 2018*

| Author | Problem | Objective Function | Solution Approximation | Methodology | Truck | Drone | Warehouse |
|---|---|---|---|---|---|---|---|
| Ha et al. [7] | FSTSP | minimize total travel time, maximize the profit | TSP | Heuristics | 1-Truck | 1-Drone | 1-Depot |
| Mathew et al. [12] | Heterogeneous Delivery Problem (HDP), Multiple Warehouse Delivery Problem (MWDP) | Total cost | TSP | Heuristics | 1-Truck | 1-Drone | 1-Depot |
| Murray and Chu [3] | FSTSP, PDSTSP | Makespan | TSP, parallel machine scheduling | Heuristics | 1-Truck | 1-Drone | 1-Depot |
| Ferrandez et al. [11] | PDSTSP | The optimal number of launch locations and the optimal total time of delivery. | TSP | Heuristics | 1-Truck | d-Drone | 1-Depot |
| Ponza [10] | FSTSP | Makespan | TSP | Heuristics | 1-Truck | 1-Drone | 1-Depot |
| Bouman et al. [5] | TSP-D | minimum cost tour | Bellman-Held-Karp dynamic programming algorithm | Exact | 1-Truck | 1-Drone | 1-Depot |
| Campbell et al. [22] | FSTSP | Minimize operational cost | continuous approximation (CA) | | 1-Truck | d-Drone | 1-Depot |
| Carlsson and Song [23] | FSTSP | minimum completion time | continuous approximation (CA) | Heuristics | 1-Truck | 1-Drone | 1-Depot |
| Daknama and Kraus [24] | Vehicle Routing with Drones (VRD) | average delivery time | VRPD | Heuristics | m-Truck | d-Drone | 1-Depot |
| Luo et al. [18] | a two-echelon GV and UAV cooperated routing problem (2E-GU-RP) | minimizes the total routing time | VRPD | Exact, Heuristics | 1-Truck | 1-Drone | 1-Depot |
| Poikonen et al. [9] | VRPD | minimize the completion time | close enough routing, VRPD | Theorems with worst case scenarios | m-Truck | d-Drone | 1-Depot |
| Wang et al [8] | VRPD | minimize the completion time | worst case analysis | Theorems with worst case scenarios | m-Truck | d-Drone | 1-Depot |
| Agatz et al. [25] | TSP-D | minimum cost tour | Eulerian cycle,TSP | Exact, Heuristics | 1-Truck | d-Drone | 1-Depot |
| Chang and Lee [26] | TSP-D | total delivery time | K-means clustering, TSP, Nonlinear programming | Heuristics | 1-Truck | d-Drone | 1-Depot |
| Cheng et al. [27] | MTDRP-EC | Total Transportation cost | Nonlinear | CUTS | 0-Truck | d-Drone | 1-Depot |
| Ha et al. [6] | TSP-D | Minimize operational cost | TSP | Exact, Heuristics | 1-Truck | 1-Drone | 1-Depot |
| Ham [28] | PDSTSP | Minimize maximum completion time | Constraint programming | Exact | m-Truck | d-Drone | n-Depot |
| Yurek and Ozmutlu [29] | TSP-D | delivery completion time | TSP | Exact, Heuristics | 1-Truck | 1-Drone | 1-Depot |
| Ulmer and Thomas [21] | SDDPHF (same-day delivery routing problem with heterogeneous fleets) | maximize the expected number of customers | stochastic dynamic vehicle routing | Heuristics | m-Truck | d-Drone | 1-Depot |

***Table 1b.*** *Classification of Studies of After 2018*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Yurek and Ozmutlu [29] | TSP-D | delivery completion time | TSP | Exact, Heuristics | 1-Truck | 1-Drone | 1-Depot |
| Hu et al. [30] | Vehicle-Assisted Multi-UAV inspection (VAMU) | Time wastage | Joint Routing and Scheduling | Heuristics | 1-Truck | d-Drone | 1-Depot |
| Jeong et al. [31] | FSTSP that implements energy consumption and no fly zone | Total time | TSP | Exact, Heuristics | 1-Truck | 1-Drone | 1-Depot |
| Karak and Abdelghany [32] | VRPDERO (en route operations) | Makespan | VRPD | Exact, Heuristics | m-Truck | d-Drone | 1-Depot |
| Kitjacharoenchai et al. [33] | mTSPD | Total time | TSP | Exact, Heuristics | m-Truck | d-Drone | 1-Depot |
| Peng et al. [34] | FSTSP | total time consumption | Location, VRP, and Bin Packing | Heuristics | 1-Truck | d-Drone | 1-Depot |
| Roberti and Ruthmair [35] | TSP-D | Makespan | TSP | Heuristics | m-Truck | d-Drone | 1-Depot |
| Sacramento et al. [36] | VRPD | Operational cost | VRPD | Exact, Heuristics | m-Truck | d-Drone | 1-Depot |
| Sah [37] | DTCO, MDTCO | Makespan | TSP | Exact, Heuristics | m-Truck | d-Drone | 1-Depot |
| Schermer et al. [38] | mTSPD | Makespan | TSP | Exact, Heuristics | m-Truck | d-Drone | 1-Depot |
| Schermer et al. [39] | VRPD | Makespan | VRPD | Exact, Heuristics | m-Truck | d-Drone | 1-Depot |
| Wang et al. [40] | CVRP | Makespan | TSP | Dynamic programmi ng, Branch, and price algorithm | 1-Truck | 1-Drone | 1-Depot |
| Wang, Z. and Sheu [41] | VRPD | Operational cost | VRPD | Exact | m-Truck | d-Drone | 1-Depot |
| Kitjacharoenchai et al. [42] | 2EVRPD | Total Time of Trucks | VRPD | Exact, Heuristics | m-Truck | d-Drone | 1-Depot |
| Murray and Raj [43] | mFSTSP | Makespan | TSP | Exact, Heuristics | 1-Truck | d-Drone | 1-Depot |
| Poikonen and Golden [44] | k-MVDRP | Makespan | TSP | Exact, Heuristics | 1-Truck | d-Drone | 1-Depot |

Kitjacharoenchai et al. [33, 42] investigated using multiple trucks and drones, disregarding flight limits, and presented models to minimize route times and consider truck capacity and drone visitations. Sacramento et al. [36] and Wang et al. [40] developed heuristic and parallel operation models for trucks and drones, respectively. Wang and Sheu [41] offered a mixed-integer model addressing complex scenarios with multiple drones and trucks, utilizing a branch and price algorithm. Sah [37] provided a comprehensive classification of drone-truck operations, presenting models for various routing challenges. Schermer et al. [39], Murray and Raj [43], and Karak and Abdelghany [32] explored multi-drone and truck combinations, focusing on service times and drone collection from non-customer points. Poikonen and Golden [44] introduced the multi-visit drone routing problem (MVDRP), a novel VRP variant.

Tamke and Buscher [46] tackled the VRP with Drones and Drone Speed Selection (VRPD-DSS), proposing a model that considers speed-dependent energy consumption for cost-saving in rural deliveries. Zhou et al. [47] focused on the Two-Echelon VRP with Drones (2E-VRP-D), optimizing last-mile deliveries with a collaborative truck-drone system. Xia et al. [48] proposed the VRP with Load-Dependent Drones (VRPLD), emphasizing the importance of energy consumption modeling for routing and hub placement.

These studies collectively advance the field of drone-assisted vehicle routing, exploring various models and heuristics to improve delivery efficiency and cost-effectiveness.

## 4. THE MODEL

### 4.1. Problem Description

While defining the problem, it is necessary to examine the usage areas and purposes of the drone. Today, many operations can be performed with drones. As seen in Figure 3, these operations are classified according to drone capabilities. In Figure 3, there are 6 main areas in drone operations. These are area coverage, search operations, routing for a set of locations, data gathering and recharging in a wireless sensor network, allocating communication links and computing power to mobile devices, and operational aspects of a self-organizing network of drones (Otto et al. [4]).

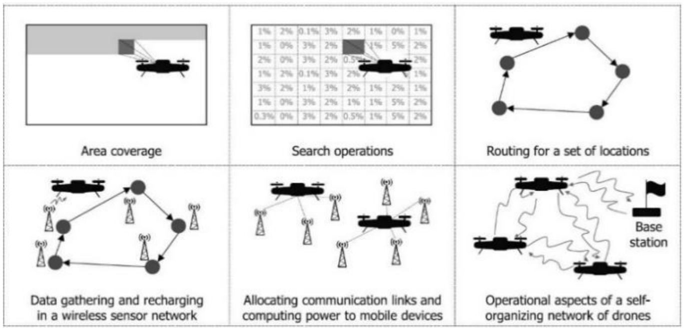


***Figure 3.*** *Classification of drone operations, (Otto et al. [4])*

In this study, the VRP is discussed to realize the integrated distribution of drones with other vehicles. The study, apart from the uniform vehicle routing in the classical TSP or VRP, is aimed at planning combined operations of drones and other vehicles as synchronized working units.

Figure 4 illustrates the efficiency of combining drones with vehicles in routing problems, contrasting traditional vehicle routing (VRP) with TSP-DRONE solutions. In classical TSP, only trucks are routed, while TSP-DRONE integrates both truck and drone routes, significantly improving efficiency (classical TSP objective value is 1500 vs. TSP-DRONE's 1001).

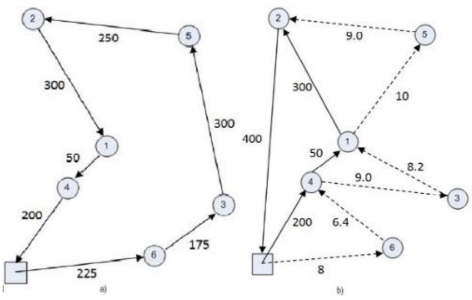


***Figure 4.*** *(a) Optimal solution of TSP, (b) Optimal TSP-Drone solution (Ha et al. [6])*

The essence of this analysis is the collaborative utilization of drones with vehicles for transportation, as discussed by Murray and Chu [3]. The integration alters the problem's dynamics and goals based on the drones and vehicles' capabilities. Figure 5 compares service times between conventional transportation and drone-vehicle combinations. It shows that drone-assisted solutions, despite drones' flight time and distance limitations, are more cost-effective than traditional methods, highlighted by Gantt charts depicting the operational efficiency gains.

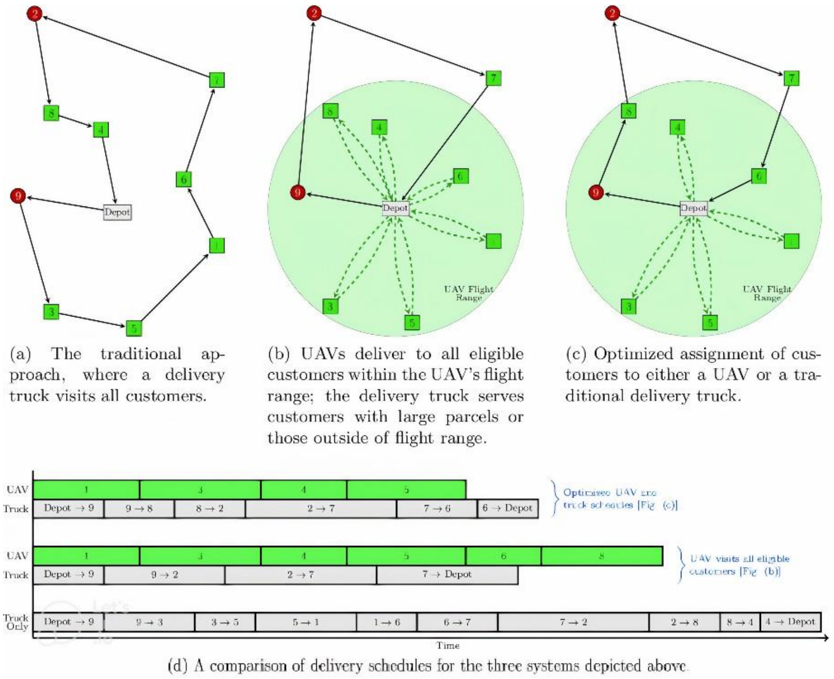


***Figure 5.*** *Classical method in transportation, the method using a drone and its optimized version (Murray and Chu [3])*

The technological and physical limitations of drones also form the assumptions of the problem in a way. Considering the problem of Murray and Chu [3], drones are flying from the warehouse and returning to the warehouse, as given in Figure 6, the comparison of the drones flown from the truck and the classical method in terms of total service time due to the flight time and battery life is shown in Figure 7.

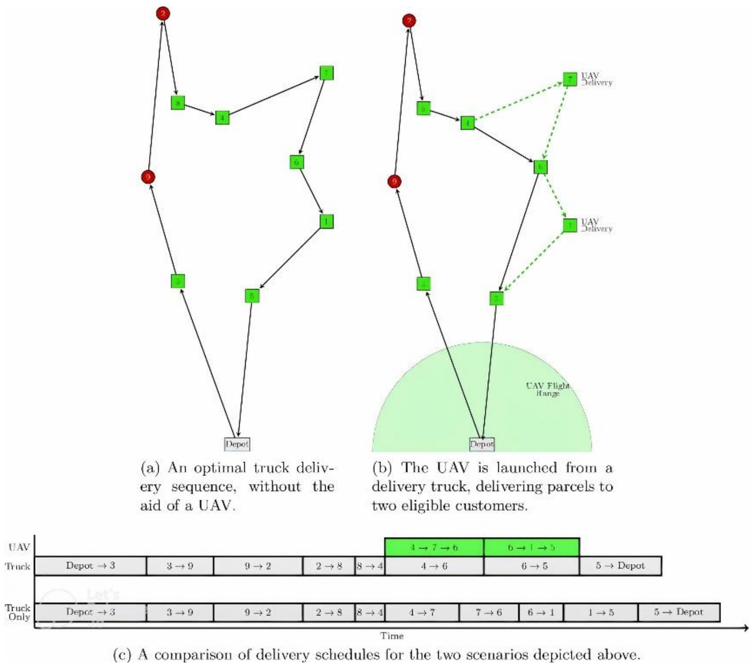


***Figure 6****. Transport of drones with trucks and the classical method (Murray and Chu [3])*

Meeting points between trucks and drones, beyond flight times and battery life, significantly impact problem dynamics. As shown in Figure 6, drones rendezvous with the truck at customer service points, introducing a key variable in problem-solving strategies.

This study focuses on optimizing the synchronized routes of vehicles and drones. Traditional TSP-DRONE approaches often use a single drone per vehicle, but as Kitjacharoenchai et al. [33] demonstrate, employing multiple drones can further reduce service times, as evidenced in Figure 7.

Another assumption revisited is the vehicle's capacity to serve all customers, transforming the problem into a capacitated vehicle routing problem (CVRP). This study distinguishes itself by considering the capacity of both vehicles and drones and by not requiring drones to return to the same vehicle from which they launched, simplifying the complex NP-Hard problem.

The aim is to minimize the return time of the last vehicle to the warehouse by synchronizing the routes of capacity-equipped trucks and drones, ensuring each customer is served by one vehicle type. This led to the development of a mathematical model for the Capacitated Multiple Drone Assisted Vehicle Routing Problem (mDroneCVRP), addressing these nuanced challenges.

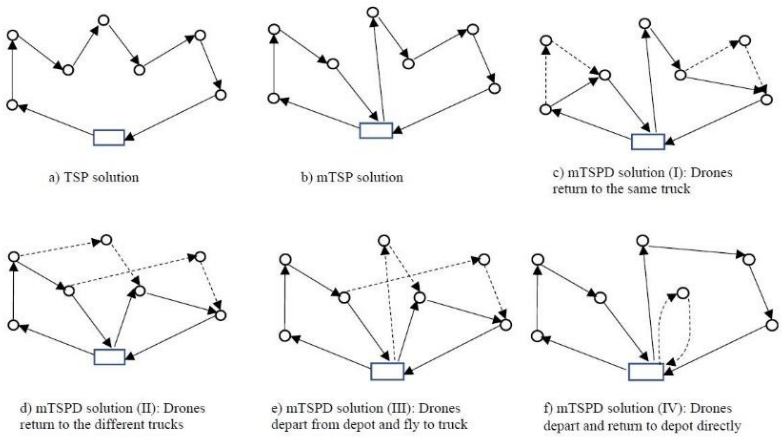


***Figure 7.*** *TSP, multi-TSP, and multi-TSP-DRONE feasible solutions are shown (Kitjacharoenchai et al. [33])*

### 4.2. Assumptions

The assumptions of the problem are listed below.
1. Drones and trucks can serve together.
2. Drones can take off over trucks, and the customer cluster it will visit may vary depending on flight time and battery life.
3. The meeting points of the vehicles must be at a service point, that is, a point where any customer is or is on the route to the customer.
4. The meeting point can be a return depot only after all service points have been called.
5. Drones can land on vehicles other than the one from which they take off.
6. More than one drone can take off and land at a time, within the limit of the number of take-offs and landings from the trucks.
7. The amount that the drone will take to the customer is included in the total capacity of the truck.
8. While the drone can only visit one customer per flight, the truck can visit multiple customers.
9. A truck or drone that returns to the warehouse cannot return to service.
10. The drone's battery is replaced without charging, and the replacement time is added to the flight start time and the truck landing time.
11. Drones are flying at a constant speed and the fluid density of air and gravity is assumed to be constant.
12. Speed coefficients are used between trucks and drones. The distance between the nodes is calculated as the Euclidean distance. Drones have always been assumed to be faster than trucks.

Considering these situations, a mathematical programming model of the problem is created as a mixed integer programming model.

### 4.3. Mathematical Model

In this section, considering the assumptions described above the proposed mathematical programming model is explained. Models in Murray and Chu [3], Kitjacharoenchai et al. [33], and Schermer et al. [38], [39], are used as base models for this study.

The index sets and parameters are given in Appendix A. In total, *c* customers are belonging to set C (customer cluster). In addition, there is one warehouse node in the network. The storage point is defined as 0 for the output. It is also defined as *c+1* as the return node. The set of all nodes in the road network is defined as N and there are a total of *c+2* node elements. Output and input nodes are defined as separate sets, respectively, as $N_0$, and $N_+$. *TR* and *DR* sets are defined for vehicles and drones, respectively. Since drones can only make a flight in three sequential points, a set of triple *S* has been created by the flight limits.

***Decision Variables***

$$X_{ijv} = \begin{cases} 1, \text{ if vehicle } v \text{ goes from node } i \text{ to node } j \\ 0, \text{ otherwise} \end{cases}$$

$$FTD_{ijvd} = \begin{cases} 1, \text{ if drone } d \text{ takes off over vehicle } v \text{ at node } i \\ \quad \text{and lands on customer node } j \\ 0, \text{ otherwise} \end{cases}$$

$$ATD_{jkvd} = \begin{cases} 1, \text{ if drone } d \text{ takes off from customer node } j \\ \quad \text{and lands on vehicle } v \text{ at customer node } k \\ 0, \text{ otherwise} \end{cases}$$

$$PT_{ijv} = \begin{cases} 1, \text{ if node } i \text{ precedes node } j \text{ in the route of} \\ \quad \text{vehicle } v \\ 0, \text{ otherwise} \end{cases}$$

$$PD_{ijd} = \begin{cases} 1, \text{ if node } i \text{ precedes node } j \text{ in the route of} \\ \quad \text{drone } d \\ 0, \text{ otherwise} \end{cases}$$

$TT_{iv} =$ Arrival time of vehicle *v* to node *i*

$TD_{id} =$ Arrival time of drone *d* to node *i*

$UT_{iv} =$ The sequence number of node *i* in the route of vehicle *v*

$UD_{id} =$ The sequence number of node *i* in the route of drone *d*

$CT_{\max} =$ Maximum service completion time of working units (makespan)

$$Y_{ijd} = \begin{cases} 1, \text{ if drone } d \text{ moves from node } i \text{ to node } j \\ 0, \text{ otherwise} \end{cases}$$

$$SA_{jkd} = \begin{cases} 1, \text{ if drone } d \text{ depart from customer node } j \text{ and} \\ \quad \text{land on node } k \\ 0, \text{ otherwise} \end{cases}$$

$$R_{ijvd} = \begin{cases} 1, \text{ if drone } d \text{ is trasported from node } i \text{ to node } j \\ \quad \text{by vehicle } v \\ 0, \text{ otherwise} \end{cases}$$

$$SD_{ijd} = \begin{cases} 1, \text{ if drone } d \text{ depart from node } i \text{ and land on} \\ \quad \text{customer node } j \\ 0, \text{ otherwise} \end{cases}$$

**Objective function:**

$$\text{Minimize } CT_{\max} \tag{1}$$

**Constraints:**

$$CT_{\max} \geq TT_{jv} \qquad \forall j \in N_+, v \in TR \tag{2}$$

$$CT_{\max} \geq TD_{jv} \qquad \forall j \in N_+, d \in DR \tag{3}$$

$$\sum_{v\in TR}\sum_{i\in N_0} X_{ijv} + \sum_{d\in DR}\sum_{\substack{i\in N_0\\ i\neq j}} SD_{ijd} = 1 \qquad \forall j \in C \tag{4}$$

$$\sum_{v\in TR}\sum_{i\in N_0} X_{ijv} + \sum_{d\in DR}\sum_{\substack{k\in N_+\\ k\neq j}} SA_{jkd} = 1 \qquad \forall j \in C \tag{5}$$

$$\sum_{i\in N_0} X_{i,c+1,v} \leq 1 \qquad \forall v \in TR \tag{6}$$

$$\sum_{j\in N_+} X_{0jv} \leq 1 \qquad \forall v \in TR \tag{7}$$

$$\sum_{i\in N_0} X_{ijv} = \sum_{k\in N_+} X_{jkv} \qquad \forall j \in C, v \in TR \tag{8}$$

$$\sum_{i\in N_0} Y_{ijd} = \sum_{k\in N_+} Y_{jkd} \qquad \forall j \in C, d \in DR \tag{9}$$

$$\sum_{i\in N_0} SD_{ijd} = \sum_{k\in N_+} SA_{jkd} \qquad \forall j \in C, d \in DR \tag{10}$$

$$\sum_{j\in N_+} Y_{ijd} \leq 1 \qquad \forall i \in N_0, d \in DR \tag{11}$$

$$\sum_{i\in N_0} Y_{i,j,d} \leq 1 \qquad \forall j \in N_+, d \in DR \tag{12}$$

$$SD_{1jd} + \sum_{v\in TR} R_{1jvd} = Y_{1jd} \qquad \forall j \in C, d \in DR \tag{13}$$

$$SA_{j,c+1,d} + \sum_{v\in TR} R_{j,c+1,vd} = Y_{j,c+1,d} \qquad \forall j \in C, d \in DR \tag{14}$$

$$SD_{ijd} + SA_{ijd} + \sum_{v\in TR} R_{ijvd} = Y_{ijd} \qquad \forall i \in C,\ j \in \{C : j \neq i\}, d \in DR \tag{15}$$

$$\sum_{\substack{i\in C\\ i\neq j}} SA_{ijd} \leq \sum_{\substack{k\in C\\ j\neq k}} SD_{jkd} + \sum_{v\in TR}\sum_{\substack{k\in N_+\\ j\neq k}} R_{jkvd} \qquad \forall j \in C, d \in DR \tag{16}$$

$$\sum_{\substack{k\in C\\ j\neq k}} SD_{jkd} \leq \sum_{\substack{h\in C\\ h\neq j}} SA_{hjd} + \sum_{v\in TR}\sum_{\substack{i\in N_0\\ i\neq j}} R_{ijvd} \qquad \forall j \in C, d \in DR \tag{17}$$

$$\sum_{\substack{i\in N_0\\ i\neq j}} R_{ijvd} \leq \sum_{\substack{k\in C\\ k\neq j}} SD_{jkd} + \sum_{\substack{k\in N_+\\ k\neq j}} R_{jkvd} \qquad \forall j \in C, v \in TR, d \in DR \tag{18}$$

$$\sum_{\substack{k\in N_+\\ k\neq j}} R_{jkvd} \leq \sum_{\substack{h\in C\\ h\neq j}} SA_{hjd} + \sum_{\substack{i\in N_0\\ i\neq j}} R_{ijvd} \qquad \forall j \in C, v \in TR, d \in DR \tag{19}$$

$$R_{ijvd} \leq X_{ijv} \qquad \forall i \in N_0,\ j \in \{C : j \neq i\}, v \in TR, d \in DR \tag{20}$$

$$\sum_{\substack{j\in C\\ j\neq i}} SD_{ijd} \leq \sum_{v\in TR}\sum_{\substack{h\in N_0\\ h\neq i}} X_{hiv} \qquad \forall i \in C, d \in DR \tag{21}$$

$$\sum_{\substack{j\in C\\ j\neq k}} SA_{jkd} \leq \sum_{v\in TR}\sum_{\substack{h\in N_0\\ h\neq k}} X_{hkv} \qquad \forall k \in C, d \in DR \tag{22}$$

$$\sum_{\substack{h\in N_0\\ h\neq i, h\neq j}} X_{hiv} + SD_{ijd} \leq FTD_{ijvd} + 1 \qquad \forall i \in C,\ j \in \{C : j \neq i\}, v \in TR, d \in DR \tag{23}$$

$$\sum_{\substack{h\in N_0\\ h\neq k, h\neq j}} X_{hkv} + SA_{jkd} \leq ATD_{jkvd} + 1 \qquad \forall j \in C, k \in \{C : k \neq j\}, v \in TR, d \in DR \tag{24}$$

$$\sum_{\substack{j\in C\\ j\neq i}} FTD_{ijvd} \leq \sum_{\substack{h\in N_0\\ h\neq i}} X_{hiv} \qquad \forall i\in C, v\in TR, d\in DR \tag{25}$$

$$\sum_{v\in TR} FTD_{ijvd} \leq SD_{ijd} \qquad \forall i\in C,\ j\in\{C:\ j\neq i\}, d\in DR \tag{26}$$

$$\sum_{\substack{j\in C\\ j\neq k}} ATD_{jkvd} \leq \sum_{\substack{h\in N_0\\ h\neq k}} X_{hkv} \qquad \forall k\in C, v\in TR, d\in DR \tag{27}$$

$$\sum_{v\in TR} ATD_{jkvd} \leq SA_{jkd} \qquad \forall j\in C, k\in\{C: k\neq j\}, d\in DR \tag{28}$$

$$\sum_{v\in TR}\sum_{\substack{i\in C\\ i\neq j}} FTD_{ijvd} \leq \sum_{v\in TR}\sum_{\substack{k\in C\\ k\neq j}} ATD_{jkvd} + SA_{j,c+1,d} \qquad \forall j\in C, d\in DR \tag{29}$$

$$\sum_{\substack{i\in N_0\\ i\neq j}} R_{ijvd} + \sum_{\substack{h\in C\\ h\neq j}} ATD_{hjvd} = \sum_{\substack{k\in N_+\\ k\neq j}} R_{jkvd} + \sum_{\substack{k\in C\\ k\neq j}} FTD_{jkvd} \qquad \forall j\in C, v\in TR, d\in DR \tag{30}$$

$$TD_{id} \geq TT_{iv} - M(1-\sum_{\substack{j\in C\\ j\neq i}} FTD_{ijvd}) \qquad \forall i\in C, v\in TR, d\in DR \tag{31}$$

$$TD_{id} \leq TT_{iv} + M(1-\sum_{\substack{j\in C\\ j\neq i}} FTD_{ijvd}) \qquad \forall i\in C, v\in TR, d\in DR \tag{32}$$

$$TD_{kd} \geq TT_{kv} - M(1-\sum_{\substack{j\in C\\ j\neq k}} ATD_{jkvd}) \qquad \forall k\in C, v\in TR, d\in DR \tag{33}$$

$$TD_{kd} \leq TT_{kv} + M(1-\sum_{\substack{j\in C\\ j\neq k}} ATD_{jkvd}) \qquad \forall k\in C, v\in TR, d\in DR \tag{34}$$

$$TD_{id} \geq TT_{iv} - M(1-\sum_{\substack{j\in C\\ j\neq i}} R_{ijvd}) \qquad \forall i\in N_0, v\in TR, d\in DR \tag{35}$$

$$TD_{id} \leq TT_{iv} + M(1-\sum_{\substack{j\in C\\ j\neq i}} R_{ijvd}) \qquad \forall i\in N_0, v\in TR, d\in DR \tag{36}$$

$$TD_{c+1,d} \geq TT_{c+1,v} - M(1-\sum_{j\in C} R_{j,c+1,vd}) \qquad \forall v\in TR, d\in DR \tag{37}$$

$$TD_{c+1,d} \leq TT_{c+1,v} + M(1-\sum_{j\in C} R_{j,c+1,vd}) \qquad \forall v\in TR, d\in DR \tag{38}$$

$$TT_{kv} \geq TT_{hv} + tTim_{hk} + s_R(\sum_{d\in DR}\sum_{\substack{j\in C\\ j\neq k,\, j\neq h}} ATD_{jkvd}) + s_L(\sum_{d\in DR}\sum_{\substack{l\in C\\ l\neq k, l\neq h}} FTD_{klvd}) - M(1-X_{hkv}) \qquad \forall h\in N_0, k\in\{N_+: k\neq h\}, v\in TR \tag{39}$$

$$TT_{kv} \leq M(\sum_{h\in N_0} X_{hkv}) \qquad \forall k\in N_+, v\in TR \tag{40}$$

$$TT_{0v} \geq TT_{kv} - [tTim_{0k} + s_R(\sum_{d\in DR}\sum_{\substack{j\in C\\ j\neq k}} ATD_{jkvd}) + s_L(\sum_{d\in DR}\sum_{\substack{l\in C\\ l\neq k}} FTD_{klvd})] - M(1-X_{0kv}) \qquad \forall k\in C, v\in TR \tag{41}$$

$$TD_{jd} \geq TD_{id} + dTim_{ijd} - M(1-SD_{ijd}) \qquad \forall i\in N_0,\ j\in\{C':\ j\neq i\}, d\in DR \tag{42}$$

$$TD_{kd} \geq TD_{jd} + dTim_{jkd} + s_R - M(1-SA_{jkd}) \qquad \forall j\in C', k\in\{N_+: k\neq j\}, d\in DR \tag{43}$$

$$TD_{jd} \leq M(\sum_{\substack{i\in N_0\\ i\neq j}} Y_{ijv}) \qquad \forall j\in N_+, v\in TR \tag{44}$$

$$TD_{kd} - (TD_{jd} - dTim_{ijd}) \leq M(2 - SD_{ijd} - SA_{jkd}) + \sum_{\substack{d'\in DR\\ d'\neq d}}\sum_{v\in DR}\sum_{\substack{l\in C\\ l\neq i}} FTD_{klvd}\, s_L + \sum_{\substack{d'\in DR\\ d'\neq d}}\sum_{v\in DR}\sum_{\substack{h\in C\\ h\neq k}} ATD_{hkvd}\, s_R + e_d \qquad \forall i\in N_0,\ j\in\{C:\ j\neq i\},\ k\in\{N_+: <i,j,k>\in S\}, d\in DR \tag{45}$$

$$dTim_{ijd} SD_{ijd} + dTim_{jkd} SA_{jkd} \leq s_R + e_d \qquad \forall i\in\{N_0: i\neq j, i\neq k\},\ j\in\{C:\ j\neq k\},\ k\in N_+, d\in DR \tag{46}$$

$$\sum_{\substack{j\in C\\ j\neq i}}\sum_{d\in DR} FTD_{ijvd} \leq dronCap_v \qquad \forall i\in C, v\in TR \tag{47}$$

$$\sum_{\substack{j\in C\\ j\neq k}}\sum_{d\in DR} ATD_{jkvd} \leq dronCap_v \qquad \forall k\in C, v\in TR \tag{48}$$

$$\sum_{i\in N_0}\Big(\sum_{\substack{j\in C\\ j\neq i}} d_j X_{ijv} + \sum_{\substack{l\in C\\ l\neq i}}\sum_{d\in DR} d_l FTD_{ilvd}\Big) \leq capT_v \qquad \forall v\in TR \tag{49}$$

$$\sum_{\substack{i\in N_0\\ i\neq j}} d_j SD_{ijd} \leq capD_d \qquad \forall j\in C, d\in DR \tag{50}$$

$$UT_{iv} - UT_{jv} + 1 \leq (c+2)(1-X_{ijv}) \qquad \forall i\in C, j\in\{N_+ : j\neq i\}, v\in TR \tag{51}$$

$$UT_{iv} - UT_{jv} \geq 1-(c+2)(PT_{ijv}) \qquad \forall i\in C, j\in\{N_+ : j\neq i\}, v\in TR \tag{52}$$

$$UT_{iv} - UT_{jv} \leq -1+(c+2)(1-PT_{ijv}) \qquad \forall i\in C, j\in\{N_+ : j\neq i\}, v\in TR \tag{53}$$

$$PT_{ijv} + PT_{jiv} = 1 \qquad \forall i\in C, j\in\{N_+ : j\neq i\}, v\in TR \tag{54}$$

$$UD_{id} - UD_{jd} + 1 \leq (c+2)(1-Y_{ijd}) \qquad \forall i\in C, j\in\{N_+ : j\neq i\}, d\in DR \tag{55}$$

$$UD_{id} - UD_{jd} \geq 1-(c+2)(PD_{ijd}) \qquad \forall i\in C, j\in\{N_+ : j\neq i\}, d\in DR \tag{56}$$

$$UD_{id} - UD_{jd} \leq -1+(c+2)(1-PD_{ijd}) \qquad \forall i\in C, j\in\{N_+ : j\neq i\}, d\in DR \tag{57}$$

$$PD_{ijd} + PD_{jid} = 1 \qquad \forall i\in C, j\in\{N_+ : j\neq i\}, d\in DR \tag{58}$$

$$PT_{0jv} = 1 \qquad \forall j\in C, v\in TR \tag{59}$$

$$PT_{i,c+1,v} = 1 \qquad \forall i\in N_0, v\in TR \tag{60}$$

$$PD_{0jd} = 1 \qquad \forall j\in C, d\in DR \tag{61}$$

$$PD_{i,c+1,d} = 1 \qquad \forall i\in N_0, d\in DR \tag{62}$$

$$1 \leq UT_{iv} \leq (c+2) \qquad \forall i\in N, v\in TR \tag{63}$$

$$1 \leq UD_{id} \leq (c+2) \qquad \forall i\in N, d\in DR \tag{64}$$

$$X_{ijv} \in \{0,1\} \qquad \forall i\in N_0, j\in\{N_+ : j\neq i\}, v\in TR \tag{65}$$

$$Y_{ijd} \in \{0,1\} \qquad \forall i\in N_0, j\in\{N_+ : j\neq i\}, d\in DR \tag{66}$$

$$SD_{ijd} \in \{0,1\} \qquad \forall i\in N_0, j\in\{C : j\neq i\}, d\in DR \tag{67}$$

$$SA_{jkd} \in \{0,1\} \qquad \forall j\in C, k\in\{N_+ : k\neq j\}, d\in DR \tag{68}$$

$$R_{ijvd} \in \{0,1\} \qquad \forall i\in N_0, j\in\{N_+ : j\neq i\}, v\in TR, d\in DR \tag{69}$$

$$FTD_{ijvd} \in \{0,1\} \qquad \forall i\in C, j\in\{C : j\neq i\}, v\in TR, d\in DR \tag{70}$$

$$ATD_{jkvd} \in \{0,1\} \qquad \forall j\in C, k\in\{C : k\neq j\}, v\in TR, d\in DR \tag{71}$$

$$PT_{ijv} \in \{0,1\} \qquad \forall i\in N, j\in\{N : j\neq i\}, v\in TR \tag{72}$$

$$PD_{ijd} \in \{0,1\} \qquad \forall i\in N, j\in\{N : j\neq i\}, d\in DR \tag{73}$$

$$TT_{iv} \geq 0 \qquad \forall i\in N, v\in TR \tag{74}$$

$$TD_{id} \geq 0 \qquad \forall j\in N, d\in DR \tag{75}$$

Equation (1) is the objective function that minimizes the service time in the system. Constraint sets 2 and 3 find the arrival time of the last vehicle to the warehouse. Constraint sets 4 and 5 allow each customer node to be serviced once by truck or drone. Constraint sets 6 and 7 ensure that each truck enters and exits the warehouse at most once, respectively. Constraint set 8 allows each truck to exit the customer node it entered. Constraint set 9 provides entry and exit for each drone from the customer node on its route. Constraint set 10 also provides entry and exit for each drone from the customer node it serves. Constraint sets 11 and 12 ensure that each drone has a maximum of 1 input and output for each node, respectively (see Figure 8). In Figure 8, while drones are in any node, they cannot land (A) or take off (B) from any other node.

Constraint sets 13 and 14 allow each drone to enter and exit the warehouse by flight or on a truck, respectively (see Figure 9). Constraint set 15 is the motion balance constraint of the drone's customer nodes (see Figure 8).

The constraint set shows the movements of the drone at the customer node. These movements should be arriving at the customer by flight or truck or leaving the customer by flight. Constraint sets 16, 17, 18, and 19 show what the previous and next movements of the drone should be according to the movement of the customer node. When arrivals are by flight or truck, it ensures that exits from the customer node are by flight or truck. Likewise, if there is a departure from the customer node, they guarantee that the arrival to that customer is by flight or by truck. Constraint set 20 enables to add the relevant customer to the route of the truck and the drone, in case of carrying out the transport operation of the drone on the truck. The same constraint group guarantees that if there are customers who are not on the route of the truck or drone, they will not be transported to those customers by truck. Constraint sets 21 and 22 ensure that there is 1 truck at that node so that the drone can take off and land at each customer node, respectively, otherwise, it cannot fly. Constraint set 23 activates the decision variable $FTD_{ijvd}$ which shows which customer, from which truck, and where the drone takes off for a flight when it takes off from a truck at any customer. Constraint set 24 activates the decision variable $ATD_{jkvd}$, which shows from which customer, which truck, and where the drone lands when it lands on a truck of any customer. Constraint sets 25, 26, 27, and 28 set $FTD_{ijvd}$ and $ATD_{jkvd}$ variables are zero if the truck or drone is not in that node.

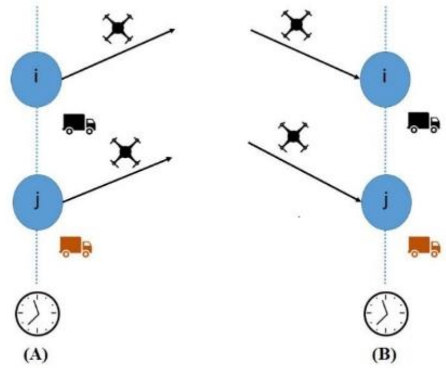


***Figure 8.*** *A) The drone cannot fly from different points at the same time. B) The drone cannot land at different points at the same time*

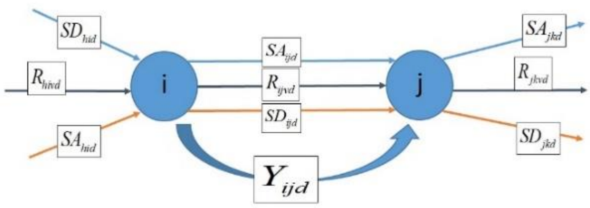


***Figure 9.*** *Illustration by decision variables of movements of drone between warehouse and customers or only customers*

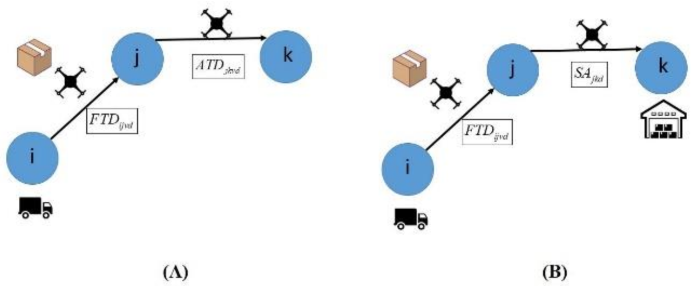


***Figure 10.*** *Movements of the drone after servicing a customer: A) land on any truck at another customer, B) return to the warehouse*

In constraint set 29, if a drone takes off from any truck to serve a customer, it is ensured that after that customer it either lands on another customer's truck or returns to the warehouse (see Figure 10). Movements blocked by these constraints set can be seen in Figure 11.

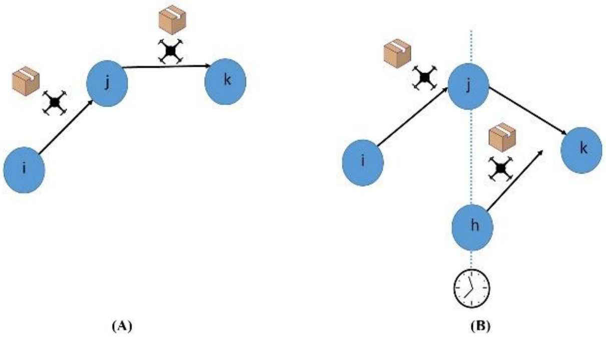


***Figure 11.*** *A) The drone cannot go to the service of another customer by flying from the customer it serves. B) Once the drone is flying for service to one customer, it cannot take off from another customer*

Constraint set 30 guarantees a flight to another customer node or transport by the same truck from the truck where a drone is transported or landing at a customer node (see Figure 12).

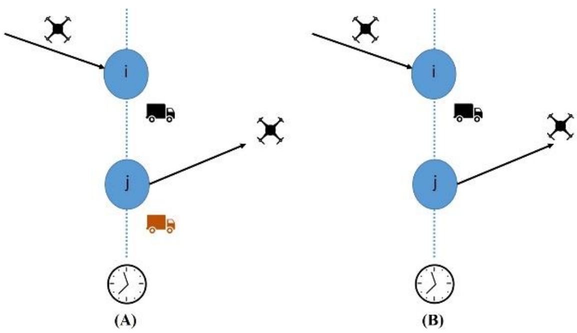


***Figure 12.*** *A) If the drone lands on a truck, it cannot make the next flight over another truck. B) If the drone has landed on a truck, it cannot take off from another customer*

Constraint sets 31-36 synchronize the arrival time of the drone with the arrival time of the trucks. Constraint sets 37 and 38 synchronize their time with trucks when the drones are transported to the warehouse by trucks. Constraint sets 39 and 40 calculate the departure and operation times of the trucks and the arrival time at the nodes, and if the truck does not go to the node, the arrival time is reset. In Constraint set 41, the exit time of the truck from the warehouse is calculated. Constraint sets 42, 43, and 44 calculate the arrival times of the drones to the nodes and the entry-exit times to the warehouse, and if the drone does not go to the node, the arrival time is reset. Constraint sets 45 and 46 ensure that the drones do not exceed the flight limit. Constraint sets 47 and 48 limit the operation of landing and take-off for trucks. Constraint sets 49 also limits the amount of transport of trucks. Constraint set 50 ensures that drones do not exceed load capacity. Constraint sets 51, 52, and 53 are sub-tour elimination constraints for trucks. Constraint set 54 establishes the priority relationship between each other for the nodes that each truck visits. Constraint sets 55, 56, and 57 are sub-tour elimination constraints for drones. Constraint set 58 establishes the priority relationship between each other for node visited by each drone. Constraint sets 59-64 provide the lower and upper limits of the priority relationship values for trucks and drones from warehouse to nodes, from nodes to warehouses, and the order values of nodes. Constraint sets 65-75 determine the type and sign of decision variables.

### 4.4. Conversion of the Proposed Mathematical Model to the Models that mDroneCVRP is Based on

By changing some parameters in the proposed mathematical model, it can be converted to the models in the literature based on the proposed mathematical model. In the model in this study, if only one truck with unlimited capacity is used and the capacity of the drones is assumed to be zero, the model is converted as a traveling salesman problem. Likewise, if the drone capacities are taken as zero and only trucks are used, it turns into a multi-traveling salesman problem. If one drone and one truck with unlimited capacities are taken in the proposed mathematical model, the model turns into the drone-assisted traveling salesman problem in Murray and Chu [3]. If more than one truck with unlimited capacity and more than one drone is used and it must be for the drones to land on the truck on which the drones are only flying, the proposed mathematical model turns into a multiple traveling salesman problem with the help of drones in the study of Kitjacharoenchai et al. [33]. The transformations are summarized in Table 2.

***Table 2***. *Models in the literature are converted from the proposed model*

| | **Changed Parameters** | | | | | **Converted Problem** | |
|---|---|---|---|---|---|---|---|
| **Proposed Model** | **Truck** | **Drone** | **Truck capacity** | **Drone Capacity** | **Constraint** | **Problem** | **Author** |
| VRPmD | 1-Truck | 1-Drone | **no limit** | **0** | | **TSP** | |
| VRPmD | **m-Truck** | **1-Drone** | **no limit** | **0** | | **mTSP** | |
| VRPmD | **m-Truck** | **1-Drone** | **limited** | **0** | | **CVRP** | |
| VRPmD | 1-Truck | 1-Drone | limited | no limit | | FSTSP, PDSTSP | Murray and Chu [3] |
| VRPmD | m-Truck | d-Drone | no limit | no limit | The drone can land on and take off from the same truck. | mTSPD | Kitjacharoenchai et al. [33] |

## 5. EXPERIMENTAL STUDY

### 5.1. Data and Parameter Estimation

Data sets are randomly generated for five scenarios and different versions of these scenarios. Diversification is used in the data sets to validate the model and measure solution quality.

### 5.2. Data and Parameter Estimation

In creating test data, the network is designed as a 2-dimensional space, with a warehouse and c customers randomly placed within a 60x60 unit area. This setup is varied across five different scenarios by altering the warehouse location and the positioning of customers relative to the warehouse and each other. Each scenario generates datasets with four varying customer counts, exploring different logistical challenges. For instance, in the first scenario (Figure 13), customers are positioned within drone reach from the warehouse, whereas in the second scenario (Figure 14), the warehouse is placed too far for direct drone access to customers. Other scenarios (Figures 15 and 16) further diversify the test conditions to analyze drone and truck dynamics under various operational circumstances. Speed ratios are established, with drones consistently faster than trucks, and Euclidean distances calculate drone travel between points. Truck distances are adjusted by these speed coefficients to reflect their relative slowness.

Three different velocity coefficients are used. If truck speed is $v_{truck}$ and drone speed is $v_{drone}$, $\alpha v_{truck} = v_{drone}$ is defined as the relationship between them. The values of α are taken as 1.5, 2, and 3, respectively.

In Scenario 1 (Figure 13), customer nodes are created where drones can go from the warehouse and it is observed whether the drone can exit the warehouse without a truck. Scenario 1 is called as General Network Structure in terms of observed movements to validate the model.

In Scenario 2 (Figure 14), the challenge stems from all customer nodes being positioned too far from the warehouse for direct drone access, differing from Scenario 1's closer proximity. The focus here is on whether drones can depart the warehouse alongside trucks and if they can launch from trucks near customers after the initial departure. This setup, named the Far Dense Network Structure, tests the model's handling of drone movements in densely located points beyond their direct flight range from the warehouse.

Scenario 3 (Figure 15) mirrors Scenario 2's constraint of nodes being out of direct drone range from the warehouse. It differentiates by testing drone efficiency as certain customer points are spaced further apart, potentially aligning closer with truck routes. This scenario also explores how varying the number of trucks affects outcomes, leading to its designation as the Far Irregular Network Structure due to the strategic placement of distant points.

In Scenario 4 (Figure 16), the nodes remain out of direct drone range from the warehouse, with even greater distances than those in the previous scenarios. Customers are grouped into clusters, significantly spaced apart, to assess the impact of speed, the number of trucks, and drone assistance within these clusters. Observations on drones' ability to serve clustered customers inform the naming of this setup as the Far Cluster Network Structure, focusing on logistical strategies for dispersed customer groups.

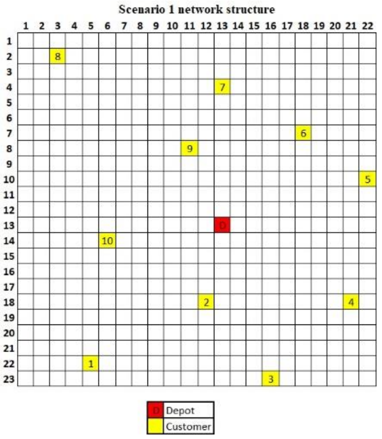


***Figure 13.*** *Scenario 1 network structure*

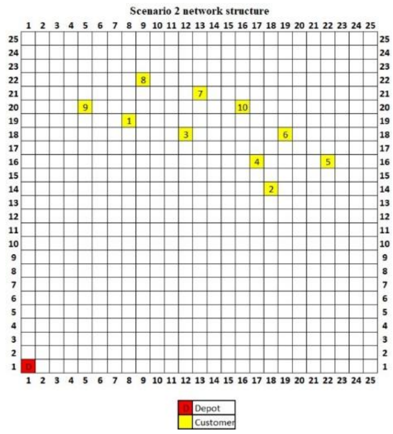


***Figure 14.*** *Scenario 2 network structure*

Scenario 5 (Figure 17) extends the distances observed in Scenario 2, focusing on the impact of speed differences and the number of trucks on logistical outcomes. Here, customers are arranged symmetrically around the warehouse, testing whether drones can launch from and land on vehicles situated on the opposite side of their initial take-off point. This setup, due to its emphasis on symmetrical distribution and strategic movement validation, is termed the Symmetric Far Network Structure.

For the test scenarios, both vehicles and drones are assigned a capacity of 1000 units, effectively treating it as unlimited relative to the demand in these scenarios. Each vehicle is capable of supporting operations for up to 4 drones simultaneously, allowing for concurrent flight and landing activities. The maximum flight duration for drones is set at 20 units. The test data vary by including 9, 8, 7, and 6 customers, along with a single warehouse, resulting in a total of 360 unique data sets. These datasets are meticulously detailed in Table 3, indicating the variety and specifics of the data (e.g., the number of scenarios and the corresponding customer counts) to provide a comprehensive overview for analysis.

***Table 3.*** *Summary of data sets*

| Customer Number | Depot | Truck Number | Drone Number | Flight Limit | Drone Velocity/Truck Velocity | Scenario | Total number of data set |
|---|---|---|---|---|---|---|---|

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 4 (6,7, 8, and 9) | 1 | 2 (1 and 2) | 3 (1,2 and 4) | 1 (20 units) | 3 (1.5, 2, and 3) | 5 (1,2,3,4, and 5) | 4x1x2x3x1x3x5=360 |

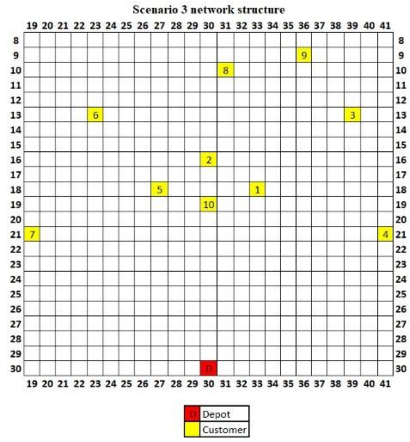


***Figure 15.*** *Scenario 3 network structure*

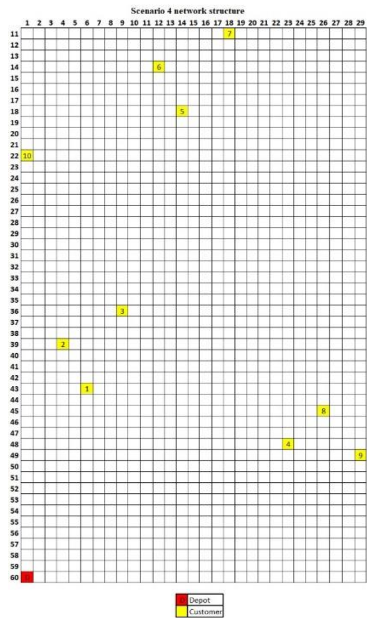


***Figure 16.*** *Scenario 4 network structure*

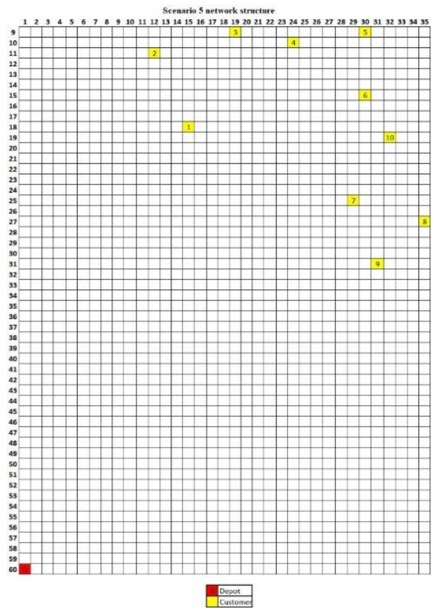


***Figure 17.*** *Scenario 5 network structure*

## 5.3. Numerical Result

The model was solved using CPLEX on a 16 GB RAM computer through GAMS. Figure 18 shows a scenario with 6 customers, 2 trucks, and 2 drones, where drones efficiently switch between trucks to serve customers, illustrating the model's operational dynamics. Figure 19, from scenario 1 with 9 customers, highlights drones' flexibility, showing they can independently serve customers or be transported by trucks, showcasing strategic deployment options.

Table 5 presents the objective function values and solution times for scenario 1's test data, revealing an increase in solution times with the number of nodes, sometimes surpassing the 8000-second limit due to the NP-Hard nature of the problem. Conversely, the objective function value decreases with the addition of drones in scenarios with the same number of nodes, indicating that employing multiple resources enhances efficiency.

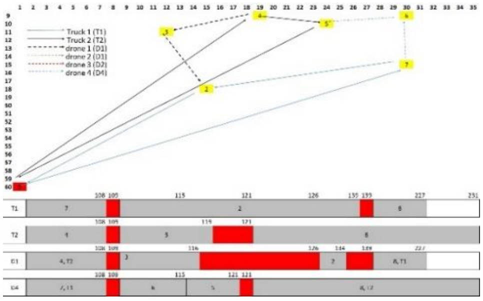

***Figure 18.*** *Solution of the model with test data with 6 customers, 2 trucks, and 2 drones*

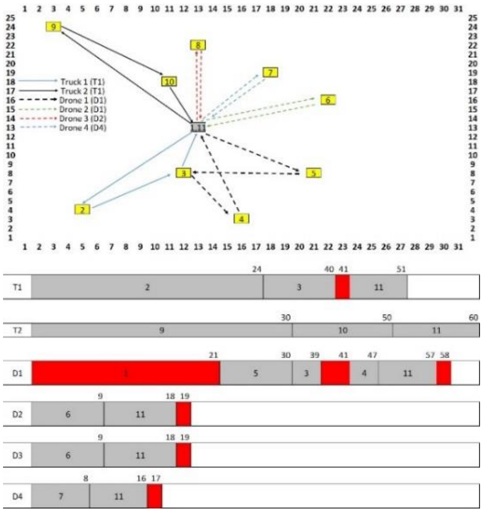

***Figure 19***. *Model solution with test data with 9 customers, 2 trucks, and 4 drones*

As can be seen in Figure 20, the positive effects of the increase in number of trucks and drones on the objective function are seen in the solutions obtained by taking the velocity coefficient of the scenario 1 dataset as 2 ( $\alpha = 2$ ). This is because the scenario 1 network structure is suitable for both vehicle types and drones can serve the majority of customers.

In Scenario 2 solutions with a speed coefficient of 2, as illustrated in Figure 21, adding more drones proves more beneficial than in Scenario 1. Doubling trucks from 1 to 2 enhances solution quality, due to the network's design necessitating trucks to depart from the warehouse, and drones' ability to serve customers while being transported. However, in scenarios with 2 trucks and 9 or 8 customers, adding more drones didn't improve the objective function value within the time limit, indicating a challenge in finding optimal solutions quickly.

Figure 22 reveals that in Scenario 3's network structure, with a speed coefficient of 2, increasing the number of drones has a negligible effect with 1 truck and only slightly impacts outcomes with 2 trucks. This minimal influence is attributed to limited truck routing options and the restricted reachability of customers by drones, reflecting the network's configuration. Similar observations apply to Scenario 4 solutions in Figure 23, where the network and limited customer accessibility similarly constrain drone effectiveness.

Figure 24 compares scenario data to highlight the impact of using 2 trucks and increasing the number of drones within the same dataset. The deployment of 2 trucks proves significantly more effective in this network structure, particularly for scenario 5, where customer nodes are symmetrically distributed across two regions. However, in the scenario with 9 customers and 2 trucks, adding more drones does not improve the objective function value, as the model fails to find a solution within the allocated time limit. This underscores the complexity of balancing resources in drone-assisted delivery systems, especially in symmetrically structured networks.

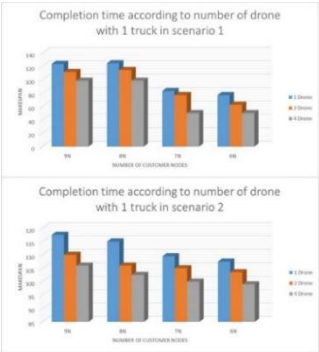


***Figure 20.*** *Comparison of objective function values of problems with 1 truck and 2 trucks in scenario 1 dataset according to the number of drones*

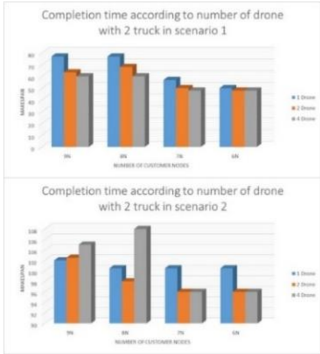


***Figure 21****. Comparison of objective function values of problems with 1 truck and 2 trucks in scenario 2 dataset according to the number of drones*


***Figure 22****. Comparison of objective function values of problems with 1 truck and 2 trucks in scenario 3 dataset according to the number of drones*

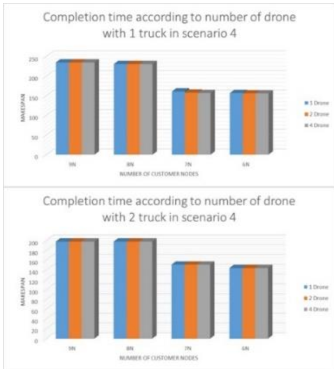


***Figure 23****. Comparison of objective function values of problems with 1 truck. and 2 trucks in scenario 4 dataset according to the number of drones*

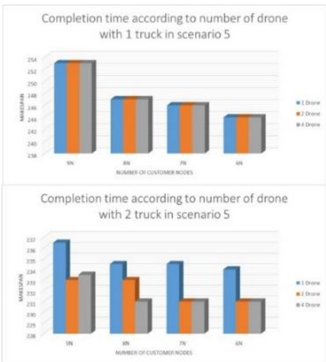


***Figure 24.*** *Comparison of objective function values of problems with 1 truck and 2 trucks in scenario 5 dataset according to the number of drones*

## 6. THE PROPOSED ALGORITHM

### 6.1. Greedy Algorithm

The greedy algorithm for an initial solution to minimize the longest vehicle time in a vehicle routing problem with multiple trucks and drones follows these steps:

1. **Identify Sorties:** Determine three-point sorties within drone flight limits from the distance matrix, defining potential customer service points for drones.
2. **Initialize Vehicles:** Assign all vehicles to start from the depot with an initial arrival time of zero.
3. **Calculate Travel Times:** Use the distance matrix to calculate and average truck travel times for each destination, then sort these averages in descending order.
4. **Assign Drone Sorties:** Assign the first suitable sortie from the sorted list to a drone and calculate its route end arrival time. If no sortie is suitable, proceed to Step 14 for warehouse exits or Step 11 for non-warehouse points.
5. **Update Sets:** Refresh the list of destinations and drone sorties according to the chosen route.
6. **Assign All Drones:** Repeat Step 4 for each drone, then move to Step 7.
7. **Route Trucks:** Send trucks to the nearest destination point, calculating arrival times.
8. **Refresh Lists:** Update destination and sortie lists based on chosen points.
9. **Synchronize Arrivals:** If a drone and truck meet, align their arrival times to the later of the two.
10. **Repeat for Trucks:** Continue routing trucks as in Step 7, then return to Step 4.
11. **Match Drones to Trucks:** Update drone routes and times to align with matching trucks.
12. **Loop Until Done:** Repeat Step 4 until all destinations are covered.
13. **Return to Warehouse:** Send all vehicles back to the warehouse, updating arrival times accordingly.
14. **Reassign Drones:** For drones without sorties, assign them to the route starting at a truck point with the highest average truck time, updating the arrival time at the route's end.

This algorithm systematically assigns sorties to drones and routes to trucks, updating times and positions to ensure efficient coverage of all customer points and synchronization between vehicle types.

The notations of the parameters and variables used in the problem solving of the algorithm are defined below. Most of the notations are the same as those used in the mathematical model.

**Notation:**
**Parameters**

| | |
|---|---|
| $C$ | Set of customer, $C = \{1,...,c\}$ |
| $N_0$ | Set of nodes to which vehicles can be departed , $N_0 = \{0,1,...,c\}$ |
| $N_+$ | Set of nodes to which vehicles can arrive, $N_+ = \{1,...,c+1\}$ |
| $N_{ort}$ | Node set ordered according to average of travel truck times of each node , $N_{ort} = \{i,j,...,k\},\ \frac{\sum_h tTim_{hi}}{c+2} \geq \frac{\sum_h tTim_{hj}}{c+2} ... \geq \frac{\sum_h tTim_{hk}}{c+2}$ |
| $N_D$ | Set of nodes to which drones can flight , $N_D = \{1,...,c+1\}$ |
| $N_T$ | Set of nodes to which vehicles can serve , $N_T = \{1,...,c+1\}$ |
| $TR$ | Highway vehicle groups, $TR = \{1,...,v,...,VN\}$ |
| $DR$ | Set of drones, $DR = \{1,...,d,...,DN\}$ |
| $tTim_{ijv}$ | Time to go from node *i* to node *j* by highway vehicle *v* $i \in N_0, j \in \{N_+ : j \neq i\}, v \in TR$ |
| $dTim_{ijd}$ | Flight time of drone *d* from node *i* to *j*, $i \in N_0, j \in \{N_+ : j \neq i\}, d \in DR$ |
| $S$ | Set of sorties which include three nodes of drone flight $<i,j,k>$, $i \in N_0, j \in \{C : j \neq i\}, k \in \{N_+ : k \neq i, k \neq j, dTim_{ij} + dTim_{jk} \leq e_d\}$ |
| $e_d$ | Maximum flight limit of drone $d$ at once, $d \in DR$ |

**Decision variables**

$$A_{ivd} = \begin{cases} 1, \text{ if drone } d \text{ rendezvous with truck } v \text{ at node } i \\ 0, \text{ otherwise} \end{cases}$$

$$X_{ijv} = \begin{cases} 1, \text{ if vehicle } v \text{ goes from node } i \text{ to } j \text{ düğümüne} \\ 0, \text{ otherwise} \end{cases}$$

$$Y_{ijkd} = \begin{cases} 1, \text{ if drone } d \text{ take off from node } i \text{ to node } j \text{ for serving then landing on node } k \\ 0, \text{ otherwise} \end{cases}$$

$TT_{iv} =$ Arrival time of vehicle $v$ to node $i$

$TD_{id} =$ Arrival time of drone $d$ to node $i$

$BN_d^D =$ Current node of drone $d$

$BN_v^T =$ Current node of vehicle $v$

$R_d^D =$ Route of drone $d$

$R_v^T =$ Route of vehicle $v$

$C_{\max} =$ Maximum arrival time of vehicles at depot

**Objective Function**

Calculation of arrival time of vehicles at a node:

$$TT_{jv} = TT_{iv} + tTim_{ijv} \quad i: \text{previous location}, \; TD_{jd} = TD_{id} + dTim_{ijd} \quad i: \text{previous location}$$

If drone and vehicle rendezvous at node j, arrival time calculation as follows:

$$A_{jvd} = \begin{cases} 1, \; TT_{jv} = TD_{jd} = \max\{TT_{jv}, TD_{jd}\} \\ 0, \text{ there is no change} \end{cases} \qquad C_{\max} = \max_{\substack{d \in DR \\ v \in TR}} \{TT_{c+2,v}, TD_{c+2,d}\}$$

Parallel machine scheduling problems are used while constructing the algorithm. Trucks and drones are assumed as machines and destinations are assumed as jobs assigned to that machines. The Longest Processing Time (LPT) rule, which reduces the maximum completion time in identical parallel machine scheduling problems, formed the basis for this algorithm. The drones are assigned to points according to the average truck times of the points, starting from the largest. The assignment method saves time by finding where the drones can go through the points with a large average time value for the truck and benefit from the speed of the drone. Therefore, trucks are assigned to nodes whose average time value is small. When assigning trucks, the machine in the LPT rule is started with the minimum machine time, that is, the vehicle time. While assigning trucks, the shortest tour time is aimed for each truck by using the nearest neighbor algorithm used in the traveling salesman problem solutions.

### 6.1. Solving Sample Problem

The application of the greedy algorithm on a sample problem is explained below. The 1st scenario, which is also used in the mathematical model solutions proposed as sample problem data, is used as a data set with 1 warehouse and 9 customers. The time matrix for the truck and the average truck times of the nodes are given in Table 4. The time relationship between the truck and the drone is formed as by taking the velocity coefficient α = 2. The number of trucks is 2, and the number of drones is 3. The drone flight limit is taken as e = 20.

***Table 4.*** *1 Warehouse with 9 customers Scenario 1 (General Network Structure) truck transportation times between nodes and average truck times of nodes*

| | | To | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Warehouse | 2. Customer | 3. Customer | 4. Customer | 5. Customer | 6. Customer | 7. Customer | 8. Customer | 9. Customer | 10. Customer | Warehouse |
| From | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| Warehouse | 1 | 0 | 24 | 10 | 20 | 18 | 18 | 16 | 18 | 30 | 10 | 0 |
| 1. Customer | 2 | 24 | 0 | 16 | 22 | 32 | 42 | 40 | 40 | 40 | 30 | 24 |
| 2. Customer | 3 | 10 | 16 | 0 | 12 | 18 | 26 | 26 | 28 | 36 | 20 | 10 |
| 4. Customer | 4 | 20 | 22 | 12 | 0 | 14 | 28 | 32 | 38 | 50 | 32 | 20 |
| 5. Customer | 5 | 18 | 32 | 18 | 14 | 0 | 16 | 22 | 32 | 48 | 28 | 18 |
| 6. Customer | 6 | 18 | 42 | 26 | 28 | 16 | 0 | 10 | 22 | 42 | 22 | 18 |
| 7. Customer | 7 | 16 | 40 | 26 | 32 | 22 | 10 | 0 | 12 | 32 | 14 | 16 |
| 8. Customer | 8 | 18 | 40 | 28 | 38 | 32 | 22 | 12 | 0 | 20 | 8 | 18 |
| 9. Customer | 9 | 30 | 40 | 36 | 50 | 48 | 42 | 32 | 20 | 0 | 20 | 30 |
| 10. Customer | 10 | 10 | 30 | 20 | 32 | 28 | 22 | 14 | 8 | 20 | 0 | 10 |
| Warehouse | 11 | 0 | 24 | 10 | 20 | 18 | 18 | 16 | 18 | 30 | 10 | 0 |
| Average time | | 13.75 | 26.00 | 17.08 | 22.67 | 20.92 | 20.83 | 18.92 | 20.33 | 29.75 | 17.00 | 14.58 |

To solve the problem, first, a list of customers, destinations, and average time is created.
$C$ = {2,3,4,5,6,7,8,9,10}, $N_D$ = {2,3,4,5,6,7,8,9,10}, $N_T$ = {2,3,4,5,6,7,8,9,10}, $N_{ort}$ ={9,2,4,5,6,8,7,3,10}, $<i,j,k> \in S, i \in N_0, j \in \{C : j \neq i\}, k \in \{N_+ : k \neq i, k \neq j, dTim_{ij} + dTim_{jk} \leq 20\}$

The algorithm runs until the list $C$ is the empty set. First, the assignment from the drones is started.

**Iteration 1:**

**For *d* = 1;** $j$ = 2 ∈ $N_{ort}$ and first sortie <*i*,*j*,*k*> in list $S$ <*i*,*j*,*k*> is <1,2,3> this route is assigned to drone.

$Y_{1,2,3,1} = 1$, $BN_1^D$ = 3, $TD_{3,1} = TD_{1,1} + dTim_{1,2,1} + dTim_{2,3,1}$ =0+20=20 $R_1^D$ ={1,2,3} Update list, $C$ = {3,4,5,6,7,8,9,10}, $N_D$ = {4,5,6,7,8,9,10}, $N_T$ = {3,4,5,6,7,8,9,10}, $<i, j, k> : i \neq 2, j \neq 2, k \neq 2, j \neq 3$

**For *d* = 2;** $j$ = 4 ∈ $N_{ort}$ and first sortie <*i*,*j*,*k*> in list $S$ <*i*,*j*,*k*> is <1,4,11> this route is assigned to drone. $Y_{1,4,11,2} = 1$, $BN_2^D$ = 11, $TD_{11,2} = TD_{1,2} + dTim_{1,4,2} + dTim_{4,11,2}$ =0+20=20 $R_2^D$ ={1,4,11} $C$ = {3,5,6,7,8,9,10}, $N_D$ = {5,6,7,8,9,10}, $N_T$ = {3,5,6,7,8,9,10}, $<i, j, k> : i \neq 4, j \neq 4, k \neq 4$

**For *d* = 3;** $j$ = 5 ∈ $N_{ort}$ and first sortie <*i*,*j*,*k*> in list $S$ <*i*,*j*,*k*> is <1,5,7> this route is assigned to drone. $Y_{1,5,7,3} = 1$, $BN_3^D$ = 7, $TD_{7,3} = TD_{1,3} + dTim_{1,5,3} + dTim_{5,7,3}$ =0+20=20 $R_3^D$ ={1,5,7} $C$ = {3,6,7,8,9,10}, $N_D$ = {6,8,9,10}, $N_T$ = {3,6,7,8,9,10}, $<i, j, k> : i \neq 5, j \neq 5, k \neq 5, j \neq 7$

**Nodes are assigned to truck according to current truck time and destination time (SPT rule is applied).**

**For *v* = 1;** $BN_1^T$ = 1, $\min_{j \in N_T}\{tTim_{1,j,1}\} = tTim_{1,3,1}$ = 10 therefore $j$ = 3 is assigned to truck. $X_{1,3,1}$ = 1, $BN_1^T$ = 3, $TT_{3,1} = TT_{1,1} + tTim_{1,3,1}$ =0+10=10 $R_1^T$ ={1,3} $C$ = {6,7,8,9,10}, $N_D$ = {6,8,9,10}, $N_T$ = {6,7,8,9,10}, $< i, j, k > : j \neq 3$

**For *v* = 2;** $BN_2^T$ = 1, $\min_{j \in N_T}\{tTim_{1,j,2}\} = tTim_{1,10,2}$ = 10 therefore $j$ = 10 k is assigned to truck. $X_{1,10,2}$ = 1, $BN_2^T$ = 10, $TT_{10,2} = TT_{1,2} + tTim_{1,10,2}$ =0+10=10 $R_2^T$ ={1,10} $C$ = {6,7,8,9}, $N_D$ = {6,8,9}, $N_T$ = {6,7,8,9}, $< i, j, k > : j \neq 10$

**Detection of the meeting point of the drone and the truck**

$BN_1^D$ = $BN_1^T$ = 3 Therefore $A_{3,1,1}$= 1, $TD_{3,1}$ = 20, $TT_{3,1}$ = 10 therefore $TD_{3,1} = TT_{3,1}$ =max{20,10}=20

**The drone moving on the truck is not found. The drone that could not exit the warehouse is not found.**

**Iteration 2:**

**For *d* = 1;** $j$ = 6 ∈ $N_{ort}$ and first sortie <*i,j,k*> in list *S* <*i,j,k*> is <3,6,7> this route is assigned to drone.

$Y_{3,6,7,1} = 1$, $BN_1^D$ = 7, $TD_{7,1} = TD_{3,1} + dTim_{3,6,1} + dTim_{6,7,1}$ =20+18=38 $R_1^D$ ={1,2,3,6,7} $C$ = {7,8,9}, $N_D$ = {8,9}, $N_T$ = {7,8,9}, $< i, j, k >: i \neq 6, j \neq 6, k \neq 6, j \neq 7$

**For *d* = 2;** $BN_2^D$ = 11 Drone arrived to warehouse thus route is determined for next drone .

**For *d* = 3;** A common node is not found for list of $N_{ort}$ and *S* <*i,j,k*>

**Nodes are assigned to truck according to current truck time and destination time (SPT rule is applied).**

**For *v* = 2 (it has minimum current time between each other);** $BN_2^T$ = 10, $\min_{j \in N_T}\{tTim_{10,j,2}\} = tTim_{10,8,2}$ = 8 therefore $j$ = 8 is assigned to truck. $X_{10,8,2}$ = 1, $BN_2^T$ = 8, $TT_{8,2} = TT_{10,2} + tTim_{10,8,2}$ =10+8=18 $R_2^T$ ={1,10,8}

$C$ = {7,9}, $N_D$ = {9}, $N_T$ = {7,9}, $< i, j, k >: j \neq 8$

**For *v* = 1;** $BN_1^T$ = 3, $\min_{j \in N_T}\{tTim_{3,j,1}\} = tTim_{3,7,1}$ = 26 therefore $j$ = 7 is assigned to truck. $X_{3,7,1}$ = 1, $BN_1^T$ = 7, $TT_{7,1} = TT_{3,1} + tTim_{3,7,1}$ =20+26=46 $R_1^T$ ={1,3,7} $C$ = {9}, $N_D$ = {9}, $N_T$ = {9 }, $< i, j, k >: j \neq 7$

**Detection of the meeting point of the drone and the truck**

$BN_1^D$ = $BN_3^D = BN_1^T$ = 7 therefore $A_{7,1,1}$ = 1, $A_{7,1,3}$ = 1 $TD_{7,1}$ = 38, $TD_{7,3}$ = 20 $TT_{7,1}$ = 46 ise $TD_{7,1} = TD_{7,3} = TT_{7,1}$ = max{38,20,46} = 46

**The drone moving on the truck is not found. The drone that could not exit the warehouse is not found.**

**Iteration 3:**

**For *d* = 1 ;** A common node is not found for list of $N_{ort}$ and *S* <*i,j,k*>

**For *d* = 2 ;** $BN_2^D$ = 11 Drone arrived to warehouse thus route is determined for next drone .

**For *d* = 3 ;** A common node is not found for list of $N_{ort}$ and *S* <*i,j,k*>

**Nodes are assigned to truck according to current truck time and destination time (SPT rule is applied).**

**For *v* = 2;** $BN_2^T$ = 8, $\min_{j \in N_T}\{tTim_{8,j,2}\} = tTim_{8,9,2}$ = 20 therefore $j$ = 9 is assigned to truck. $X_{8,9,2}$ = 1, $BN_2^T$ = 9, $TT_{9,2} = TT_{8,2} + tTim_{8,9,2}$ =20+18=38 $R_2^T$ ={1,10,8,9} $C$ = {}, $N_D$ = {}, $N_T$ = {}, $< i, j, k >: j \neq 9$

**For *v* = 1;** $N_T$ = {} therefore go to next step.

A node that is a meeting point of drone and truck, is not found. The drone moving on the truck is not found. The drone that could not exit the warehouse is not found.

**Finalization: *C* = {} therefore all vehicles are returned to warehouse**

**First trucks are returned.**

**For *v* = 1;** $BN_1^T$ = 7, $tTim_{7,11,1}$ = 16 therefore $j$ = 11 is assigned to truck. $X_{7,11,1}$ = 1, $BN_1^T$ = 11, $TT_{11,1} = TT_{7,1} + tTim_{7,11,1}$ = 46 + 16 = 62 $R_1^T$ ={1,3,7,11}

***Table 5.*** *Comparison of greedy heuristics and proposed model*

| | | | | | | mDroneCVRP | | Greedy Heuristics | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Number of Customer** | **Number of Truck** | **Number of Drone** | **Flight Limit** | **Scenario** | **Drone Velocity / Truck Velocity** | **Execution Time** | **Make-span** | **Execution Time** | **Make-span** | **GAP** |
| 6 | 2 | 2 | 1 | 1 | 2 | 15.19 | 36.00 | 0.051 | 48.00 | 33.33 |
| 7 | 2 | 2 | 1 | 1 | 2 | 20.28 | 44.00 | 0.026 | 50.00 | 13.64 |
| 8 | 2 | 2 | 1 | 1 | 2 | 54.72 | 60.00 | 0.03 | 84.00 | 40.00 |
| 9 | 2 | 2 | 1 | 1 | 2 | 167.24 | 55.00 | 0.062 | 66.00 | 20.00 |
| 6 | 2 | 2 | 1 | 2 | 2 | 44.50 | 95.00 | 0.063 | 112.00 | 17.89 |
| 7 | 2 | 2 | 1 | 2 | 2 | 473.93 | 95.00 | 0.118 | 118.00 | 24.21 |
| 8 | 2 | 2 | 1 | 2 | 2 | 8012.46 | 100.00 | 0.164 | 120.00 | 20.00 |
| 9 | 2 | 2 | 1 | 2 | 2 | 8041.96 | 100.00 | 0.196 | 124.00 | 24.00 |
| 6 | 2 | 2 | 1 | 3 | 2 | 24.59 | 68.00 | 0.017 | 76.00 | 11.76 |
| 7 | 2 | 2 | 1 | 3 | 2 | 82.72 | 70.00 | 0.025 | 82.00 | 17.14 |
| 8 | 2 | 2 | 1 | 3 | 2 | 366.26 | 82.00 | 0.053 | 82.00 | 0.00 |
| 9 | 2 | 2 | 1 | 3 | 2 | 3136.06 | 62.00 | 0.061 | 95.00 | 53.23 |
| 6 | 2 | 2 | 1 | 4 | 2 | 32.42 | 144.00 | 0.05 | 216.00 | 50.00 |
| 7 | 2 | 2 | 1 | 4 | 2 | 165.54 | 150.00 | 0.08 | 221.00 | 47.33 |
| 8 | 2 | 2 | 1 | 4 | 2 | 1039.58 | 197.00 | 0.11 | 240.00 | 21.83 |
| 9 | 2 | 2 | 1 | 4 | 2 | 2276.12 | 197.00 | 0.03 | 197.00 | 0.00 |
| 6 | 2 | 2 | 1 | 5 | 2 | 48.66 | 230.00 | 0.06 | 254.00 | 10.43 |
| 7 | 2 | 2 | 1 | 5 | 2 | 182.21 | 230.00 | 0.09 | 254.00 | 10.43 |
| 8 | 2 | 2 | 1 | 5 | 2 | 1914.23 | 232.00 | 0.03 | 254.00 | 9.48 |
| 9 | 2 | 2 | 1 | 5 | 2 | 8012.66 | 232.00 | 0.04 | 254.00 | 9.48 |
| 6 | 2 | 4 | 1 | 1 | 2 | 16.92 | 32.00 | 0.11 | 37.00 | 15.63 |
| 7 | 2 | 4 | 1 | 1 | 2 | 25.55 | 32.00 | 0.06 | 48.00 | 50.00 |
| 8 | 2 | 4 | 1 | 1 | 2 | 33.00 | 60.00 | 0.08 | 69.00 | 15.00 |
| 9 | 2 | 4 | 1 | 1 | 2 | 341.01 | 48.00 | 0.09 | 66.00 | 37.50 |
| 6 | 2 | 4 | 1 | 2 | 2 | 168.30 | 95.00 | 0.10 | 112.00 | 17.89 |
| 7 | 2 | 4 | 1 | 2 | 2 | 6927.98 | 95.00 | 0.12 | 112.00 | 17.89 |
| 8 | 2 | 4 | 1 | 2 | 2 | 8010.62 | 96.00 | 0.17 | 118.00 | 22.92 |
| 9 | 2 | 4 | 1 | 2 | 2 | 8043.51 | 100.00 | 0.25 | 112.00 | 12.00 |
| 6 | 2 | 4 | 1 | 3 | 2 | 40.55 | 68.00 | 0.02 | 79.00 | 16.18 |
| 7 | 2 | 4 | 1 | 3 | 2 | 121.27 | 70.00 | 0.03 | 80.00 | 14.29 |
| 8 | 2 | 4 | 1 | 3 | 2 | 792.81 | 70.00 | 0.06 | 82.00 | 17.14 |
| 9 | 2 | 4 | 1 | 3 | 2 | 8012.91 | 57.00 | 0.09 | 105.00 | 84.21 |
| 6 | 2 | 4 | 1 | 4 | 2 | 36.86 | 144.00 | 0.05 | 216.00 | 50.00 |
| 7 | 2 | 4 | 1 | 4 | 2 | 303.44 | 150.00 | 0.09 | 221.00 | 47.33 |
| 8 | 2 | 4 | 1 | 4 | 2 | 364.60 | 197.00 | 0.11 | 240.00 | 21.83 |
| 9 | 2 | 4 | 1 | 4 | 2 | 8012.86 | 197.00 | 0.03 | 197.00 | 0.00 |
| 6 | 2 | 4 | 1 | 5 | 2 | 71.50 | 230.00 | 0.07 | 254.00 | 10.43 |
| 7 | 2 | 4 | 1 | 5 | 2 | 269.30 | 230.00 | 0.09 | 254.00 | 10.43 |
| 8 | 2 | 4 | 1 | 5 | 2 | 4744.57 | 230.00 | 0.04 | 254.00 | 10.43 |
| 9 | 2 | 4 | 1 | 5 | 2 | 8020.45 | 232.00 | 0.06 | 254.00 | 9.48 |

**For *v* = 2;** $BN_2^T$ = 9, $tTim_{9,11,2}$ = 30 therefore $j$ = 11 is assigned to truck. $X_{9,11,2}$ = 1, $BN_2^T$ = 11, $TT_{11,2} = TT_{9,2} + tTim_{9,11,2}$ =38+30=68 $R_2^T$ ={1,10,8,9,11}

**For *d* = 1;** $A_{7,1,1}$ = 1 therefore $TD_{11,1} = TT_{11,1}$ = 62, $R_1^D$ ={1,2,3,6,7,11}

**For *d* = 2;** $A_{11,v,2}$ = 0 therefore $TD_{11,2} = 20$ , $R_2^D$ ={1,4,11}

**For *d* = 3;** $A_{7,1,3}$ = 1 therefore $TD_{11,3} = TT_{11,1}$ = 62, $R_3^D$ ={1,5,7,11}

**If all vehicles are arrived to warehouse;** $C_{\max} = \max \{ TD_{11,1}, TD_{11,2}, TD_{11,3}, TT_{11,1}, TT_{11,2} \} = \{ 62, 20, 62, 62, 68 \}$ According to the solution of the greedy algorithm, the highest service end time is found to be 68. The solution of the algorithm is given in Figure 25. A comparison is made between the heuristics algorithm and the proposed mathematical model in terms of execution time and makespan of problems (see in Table 5). Some heuristics solutions are better than mathematical model solutions because of the solution time limit of the mathematical model.

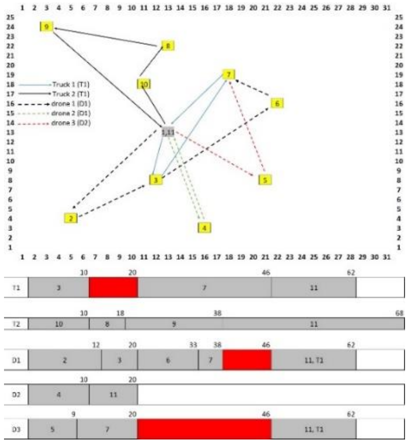

***Figure 25.*** *Solution presentation of the greedy algorithm (Scenario 1, 9 customers, 2 trucks, 3 drones, α=2)*

## 7. CONCLUSIONS

Drone applications, increasingly prevalent across various sectors, introduce a diverse array of challenges, with many centered around drones as the primary factor. While technological advancements enhance drone capabilities, the associated costs can be prohibitive, especially for high-capacity vehicles. This study explores the synergistic use of drones with another vehicle, broadening operational capabilities and optimizing efficiency. The objective is to facilitate a more effective and economical deployment of drones in conjunction with other vehicles.

Existing literature has predominantly focused on drone-centric scenarios, emphasizing combined operations as a distinct area of inquiry. This study delves into routing problems to strategically plan synchronized operations involving drones and other vehicles as cohesive working units.

The study underscores its motivation by illustrating that diverse vehicle types working in tandem prove more efficient than relying solely on a single vehicle type, as commonly observed in classical Vehicle Routing Problem (VRP) and Traveling Salesman Problem (TSP) methods. The application of multiple vehicles in collaborative routing scenarios significantly outperforms studies employing a single drone and a single vehicle. Consequently, the study aims to determine routes for capacitated vehicles and drones, minimizing the maximum arrival time at the warehouse. To address this, a mixed-integer mathematical model is formulated, termed the Multiple-Drone Assisted Capacitated Vehicle Routing Problem (mDroneCVRP).

Several assumptions in the mDroneCVRP model differentiate it from other studies. A crucial and distinctive aspect is the flexibility introduced into rendezvous operations between drones and vehicles, mirroring real-world scenarios. This flexibility allows drones to land on vehicles other than the one they took off from, with specific limits imposed on the number of take-off and landing operations on a given vehicle.

To validate and verify the model, a systematic approach is employed, generating 360 data sets comprising five scenarios, varying numbers of nodes, and different velocity coefficients between drones and trucks. These instances are solved within an 8000-second time limit. The solutions demonstrate the efficiency gains achieved by employing multiple drones, including the drone's ability to land on various trucks, conduct direct flights from customers to the warehouse, and return independently to the warehouse while servicing customers. Furthermore, the study explores the transformation of the model into representations found in other studies, indicating its adaptability to solving diverse problems.

As evident in the solutions, the NP-Hard nature of the problem results in solution times surpassing limits as the number of nodes increases. Future research should focus on developing heuristic methods to address larger-scale problems. To this end, a solution construction heuristic algorithm is introduced, providing initial solutions for forthcoming metaheuristic algorithms. This algorithm, grounded in familiar heuristics from scheduling problems, proves effective in producing consistent and quality solutions.

The burgeoning interest in drone studies gives rise to novel problem variants. One such variant involves the delicate balance between drone load and energy consumption, especially pertinent to optimizing battery weight. Future studies should explore these challenges, alongside the evolution of rendezvous points and the incorporation of multiple warehouses.

As drones play an increasingly vital role in humanitarian logistics, the study's application of routing problems to drone and vehicle combined operations signifies broader implications. While many studies aim to reduce costs, this research underscores the paramount importance of time over cost in real-world scenarios, offering valuable insights into optimizing time utilization within diverse environments.

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## CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

#### Appendix A Parameters Used in the Mathematical Model

***Sets***

| | |
|---|---|
| $C$ | Set of customer, $C = \{1,...,c\}$ |
| $N_0$ | Set of nodes to which vehicles can be departed , $N_0 = \{0,1,...,c\}$ |
| $N_+$ | Set of nodes to which vehicles can arrive, $N_+ = \{1,...,c+1\}$ |
| $N_{ort}$ | Node set ordered according to average of travel truck times of each node , $N_{ort} = \{i,j,...,k\}$, $\frac{\sum_h tTim_{hi}}{c+2} \geq \frac{\sum_h tTim_{hj}}{c+2} ... \geq \frac{\sum_h tTim_{hk}}{c+2}$ |
| $N_D$ | Set of nodes to which drones can fly , $N_D = \{1,...,c+1\}$ |
| $N_T$ | Set of nodes to which vehicles can serve , $N_T = \{1,...,c+1\}$ |
| $TR$ | Highway vehicle groups, $TR = \{1,...,v,...,VN\}$ |
| $DR$ | Set of drones, $DR = \{1,...,d,...,DN\}$ |
| $tTim_{ijv}$ | Time to go from node *i* to node *j* by highway vehicle $v$ $i \in N_0, j \in \{N_+ : j \neq i\}, v \in TR$ |
| $dTim_{ijd}$ | Flight time of drone *d* from node *i* to *j*, $i \in N_0, j \in \{N_+ : j \neq i\}, d \in DR$ |
| $S$ | Set of sorties which include three nodes of drone flight $<i,j,k>$, $i \in N_0, j \in \{C : j \neq i\}, k \in \{N_+ : k \neq i, k \neq j, dTim_{ij} + dTim_{jk} \leq e_d\}$ |
| $e_d$ | Maximum flight limit of drone $d$ at once, $d \in DR$ |

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