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Abstract

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Research Article

GeoGebra Software on the Mathematical Language Developments and Self-Efficacy Perceptions of Students*

Nimet BARÇIN [1] **Arzu AYDOĞAN YENMEZ** [2]

Abstract

The aim of the study is to investigate the effects of using GeoGebra in teaching functions on mathematical language development and self-efficacy perceptions of tenth grade students. The study, which used the action research method, changes in participants' language structures were examined with the worksheets, mathematical language questions, researcher's logs and participant's logs; participants' self-efficacy perceptions were also examined with the self-efficacy perception scale. The ability of the participants to switch between the sub-dimensions of mathematical language was observed. The research showed that GeoGebra-Assisted Education improved the participants' perceptions of mathematical self-efficacy and positively affected their mathematical language skills. Since the effective use of mathematical language is an important component of mathematics lessons, the results present important findings.

Keywords: : Mathematical language, self-efficacy perceptions, geogebra, action research

1. INTRODUCTION

Mathematical educators have long conducted research on the role of language in mathematical teaching (Austin & Howson 1979; Pimm 1987, Wilhelm, Büchter, Gürsoy, & Benholz, 2018). Language that enables individuals to communicate and that carries a certain systematic structure plays an effective role in structuring and understanding mathematics (Arguen, Yazgan Sağ, & Gülkılık, 2010). This universal structure that contains mathematical concepts, symbols, and grammar in its unique structure and enables communication is called mathematical language (Bali, 2003). Mathematical language is a skill that should be developed by using mathematical concepts, symbols, operations, and problems that enable students to use mathematical thinking skills in the process of structuring mathematics (Akarsu, 2019; Canbazoğlu & Tarım 2019). Studies have shown that students' mathematical language skills have a great impact on their success in mathematics (Barwel, 2018; Xu, Lafay, & Douglas, 2022). Therefore, mathematical language needs to be promoted with new instructional approaches (Haag, 2013; Marshman, 2015; Prediger, 2019).

One of the fundamental elements for the acquisition of concepts and knowledge about mathematics and for the development of mathematical thinking is the correct use of the language of the subject. For conceptual learning to occur, teachers are expected to design classroom activities to support students' mathematical language development (Yeşildere, 2007). Language use plays an important role in students' understanding of the concepts presented. It is very important that the concepts used by the teacher in mathematics class have the same meaning for the students. The terms

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and concepts used in mathematics are sometimes not familiar to students; if these concepts and terms are not used with the correct content, they may have different meanings (Çalıkoğlu-Bali, 2002). The technical language that teachers do not use correctly leads to unhealthy communication over time and creates problems in constructing mathematical concepts in the long run (Emre, Yazgan Sağ, Gülkılık, & Arguen, 201; Yeşildere, 2007). For this reason, it is necessary to communicate in accordance with mathematical principles and information in mathematics classes. Students who use mathematical language effectively can correctly switch between symbols, verbal expressions, and graphs (Çakmak, 2013). The ability to use mathematical language effectively is the correct use of mathematical symbols and sub-dimensions of mathematical language when using a mathematical concept. For students, this transition between sub-dimensions is usually not easy (Güner, 2012). During instruction, teachers should prepare appropriate environments where students can share, express, and justify their ideas using mathematical language to establish a relationship between concepts (Ministry of National Education (MoNE), 2018). National Council of Teachers of Mathematics [NCTM] (2001) stated that technology is a necessary element for teaching mathematics and enriches learning. Using technology in the classroom helps students acquire skills such as critical thinking, problem solving, and creative thinking (Saavedra & Opfer, 2012). GeoGebra was developed in 2001 as a master's thesis by Markus Hohenwarter at the University of Salzburg, Austria (Hohenwarter & Preiner, 2007). GeoGebra is a dynamic open-source mathematics software that combines geometry, algebra, and statistics and can be easily used today at any level of mathematics education (Hohenwarter & Lavicza, 2007). GeoGebra helps students understand mathematical relationships by allowing them to see diagrams in two and three dimensions. GeoGebra allows students to examine the reflections of changes in equations on figures and graphs (Gökçe & Güner, 2022). Students can develop a positive perspective and sense of self-efficacy toward mathematics by encouraging them to actively participate in class and use different representational systems through technology-enhanced mathematics instruction (Köysüren & Uzel, 2018). Self-efficacy is one of the most important concepts in social learning theory, which represents the need for a sense of confidence that individuals can effectively use their abilities to successfully perform certain tasks (Bandura, 1997). Self-efficacy refers to an individual's conclusion and personal belief that he or she will succeed or fail at a particular task, rather than his or her actual abilities (Thumb & Barzel, 2021). A low perception of self-efficacy is also evident in students who are unsuccessful in mathematics class (Kohen, Amram, Dagan, & Miranda, 2022). Individuals with positive self-efficacy perceptions persist in their decisions without giving up in the face of difficult situations, whereas individuals with negative self-efficacy perceptions abandon their actions because they become distressed after negative experiences (Can & Gündüz, 2021). At the same time, research on self-efficacy in mathematics education shows that there is a positive relationship between students' mathematics achievement and mathematics self-efficacy (Cheema & Poulou, 2021). Studies on self-efficacy have found a significant relationship between mathematics achievement and self-efficacy.

Function is one of the concepts that form the foundation of mathematics and plays a key role in expressing many concepts and making connections between concepts (Bayturan, 2011). The reasons for students' difficulties in understanding the object of function, different representations of function and transitions between representations, notations about function, symbolic writings, inverse function, and resultant functions are discussed (Kul, 2020). GeoGebra provides the ability to convert mathematical expressions into each other, solve equations and statistical calculations, represent functions in two or three dimensions, and perform graphical operations (Hohenwarter & Jones, 2007). Dynamic Geometry software allows mathematical concepts to be explored and interpreted in a variety of ways, such as dynamic multiple representations and mathematical modelling. For this reason, GeoGebra was preferred for teaching functions in this study. The purpose of this study is to investigate the effects of using GeoGebra in teaching functions on tenth grade students' mathematical language development and perceptions of self-efficacy. The research was examined within the framework of the following research questions.

RQ1. How do 10th grade students' perceptions of self-efficacy change before and after using GeoGebra to teach functions?

RQ2. How do 10th grade students' mathematical language structures formed during the application process involving the use of GeoGebra in teaching about functions?

1.1. Theoretical Framework

Mathematical language was examined in several sub-dimensions, and the difficulties students have in using the language were examined in detail. Pirie in his studies treated mathematical language in 6 dimensions (Pirie, 1998). These sub-dimensions are: symbolic language, colloquial language, mathematical verbal language, non-verbal language, visual language and semi-mathematical language. Marzano (2004) explained the elements of the language of mathematics. These elements are: informal explanations, repetition of the situation by students in their own words, pictures, diagrams, and drawings, continuous improvement of knowledge, thinking about the meanings of concepts, playful activities (Riccomini, Smith, Hughes, & Fries, 2015). Pimm (1987), who considers mathematics as a language, includes the sub-dimensions of mathematical language; the language spoken by the students, the language spoken by the students and the teacher in the classroom, the written language, the written language used by the students and the teacher were determined as the syntax of mathematics. Goslin (2016) studied mathematical language by dividing it into 4 subgroups: spoken language, written language, symbolic language, and mimic language. Baykul (2009) accepted mathematical communication as a necessary structure to express mathematical ideas and treated it in four sub-dimensions. These are: expressing mathematical ideas with representations such as concrete models, figures, diagrams, and tables; expressing ideas about mathematics and problems orally and in writing; connecting daily language with mathematical language and symbols; being aware of the importance of speaking, writing, discussing, and reading about mathematics.

In the study, 4 sub-dimensions of mathematical language were created based on the studies. The sub-dimensions mentioned in the study were created by the researchers after reviewing the literature on the subject. The sub-dimensions are: 1. Verbal language, 2. Symbolic language 3. Visual language, 4. Problem posing with everyday situation. In this study, the effects of using GeoGebra on the development of mathematical language in the classroom on the topic of functions, the sub-dimensions of mathematical language and the design of mathematical language structures when switching between sub-dimensions were investigated.

2. METHOD

Under this heading, information about the research model, study group, data collection, and data analysis are presented.

2.1. Research Model

This study was designed using the action research method. In this study, the action research model was chosen because the second researcher has a teaching role.

2.2. Participants

The sample of the study consists of 10th grade students of a high school in a province of the Central Anatolian region of Turkey. There are 20 participants in the study. The participants were selected using the purposive sampling method in accordance with the purpose of the problem. None of the participants have prior knowledge about GeoGebra and its functions. The participants are not in a school that admits students after exams, and according to the language questions on applied mathematics, all of them have low achievement in mathematics.

2.3. Data Collection Tools

The data collection instruments used in this study were the Mathematics Self-Efficacy Perception Scale, worksheets, video recordings, researcher logs, participant logs, and clinical interviews. Attempts were made to address the weaknesses of each measurement tool by using different data collection instruments that were reported throughout the study. Consistency of data was checked by comparing the data obtained. The mathematical language test prepared by the researchers was also used with the students in the study. In the study, a pilot application was conducted before the actual application to obtain information about the application. The pilot application was conducted with students who were at the same level as the participants in the original application. During the first semester of the 2020-2021 academic year, data collection was scheduled to occur during the 6-week period allotted for the functions topic in the curriculum, which lasted 8 weeks due to interruptions caused by distance learning.

2.3.1. Self-efficacy perception scale against mathematics

The mathematical self-efficacy perception scale developed by Umay (2001) consists of 14 items. The Cronbach alpha reliability coefficient of the scale was calculated to be 0.823 for the pre-test and 0.840 for the post-test. The scale consists of 8 positive and 6 negative items.

2.3.2. Participant logs

Participant logs; they allow the evaluation of the data obtained from the students' thoughts (Atasoy, 2012). Photographs of the logs were taken at the end of each activity, and the logs were collected when the students came to school in January for the written process. Detailed explanations were given since this will be the first time that the participants will make such a request. The goal is to determine the opinions of the participants based on their own expressions

2.3.3. Clinical interview

The clinical interview is a technique that aims to reveal students' thinking styles. In this research, considering distance learning, the times when students could be isolated from distracting situations and in a quiet environment were conducted via Zoom. Academic achievement level was considered in the selection of participants for the clinical interview. A total of six individuals with good, average, and poor performance were selected for the clinical interviews. The academic grade point average of two students with good level is above 80, the average of three students with medium level is between 60-80, and the average of students with poor level is between 40-60. The interviews were recorded and code names were assigned to the students in the interview samples. Among the students in the range, the students to be interviewed were randomly selected.

2.3.4. Worksheets

The researcher has prepared worksheets suitable for any acquisition to teach the topic of functions. Two worksheets were prepared for the pilot application and six worksheets for the main application. The worksheets were prepared with the inclusion of expert opinions to cover all the achievements of the topic of functions in the tenth grade according to the weeks. The participants followed the instructions on the worksheet throughout the application and answered the corresponding questions using GeoGebra software. The necessary arrangements were made with the opinions of a math teacher and a math education specialist.

The question related to verbal language in the first week of GeoGebra applications is given below.

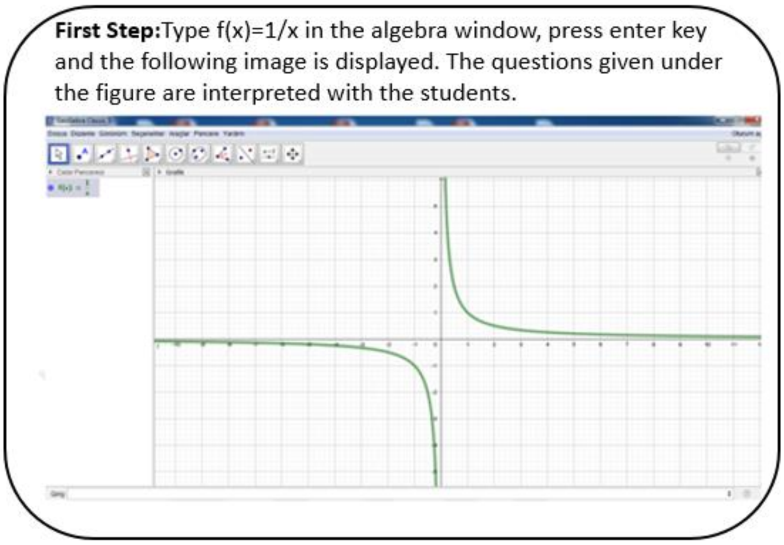

Figure. 1 The first step of application in the first week

The question about graph interpretation in the third week of GeoGebra applications is given below.

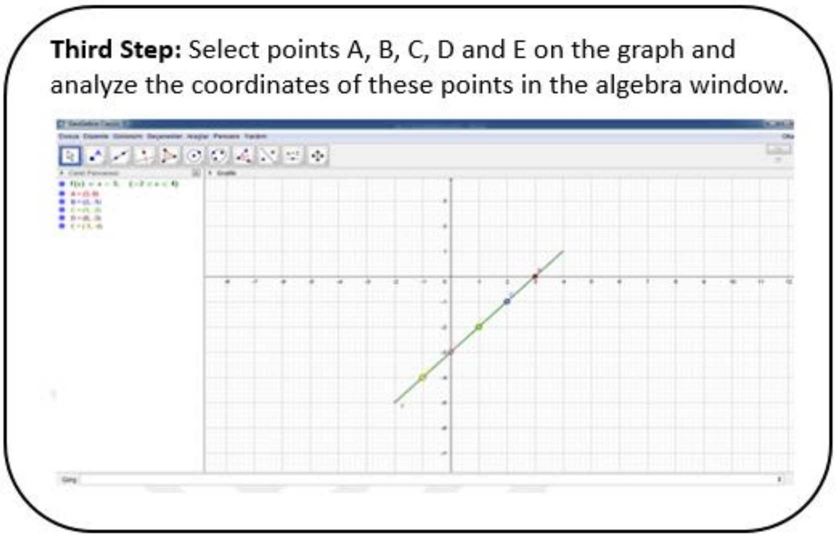

Figure. 2 The third step of application in the third week

2.3.5. Researcher logs

The researcher’s log is the notebook used to record the researcher’s observations and thoughts throughout the investigation. The researcher may also use a recording device (Johnson, 2015). The researcher’s log, which is commonly used in qualitative research, is used to examine behaviors and phenomena in a particular setting in depth. In this study twenty 10th grade students were observed during activities and detailed notes were taken of students’ verbal expressions, changes in those expressions, abuse, questions asked, changes in mathematical language, and any items deemed important by the researchers. Since the observers were directly involved with the participants, they observed as full participants and, after their observations, recorded the items they considered important and decided what to observe at the beginning of the study.

2.3.6. Mathematical language test

The mathematical language test, which is one of the data collection instruments of the study, was used as a pre-test and post-test. For each sub-dimension identified in this test, four times as many questions were elaborated, which were evaluated by two experts (mathematics educators), and it was decided to include the common items in the mathematical language test. The mathematical language test was used for qualitative analysis. In the mathematical language test: 8 questions of daily verbal language sub-dimension, 5 questions of verbal explanation of symbolic expressions, 4 questions of verbal explanation of graphs; in the visual language dimension, there are 6 questions belonging to the graphical design sub-dimension, 5 questions belonging to the graphical interpretation dimension, 2 questions belonging to the symbolic language dimension and 3 questions belonging to the problem posing sub-dimension.

2.4. Data Analysis

In the research, the analyzes were carried out within the framework of the dimensions discussed in the theoretical framework. During the process in which the worksheets were used, the content analysis method was used to study the qualitative data, observing the change in the mathematical language structure of the participants. In content analysis, words in the text are grouped into code structures according to certain rules. In this way, data with similar structure are grouped under certain concepts and themes to make them more understandable. Data obtained from observations and interviews are analyzed (Büyüköztürk, 2020). In the content analysis of the study, participants were coded as P1, P2..., and these codes were used throughout the analysis. The obtained data were analyzed in two ways: retrospective and prospective. In prospective analysis, each application is analyzed and guides the next application. Retrospective analysis was done by analyzing all data at the end of the study. Based on this data, changes in the implementation process were made as appropriate. The extent to which the study's action plan was implemented was determined in the committee meetings that took place with the participation of two experts in mathematics education after each lesson. The study collected data through the self-efficacy scale, worksheets, researcher logs, participant logs, clinical interviews, and audio and video recordings. Although one of these instruments, the self-efficacy scale, is a quantitative research instrument used for data collection, it was used for data diversity in this study. Analysis of the self-efficacy perception scale data in comparison to mathematics was conducted using the SPSS 20.0 program. Normality of the data was determined using the "Kolmogorov-Smirnov" and "Shapiro-Wilk" tests. Although the results of the pre-test and post-test meet the assumption of normality according to the results of the Kolmogorov-Smirnov and Shapiro-Wilk tests, the Wilcoxon Signed Rank Test was used to analyze whether the results differ before and after the application, since the sample size is less than 30. The assumptions of the test are met because the dependent variable is at least one variable of the rank scale and two repeated measures belong to the same group.

2.5. Validity and Reliability

In this study, the following validity and reliability measures were taken. The opinions of experts and teachers were obtained in the development of the measurement instruments. The study detailed the participants, the environment, the data collection instruments, and the application process. Immediately after the clinical interviews, the statements were transcribed without modification. These data were read to and confirmed by the respective participants. In analysing the worksheets, direct quotes, video recordings, and statements in the researcher's and participants' transcripts were transcribed unaltered. The consistency of the data obtained from the research was checked. All clinical interviews were recorded.

3. RESULTS

The findings obtained from the research are discussed under separate headings to answer the research questions. Analysis and interpretations of the data obtained with the data collection tools, using methods and techniques in line with the purpose of the research, are included. The results were reflected in the form of tables, analyzed and interpreted through codes.

3.1. Students’ Self-Efficacy Perceptions

In this section, Wilcoxon signed-rank test results on whether students' scores on the mathematics self-efficacy perception scale differ before and after the application are included. The pre-test and post-test scores of the students participating in the study from the mathematics self-efficacy perception scale are given in Table 1.

Table 1. Wilcoxon signed-rank test results regarding the mathematics self-efficacy perception scale before and after the application *

Post test-Pre test		Rank Average	Rank Sum	z	p
Negative Rank	3	7,33	22.00	2.959	.003
Pozitive Rank	16	10,50	168.00		
Eşit	3	-	-		

* Based on negative ranks

According to the test results given in Table 1, it is seen that there is a significant difference between the pre-test and post-test scores of the students participating in the study from the mathematics self-efficacy perception scale ($p<0.01$). When the rank totals of the difference scores are taken into account, it is seen that this observed difference is in favor of positive ranks and post-test scores. According to these results, it was observed that the GeoGebra-supported training applied in teaching the subject of functions had a significant effect on developing students' mathematical self-efficacy perceptions.

3.2. Students’ Mathematical Language

In the second research question it was investigated that “*How are the mathematical language structures of 10th grade students formed during the application process in which the use of GeoGebra is included in the teaching of the subject of functions?”*. This structure was examined under the sub-dimensions.

3.2.1. Verbal language sub-dimension

Verbal language is the written or verbal explanations that are used in daily language, sometimes by adding the terminology specific to the language of mathematics (Eroğlu & Deniz, 2020). Mathematical language; in addition to the unique expressions of mathematics, it also includes the words used in daily communication, this part of the mathematical language is named as verbal language (Aydın & Yeşilyurt, 2007). The verbal language findings are given under the sub-headings of daily language, verbal explanation of symbolic expression, verbal explanation of graphics.

Daily Language

While the students had difficulty in expressing their thoughts in the first weeks, they did not avoid using some definitions and terms in the following weeks. At the same time, the daily life examples they gave related to the question were mostly compatible with the question. In these respects, the developments of the participants in their daily language use were determined. The reflections of this development process in different applications are exemplified below.

For example, in worksheet 1 applied in the 1st week; The participants were asked to create the graphic of f(x)=1/x in GeoGebra, (see Figure 3). Then, the question was interpreted together with the participants.

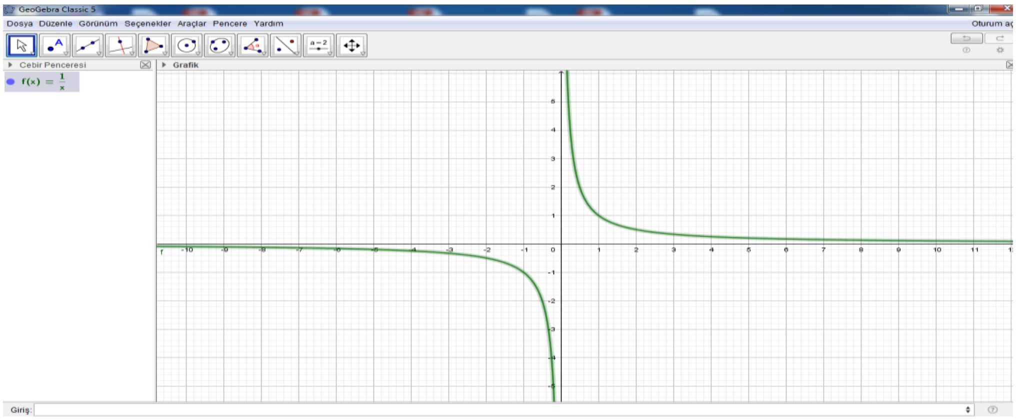

Figure. 3 The graphic of the verbal language sub-dimension in Worksheet-1

The participants were asked how the *function moves along the x-axis*, and the codes of the answers given by the participants using the daily language structure are presented in Table 2.

Table 2. Codes for daily language use in the first question

Codes	Participants
Fallen to zero	P1, P20
It vanished	P3, P12
It has two pieces	P9, P15
Took a break	P10

According to Table 2, it was determined that almost all of the students used expressions such as figure-line instead of the word graphic. It has been seen that they use the words down, go down, go without mentioning the positive part of the x axis, the negative part or the regions in the coordinate system. This showed that the students could not use the correct definitions and terms in their verbal expressions or they did not prefer to use these words at all. The student statement obtained from the video recordings supporting this finding is given below.

"The figure has come down. The first piece also went down, the second piece went down and to the side." (P3)

On the other hand, in worksheet 6, the development in daily language can be followed. When you examine the operations applied to the variable in the functions given as f(x)= 2x+1, g(x)=(x-1)/2 the answers given by the participants to the question of what do you think about these operations are examined, and the code structures are presented in Table 3.

Table 3. Codes for the question in worksheet 6 in the sub-dimension of daily language use

Codes	Participants
The same procedures were done	P1
Reverse transactions have been made	P15, P10
Transactions were performed in reverse order	P8, P19, P12

When the table 3 is examined, although one of the participants said that the same operations were performed, most of the participants noticed the operations applied to the variable during the activity. Therefore, it can be said that the participants were able to interpret the graph verbally.

Verbal Explanation of Symbolic Expression

In the process of verbally explaining the symbolic expression, the answers of the participants were examined for 6 weeks and little improvement was observed during the weeks. The reflections of this situation in different applications of the process are exemplified below.

The code structures obtained by examining the answers given to the questions "if $x_1 \neq x_2$,then $f(x_1) \neq f(x_2)$ "in the context of the verbal explanation of the symbolic expression which was discussed in worksheet 2, are presented in Table 4.

Table 4. Participant codes of the second question in the verbal explanation of symbolic expression

Codes	Participants
x_1 is not equal to x_2	P3, P12, P18
I do not know	P6, P12, P15

For example, when the code structures of Table 4 were examined in worksheet 2 applied in Week 1, it was seen that the participants were insufficient to explain the expression "if $x_1 \neq x_2$,then $f(x_1) \neq f(x_2)$".

Participants mostly avoided answering. The majority of the participants said that this statement "if $x_1 \neq x_2$,then $f(x_1) \neq f(x_2)$"did not mean anything. They have deficiencies in verbally expressing a symbolic expression. Examples from the researcher's logs explaining this situation are presented below.

…

P7: x_1 and x_2 are not equal.

R: What do you think about " $f(x_1) \neq f(x_2)$ " the rest of the statement?

P7: I don't know

R: What could " $f(x_1)$ " mean?

P7: It can be a function of x_1.

…

According to worksheet 3, when the question regarding the verbal explanation of symbolic expressions was examined, it was observed that the participants had difficulties in verbally explaining the symbolic expression.

The answer given to the question in worksheet 3, when A→B f(x)=2x-1, the symbolic expression f(A) = B was asked to be explained verbally and the answers was examined in the context of verbal explanation of the symbolic expression and the code structures obtained are presented in Table 5.

Table 5. Participant codes of the third question in the verbal explanation of symbolic expression sub-dimension

Codes	Participants
Set A is equal to set B	P1
f(A) is the same as B	P2, P13
The image set is set B	P19

When the code structures of Table 5 were examined, it was seen that the participants had a lot of difficulty in verbally explaining the symbolic expression. Similar to the second week, many students avoided answering or frequently used expressions such as I can't explain. In the answers of the

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participants, the word equality was mostly used, but it could not be explained what the equal sets mean.

Verbal explanation of graphics

Considering the verbal interpretation of the graphics, while the participants had difficulty in interpreting the graphics in the first weeks in the implementation process, the improvement in the interpretation of the graphics was determined in the following weeks. The reflections of this development process in different implementations are exemplified in this heading. For example, in the worksheet 1 applied in the 1st week, the examples of the situations in which the participants interpreted the graphics verbally are given below.

In Worksheet 1, the answers given in the frame of the question "If the graph is defined considering the conditions of being a function, is it a function graph?" in Figure 1 were examined in the context of verbal interpretation of the graph. The example expressions in the answers that the participants expressed verbally in the graphics are given in Table 6.

Table 6. Participants' sample expressions of the second question in the verbal interpretation of graphics sub-dimension

Sample Expressions	Participants
It is not, because it is in two parts	P4, P14
It is not, it does not pass through the zero point	P3, P15
It's not, the first piece is going down, the other piece is going towards zero.	P19

When the sample expressions in Table 6 were examined, although the participants knew the definition of the function, they failed to interpret the graph verbally. They mostly saw the graph's non-continuity as a problem. In addition, while interpreting the graph, they interpreted the domain and value set without taking into account. There are serious problems in the verbal interpretation of the graph. An example of this situation taken from the video recording is given below.

"It is not a function graph because it is in two parts." (K2)

In Worksheet 5 applied in the fifth week, a positive change is observed in the verbal interpretation of the graphics by the participants.

The answers given within the framework of the question "For the graph of the function $f(x) = x^2+3$, where is the symmetry axis of the graph and what does this mean?", which was discussed in Worksheet 5, the answers were examined in the context of verbal interpretation of the graph. The code structures of the sentences used by the participants while expressing the graphics verbally are given in Table 7.

Table 7. Participant codes of the sixth question in the verbal interpretation of graphics sub-dimension

Codes	Participants
center/y/y axis	P8, P5, P13, P19
I don't know	P17, P16

When the code structures of Table 7 are examined, it is seen that the participants generally say the symmetry axis of the graph correctly. It was observed that the number of mathematical terms used increased. There are two participants who answered I don't know, and one participant said they could not answer. Although there were answers such as the right one going up, the participants correctly sensed the axis of symmetry and answered. In these respects, it can be said that there is a positive change in the verbal interpretation of the graph.

3.2.2. Visual language sub-dimension

Verbal expressions and symbols are not sufficient to understand abstract concepts. For this reason, visual expressions that enable data to be classified and made concrete are also used (Van De

Walle, Karp, & Bay-Williams; 2014). Elements of visual language; graphs, tables, diagrams, schema, models. Examples of situations where the participants created function graphs and interpreted function graphs are given below.

Creating a Function Graph

While the frequency of incorrect, erroneous, or empty responses to questions about creating a function graph was significant in the initial weeks of the change, more accurate responses to the questions about creating a function graph were provided in the weeks that followed. Below are some examples of how this development approach is reflected in various applications.

For instance, the first week's worksheet 1 requested the participants to "What is f (2) according to the function graph?" By posing the query, their responses were looked over, and the code structures were provided in Table 8.

Table 8. Participant codes of the first question in the graphing sub-dimension

Codes	Participants
f (2) = 2	P1, P20
We can't know	P3, P12
It's a match	P9, P15
Wrong answers like 0.3	P10, P16, P8

When the data in Table 8 is examined, the answers to the question that "What is f (2) according to the function graph?", it was seen that more than half of the answers given by the participants to the question "We can't know". The participants also gave the answer "it is a match" by making use of their knowledge in the function definition, but they could not answer the question of which number matched which number or what kind of a match was made. Two of the participants stated that the number 2 would match 2 and they explained the reason by saying "every number matches itself". Some students gave wrong answers by saying any number. None of the participants have the knowledge to tell the coordinate of the point on the graph, which is necessary to create a function graph, that is, they do not have the necessary information to create a graph. An example of this situation, taken from the researchers' log, is presented below.

…

P3: The number 2 matches 2 because it is a match.

R: Why does it match 2?

P3: Because every number must match in matching.

R: So why does the number 2 match 2?

P3: I don't know.

…

In worksheet 5, the progress in the process of creating a function graph can be followed. When the question in worksheet 6 below is examined, it can be observed that the participants have the necessary information to create a function graph.

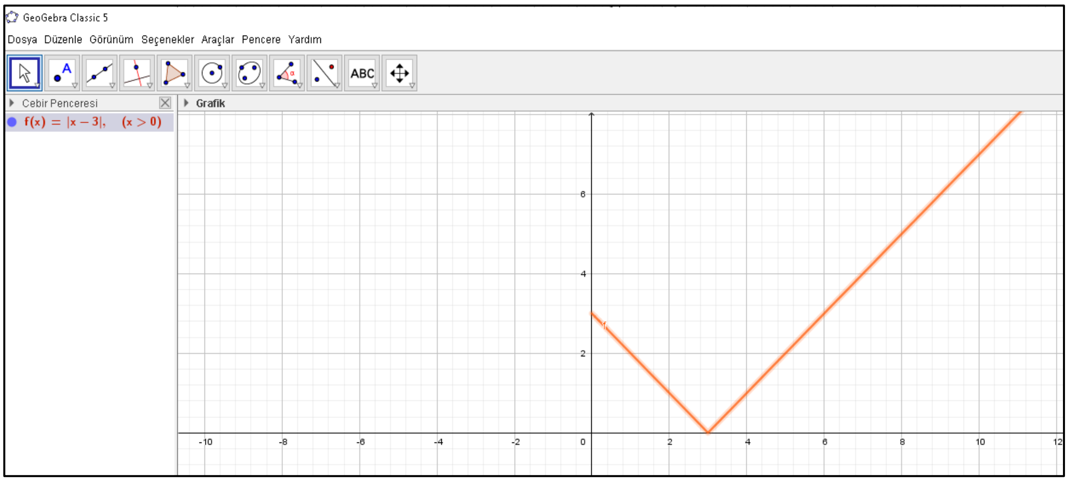


Figure. 4 Worksheet-5's graph for the subdimension of graphic creation

Within the framework of the question “Change the domain appropriately so that the given graph belongs to the even function” based on the graph that take place in Figure 4, the answers given by the students were examined in the context of creating graphics. The codes of the answers given by the participants are given in Table 9.

Table 9. Participant codes for the ninth question in the sub-dimension of chart creation

Codes	Participants
The graph should also be on the left	P16, P8
Other side of x/ x axis	P21, P10
Domain set	P4
All numbers	P18

According to the code structures of table 9, some of the participants gave non-significant answers by saying the domain or some numbers. The majority of them sensed that negative real numbers should be included in the domain, since the graphs of even functions should be symmetrical about the y-axis. It was observed that there was an increase in the number of terms used by the participants in their answers. Although they cannot express this situation appropriately, it can be said that they understand the necessary condition for the creation of an even function graph.

Interpreting Function Graphs

While the majority of queries regarding the creation of a function graph were irrelevant, incorrect, and left unanswered in the initial weeks, they were more receptive to interpreting the function graphs in the ensuing weeks and provided more accurate responses to the questions in question. The participants' progress in drawing a function graph was tracked throughout the study. Below are some examples of how this development approach is reflected in various applications.

For example, in Worksheet 2 applied in the second week, students are asked to create a graph of f(x)= x-3 (Figure 5) in the GeoGebra. Then, the questions about Figure 5 are interpreted together with the participants.

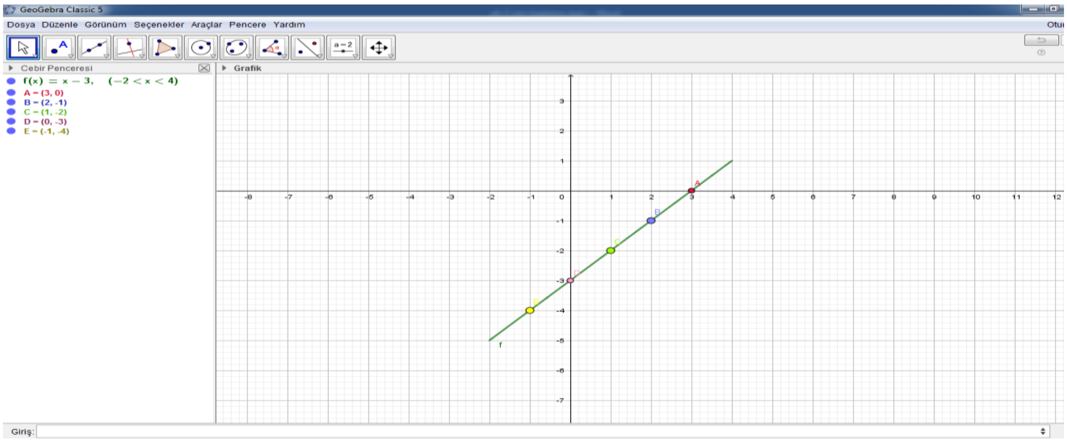


Figure. 5 The graph belonging to the worksheet-1 in graph interpretation sub-dimension

"What could be the value of *f(4)*?" according to the graph in Figure 5. Within the framework of the question, the answers given by the students were examined in the context of graphic interpretation. The answers given by the participants to this question were examined and the code structures are presented in Table 10.

Table 10. Codes for the second question of the function graph interpretation

Codes	Participants
There is a point above	P4, P13
Goes in order	P2, P11

According to the data in Table 10, the participants could not predict what the other point should be. Although a few of the participants showed their location on the screen, no one could say the coordinate of the point. The other participants gave answers as "just go up, go up". According to these data, the participants cannot interpret the graph. In worksheet 5, the development of the interpretation of the graphics can be followed. Examining the question in worksheet 5 below, it was concluded that the majority of the participants interpreted the graph correctly. Belonging to worksheet 5, "How can one interpret the slopes of inverse functions?" The question was asked to the participants and their answers were examined within the framework of the graphic interpretation sub-dimension. The answers given by the participants to this question were examined and the code structures were presented in Table 11.

Table 11. Codes for the eighth question of the function graph interpretation

Codes	Participants
Slopes are the same	P1
Slopes are opposite of each other	P3, P12, P15
Slopes are opposite of each other with respect to multiplication	P9, P15

According to Table 11, although P1, one of the students with low academic achievement, thought that there was no difference between the slopes, the other participants concluded that the slopes of inverse functions are inverses of each other according to the multiplication operation. The participants were mostly able to interpret the graph about the slope correctly.

3.2.3. Symbolic language sub-dimension

Symbolic language is one of the most used forms of mathematical language in mathematics (Emre, Yazgan-Sağ, Gülkılık, & Argün, 2017). Students should be able to make sense of what symbols mean, and should not see mathematical symbols as meaningless shapes that everyone perceives differently (Boz, 2008). Symbolic language is expressing a mathematical expression through mathematical symbols that everyone attributes the same meaning (Pirie, 1998). Examples of situations where participants use symbolic language are given below. In this study, practices related to symbolic language were the most difficult part of the participants and developed more slowly than the other sub-dimensions. The reflections of this process in different applications are exemplified below.

The following dialogue was held with the participant P18 for the question, "Do you think there is a change in transferring the expressions given verbally in the activities to symbolic language? Could you explain this change?" which was included in the clinical interview questions.

…

P18: "I understand what you write, but I can't write like that."

R: "How can you not write like that?"

P18: "Using such signs"

R: "Do you think there was a change in using symbolic language during the activities?"

P18: "I understand what you write, but I can't write it"

…

For example, in Worksheet 3, the answers given by the participants to the question "Write the domain and image set of the function that is written and graphed in the algebra window of the GeoGebra screen using symbolic language" were examined and the code structures are presented in Table 12.

Table 12. Codes for the third question of using symbolic language

Codes	Participants
R	P2, P14
All Numbers	P16, P3
Natural Numbers	P17
Domain	P2
X axis	P5

According to Table 12, some of the participants said that they did not understand what is meant by the domain of a function, and then gave the domain set, x-axis answers. Then, it was discussed again about what is meant by domain set, value and image set. The participant named P17 stated that the numbers in the domain are 0 and positive numbers on the x-axis of the graph and stated these as natural numbers. Although most of the participants gave completely wrong answers, a few came close to the correct answer. An appropriate example of this situation, taken from the researcher's log, is given below.

…

P5: When I say domain set, x-axis comes to mind, domain set is related to the x-axis.

R: Let's look at the graph together, which numbers on the x-axis do you think are in the domain?

P5: There are infinite numbers on the x-axis, they are all in the domain of the function.

R: Well, let's look at the graph again, can you tell which number -3 on the x-axis matches?

P5: If we choose a point from GeoGebra, it doesn't work. No numbers matched.

…

In the answers given for the question in worksheet 4, it is seen that it is difficult for the participants to use symbolic language in the following weeks. This example is given below. The graph of (2,4)→(2,4) f(x)=x, ie unit function, is plotted on GeoGebra's algebra screen (Figure 6). Then, the questions about the graph in Figure 6 were interpreted together with the participants.

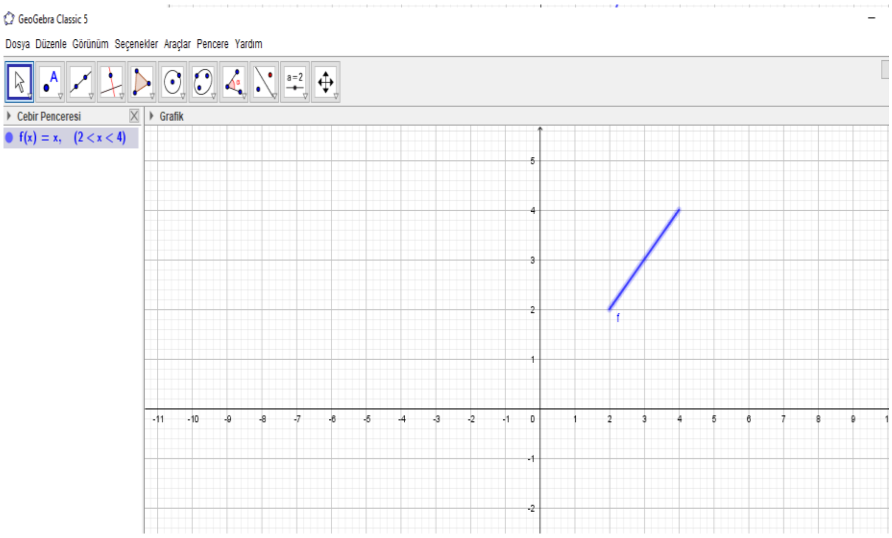


Figure. 6 Worksheet-1 graphic of symbolic language sub-dimension

For the graph in Figure 6, "Write the number range of the appropriate value set so that the graph of the specified function can be onto" The answers given by the participants to the question were examined in the context of symbolic language. The answers given by the participants to this question were examined and the code structures are presented in Table 13.

Table 13. Codes for the fifth question of using symbolic language

Codes	**Participants**
All Numbers	P5
X axis	P7
2 and 4	P11
(2,4)	P14, P17

According to the data in Table 13, although the majority of the participants were able to explain the onto function using everyday language and give appropriate examples from daily life, when they were asked to write down a set of values symbolically for the definition of the onto function of the graph, they mostly gave non-significant answers such as the x-axis and y-axis. Most of the participants gave the answers 2 and 4, (2,4). When asked what the parenthesis in the given answer meant, they could not answer. This situation shows that the participants are insufficient in using symbolic language. An appropriate example of this situation, taken from the researcher's log, is given below.

This situation shows that the participants are inadequate in using symbolic language.

…

R: "Which numbers do you mean by (2,4)."

P17: "The numbers between 2 and 4."

R: "Well, can you give examples of numbers in the range you specified?"

P17: "Three, but other numbers are also possible. Real numbers are also possible."

R: "So, should the numbers 2 and 4 be included in the definition set?"

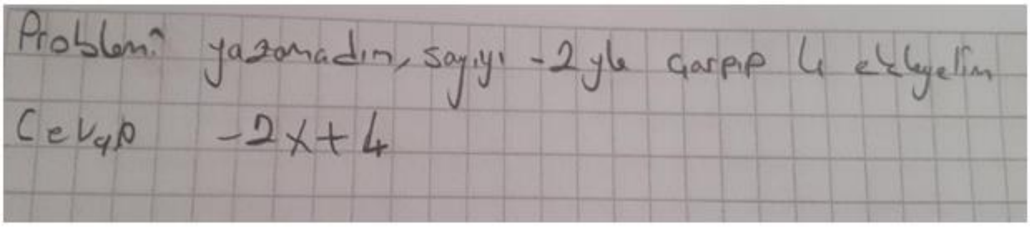
P17: "They should be included, we had these topics last year."

…

3.2.4. Problem posing sub-dimension

Problem posing can be expressed as a process that includes creating meaningful mathematical problems related to concrete situations with individual comments (Özdişçi & Katrancı, 2020). With problem posing, the ability to express mathematical situations in writing and verbally is gained, and students are enabled to discover mathematical situations (Akay, Soybaş, & Argün, 2006). An example of the participants' problem posing situations is given below.

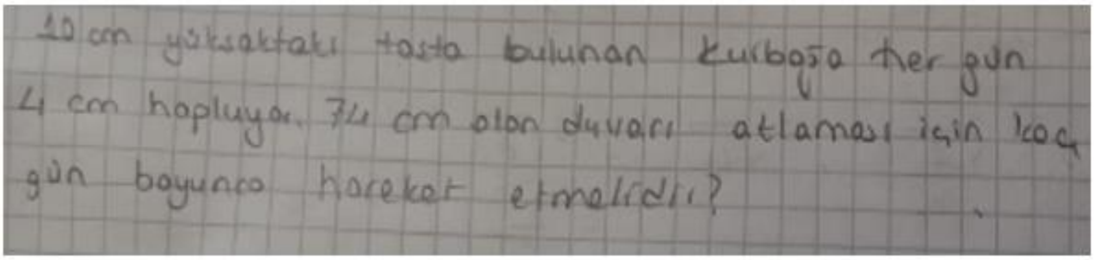
During the Problem Posing application process, the participants often left the problem posing sections blank or tried to compose relatively complex problem sentences related to daily life situations in the following weeks, while they were quite short. For these reasons, progress has been detected in the problem posing process. The reflections of this development process in different applications are exemplified below.



I couldn't write the problem however multiply the number by -2 and add 4
Answer -2x+4

Figure. 7 Structured problem posing example of participant coded P11

On Worksheet 5, "Write a problem statement for the inverse of a function, identifying appropriate definitions and sets of images." The question was asked to the participants and the sample of the participant is given below.



The frog, which is found on a stone 10 cm high, jumps 4 cm every day. For how many days must it move in order to pass the 74 cm wall?

Figure 8. Structured problem posing example of participant P5

In the researcher logs, it was noted that the participants tried to create problem sentences for structured and semi-structured problem sentences, but they tended to ask a question with simple sentences. It was observed that the participants tried to include mathematical terms while constructing problem sentences and tried to construct more complex sentences.

4. DISCUSSION and CONCLUSION

The study investigated how the use of GeoGebra in teaching functions affects students' mathematical language development and perceptions of self-efficacy. In the section examining how participants' self-efficacy perceptions change before and after the instructional process, there is a significant difference between the pre-test and post-test scores of the students participating in the study on the mathematical self-efficacy perception scale. Looking at the rank sums of the difference scores, it was found that this difference was in favor of the positive ranks and posttest scores. According to these results, the applied GeoGebra-based training had a significant effect on the development of students' mathematical self-efficacy perception. Orçanlı (2016), in his study investigating the effects of computer-based geometry instruction on students' perceptions of geometry self-efficacy, found a significant difference before and after the application. These results show similarities that the method of computer-assisted geometry instruction used in teaching geometry has a significant effect on students' self-efficacy. Balcı-Şeker and Erdoğan (2014) concluded in their research that GeoGebra-supported mathematics instruction causes a significant difference in terms of self-efficacy in the experimental and control groups. This result is consistent with the research findings. In studies investigating the relationship between different subjects of mathematics and computer-assisted instruction with self-efficacy, or more specifically, the relationship between GeoGebra-assisted mathematics instruction and self-efficacy in the literature, it is found that GeoGebra-assisted instruction has a significant effect on improving students' mathematical self-efficacy perceptions, as in this study.

During the GeoGebra lesson on functions, participants were asked questions designed to enable them to use all the sub-dimensions of the mathematical language. However, participants tended to answer the questions using mainly the sub-dimensions of the verbal language. Although participants used visual and symbolic language as the weeks progressed, they indicated that they were most comfortable with verbal language during the first few weeks. In Akarsu's (2013) study, students transformed the dimensions of mathematical language when answering mathematical questions. They used both verbal and written language to explain the required expressions. However, he concluded that students generally used spoken language first. This could be due to the fact that students are used to responding with verbal language because of their previous experiences. The results of the study are completely consistent with this finding. When asked to explain a graphical or symbolic expression using verbal language, participants responded that they did not know the answer, that they could not find the result, rather than making comments and verbally expressing what they understood. When participants were reminded again what to do and the explanation that they did not have to reach a numerical value was repeated, they tried to answer in verbal form. This result is consistent with Yalvaç's (2019) finding that they were more likely to find a numerical value than to explain using mathematical language. In this case, it is assumed that participants focus on solving rather than expressing what they understood from the questions. It was found that the worksheets used during the study had positive effects on participants' verbal language use. This is because during the lessons, the rate of the participants to use the definitions and terms mathematically in their daily language use, to express the graphs correctly, and to explain the symbolic expressions verbally increased. It was observed that prior to the implementation of the action plan, participants used mathematical definitions and concepts inadequately. After the implementation of the action plan, it was found that the participants were able to achieve the results with the prepared worksheets, and the number and quality of mathematical definitions and concepts used increased. While the increase in the number of correct uses of definitions and concepts is lower for P1 and P11 participants with lower academic achievement, the rate of effective use of verbal language is higher for students with medium and high academic achievement. This result is consistent with the findings of Akarsu's (2019) study that there is a moderate, positive, and significant relationship between students' ability to understand and use mathematical language and their achievement in mathematics. Zengin (2017) addressed mathematical language as a whole in his study and defined mathematical language as the effective use of mathematical terminology and concluded that the use of GeoGebra has a positive effect on the use of mathematical language. This study also supports the result of the research. From the research conducted by Gökçe, Yenmez, and Özpınar (2016), in which they investigated mathematics teachers' opinions about the worksheets created with GeoGebra, it was found that before using the computer, the teachers only saw an advantage in speed and time. After use, they indicated that GeoGebra can be used to review concepts, and they agreed that the software can help students understand concepts that are difficult to convey verbally. Studies consistently show that numerous ideas can be explored using worksheets created with GeoGebra, that children can understand concepts that they have difficulty describing verbally and on the board, and that the software has a positive impact on verbal language use.

Symbolic language was the sub-dimension that participants had the most difficulty answering during the 6-week study. When asked to express a particular statement symbolically, participants were reluctant to respond. Even when they responded more in the following weeks, they initially tended to respond verbally. Argün (2016), in his study of prospective teachers' use of mathematical language, found that they tended to use verbal language rather than symbolic language when trying to understand a concept. As a result of the study, it was found that it would be beneficial for instructors and teachers to use not only symbolic language but also verbal language when trying to understand a concept while teaching the concept. This result is consistent with the finding that students in the study used verbal language instead of symbolic language. Altıner and Önal (2022), in their study in which they investigated the visual and non-visual representations used by students in solving verbal problems, concluded that although the success rate was higher for the answers in which they used visual structures, they used symbolic structures to a greater extent. The investigation with 10th grade students revealed that the participants avoided symbolic expressions most of all. The results of the study are not consistent in this regard. The reason for this is probably that the students participating in the study were not in situations where they would use symbolic language. Although the students intended to respond in symbolic language, examination of their responses revealed that they used meaningless symbols and that what they intended to say and what they rendered were not parallel. Capraro and Joffrion (2006) conducted a study on symbolic language and verbal language with seventh and eighth grade students. In the study, students were asked to write down mathematical expressions given verbally in algebraic form. It was found that few of the students gave correct answers, and it was concluded that the students were underusing symbolic language. The results of this study are consistent with the findings of Capraro and Joffrion's (2006) study. According to other research findings that support this result, the transitions between algebraic symbolization and verbal representation present many difficulties for students. Students with weak verbal language structures were also very unsuccessful in using symbolic language. Çakmak (2013) concluded in his research that these two languages were significantly and highly correlated in terms of the dimensions of verbal language and symbolic language, which are two important components of mathematical language. The conclusion that symbolic and verbal language are very important for the development of mathematical language and that they are interrelated is common to both studies.

Visual language structures were examined in the sub-dimensions of graphic design and graphic interpretation. Participants had difficulty interpreting the diagram. Instead of examining the relationship between the variables in the given diagram, they saw the diagram as a figure. Previous studies have noted that one of the difficulties encountered with graphs is the 'error of perceiving graphs as a picture' (Bell & Janvier, 1981). In this misconception, which is seen in students at different levels, the structure at hand is drawn as a picture on the plane or the given picture is simply transferred to the analytic plane (Slavit, 1994). Students with this misconception could not understand the relationship between variables (Bayazıt, 2011). The results of the study showed that one of the problems that students had in interpreting the graphs was a common misconception. When asked whether a particular graph belonged to a function, the participants indicated that when the function graph was mentioned, it should be a linear function graph. In the studies conducted at different times, it was concluded that the students' graph should be linear or increasing only. Another misconception is that students focus on linear graphs (Karataş & Güven, 2004). In their study, students put a certain shape as a graph without understanding the relationship between variables and concluded that they interpreted accordingly. The results of the study show that participants have an image of a graph of a function in their minds, and this image is usually a linear geometric shape.

In this study, the problems posed by students are generally those that do not have much to do with the daily life problems that participants have encountered before. The participants kept the problem posing as short as possible and did not tend to add any difficulty elements. The results of the research show different results on these questions. In this regard, the results of the study are contradictory. In general, the tasks given by the participants do not have an original structure, are not mathematically or linguistically complex, consist of simple sentences, and cannot always be related to situations from daily life. This is true even though the participants' performance in each of the three sections was different. In his study, Güç (2021) investigated teachers' performance in task setting related to correct use of mathematical language, appropriate task setting, solvable task setting, original task setting, and task setting in which GeoGebra can be functionally used to solve. As a result of the study, it was found that participants' performance in solving problems was generally low. Although it is still difficult for the participants to pose problems, it was found that progress was made over weeks in structured and semi-structured problem posing.

Activities related to problem-posing, symbolic language, visual language, and verbal language skills, which are the sub-dimensions of mathematical language, can be incorporated into mathematics instruction to strengthen students' mathematical language and sense of self-efficacy. Students' development in mathematical language can be studied longitudinally so that in-depth information about the durability of change in mathematical language structure can be obtained. In the GeoGebra classroom, the reasons why students who have not achieved positive change in their mathematical language structures have not been able to make changes can be explored in greater depth by adding factors such as attitude and readiness. Studies can be conducted to separately investigate the change in mathematical language structures of students with different achievement levels. The study was conducted with 10th grade students. Working with a different mathematics subject at different grade levels with appropriate data collection instruments can provide detailed information about how students' mathematical language structures change. The study was conducted with 20 students. Although the number of students is considered sufficient for action research, the results of the study with more students may provide different information. Teachers can be trained in this area in education departments to facilitate extensive communication in mathematics classrooms and to develop students who use mathematical language successfully.

GeoGebra software has been observed to facilitate students' conceptual understanding as it provides the opportunity to examine different representations of concepts (Zengin, 2017). Between representations transformation becomes easier with the use of GeoGebra (Zengin, 2017). It supports the framework in which conceptual understanding is associated with the transformation between representations. The participants' self-efficacy increased when they saw how the changes they made in the function equations changed the function graphs. GeoGebra allows students to see diagrams in two and three dimensions, helping users to understand mathematical relationships. GeoGebra allows students to examine the reflections of changes in equations on figures and graphs. Participants have the opportunity to control the changes they make (Gökçe & Güner, 2022). Via GeoGebra, students can actively participate in the lesson and develop a positive perspective on mathematics and a sense of self-efficacy by using different representation systems (Köysüren & Uzel, 2018). Similar to other studies, the features related to GeoGebra increased the participants' self-efficacy perceptions.

Ethics Committee Decision

Ethical approval and written permission for this study were obtained from the Social and Human Sciences Scientific Research and Publication Ethics Committee of Niğde Ömer Halisdemir University with the decision numbered 69972237-302.08.01-E.52412.

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