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Pre-service Teachers' Imaginary Creative Approaches to Address Students' Erroneous Understanding of Algebraic Expressions

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Abstract

This study examines the creative responses of pre-service mathematics teachers in their lesson plays designed to address sixth-grade students' misconceptions about algebraic expressions. This research employs a qualitative descriptive research design, involving 78 third-year students enrolled in an elementary mathematics education program. Using lesson plays, the pre-service teachers developed hypothetical lessons that demonstrated how dialogues between teachers and students could unfold in a classroom. The research revealed that the pre-service teachers exhibited pedagogical and mathematical flexibility in addressing students' misconceptions in algebraic expressions. While the participants did not display mathematical and pedagogical originality, they were able to create a variety of hypothetical instructional settings. This study highlights the potential of lesson plays as an effective tool to examine pre-service teachers' creativity and explores various pedagogical approaches in their hypothetical instruction. The findings suggest that teacher education programs should include more opportunities for pre-service teachers to develop their creativity using lesson plays and for preparing them to effectively and originally address students' misconceptions about algebraic expressions.

Key Words

Algebra • Creativity • Lesson play • Pre-service matematics teachers

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Introduction

Several research studies have concentrated on students' conceptions and misconceptions about algebraic symbols (e.g., Bush & Karp, 2013; Knuth et al, 2005; Mac Gregor & Stacey, 1997; Usiskin, 1988). In these studies, students' conceptions were classified as (*i*) seeing algebraic letters as mere labels or abbreviations instead of representatives for quantities (Weinberg et al. 2004), (*ii*) believing that different letters in an algebraic expression must signify distinct numerical values, (*iii*) being unable to interpret algebraic letters as generalised numbers, (*iv*) overlooking the algebraic letters and substituting them with actual numbers (MacGregor & Stacey, 1997), (*vi*) finding the concept of 'equality' complex and challenging to grasp (Knuth et al., 2005), and (*vii*) recognizing students' tendency to combine or complete open expressions (Tirosh et al., 1998). One fundamental concept that is a prerequisite for success in algebra is understanding what algebraic symbols represent (National Council of Teachers of Mathematics [NCTM], 2000). In teaching algebraic expressions, teachers employ various approaches such as practicing the manipulation of terms, solving for unknowns, and encouraging students to perceive variables as concise means to express their understanding of varying quantities (Stephens, 2005), improving students' understanding of the equal sign may require changes in teachers' instructional practices (Knuth et al, 2005), using the 'like terms' and the 'fruit salad' teaching approaches (Tirosh et al., 1998). Traditional pedagogical approaches fail to help students develop accurate conceptual understandings of algebraic expressions (Hiebert & Grouws, 2007). However, creative teachers can suggest mathematically and pedagogically innovative ways to support students' learning and address students' misconceptions (Lev Zamir & Leikin, 2011). Teachers need to suggest innovative ways, both mathematically and pedagogically, to nurture creativity in the classroom. Creative teachers can leverage strategies like open-ended tasks and multiple solution problems to address students' misconceptions and promote deeper learning of algebra concepts. By valuing creative thinking, teachers enable students to solve problems in diverse ways, gain new perspectives, and construct robust conceptual knowledge. Creativity is an important concept in mathematics education, though it has often been overlooked in traditional pedagogical approaches (Levenson, 2011; Sriraman, 2009). Recent research has highlighted the value of mathematical creativity - producing unusual yet insightful solutions - in developing students' conceptual understanding and problem-solving abilities (Silver, 1997). By encouraging creative approaches, teachers can support students in developing more flexible and accurate understandings of algebraic expressions.Creativity in mathematics education is an underexplored topic, and teachers' ideas about creativity are limited (Bolden et al., 2010). The limited research on creativity in mathematics education reveals that pre-service and in-service teachers lack creativity in problem-solving approaches (Leikin & Pitta-Pantazi, 2013), instructional strategies (Zazkis & Leikin, 2010), adapting to students' needs (Chamberlin & Moon, 2005), and implementing original tasks beyond the curriculum (Liljedahl, 2016). Using a creative approach when teaching the challenging concept of algebraic expressions to students is considered to be a way to overcome the problems encountered with this concept. In light of the challenges students face in understanding algebraic expressions and the limited creativity demonstrated by teachers and pre-service teachers in mathematics education, Zazkis (2017) provided a valuable insight for the importance of considering the role of teacher educators in fostering creativity among pre-service teachers. By incorporating the lesson play method into the creativity in mathematics teaching model, she highlighted the potential for recognizing and promoting various conceptions of creativity in mathematics teaching. In line with this

suggestion, the present study examined the creative pedagogical approaches of pre-service mathematics teachers in the lesson plays they prepared to eliminate sixth grade students' misconceptions about algebraic expressions. Identifying pre-service teachers' pedagogical approaches to address students' misconceptions will assist teacher educators in revising tasks related to creativity in teacher education. Additionally, the findings may help mathematics teacher educators foster pre-service teachers' creativity in teaching algebraic expressions by incorporating innovative pedagogical approaches to teacher education courses involving the teaching and learning of algebra. Preparing future teachers better with these innovate pedagogical approaches can pave the way for increased student understanding and success in algebra.

Lesson Play

Lesson plays are scenarios that present an alternative to traditional lesson plans by constructing imaginary dialogues between teacher-student or student-student interactions (Zazkis et al., 2009). When creating lesson plays, it is important to focus on student concepts that may arise in the classroom and teaching strategies that will facilitate the intended student thinking (Zazkis et al., 2013). For example, a lesson play could focus on a potential misconception students may have in an algebra lesson, the explanations revealing this misconception, the underlying reasons, and creative teacher approaches to help students overcome this misconception. Relevant literature has documented that lesson play tasks greatly assist pre-service teachers in better understanding mathematics and mathematics teaching (e.g., Shure & Rösken-Winter, 2022; Shure et al., 2022; Zazkis et al., 2013; Zazkis & Zazkis, 2014). Developing a repertoire of lesson plays on various mathematical topics not only supports pre-service teachers' professional development, but also serves as a tool for researchers to examine pre-service teachers' creative pedagogical approaches (Zazkis et al., 2013; Zazkis, 2017). Zazkis (2017) emphasized the potential to acknowledge and encourage diverse understandings of creativity in mathematics teaching by integrating the lesson play method into the Lev Zamir and Leikin's (2011) model of creativity in mathematics teaching. She emphasized that the lesson play task presents opportunities to identify instances of the task designer's mathematical and pedagogical creativity, as well as creative implementation opportunities for teachers using the task. In this study, the aim was to examine the creative pedagogical approaches that pre-service teachers proposed in the lesson plays they created about teaching algebraic expressions.

Creativity in Mathematics Teaching and Algebraic Expressions

The theoretical framework of this research is based on the work of Lev Zamir and Leikin (2011) on creativity in mathematics teaching. This model adopts Torrance's (1974) definition of creativity, characterized by flexibility, originality, and elaboration. The model shows how creativity differs in instructors' descriptions from how it is used in the classroom. Lev Zamir and Leikin (2011) further differentiated between the creativity manifested in teachers' actions, referred to as "teacher-directed conceptions of creativity," and the creativity exhibited in student actions, known as "student-directed conceptions of creativity." Regarding teacher-directed creativity, they identified four key aspects: mathematical flexibility, which they defined as transforming mathematical problems and presenting various problem-solving approaches; mathematical originality, which pertains to the generation of tasks that go beyond the standard curriculum; pedagogical flexibility, which encompasses the adapting to students' needs and

responses; and pedagogical originality, which is characterized by the utilization of diverse instructional settings, strategies, and tools.

According to Lev Zamir and Leikin (2011), creativity in mathematics teaching can be achieved by incorporating problem-solving tasks into classrooms, promoting mathematical discourse, and creating a positive learning environment. Applying these principles to the context of algebra, pre-service teachers can develop pedagogical approaches that go beyond traditional methods to more innovatively engage students in learning and facilitate their understanding of algebraic symbols. While traditional approaches have been criticized for being too rooted in theory and not meeting most students' needs (Abaté & Cantone, 2005), recently there have been calls for reform through more innovative, constructivist-based teaching approaches that promotes conceptual understanding (Chang, 2011; Mokhtar et al., 2010). By drawing on these more innovative principles, pre-service teachers can create instructional approaches tailored to engage today's algebra students. Literature review provides an overview of pedagogical strategies for pre-service teachers to teach algebraic expressions effectively (e.g., Kaput, 1999; Leikin, 2009). One approach to teaching algebraic expressions is through the use of *multiple representations* (Kaput, 1999). Research suggests that engaging students in multiple representations (e.g., graphical, numerical, and verbal) can help them develop a deeper understanding of algebraic concepts (Ainsworth, 2006; NCTM, 2000). For example, pre-service teachers can employ function tables, graphs, and verbal descriptions to illustrate algebraic relationships (Moschkovich, 1999). The utilization of technological tools such as graphing calculators and computer software can also facilitate the exploration of multiple representations (Trouche & Drijvers, 2010). Another approach to teaching algebraic expressions involves the use of contextualized problems (Kieran, 2007). By presenting algebraic concepts within real-world contexts, pre-service teachers can help students make connections between abstract algebraic symbols and their applications (Lesh & Zawojewski, 2007). Research indicates that contextualized problems can improve students' motivation and engagement, as well as their ability to apply algebraic expressions in various situations (e..g., Haines & Crouch, 2001).

Promoting algebraic reasoning through pattern generalization and functional thinking represents another effective pedagogical approach for pre-service teachers (Radford, 2006). By engaging students in activities that require them to analyze and generalize patterns, pre-service teachers can foster their ability to think abstractly and manipulate algebraic symbols (Lee & Freiman, 2006; Warren, 2003). Moreover, focusing on functional relationships between variables can help students develop a better understanding of algebraic expressions and the role of variables in representing real-world situations (Kaput, 2008). Research suggests that providing clear explanations and step-by-step guidance can support students' understanding of algebraic symbols and their manipulations (e.g., Nathan & Koedinger, 2000; Warren, 2003).

Method

Research Design

This research employs a qualitative descriptive research design, known for its intention to offer a comprehensive summary of events (Lambert & Lambert, 2012). This approach is particularly fitting for this study, as it allows for a rich exploration of the creative pedagogical approaches used by pre-service mathematics teachers, investigating the

approaches in which they address misconceptions in algebraic expressions. Within this framework, the study explores the creative strategies of pre-service mathematics teachers by examining lesson plays created by third-year students enrolled in an elementary mathematics education program.

Participants

The research participants were 78 third-year pre-service middle school mathematics teachers (n_{female} = 69, n_{male} = 9) enrolled in a teacher education program at a university in Turkey. The participants were enrolled in a compulsory mathematics education course titled "Algebra Instruction" during the course of this research. The algebra teaching course consists of three 45-minute lecture hours per week for 14 weeks. In this course, pre-service teachers acquire information about the concepts and the teaching of these concepts in the field of algebra within the secondary school mathematics curriculum (Ministry of National Education [MoE], 2018). The course primarily focuses on the mathematical meaning of the concepts, and then is based on the implementation of activities to be used in teaching the subject in accordance with the grade level. The course proceeded in a student-centered structure, where pre-service teachers participated in activities and held discussions in real teaching environments of the activities. Within the scope of this research, pre-service teachers created the lesson plays at the end of the seventh week of the course, after learning the subject of algebraic expressions. The focus of the research was on what kind of situations pre-service teachers proposed in the context of creativity when their knowledge of algebraic expressions was at a sufficient level.

The selection of participants in this research was carried out using the criterion sampling method (Patton, 2001). This method involves selecting cases that meet predetermined criteria of importance and can be useful for identifying and understanding information-rich cases. The criteria for participation in this research were being a third-year student in the elementary mathematics education program and are currently enrolled in the algebra teaching course. Participants' involvement in the study was voluntary. These pre-service teachers entered the department after taking the high-stakes national examination entitled YKS (Higher Education Institutions Examination). All high school graduates must enter this examination in order to study at a university. After four years (i.e., eight semesters) of education in teacher education programs, the pre-service middle school mathematics teachers earn a Bachelor's degree that licences them to work public or private schools as a mathematics teacher.

It is important to note that these pre-service teachers had not taken any courses directly related to creativity in mathematics teaching. Participants provided informed consent prior to their participation in the study. To guarantee participant anonymity, all identifying information was deleted from participants' scripts and each participant was given a special code. The participants in the study were informed that they could voluntarily withdraw from the study at any time.

Data Collection

Data were collected via the prompts for the lesson play task (Zazkis, 2017). Lesson plays consist of student-student or student-teacher dialogues called prompts (Zazkis et al., 2009). Zazkis (2017) suggests that lesson play tasks can be used in addition to alternative methods such as interviews, observation or lesson planning to examine

the knowledge, skills and creativity of pre-service teachers. Since the creative pedagogical approaches of pre-service teachers were investigated in this research, lesson play tasks were deemed appropriate to carry out this analysis. The prompts in lesson play were developed based on research on students' misconceptions on algebraic expressions (Kieran, 2007; Knuth et al., 2005; MacGregor & Stacey, 1997; Usiskin, 1988; Weinberg et al., 2004). These misconceptions, which were selected to create the lesson play task, were included in the course content as misconceptions that students may have in teaching algebraic expressions in the algebra teaching course. During the algebra teaching course, pre-service teachers discussed the reasons underlying these misconceptions and how they could eliminate these misconceptions. Therefore, it was assumed that pre-service teachers can demonstrate their creativity better when their knowledge about the subject is adequate. These misconceptions demonstrated students' erroneous thoughts on different meanings of algebraic symbols. The prompt was based on Zazkis's (2017) task that states an error, but does not include a reason for this error. The task used as a data collection tool is presented below:

> There is a conversation between a teacher and his/her students. There are 25-30 students in the classroom. Three students in the classroom have the following conceptions about algebraic symbols:
>
> Teacher: What is "a" in an algebraic expression $3a$?
>
> Student 1: For example, if $3a$ is equal to 32, then a is equal to 2. That is, digit of ones.
>
> Student 2: a is the variable. It can represent any number.
>
> Student 3: "a" denotes the apples.

The data collection tool asked the participants to develop a lesson play that these prompts could be included in somewhere in the script and to answer the following tasks: a. Choose a creative mathematical task and create a play where these imagined students' answers could be occurred, b. Choose a creative pedagogical approach if your student has a misconception, then choose an approach to overcome your students' erroneous conceptions about algebraic symbols. The task was administered to pre-service teachers at the end of the seventh week of the course at the university, and pre-service teachers' lesson plays were collected after one week on an online platform that belongs to the university. Since pre-service teachers were asked to prepare their lesson scenarios on the computer, an online platform was preferred to collect their lesson plays. There were 78 lesson plays concerning algebraic expressions, resulting in a total of 194 single-spaced pages of data in Times New Roman 12 pt font. All of the lesson plays were prepared by pre-service teachers on A4 paper and included 2-4 photographs of mathematical models explaining the subject. All of the pre-service teachers returned their lesson plays on the due date.

Data Analysis

The data analysis was based on creativity in mathematics teaching framework, concentrating on mathematical flexibility, mathematical originality, and pedagogical flexibility, and pedagogical originality (Leikin, 2013). A qualitative content analysis method was used for data analysis (Creswell & Poth, 2018). This method was chosen to carefully review the lesson plays in order to find codes that emerged from the data. The data analysis process was carried out in several steps. First, the researcher read all the lesson plays carefully to be familiar with the data and

obtain an overall understanding of the content. Then, the lesson plays were re-read, and initial codes were generated based on the codes in the creativity in mathematics teaching framework. After generating the initial codes, the researcher engaged in a constant comparison process, in which the codes were compared within and across the lesson plays (Glaser & Strauss, 1967). The researcher was able to categorize the codes and themes through this procedure and to establish connections between them (Saldaña, 2015). Table 1 provides a summary of the codes and explanations of excerpts from the lesson plays.

Table 1.

Codes and Explanations for Data Analysis

Theme	Code	Explanation
Pedagogical flexibility	1. Adjusting the planned learning trajectory to students' needs and responses	The ability of pre-service teachers to modify their planned instruction based on students' responses and needs during the lesson play.
	2. Creating an instructional setting	Pre-service teachers' ability to design a hypothetical teaching environment that facilitates students' learning and understanding of algebraic expressions, including the use of dialogues, group work, or problem-solving activities.
	3. Switch planning	The ability of pre-service teachers to alter their initial lesson plan during the lesson play, based on hypothetical students' needs, misconceptions, or unexpected situations that arise in lesson plays.
	4. Suiting the content to students at different stages of learning	The ability of pre-service teachers to adapt their hypothetical instruction to accommodate the diverse needs and abilities of students, by providing differentiated tasks or support.
Mathematical flexibility	1. Transforming a mathematical task	The ability of pre-service teachers to modify or adapt mathematical tasks in their lesson plays by changing the numbers, variables, or contexts of the problems.
	2. Using mathematical models	The incorporation of mathematical models (e.g., diagrams, tables, or manipulatives) in the lesson plays to support students' understanding of algebraic expressions and address their misconceptions.

Table 1 provides an overview of the codes, explanations, and example excerpts from the lesson plays that were identified during the data analysis process. The codes are organized under the broader categories of pedagogical flexibility and mathematical flexibility, which emerged from the analysis as the primary dimensions of creativity exhibited by pre-service teachers in their lesson plays. In the analysis of lesson plays, no codes were identified within the categories of mathematical originality and pedagogical originality.

To ensure the reliability of the coding process, two researchers were involved. The primary researcher initially coded the qualitative data derived from the lesson plays. Subsequently, a second expert, experienced in coding both

lesson plays and creativity, independently reviewed 20% of the coded data to ensure consistency and accuracy (O'Connor &Joffe, 2020). Any disagreements in coding process were addressed through discussion and by revisiting the data, aiming to achieve a consensus on data interpretation. Finally, the researcher reported the findings to generate a comprehensive understanding of the creativity exhibited by the pre-service teachers in their lesson plays, and to provide insights into the ways in which they addressed students' misconceptions about algebraic expressions. During the data analysis, each scenario was assigned a number. Once the analysis was complete, examples from chosen scenarios were hierarchically reorganized to maintain coherence in the findings section. Subsequently, examples from six different scenarios were featured in the findings. The findings are presented in the following section with representative excerpts from the lesson plays to illustrate the key themes and categories that emerged from the data analysis.

Results

In this section, we present the results obtained from the analysis of the lesson plays created by pre-service teachers. These lesson plays were examined utilizing the creativity in mathematics teaching framework, with a particular focus on themes of mathematical flexibility, mathematical originality, pedagogical flexibility, and pedagogical originality . The key insights and patterns observed in the data are outlined and discussed below.

Upon analyzing the lesson plays created by the pre-service teachers according to the given task, using creativity in mathematics teaching framework, no findings were identified within the themes of mathematical originality and pedagogical originality. However, there were instances of mathematical flexibility and pedagogical flexibility within the pre-service teachers' lesson plays. Consequently, the pedagogical approaches that the pre-service teachers employed creatively in their lesson plays are summarized in Table 2.

Table 2.

Pre-service Teachers' Creative Pedagogical Approaches in Lesson Plays

Pedagogical flexibility	**Mathematical flexibility**
Adjusting the planned learning trajectory to students' needs and responses	Transforming a mathematical task
Creating an instructional setting	Using mathematical models
Switch planning	
Suiting the content to students at different stages of learning	

Pedagogical Flexibility

Under the theme of pedagogical flexibility, pre-service teachers displayed four distinct examples of creativity in their lesson plays. These include adjusting the planned learning trajectory to students' needs and responses, suiting the content to students at different stages of learning, and switch planning. The data reveal that pre-service teachers adjusted the planned learning trajectory according to students' needs and responses. In the imaginary script, the teacher began with simpler examples and gradually increased their complexity to assess students' understanding. Here is one of the excerpts from the scripts:

Scenario 1:

Teacher: Let's look at the operations I wrote on the board. First, let's find a in the operation 9 + *a* = 17. What is *a*?

Feray: *a* is equal to 8.

Ilayda: 8.

Teacher: Yes, you found it right, *a* is equal to 8. So, what is a in the operation $\begin{array}{r} 5 \\ +\ a \\ \hline 9 \end{array}$ now?

Şirin: 4.

Beyza: Very easy, teacher, *a* is equal to 4.

Teacher: Yes, these were a bit easy, you are right, Beyza, let's make it a bit harder. What is *a* in this example? $\begin{array}{r} 21 \\ +\ 2a \\ \hline 48 \end{array}$

Şirin: *a* is seven.

Beyza: Teacher, it is very easy again, *a* is equal to seven.

Teacher: Yes, *a* is equal to seven. Then, let's see, what is *a* if 43 + 2a = 75?

İlayda: *a* is three.

Teacher: Why did you say 3? İlayda, can you explain?

İlayda: Three plus two equals five. Then four plus "*a*" equals seven, so *a* is three from here.

Feray: İlayda's answer is wrong, *a* is two.

Teacher: Why do you think it is wrong Feray?

Feray: İlayda is adding the wrong digits. Since the ones digit is 3 and *a*, if three plus "*a*" equals five, then *a* is two.

Teacher: Yes, Feray, you are right that the same digits should be added together. Can you establish the equality you said for the ones digit and for the tens digit?

Feray: Of course, we can. There are four and two in the tens digit. Their sum should be equal to six. But the question says five, I think there is a mistake.

Şirin: No, I think what is written in the question is correct. I think what you said is wrong. The sum of four and two should equal six, but since the answer is seven, three plus *a* should equal fifteen. Thus, *a* is equal to 12.

This excerpt (from Scenario 1) demonstrates that the pre-service teacher selected examples for students to interpret various uses of symbols in the imaginary lesson, using the symbol "*a*" in all of these examples. It is evident that the pre-service teacher initially focused on the use of symbols in arithmetic within the chosen learning trajectory before progressing to their application in algebra. This example falls under the theme of pedagogical flexibility and is the most frequently occurring instance in the lesson plays.

Secondly, pre-service teachers constructed problem situations to enable students to grasp the meanings of algebraic symbols in the scenarios they devised, creating an imaginary teaching environment. The pre-service teacher initiated the scenario with a problem and anticipated potential errors that might arise in a problem situation. To solve the problem, the teacher created dialogues for the student characters, which included various errors such as

treating a variable as a label, adding a number to an algebraic expression, and using place value in a two-digit number. A direct quotation from the sample script is as follows:

Scenario 2:

Problem: "Emre is preparing red and white balloons for the April 23rd celebrations. After starting the process, he takes a break to help Mehmet set up a sound system. At that moment, the number of white balloons is four more than three times the number of red balloons. Given that there are 34 white balloons, how many red balloons are there?"

Teacher: How do you solve this problem?

Student 1: There are 34 white balloons, and 3 times the number of red balloons plus 4 will give this number, so I subtract 4 first. After subtracting 4, 30 is left. Then, I thought I could multiply 3 by a number to get 30. Here, I found 10.

In the second scenario, the pre-service teacher devised a different problem, yet the scenario was similar in terms of creating an imaginary teaching environment. The teacher tackled the students' confusion about the value of variables in various algebraic expressions. A part of the lesson play is as follows:

Scenario 3:

Ayşe: There is "*a*" in these two operations, but why do the values of "*a*" differ?

Fatma: Let's first convert the mathematical expression $3a + 23 = 56$ into a verbal expression. Let the number of marbles "*a*" represent the unknown. "23 more than 3 times Ali's marbles is equal to the number 56". According to this sentence, how many marbles does Ali have?

Ayşe: In this case, Ali has 11 marbles.

Ali: Sir, I don't quite understand.

Both scenarios demonstrate the pre-service teachers' ability to imagine an instructional setting that fosters students' understanding of algebraic symbols in a problem context. By addressing students' misconceptions and encouraging them to engage in thinking about algebraic expressions in different contexts, the pre-service teachers created an imaginary learning environment that promoted students' understanding.

In the pedagogical flexibility theme, another common aspect found in the lesson plays was the use of switch planning. It is observed that pre-service teachers constructed their scripts by initially presenting a mistake in algebraic expressions. They hypothesized that students might make errors in these expressions, such as misunderstanding variables as placeholders. Consequently, they incorporated additional excerpts into their scripts to address and overcome students' mistakes before returning to the algebraic expressions. One example of such a script is as follows:

Scenario 4:

Teacher: I'm going to ask you a question now. What is the answer to this question?

$$\begin{array}{r} 12 \\ +\ 5 \\ \hline \end{array}$$

Burcu: When we add 12 and 5, we get 17.

Teacher: Well, done, you got the result right. So, according to this question, what will be the value of □?

$$\begin{array}{r} 12 \\ +\ \square \\ \hline 17 \end{array}$$

Burcu: Subtract 12 from 17. 5 is written instead of □.

Teacher: Well, done, you got the result right. So, according to this question, what will be the value of □?

$$\begin{array}{r} 1\square \\ +\ 5 \\ \hline 17 \end{array}$$

Burcu: What do I add with 5 to make 17? **17-5=12** Accordingly, □ should be replaced by 2.

$$1\square = 12$$

Teacher: Well, done, you got the result right. Now let's go back to the question I asked first. What is the value of "*a*" here?

$$\begin{array}{r} 3a \\ +\ 23 \\ \hline 56 \end{array}$$

In this scenario, the pre-service teacher responsible for scenario 4 anticipated that the student might make an error during a two-digit addition operation. The teacher adapted the lesson by simplifying the questions and employing various problem-solving approaches (guiding the student to use addition in the final step of the subtraction operation). This demonstrates creativity in the form of flexibility.

Lastly, in the lesson plays, pre-service teachers demonstrate pedagogical flexibility by suiting the content to students at different stages of learning. In the scenarios, pre-service teachers identified a sequence that accommodated the varying learning levels of students and created their scenarios accordingly. For example, in scenario 5, the pre-service teacher transformed a beginning algebraic problem into another problem and created a new one. The problems, in sequence, are as follows: (*i*) 3 times the number of Ayça's pencils and two more pencils equals 32. Based on this, how many pencils does Ayça have ?(*ii*) Her father gave Ayça a piggy bank for her birthday. Her mother gave Ayça 10 TLs to put in her piggy bank. Her father regularly gives Ayça 3 TLs every day to deposit in her piggy bank. According to this, if Ayça initially had 10 TLs in her piggy bank, how much money will there be in total at the end of 4 days? As the scenario continues, this pre-service teacher gradually guides the script towards a pattern-based solution and anticipates that the student (the character named Ümit) may understand the concept of a variable. The part of the lesson play in which the pattern approach was used is provided below:

Scenario 5:

Teacher: How can we solve this question?

Ümit: Since Ayça puts 3 TLs into her piggy bank in one day, we can multiply 3 by 4 to find out how much she saved in 4 days.

Ceylin: But in the beginning, she already had 10 TLs in her piggy bank. If we add the amount accumulated in 4 days to the initial 10 TLs, we can find the total amount in the piggy bank.

Teacher: So how do we express this mathematically?

Tülin: The initial amount was 10 TLs. We express the amount accumulated in 4 days as 4×3. From here, we can write $10 + 4 \times 3 = 22$.

The teacher writes the solution next to the first question. Then, they proceed to the next step.

Teacher: So how much does Ayça save in total at the end of a week?

Berkay: Since Ayça puts 3 TLs in her piggy bank each day, she will save a total of 7×3 TLs, or 21 TLs, in a week. If we add the initial 10 TLs in her piggy bank to the 21 TLs, she saved in 7 days, we can find the total amount in the piggy bank, which is 31 TLs.

The teacher writes the expression $10 + 7 \times 3$ next to the second question.

Teacher: If we look at the answers to these two questions, can we establish a relationship between them?

Ceylin: We added the initial 10 TLs in both questions.

Ümit: Only the number of days changes. The rest is the same. So, the only difference between the first statement and the second one is that in the first question we write 4 because we express the money 4 days later, and in the other, we write 7 because we talk about it a week later.

The teacher writes the following statements on the board and draws attention to the part that changes as the number of days changes:

1st day: $10 + 1 \times 3$

2nd day: $10 + 2 \times 3$

…

7th day: $10 + 7 \times 3$

Teacher: So, without knowing how many days have passed, can we express the amount that accumulates in Ayça's piggy bank after a certain period of time?

Berkay: The numbers we multiplied by 3 show the number of days passed.

Teacher: Yes. Whatever the number of days passed, I write that number in the blank. Since I don't know how many days have passed in this question, I can replace the unknown with a symbol representing that number. We usually choose that symbol from letters. For example, I can express it as a day: $10 + a \times 3$.

Teacher: What does the number a represent here?

Tülin: Number of days. So, if the number of days passed is 10, a would be equal to ten.

Teacher: For example, after 70 days, how much will be accumulated in Ayça'ss piggy bank?

Ümit: We can find out that 220 TLs will be accumulated from $10 + 70 \times 3$.

Teacher: Yes, you answered correctly, Ümit.

Teacher: If I use the letter *b* instead of this letter *a*, will the result be different?

The teacher writes $10 + b \times 3$ on the board.

Teacher: I called the number of days past *b*. Can you calculate how many TLs will be accumulated at the end of 70 days?

Berkay: It will be 220 TLs again.

Teacher: Then, I can use whatever symbol I want to express the number of days, right?

Lale: Yes, the result does not change.

Mathematical Flexibility

Another aspect of creativity demonstrated by pre-service teachers was mathematical flexibility. They exhibited two types of creativity within the theme of mathematical flexibility: (*i*) transforming mathematical tasks and (*ii*) utilizing mathematical models.

The pre-service teachers demonstrated mathematical flexibility by altering the problems they used and the numbers in the algebraic expressions within their scenarios. For example, upon examining Scenario 1 mentioned earlier, the pre-service teacher adjusted the numbers and presentation of the operations in the scenarios to convey different meanings of algebraic symbols in the task. It is evident that the choice of the operation $43 + 2a = 75$ is a careful selection. This is because, in this example, the sum of 3 and 2 equals 5, making it appear as though "*a*" is equal to 2. The pre-service teacher incorporated this example into the scenario 1, anticipating that students might make a similar mistake during instruction and basing the scenario on this example. Moreover, a closer look at the first examples generated by the pre-service teacher reveals that the procedures were initially written sequentially before being altered to be shown side by side. This instance exemplifies how pre-service teachers can transform mathematical tasks by changing numbers, demonstrating mathematical flexibility. In another lesson play, pre-service teachers aim to help students' comprehension of the concept of variables and algebraic expressions by changing the numbers in the problems they are working on. This script is also shown in Scenario 5. The teacher asks students to solve a problem and then guides them through the process step by step, allowing them to recognize patterns and eventually understand the concept of variables. In the final example within this category, the pre-service teacher scripted questions that involved altering equations and numbers to help students interpret algebraic expressions differently. The following scenario exemplifies this approach:

Scenario 6:

Teacher: I want you to find the "*a*" in the sum operation below.

$$\begin{array}{r} 3a \\ +\ 23 \\ \hline 56 \end{array}$$

Student A: When we add the number "a" and 3, it becomes 6. If we add 3 and 3, we get 6.

Student B: When we add 3 and 2, we get 5. So "*a*" should be 3.

Teacher: You got it right. Now let's find "*a*" in the equation given below.

$3a + 23 = 56$

Student C: I think that whatever number I add to 23 will make 56, and if I subtract 23 from 56, I find 33. If $3a = 33$ then $a = 11$.

Teacher: Why is it 11?

Student C: To equalize, both sides must have the same number. The 3's were equal to each other, so the other 11's should be equal to each other.

Teacher: Well, let's change our equation a bit.

$3ab + 23 = 56$

If a =11 in this equation, what would b be?

Student D: I subtract 23 from 56, as my friend did, and find 33.

$3ab = 33$.

a is 11 but we cannot find b.

Teacher: Why can't you find it?

Student D: Because there is no number on the other side that I can equalize.

Teacher: Then, let's examine this example. What is b in this equation?

$4b + 5 = 85$

Student A: I subtract 5 from 85 and find it as 80. $4b = 80$.

Teacher: There is no number that we can equalize from the right side of the equation. What do you do now?

Student C: Yes, it is not equalized, but we can think of it like this. The sum of the four b's is 80. From here, $b = 20$. So, we actually did multiplication here. "What do we multiply by 4 to get 80?" We can think of it as $b = 20$.

In this scenario 6, the pre-service teacher created excerpts by questioning the placeholder meaning of the algebraic expression and then the unknown meaning. She then added a second variable to the algebraic equation and predicted that this would prevent the students from considering the unknown value as a placeholder. In other words, she believed that the students could transform the mathematical task to eliminate the misconception, and she based her scenario on this assumption.

The last creativity indicator used by the pre-service teachers in the scenarios they prepared was the use of mathematical models. To eliminate misconceptions in the lesson play, they preferred to use models in some of their scenarios. For example, in scenario 3, a pre-service teacher stated, "23 more than 3 times the number of Ali's marbles equals 56". He believed that the students would not understand the equation $3a + 23 = 56$ written for this verbal expression, so they structured their scenario accordingly. In this scenario, the pre-service teacher first explained how to use algebraic expressions with models, using a model for algebraic expressions. Then, she directed the students to use the model below to explain the given verbal expression, and as a result, the model shown in Figure 1 emerged at the end of the scenario.

Figure 1.

Model for Algebraic Expressions Used in the Scenario

3 a's + 2 tens + 3 ones = 5 tens + 6 ones

3 a's= 3 tens and 3 ones = 33 —> a= 11

Another example of using models in scenarios is the continuation of the lesson play section described in Scenario 2. The pre-service teacher (who created scenario 2) predicted that a student might have a misconception in the form of 3a + 4 = 7a and continued the scenario by using a model. Here is an example that explains this section of the lesson play.

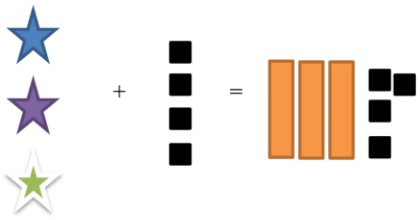
Student 2: I add 3*a* and 4 to find 7*a*. From there, it will be equal to 34.

Teacher: Now, let's revisit the problem we presented at the beginning and consider how to represent it using the model.

The pre-service teacher predicted that the students could draw the model shown in Figure 2 based on this excerpt. They continued with their scenario, believing that the student's mistake could be corrected with the help of the model formation below:

Figure 2.

Example of a Mathematical Model from the Scenario



3 a's + 4 ones = 3 tens and 4 ones

In conclusion, the majority of pre-service teachers in this study demonstrated diverse forms of creativity in their lesson plays, encompassing both pedagogical and mathematical flexibility. They used these creativity indicators to address students' misconceptions and to create instructional environments that promote students' understanding of algebraic symbols. By incorporating various problem-solving approaches, guiding students to recognize patterns, and transforming mathematical tasks, most of the pre-service teachers demonstrated their creativity in designing lesson plays. The use of models in scenarios further exemplifies their creativity in designing hypothetical instructional

design that promote students' understanding of algebraic expressions. Overall, the study provides valuable insights into the ways in which pre-service teachers can use creativity in designing lesson plays that cultivate students' comprehension of algebraic expressions.

Discussion, Conclusion & Suggestions

This study examines the creative responses of pre-service middle school mathematics teachers in their lesson plays designed to address sixth-grade students' misconceptions about algebraic expressions. One of the most important results of this research is that pre-service teachers can create lesson plays in response to the task consists of addressing students' algebraic misconceptions. While the pre-service teachers did not demonstrate creativity in terms of mathematical and pedagogical originality, all were skilled at designing hypothetical lessons. These lessons illustrated potential dialogues between a teacher and a student, highlighting another aspect of creativity. Some unrealistic student responses were also included in the created lesson plays. This result is consistent with the findings of Zazkis (2017). Nevertheless, the task of designing lesson plays empowered all pre-service teachers to envision a teaching environment and explore diverse pedagogical approaches in their hypothetical instruction. Zazkis et al. (2013) argue that lesson play is a tool for exploring pre-service and in-service teachers' mathematical knowledge for teaching. Creativity in mathematics teaching can be considered a component of teachers' knowledge (Chapman, 2013). In this regard, this study demonstrated that lesson play is an effective tool for examining the creativity of pre-service teachers.

Based on the research results provided, it appears that most of the pre-service mathematics teachers exhibited pedagogical and mathematical flexibility in their lesson plays designed to address sixth-grade students' misconceptions about algebraic expressions. Participants adjusted their planned learning trajectory to suit the needs and responses of their students. In the present study, pre-service teachers adjusted their planned learning trajectory and instructional settings based on their students' needs and responses, demonstrating their ability to modify their teaching approaches according to the evolving classroom dynamics (Daro et al., 2011). Compared to studies observing pre-service teachers frequently using similar strategies in algebraic expressions (Kaput, 2008; Kieran, 2007), this research suggests that lesson plays provide a more effective method for these teachers to diversify their teaching approaches. This diversity is essential for addressing the wide range of misconceptions and difficulties students may encounter in the learning process (NCTM, 2000).

It was found that some pre-service teachers also created instructional settings that included dialogues for student characters with various errors to help students understand algebraic expressions. Additionally, the pre-service teachers constructed their scripts by initially presenting a mistake in algebraic expressions and incorporated additional excerpts into their scripts to address and overcome students' mistakes before returning to the algebraic expressions. Some of the pre-service teachers also transformed mathematical tasks by altering the problems they used and the numbers in the algebraic expressions within their scenarios. Most of them also utilized mathematical models in some of their scenarios. Teachers can use model eliciting activities to reveal their models of students' algebraic thinking and promote the development of that model (Hallagan, 2006). Gabina (2019) found that using manipulatives in teaching and learning of algebraic expressions has a positive effect on students' achievement.

Teachers can use multi-representations to express algebra (Jao, 2013). Moreover, the pre-service teachers' ability to construct their scripts by initially presenting a mistake in algebraic expressions and incorporating additional excerpts to address and overcome students' mistakes is reflective of their problem-solving abilities, which is a crucial aspect of effective teaching (Schoenfeld, 2014). By transforming mathematical tasks and utilizing mathematical models, the pre-service teachers demonstrated their capacity to engage students in meaningful learning experiences that offered to their individual needs and misconceptions (Kilpatrick et al., 2001). Kieran (2020) also illustrates the necessity of elucidating the connections between different terms necessary to formulate sentences and create meaning. Contrary to the findings of this research, Even et al. (1993) found that only expert teachers were able to engage students in meaningful learning experiences. These teachers addressed students' individual needs and misconceptions by transforming mathematical tasks and utilizing mathematical models.

Overall, the research suggests that most of the pre-service mathematics teachers can exhibit flexibility in their lesson plays to address students' misconceptions about algebraic expressions. However, it is important to note that no findings were identified within the themes of mathematical and pedagogical originality in this study. This suggests that while most of the pre-service teachers can exhibit flexibility in their lesson plays, further research and training might be needed to help them develop original and innovative teaching approaches to address students' misconceptions about algebraic expressions (Ball et al., 2008). In conclusion, this research contributes to the growing body of literature on pre-service mathematics teachers' creativity in addressing students' misconceptions about algebraic expressions. The findings highlight the importance of developing pre-service teachers' pedagogical and mathematical flexibility to ensure that they can adapt to diverse student needs and respond to the evolving dynamics of the classroom. Future research should explore additional strategies and interventions to support pre-service teachers in developing original and innovative teaching approaches (e.g., the use of lesson plays) to enhance their effectiveness in addressing students' misconceptions about algebraic expressions.

In this study, the creativity demonstrated by pre-service teachers in the lesson plays was examined and illustrated with sample exercises. Although the pre-service teachers displayed no creativity in mathematical and pedagogical originality, the scenarios they created have potential for further development during teacher training, ultimately supporting their creativity (Zazkis, 2017). The findings obtained from this study were based on data collected at a single time. However, pre-service teachers may prepare scenarios that are richer in creativity through repeated processes and lesson plays that they develop in collaborative discussion environments (Shure et al., 2022). Shure et al. (2022) conducted a content validity study for lesson plays to be prepared by pre-service teachers on multiplication and division in fractions. Moreover, Zazkis (2017) emphasizes the importance of designing creative lesson play tasks to enhance the creativity of pre-service teachers. Pre-service teachers can enrich their scenarios and develop their own creativity knowledge through training focused on the theme of originality (Chapman, 2013). In light of these findings, teacher education programs may benefit from incorporating more opportunities for pre-service teachers to develop their creativity through the use of lesson plays and other innovative pedagogical approaches. By doing so, teacher education programs can better prepare future mathematics teachers to effectively address students' misconceptions about algebraic expressions and create more engaging and meaningful learning experiences for pre-service teachers.

Ethics

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