The Eurasia Proceedings of
Educational & Social Sciences (EPESS)
ISSN: 2587-1730

The Eurasia Proceedings of Educational & Social Sciences (EPESS), 2014

Volume 1, Pages 255-261

ICEMST 2014: International Conference on Education in Mathematics, Science & Technology

THE INVESTIGATION OF CONTENT KNOWLEDGE OF PRESERVICE ELEMENTARY MATHEMATICS TEACHERS ABOUT PROBABILITY

Gamze KURT
Middle East Technical University

ABSTRACT: Change in the middle school curriculum in Turkey necessitates the study of examination of knowledge of preservice elementary mathematics teachers about these highlighted subjects, namely probability. So, this study is significant in the needs of the Turkish mathematics education literature as well as it contributes to the consequences of curriculum efforts. In order to possess an understanding of probability and statistics concepts for preservice mathematics teachers and develop a comprehension, they must have both conceptual and procedural knowledge (Hiebert & Lefevre, 1986), where it is the main concern of this study. While using an instrument which considers probability knowledge as described in those two types of knowledge, researcher aims to understand to what extent preservice teachers are capable of conceptual and procedural knowledge of probability and statistics teaching. This paper deals with a part of data collected for the dissertation of the researcher.

Key words: Probability, content knowledge, preservice mathematics teachers

INTRODUCTION

This study aims to investigate the knowledge which is held by preservice elementary mathematics teachers in probability subjects limited with the elementary mathematics school curriculum in Turkey.
In our revised curriculum, which started to be instructed in middle-level schools in Turkey in September, 2013, the density of probability was reduced compared to previous curriculum, and its instruction is placed into the 8th grade level only with a superficial understanding of probability, such as determining the probable cases of an event, determining the cases whose probabilities are more probable, less probable or equiprobable, understanding that the probability of an event is between 0 and 1, and that of certain and impossible events, and computing the probability of a basic event. These can be called as 'basic concepts of probability'. Moore (1997, as cited in Biehler, Ben-Zvi, Bakker, & Makar, 2012) recommends some changes from the statistical point of view, in that of content (more key concepts, and data analysis, and less probability), pedagogy (fewer lectures, more active learning) and technology (for data analysis and simulations). So, the new curriculum could be identified a well-reflection of Moore's recommendation that it enhances more statistics and less probability while leaving the deeper conceptual knowledge to the high-school level, as compared with previous curricula with an integration of use of technology where available for teachers.

What earlier studies showed that preservice mathematics teachers have a less comprehension of probability compared with the other learning areas of curriculum, that is, they found probability subjects difficult to teach especially because of their lack of content knowledge related with them (Quinn, 1997; Stohl, 2005). Contemporary efforts are addressing the same issue as well so that teacher education should be enhanced while giving an attention to probability teaching for mathematics teachers (Stohl, 2005; Jones & Thornton, 2005). Change in the middle school curriculum necessitates the study of examination of knowledge of preservice elementary mathematics teachers about the highlighted subject, namely probability. Whether preservice elementary teachers have both conceptual and procedural level of understanding of probability in order to teach it has been understood (Star, 2005). So, this study is significant in the above needs of the Turkish mathematics education literature as well as it contributes to the consequences of curriculum efforts and will be a light for future considerations of this issue.
The research questions in this study are as follows: (a) to what extent are preservice elementary mathematics

- Selection and peer-review under responsibility of the Organizing Committee of the conference
*Corresponding author: Gamze KURT- e-mail: gkurt@metu.edu.tr

teachers capable of conceptual and procedural knowledge of probability subjects held in elementary mathematics curriculum in Turkey? (b) what are the main strengths and weaknesses of preservice elementary mathematics teachers in probability subjects held in elementary mathematics curriculum?

REVIEW OF RELATED LITERATURE

Before analyzing the knowledge for probability, it can be interpreted under the framework of mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008). What this framework summarized the mathematical knowledge for teaching as in the following and Groth (2012) developed the framework from a stochastic knowledge point of view.

The framework formed by Hill, et.al. (2008). has two main dimensions for mathematical knowledge for teaching: first, subject matter knowledge which includes common content knowledge (CCK), specialized content knowledge (SCK) and knowledge at the mathematical horizon (Hill, et al., 2008); second, pedagogical content knowledge which includes knowledge of content and students (KCS), knowledge of content and teaching (KCT) and knowledge of curriculum. From the statistics point of view, CCK is considered as computing and interpreting the most frequent measures of central tendency; SCK is considered as special for teaching as which is best for which statistics term; Horizon knowledge is considered as working on populations will eventually emerge the working on samples, for example. For the second dimension, KCS can help teachers to catch the common strategies which students use in developing students' statistical reasoning; KCT deals with the content-specific strategies like knowing how to explain arithmetic mean as a fair share or as a balance point; and knowledge of curriculum can help teachers about structural properties that a curriculum possess (Groth, 2012).

Therefore, Groth has developed a framework for combining above terminology and suggested the following framework:

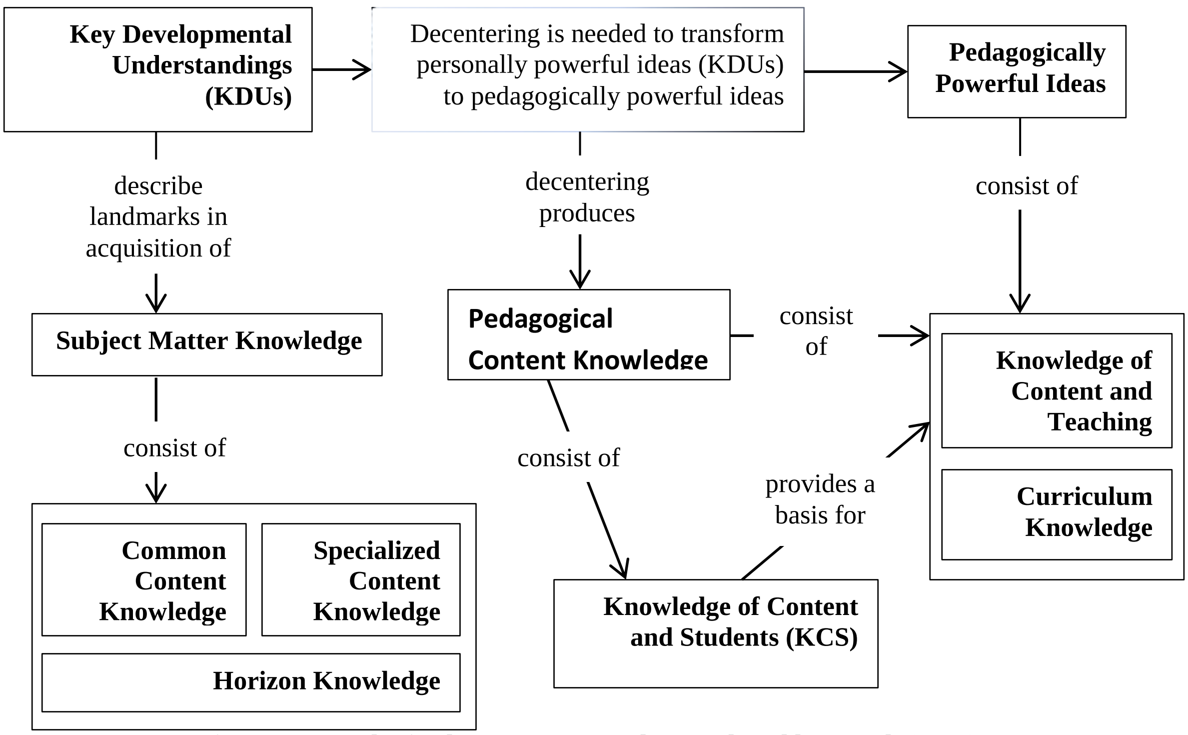

Figure1. Hypothesized SKT Framework Introduced by Groth (2013).

Based on the efforts in conducting the course which he was teaching, namely as Statistical Knowledge for Teaching (SKT) (including probability), he has developed the framework for SKT, while adding two new constructs to the statistical knowledge for teaching framework, one of which is key developmental understandings and the second one is pedagogically powerful ideas. Key developmental understandings were defined as "cognitive landmarks in the learning of fundamental ideas needed to understand content" (Simon, 2006, as cited in Groth, 2013). Pedagogically powerful ideas can be defined as ideas that occur as the result of transforming key developmental understandings into ideas that facilitate students' learning of the key developmental understandings. Groth (2013) in his hypothesized framework relates these two dimensions with the other existing dimensions of mathematical knowledge of teaching.

Another dimension highlighted in this study is the conceptual and procedural knowledge types, which they introduced by first Scheffler (1965), but expanded by Hiebert and Lefevre (1986) and Star (2005), where 'procedural knowledge is characterized most clearly as knowledge that is rich in relationships, like a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information' (p. 3). They also categorize the conceptual knowledge as primary and reflective. Apart from conceptual knowledge Hiebert & Lefevre (1986) also explains the procedural knowledge in two types: 'one kind of procedural knowledge is a familiarity with the individual symbols of the system and with the syntactic conventions for acceptable configurations of symbols; the second kind of procedural knowledge consists of rules or procedures for solving mathematical problems' (p. 7). In order to possess an understanding of probability concepts for preservice elementary mathematics teachers and develop this comprehension, they must have both conceptual and procedural knowledge (Hiebert & Lefevre, 1986). Ball (1988) described the subject matter knowledge similar to above; she names the procedural knowledge as substantive knowledge, which refers to 'understanding of particular topics, procedures, concepts and relations among them' (p. 4), and secondarily, knowledge about mathematics is named in place of conceptual knowledge. So, this type of categorization fits with the above expressions, as well.

METHODOLOGY

This study uses qualitative efforts in order to investigate its research questions. Researcher plans to collect data through face-to-face interviews (later transcribed verbatim and analyzed through data coding with the usage of qualitative data analysis techniques as Creswell (2007) outlined in his book). Participants will be determined from elementary mathematics education departments where researcher can reach. The 3rd or 4th grade preservice teachers were chosen since the courses related with teaching methods related with subject matter knowledge have been covered beginning with the 3rd year of elementary mathematics teacher education in Turkey. Then, 23 participants have been interviewed voluntarily, who 12 of them are 4^{th} year students and the rest are in their 3^{rd} year of elementary mathematics teacher education. 9 of participants are from Istanbul University and 14 students are from Boğaziçi University.

In order to investigate the understanding of preservice elementary mathematics teachers' knowledge for probability, an instrument was used. It involves open-ended questions as well as multiple-choice items which deal with both the procedural and conceptual types of knowledge of probability. Specifically, it involves questions related to topics of probability of a basic event; certain, impossible and equally-likely events; theoretical and experimental probability. This instrument is organized with the questions taken Diagnostic Teacher Assessments in Mathematics and Science developed at the University of Louisville (CRiMSTeD-Center for research in Mathematics and Science Teacher Development). These assessments have established high levels of reliability and validity (Bush, et al, under review, as cited in Jacobbe, 2007). For this research, the questions, which are related with probability subjects framed by elementary mathematics curriculum in Turkey, were taken from CRiMSTeD. The items in this test are multiple-choice and open-ended questions and have been posed to the participants with a prepared sheet during interview. Besides, participants were also asked questions related with basic concepts in probability subjects covered in curriculum. The average duration for each interview was 45 minutes.

FINDINGS

During data collection period, participants were directed questions related with probability terminology such as the definition of probability, definitions of certain, impossible and equally likely events and calculation of probability of an event as well as they were asked to solve the test including 8 items related with the above subjects and additionally the difference/relation of theoretical and experimental probability. 5 of the items in the test are multiple-choice (choosing 1 among 4 alternatives) and the rest are open-ended questions.

During the interview, some of the participants defined probability as giving a method for calculation of it, it was not a complete definition, and some gave explanations with synonym words for probability. All of them knew that the measure of probability was between 0 and 1, which was another question directed through interview. They gave also complete explanations for certain and impossible events. For the definition of equally likely

events, nearly half of the participants had a misconception that the probabilities of equally likely events are the same and ½.
Related with the question how a probability of an event is calculated, most of the participants did not use the expected terminology, such as the word 'sample space'.
It is also worth to mention here that most of the participants have identified probability as the most troublesome topic for themselves, some were said that 'I know probability, but I don't know what I do in class while I am teaching it, since I don't know the logic behind it'. Most of them have mentioned also that they found probability as the least known topic by themselves, and when the researcher asked the reason for that, probability was the topic which was accepted as dealing with abstract issues more with respect to other subjects in the elementary mathematics curriculum, according to responses of participants. They pointed that they learned probability without knowing in their elementary school years, like memorization. For this reason, nearly all of the participants considered the change in the curriculum related with the probability subject as meaningful and stressed that probability was early to teach in elementary school because of its abstract nature.

For the secondary data for this research, i.e. the test, evaluation of open-ended items was performed through a previously prepared rubric such that a full response means that participant talked about all the expected terminology and provided all the aspects of the topic covered in it; an incomplete response means that participant did not provide all of the expected discussion and did not a satisfactory response as much as expected; a wrong response means that participant responded irrelevantly and did not mention about any of the expected aspects of the topic covered in the item. The findings were summarized based on these data as in the following table:

Table 1. Findings Based on the Items

Item #	Type	Related Topic	Ratio of achievement
Item1	Multiple Choice	Impossible Event	22 of 23 are correct
Item2	Multiple Choice	Finding the probability of an event	23 of 23 are correct
Item3	Multiple Choice	Theoretical vs. Experimental Probability	17 of 23 are correct
Item4	Open-ended	Finding the probability of an event	6 of 23 responded full, 17 of 23 responded completely wrong.
Item5	Open-ended	Sample space	12 of 23 responded full, 4 of 23 responded wrong. 7 of 23 responded incomplete.
Item6	Open-ended	Theoretical vs. Experimental Probability	6 of 23 responded wrong or gave no information. 6 of 23 have responded incomplete. 11 of 23 responded full.
Item7	Multiple Choice	Types of events	17 of 23 are correct.
Item8	Multiple Choice	Sample space	20 of 23 are correct.

The items 1 and 2 were analyzing the procedural knowledge related with impossible events and finding the probability of an event. While all participants have responded correct to the second item, only one participant had a mistake in her response for the first item. Another item, which participants had higher achievement with respect to the others, was the last one, i.e. 20 participants responded correct to it. Fifth item which is related with the last have not been resulted with the similar findings as in the last one, although they cover the same topic. Nearly half of the participants (12 of 23) responded full, the rest answered incomplete or wrong to this item.

17 participants have responded correct to the third item, which is related with the relation of theoretical and experimental probability. Similar success ratio can be seen in the sixth item, which is related with the same subject. In the sixth item, participants were directed to describe a class activity showing the difference between theoretical and experimental probability. While 15 participants responded full, the rest gave incomplete or wrong

answers. Some of them had no idea about the difference between theoretical and experimental probability, some were gave irrelevant examples.
Seventh item is another item which has a higher achievement ratio among all of the items, and it is questioning the types of events, like certain events, impossible events and equally likely events. 17 of participants have correctly answered to this question.

Fourth item was dealing with the predicting the catfish population in a river through two consecutive hunts, i.e. in the first hunt biologists caught 138 catfish and they marked them and in the second one, they caught 241 catfish, 16 of them are pre-marked. The condition is that 138 marked catfish intermingled freely in the river with the unmarked ones, and during the period between these two hunts, neither new catfish added nor existing catfish died. This item was the most challenging one in the test, although the related multiple-choice item had a higher achievement, most of the participants (17 of 23) answered completely wrong, only 6 of them gave a full response. There was no incomplete response for this item.

DISCUSSION

The findings of this study show similar aspects mentioned in the above framework for content knowledge described by Groth (2013), Hiebert & Lefevre (1986) and Ball (1988). According to their categorization of content knowledge, three of the items directed as open-ended all in the test can be described as dealing with conceptual knowledge and the rest are dealing with the procedural knowledge and all of them are multiple-choice items. It can also be said that the questions directed to participants during the interview, they could not show their conceptual knowledge about probability since their answers were mostly procedural knowledge base.

In general, it can be claimed that preservice elementary mathematics teachers has a high achievement in procedural level of knowledge for probability subjects limited in Turkish national elementary mathematics curriculum. They mostly know that the concepts, such as definition of probability, types of events, difference/ relation between theoretical and experimental probability, definition of sample space. However, most of the participants have difficulty in answering the questions necessitating conceptual knowledge, which are related with the subjects of finding the probability of an event (catfish problem), sample space, and theoretical and experimental probability relationship. It can be claimed that the participants for this study have not an ability to connect what they know about probability and have not a higher-order comprehension needed for knowledge answering to the questions (Ball, 1988; Hiebert & Lefevre, 1986, Groth, 2013).

Based on the findings through interviews, definition of the probability of an event was performed procedurally, most of them used the sentence such as 'it means the division of wanted events divided by all events' although this definition has some terminological mistakes. For example, none of them used the word 'ratio' as defining it or the term 'sample space' as Green (1987) stated as one of the conditions of having an understanding of probability conceptually. For the definitions of certain and impossible events, all of them explained that a certain event has a probability of 1, and an impossible one has 0 as its probability. Some of them have provided examples for their definitions additionally and their examples were also appropriate. However, for the definition of equally likely events, nearly half of them explained that their probability is ½ and they mostly supported their explanations with the example of coin tossing, like having a tail and having a head are equally likely events. This also shows that participants have not used the knowledge for probability as conceptually since they could not use the concept in different situations, they could not relate it with higher order thinking abilities (Ball, 1988).

Findings related with considering probability as one of the most abstract issues in mathematics shows that preservice teachers have an understanding of probability as a subject roughly, not deeply, which can also be claimed the reason is that their knowledge level is in the procedural level and lack of conceptual knowledge, they found probability as abstract.

The findings based on the test tend to be similar to the findings based on interview obviously since their achievement ratios to the items regarding the type of items as conceptual or procedural knowledge for probability. There were 3 paired (one for procedural and one for conceptual) items for three subjects: sample space, finding the probability of an event and difference/relation between theoretical and experimental probability. When these pairs compared with each other, it can be seen that achievement ratio of items for procedural knowledge are higher than their pairs for conceptual knowledge.
For example, third item prepared for difference/relation between theoretical and experimental probability is

asking the true alternative based on the results of an experiment, in which colored spinner is used. Using an elimination method among the alternatives, 17 of participants made correct decision on this item. However, its paired item related with the same subject was searching for a class activity which can help to the students in order to distinguish the relation between theoretical and experimental probability. Most of the participants had difficulty in describing an activity which includes specifically increasing the number of experiment. The participants responded wrong to this item, had no idea about the difference about them. Therefore, it can again be claimed that preservice elementary mathematics teachers lack of conceptual thinking, they prefer to solve procedurally, not deepening their comprehension process (Ball, 1988; Hiebert & Lefevre, 1986).

When we consider the possible reasons of why conceptual knowledge of preservice elementary mathematics teachers have been less-developed compared with procedural knowledge, the courses offered for teacher candidates during their university education are like ‘recipe-type’ or ‘rule-bound’ courses which only deals with the calculations and lead preservice teachers to memorize the subjects while underestimating the logic behind it, as Shaughnessy (1992) stressed out previously (p.466). He claims also that preservice teachers lack of opportunity to develop their stochastic reasoning in university courses with their misunderstandings about probability. Nearly half of the participants have stressed that they feel themselves not knowing very well about probability although they have taken a course namely as probability and statistics. The other half of the students have mentioned that they have a course related with teaching probability and statistics in elementary level, however, unless they learned about probability very well, they cannot teach it so first they need to know it, as they expressed and therefore corresponds with the arguments by Shaughnessy (1992).
So on the whole, this study discussed the content knowledge for probability held by preservice elementary mathematics teachers very well. Findings implied that content knowledge assessed by the items in the test and questions directed through interviews have two dimensions, procedural and conceptual knowledge, as discussed clearly by the researchers previously (Hiebert & Lefevre, 1986; Ball, 1988; Groth; 2013) and corresponds to the framework which was bounded before.

RECOMMENDATIONS

The implications of this study will be enlightening for the future research of the content knowledge of preservice elementary mathematics teachers in Turkey. The discussion of the findings can have an impact on teacher education programs in the universities. While some universities have currently specific courses related with pedagogical content knowledge for statistics and probability (like Boğaziçi University) including content knowledge needed for those subjects as well, some of them have not. This study can have positive influences on the development of elementary mathematics education programs in nationwide, and might affect the perspectives of teacher educators, who are responsible for training the teachers, as well.

REFERENCES

Ball, D. L. (1988). *The subject matter preparation of prospective mathematics teachers: Challenging the myths*. East Lansing, MI: National Center for Research on Teacher Education.

Biehler, R., Ben-Zvi, D., Bakker, A., & Makar, K. (2012). Technology for enhancing statistical reasoning at the school level. In K. Clements, A. Bishop, C. Keitel, J. Kilpatrick, & F. Leung (Eds.), *Third international handbook of mathematics education*. New York: Springer.

Bush, W. et. al. (under review). Diagnostic mathematics assessment for middle school teachers. *Journal of Mathematics Teacher Education*.

Creswell, J. W. (2007). *Qualitative inquiry and research design: Choosing among five traditions* (2nd ed.). Thousand Oaks, CA: Sage.

Green, D. (1987). Probability concepts: putting research into practice. *Teaching Statistics, 9*(1), 8-14.

Groth, R. E. (2013). Characterizing key developmental understandings and pedagogically powerful ideas within a statistical knowledge for teaching framework. *Mathematical Thinking and Learning, 15*(2), 121-145.

Groth, R. E. (2012). The role of writing prompts in a statistical knowledge for teaching course. *Mathematics Teacher Educator, 1*(1) 23–40. 2012

Jacobbe, T. (2007). *Elementary school teachers’ understanding of essential topics in statistics and the influence of assessment instruments and a reform curriculum upon their understanding* (Doctoral dissertation, Clemson University).

Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 1-27). Hillsdale, NJ: Erlbaum

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 372-400.

Jones, G. A., & Thornton, C. A. (2005). An overview of research into the teaching and learning of probability. In *Exploring Probability in School* (pp. 65-92). Springer US.

MONE (2013). *Ortaokul Matematik Dersi (5, 6, 7 ve 8. Sınıflar) Öğretim Programı (Middle-School Mathematics Curriculum (Grades 5, 6, 7 and 8)*. Ankara: MEB.

Shaughnessy, J. M. (1992). Research in probability and statistics: Reflections and directions. In Grouws, D. A. (Ed) *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics.* (pp. 465-494). New York, NY, England: Macmillan Publishing Co. Inc.

Star, J. R. (2005). Reconceptualizing procedural knowledge. *Journal for Research in Mathematics Education, 36*(5), 404-411.

Stohl, H. (2005). Probability in teacher education and development. In *Exploring probability in school* (pp. 345-366). Springer US.

Quinn, R. J. (1997). Effects of mathematics methods courses on the mathematical attitudes and content knowledge of preservice teachers. *The Journal of Educational Research*, *91*(2), 108-114.