CATEGORIZING MATHEMATICS KNOWLEDGE TO USE ICT IN MATHEMATICS EDUCATION

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ABSTRACT: This paper makes an attempt to investigate the nature of mathematics knowledge and ICT-information and communication technology-potential. It proposes that mathematics knowledge be categorized into two parts, Meta knowledge and executive knowledge. Each type of the knowledge requires entirely different planning from the instructors, if they wanted to use ICT in mathematics education. To this end, it is proposed that the class instructor be substituted with a class director, who has been trained by mathematics education system for this purpose. The class director duties will be briefly discussed. When the class director executes the model, the mooted method, s/he is going to produce a valuable lecture which is called evolved-lecture.

Key words: Evolved-lecture, meta knowledge, executive knowledge

INTRODUCTION

This paper wants to challenge the usage of information and communication technology (ICT) in mathematics education, drawing on both Artificial Intelligence (AI) and software programming. Of course, this discussion can extend to all of science education in general. The essential question is whether ICT is a threat to the quality of mathematics education or it is an opportunity for that. Undoubtedly, ICT has found its way to every aspects of human life; however, it shouldn’t have entered the educational systems without sufficient study.

In some cases, one assumes that ICT can control the education process entirely. It has grown out of the seed of electronic and computer progresses (Frederic Bourassa, 2011). When students surrender to ICT they might be skeptical of learning mathematics and, in their opinion, it could be enough if they just learned different mathematics software. Of course this willingness is a great advantage, as the educational system won’t need to schedule how to teach this different mathematics software.

The paper tries to analyze the mathematics knowledge and to find a division in it before using ICT in mathematics education. Then it wants to propose a teaching method and the way of presenting topics so that students can trust systems of mathematics education because they have to spend a lot of time and energy learning mathematics. Teaching experiences indicate how this confidence fades away (Carmen M.Latterell, 2007).

The goal of the paper is a profound investigation of mathematics’ topics’ nature. It wants to propose which part of the knowledge of mathematics lends itself well to the facilities of ICT and which part of it may not be so.

Different modes of classroom teaching

In this section some common modes, which are widespread in universities and colleges, will be briefly reviewed.

The traditional teaching mode

The traditional teaching mode, in which instructors do not use ICT, has been being used for quite a long time. In spite of ICT progress, it is not wise to give up traditional mode entirely. In the same way, the invention of the printing industry has given the students an opportunity to have a lot of reference at their disposal with all the details of the topics but it could never take the instructor’s place rather it came to be used as means of providing support for the students, that is, thanks to the printings, students attend the classes without overly worrying about complete note-taking.

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The variety and abundance of books and libraries have not decreased the importance of traditional classes. In the new age, in spite of ICT’s progress, the most popular educational system is still the traditional one. Because, a software package cannot simulate a seasoned teacher who has accumulated his art of teaching in the course of many years.

Some advantages of the traditional mode are as follows:
- Instructors have been trained with this mode.
- Instructors can make a direct connection with student.
- The class is teacher-centered generally.
- The students are separated from splendor of virtual environment and ICT.

Some shortcomings of the traditional education are as follows:
- It is traditional.
- There is a lack of experienced instructors who has teaching talent.
- The class is teacher-centered.

Some advantages of the modern education are as follows:
- It is modern.
- There is a possibility of distance teaching.
- It can be learner-centered (Macdonald, 2009).

Some shortcomings of the modern education are as follows:
- It is technology-centered.
- It has unknown results for educational system.

The modern teaching mode

The modern teaching mode, that is named e-education, tries to take maximum help from ICT. It wants to minimize human-to-human teaching by using software, hardware and internet network.

Some advantages of the modern education are as follows:
- It is modern.
- There is a possibility of distance teaching.
- It can be learner-centered (Macdonald, 2009).

Some shortcomings of the modern education are as follows:
- It is technology-centered.
- It has unknown results for educational system.

Traditional and modern modes combined

The modes, mentioned in the previous section, are established in the education system. The traditional mode is the most well-established one. The modern mode, e-education, is used to teach in special conditions such as short time education, lack of experienced people and saving costs (Hennessy et al, 2010).

This paper does not intend to criticize the traditional or the modern education system. It basically seeks to find a way to the traditional and the modern combination modes (e.g. teaching by ICT). The traditional and the modern combination modes have entered educational system in two ways. First, they have been exercised personally by instructors; second, its industrial publicity has paved the way for its entrance into educational system. However, almost neither of them has been utilized professionally on scientifically sound bases.

Using of PowerPoint files is the most popular combination mode. These files include sentences, formulas, figures, movies etc. They are prepared to be shown to students by instructors. In this way the topics can be shown very fast. Obviously increasing the speed of showing doesn’t necessarily mean that the information is transferred to the students by instructor as fast. This method has minimally gotten help from ICT potentials. This may reduce instructor’s mental and physical effort, in the class, but it is not a suitable option for serious teaching in its strict sense.

Of course it is very important that the students be taught faster than the past, because science progresses exponentially. Not only will not the problem be solved by showing the topics faster, but also the instructor’s role will be decreased to an unskilled showman. The problems mentioned above and the subjects below made us consider some transformations in the traditional and modern combination mode.

ICT has a lot of potentials to be used in education.
There is a necessity to teach knowledge to students in short intervals.
The students are intensively attracted to ICT.

The next section of this paper wants to deliberate on mathematics knowledge. Because one has to know the road before they choose the kind of car and provisions for their trip.

MATHEMATICS KNOWLEDGE

Now there is a big question; with ICT attendance, is it necessary that whole mathematics knowledge be taught to the next generation? If the answer is not so clear, the question that follows would be which part of mathematics knowledge education needs to be changed basically?

For making this partition, we need to use progresses made in computer science and artificial intelligence, because we have to know which parts of mathematics education can be compatible with ICT. In this paper, knowledge is divided into two parts, Meta knowledge and executive knowledge. In the following sections these subjects will be elaborated on. Firstly we have to remember two definitions, because they are the foundation of computer science.

Algorithm:
A prescribed set of well defined rules or instructions for the solution of a problem, e.g. division algorithm, in a finite number of steps (Oxford Reference, 1990)

Heuristics:
A prescribed set of rules employing a self-learning approach to the solution of a problem, e.g. chess heuristics, in this case, unlike the Algorithm where the result is guaranteed you may or may not be able to solve problems (Oxford Reference, 1990)

Executive knowledge
Some part of Human knowledge that is named executive that can be transformed into algorithm or even heuristics. The following sentences are some examples to clarify the matter.

When one is able to do fundamental operations of arithmetic s/he has executive knowledge about them.
When one is able to calculate square root of a number s/he has executive knowledge about it.
When a football player kicks the ball carefully toward the goal then you can say that s/he has executive knowledge about it.
If a person knows English grammar completely then s/he can put the words in the order without any grammatical mistakes. They have the related executive knowledge.

The examples mentioned above show the abilities and some operations one can perform. At least in the above mentioned examples computers, by using algorithms and heuristics, can perform the operations faster and more accurately than human. So we must be more careful when we teach this part of knowledge, executive knowledge, because students might feel the time and energy that they spend on learning mathematics is wasted due to the fact that they are not only now but always will be far behind computers in mathematics (Heid, 1988).

Meta knowledge

Meta knowledge is knowledge about knowledge (Rich et al, 1991). This part of knowledge has not been able to be transformed into algorithms or even heuristics by artificial intelligence completely, and, it is not expected to happen by simulation of Meta knowledge, at least, in the near future. The following sentences are to provide examples to further clarify the matter.

We have twelve red apples. If we want to divide them among four persons equally. Which arithmetic operation will be needed?
In an isosceles right angled triangle, in which if we know the length of base how can we find the length of hypotenuse?
Which wing of the rival team could be more vulnerable for attacking?
What is the best sentence for a president, who wants to give speech at a present situation on a graduation day?

The following superiorities in executive knowledge will not help to solve any of the above mentioned problems. To perform quickly fundamental operations of arithmetic.
To calculate square root of a number in less than a second with arbitrary approximation.
To shoot precisely and quickly.
To make thousands of correct sentences, from grammatical point of view, in a short time.

Mathematics educational system must focus on this part of knowledge, Meta knowledge. The structure of Meta knowledge does not suit up with computer science; therefore, it is necessary for the instructors be careful when they want to use ICT in their Meta knowledge classes.
Since the present educational system is based on facilities before ICT age, the instructors and students spend the major part of their time and energy, teaching and learning the executive knowledge. For example, in the differential equation lesson the instructors and their students spend their teaching and learning time on solving techniques- executive knowledge. It is not comparable with the time allocated to Meta knowledge part which exists in the differential equations, which creates certain tendency in students toward executive knowledge rather than Meta knowledge, while the process of education can be fruitful when the students pay more attention to Meta knowledge.

Example: Suppose the aim is to teach the following second order differential equation. Therefore, an instructor wants to teach four subjects to the students.

\[ ay'' + by' + cy = g(x) \]

-The differential equation entity and the goal intention of solving that
In this case the instructor must teach the kind of category which includes these equations. S/he has to explain the differences and resemblances between the above mentioned equations and quadratic equations below (Boyce et al, 2005)

\[ ax^2 + bx + c = 0. \]

For example, when you’re solving the second order differential equation, you’ll find some functions and when you solve, the quadratic equation, you’ll find two numbers.

-The physical meaning of differential equation
The aim of this part of education, physical interpretation, is to model a phenomenon and produce the differential equation. For example, an instructor may illustrate the point, using the mass, spring and damper mechanism (See Figure 1) (Ahsan, 2005).

![Figure 1: Mass, spring and damper mechanism](image)

\[ m\ddot{x} + B\dot{x} + kx = F \]

m, B and k are the mass and constant of damper and spring, respectively. If s/he discusses constant coefficients and point out that they can be variable in practice, then they can practically provide for a deeper understanding of the problem in the students mind.

-Uniqueness and existence of solution
An instructor can expand this part of the topics by further elaboration; of course, the intensity of their explanation will depend on the level of the class (Boyce, 2005).

-Solution techniques
In this part, the instructors want to give solution techniques most of which are algorithmic (Boyce, 2005). As we know, major part of the books, lectures and educational files are allocated to teaching the solution techniques. This might have been justifiable in the absence of ICT, and the students had to exercise different methods of the differential equations, because they had to develop a skill for solving the problems and finding final solutions. Out of the four above-mentioned subjects, only the fourth one is executive knowledge while the other three are Meta knowledge. Usually students focus just on the forth one.
The presence of ICT, makes instructors rethink the teaching process, find it too slow, while it makes the students feel that mathematics education system keeps them in bondage, because they are wanted to do certainty the activities that their computers can do very much faster than them. The subjects 1, 2 and 3 do not make such impressions on the students. This paper does not propose to discard the learning of executive knowledge part in mathematics. The goal is to have mathematics Meta knowledge and executive knowledge be separated and treated differently, regarding the ICT use. In teaching executive knowledge, ICT can be utilized as mean of facilitating and speeding the teaching learning processes. However, as far as Meta knowledge concerned the optimal use of ICT can lead to evolved-lecture, to be explained in the later section.

The frontier between Meta and executive knowledge in mathematics

Although one can give a lot of examples for executive or Meta knowledge in isolation, generally speaking, there is not a clear-out boundary between them, That is, in some cases the frontier is fuzzy (see Figure 2).

Example: In calculus, when the hyperbolic parabolic function (Larson, 2006),

\[ z = f(x, y) = y^2 - x^2 \]

is brought up by instructor, her /his goals are
To present a sample of two variables function (Meta knowledge).
To sketch an example of two variables function (executive knowledge).
To introduce a new point, saddle point, on the surface (Meta knowledge).
To raise the students ability in three dimensional imagination (Meta knowledge).
In this case, if s/he wants to separate executive knowledge from Meta knowledge it, might impair the students learning process.
Example: When an instructor wants to prove the theorem about exact differential equation

\[ M(x, y)dx + N(x, y)dy = 0 \]

S/he simultaneously teaches the solution technique; while proving the theorem is a part of Meta knowledge and the solution technique is a part of executive knowledge (Boyce, 2005).
Therefore, if a topic falls into the frontier between Meta knowledge and executive knowledge, it is suggested that it be categorize in Meta knowledge part.

THE INSTRUCTOR

The fact that there were just a few good mathematic instructors during our college years can be a good reason proving that there are not a lot of good instructors to teach. However, you can find a lot of researchers in mathematics. The traditional educational style has tried to resolve this discrepancy by books. That means a good instructor has tried to write a book in her or his specialized field so that her/ his experiences would be comprehended by another instructor or students. Nonetheless, by studying different kinds of books on a special topic, it could be understood that the writer had mastered some parts better in comparison with other parts. Therefore, if the students want to understand a topic profoundly, a part from their class lectures, they have to study about it in different works. Maybe the best commentary of a topic, especially in mathematics, may belong to certain at a particular instructor from a university. Now the following questions are mooted:

Can all of the students, who are interested in the topic, benefit from the instructor?
Is making the instructor write a book about a whole topic the only possible way?
How can one find such instructors?
Will the preparation of a video and a documentary from an instructor’s class and their presentation in the internet, be suitable?
Can we use social networks for finding the instructor?
How can an instructor use such instructors’ methods in her or his class?
To answer the questions above, in the next part, a person will be suggested as the class director – rather than instructor- to manage the class.
The class director as a substitute for an instructor

This section wants to propose that a class director substitute an instructor in the presence of ICT. The class director must not only have mastery over the whole topics of a lesson but also s/he has to manage the following items.

To manage the syllabus.
To find the best innovated way of instruction in a specific topic.
To assemble and to update the resources.
To be well informed about the ways of presentation.
To control the amount of information the students are provided at a given time.

Since the experiences of an instructor are accumulated in a long time, the educational system cannot possibly train them in a short time. However, the education system can train a class director in a short time with ever higher efficiently. Of course, some of the class’s directors will be specialize in teaching certain topics. The following parts will extend the mentioned duties of the class directors.

To manage the syllabus

The syllabus will be categorized into two parts, Meta knowledge and executive knowledge, by the director of a class. To carry out such division, the instructor will have to have enough information about the topics, as well as the capabilities of different mathematics software. Only in this way will the director have the required ability to treat the two categories of knowledge differently that is through knowing different mathematics software comprehensively. In the executive knowledge this ability has particular significance, regarding (Wiwatanapataphee et al, 2010).

The division is done in ordinary differential equations in Shabestar Branch, Islamic Azad University practically, some details of which are attached as an appendix.

To find the best innovated way of instruction in a specific topic

Finding the best innovated way of instruction in a specific topic area can a matter of opinion and may depend on personal inclination or judgment and, in some cases, the class directors can find certain unique techniques in their education process. For example, the limit definition is a very difficult topic in calculus and there is no tangible interpretation in the calculus books. But the class director can find a tangible interpretation in limit definition and pliers mechanism (Hosseingholizadeh et al, 2011). For another example s/he can chooses some video from websites (see http://ocw.mit.edu/courses/) and use that in her or his class directly etc. Then, s/he can select the one that has worked best for her/his next semester class.

To assemble and to update the resources

To assemble and to update the resources in a good manner is important to make optimal use of time and energy in teaching and learning processes, and this paper proposes to maximize the usage of internet, websites and different software. Of course, the optimal point is when instructors in the universities promote their relationships and share their lectures, videos and educational files. For example, in the field of Meta knowledge if the class director knows that some parts of a lesson has been taught by a professor in a very good manner and its video or lecture is available, then her/his documents can be used in the class directly. In the field of executive knowledge the class director has to know about different mathematics software, and web sites, so that s/he can seek help with the calculation of long mathematical expressions and to sketch some functions’ graphs etc.

When the class director wants to assemble her/his class lectures, they have to prepare a lot of useful links; yet the lecture which has been prepared by the class director may not mature enough. A lecture will mature, when the number of useful links is grown and the volume of primary lectures has shrunk by crossing out some topics and substituting them with the links. This paper calls such a lecture an evolved-lecture. We propose that the volume of the lectures be reduced down to a level which approaches the ideal sample in the worldwide educational viewpoint. These lectures will be the most evolved ones.

Considering the nature of mathematics, it will be possible to generate such evolved-lectures. In this way mathematics’ education can make a progress, on the basis of ICT, by generating evolved-lectures.
**The ways of presentation**

The educational procedure, mathematics education, is a linear process. When a student commences to read a mathematics book, s/he is following the author's thinking line. The linear education process is followed clearly by the students, when they attend the class. In the traditional education, knowledge has been transferred to students in a sorted package of written, oral and body languages of the instructors simultaneously. Therefore, the students cannot move back or forward. So in a complex human relationship, the instructor’s line of thinking is followed by the students.

When we use ICT, not only can the students move in any direction in the topic, but also their minds might be strayed by ICT distraction. The class director will need to be educated and trained about the manner of setting and presenting the topics, because the linearity of education process must be maintained (Stokes, 2001).

**To control the amount of information the students are provided at a given time**

In the traditional education, the amount of information the students are provided at a given time has been controlled automatically. The information has been transferred in an acceptable time by written, oral and body languages of the instructors, so there is enough time for the students to receive the matter. Now if you let the class director be armed with powerful information systems like computer, internet and different software, the control won’t happen automatically. Therefore, the class directors should be trained in this case, Meta Knowledge part especially, so that they can control the teaching speed by debating, questioning, answering etc (Means et al, 2010).

One of the principal goals is to transfer Meta knowledge faster and more favorably. As it has already been pointed out, the educational system cannot reach this goal by increasing the speed of teaching, nevertheless, it can save time by categorizing Meta knowledge from executive knowledge and making changes in the teaching method of both knowledge types through using ICT.

**PRACTICAL RESULTS**

The mooted method has been executed four semesters, two years, in teaching Ordinary Differential Equation class. The students, who took the course, were from different field of engineering like electronic, mechanic, nutrition, civil, computer. The following table shows that the method is not disappointed (Table 1).

<table>
<thead>
<tr>
<th></th>
<th>Proponents</th>
<th>Opponents</th>
<th>In a daze</th>
<th>Neutral</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>315</td>
<td>98</td>
<td>62</td>
<td>77</td>
</tr>
<tr>
<td></td>
<td>552</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: The students’ verdict about the method

Incidentally a survey between proponents and opponents, 413 students, show most of them interested to executive knowledge part of Ordinary Differential Equation (Table 2).

<table>
<thead>
<tr>
<th>The students’ Field</th>
<th>Total</th>
<th>Proponents’ Number</th>
<th>Students’Purely interested to Meta knowledge</th>
<th>Purely interested to executive knowledge</th>
<th>Interested to both part of knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electronic</td>
<td>92</td>
<td>28</td>
<td>51</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>Mechanic</td>
<td>67</td>
<td>30</td>
<td>26</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>Nutrition</td>
<td>76</td>
<td>18</td>
<td>39</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td>Civil</td>
<td>109</td>
<td>16</td>
<td>64</td>
<td>29</td>
<td></td>
</tr>
<tr>
<td>Computer</td>
<td>69</td>
<td>10</td>
<td>32</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>413</td>
<td>102</td>
<td>212</td>
<td>99</td>
<td></td>
</tr>
<tr>
<td>Percent</td>
<td>100%</td>
<td>24.7%</td>
<td>51%</td>
<td>23.9%</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: The students’ verdict about each parts of knowledge
CONCLUSION

Information and communication technology was born by mathematics researches therefore, it is very important that mathematics educational system pays attention when it wants to use ICT because it may be possible for the educational system to be captured in a vicious circle. Investigating lesson syllabuses and the potential of ICT more thoroughly has a higher priority than equipping the class with ICT. That is to say it will be better off if ICT can provide the mathematics education system’s recommendations for teaching. That has been the main focus of the paper. Consequently it proposes categorizing mathematics knowledge into two parts, Meta knowledge and executive knowledge, and introduces the class director, who is responsible for the production of the evolved-lecture. Of course, both the duties of the class director and producing the evolved-lecture have a lot of details for future researches.

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REFERENCES


APPENDIX

Example: This example separates the syllabus of ordinary differential equations (ODE) in two parts, Meta and executive knowledge. The Frontier between Meta and executive knowledge is lodged in the Meta knowledge part.

Part one (Meta knowledge):

To introduce the kinds of differential equations and intend to solve them.
Family of curves and orthogonal trajectories.
The physical methods.
Unique existence theorem of the first-order differential equations.
Nonlinear first-order differential equations in the form of $M(x,y)dx + N(x,y)dy = 0$.
First-order differential equations applications examples.
Second-order differential equations.
Linear second differential equations and unique existence theorem of them.
Linearly independent solutions of homogeneous linear second-order differential equations (linear combination, linear independence and Wronskian).
Complementary and particular solution of nonhomogenous linear second-order differential equations and a relation between them.
Practical examples of second-order differential equations.
Power series review.
To propose using power series to solve the second-order differential equations when their coefficients are function from the independent variable.
Ordinary point and its theorem.
Regular singular point and its theorem.
Irregular singular point.
To introduce the integral transform and Laplace transform for sample of integral transforms.
Piecewise continuous functions and their Laplace transform and theorems about them.
Relation between a function Laplace transform and its derivative Laplace transform.
Step function and the theorem about inverse transformations.
Impulse function.
Examples of linearly first-order systems applications.
To remember the matrix topics.
Principle theorem of linear first-order systems.

Part two (Executive knowledge):

How can you solve linear first-order differential equations? Separable differential equations and its solutions? How can you solve Bernoulli differential equation? Riccati equation and its solution? How can you solve the exact differential equations? How can you solve homogeneous equations? What is an integrating factor? And how can you use it?
How can you find second linear independent solution of a second-order differential equation (reduction of order)? How can you solve a linear second-order homogeneous with constant coefficients? How can you solve a linear second-order nonhomogeneous with constant coefficients (undetermined coefficients method)?
What is variation of parameters method?
How can you use power series for solving second-order differential equations at neighborhood of an ordinary point?
How can you solve Euler differential equation?
How can you solve differential equations at neighborhood of regular singular point?
How can you solve Bessel differential equation?
How can you find Laplace transform of important functions?
How can you solve initial value problem by Laplace transform?
How can you evaluate piecewise continuous functions?
How can you solve a differential equation with piecewise continuous function?
How can you solve linear first-order differential equation systems?