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Pre-Service Mathematics Teachers' Attention to Tasks' Affordances While Analyzing and Designing Tasks

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ABSTRACT

The objective of this collective case study is to comprehend how pre-service mathematics teachers (PMTs) attend to mathematical and pedagogical affordances in task analysis and how their attention reflects their original task-design. To achieve this, we acquired data from written reports analyzing their selected tasks, instructor notes, and the designed tasks of five PMTs over four phases. PMTs conducted an analysis of a task during Phase 1, revised their analysis in Phase 2, had the opportunity to observe a task implementation provided by the course instructor in Phase 3, and designed an original task during Phase 4. As a result of being prompted to identify the mathematical elements of the activities, PMTs described more mathematical and pedagogical aspects of the tasks. Based on the instructor's notes, PMTs have a belief that quality tasks require intricate procedures, leading to critical instructional phases being overlooked during implementation. Furthermore, the PMTs, who paid attention to the instructional questions, appropriately designed tasks with a higher level of cognitive demand. Therefore, PMTs require assistance in evaluating and designing original tasks with regards to their mathematical and pedagogical elements.

Etkinlik Analizi ve Tasarlama Sürecinde Matematik Öğretmen Adaylarının Etkinliklerin Sunduğu Olanakları Dikkate Alma Durumları

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ÖZ

Bu kolektif durum çalışmasının amacı, ilköğretim matematik öğretmen adaylarının (MÖA) etkinlik analizi bağlamında etkinliklerin sunduğu matematiksel ve pedagojik olanakları dikkate alma durumlarını ve bu dikkate aldıkları durumları özgün etkinlik tasarımlarına nasıl yansıttıklarını anlamaktır. Bu amaç doğrultusunda veriler, beş matematik öğretmeni adayının seçtikleri etkinliklerin analizine ilişkin yazılı raporlarından, eğitmen notlarından ve adayların tasarladıkları etkinliklerden dört aşamada elde edilmiştir. MÖA'lar Aşama-1'de matematiksel bir etkinliği analiz etmiş, Aşama-2'de daha önceden yaptığı analizleri gözden geçirmiş, Aşama-3'te ders eğitmeninin yaptığı bir etkinlik uygulamasını gözlemlemiş ve Aşama-4'te özgün bir etkinlik tasarlamışlardır. MÖA'lar etkinliklerin matematiksel niteliklerini belirlemeye yönlendirildikçe, etkinliklerin matematiksel ve pedagojik yönlerini daha fazla tanımlamışlardır. Eğitmenin notlarına göre, MÖA'lar iyi etkinliklerin karmaşık süreçler içerdiğini düşünmektedirler ve bu nedenle uygulamanın önemli öğretim aşamalarını gözden kaçırmaktadırlar. Son olarak, öğretimsel sorulara dikkat eden MÖA'lar, diğerlerine göre daha yüksek bilişsel istem düzeyine sahip etkinlikleri uygun şekilde tasarlamışlardır. Sonuç olarak, MÖA'lar matematiksel ve pedagojik unsurları bakımından özgün etkinlikleri değerlendirme ve tasarlama konusunda yardıma ihtiyaç duymaktadır.

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INTRODUCTION

To execute the vision of student-centered instruction, teachers should build teaching-learning processes based on challenging mathematical tasks and create a classroom environment that encourages meaningful mathematical discussions (Ayalon et al., 2021). The tasks chosen for this objective are also beneficial in terms of what students learn (Stein et al., 1996). Thus, the selection of tasks for various purposes in mathematics at various levels and the adaptation and implementation of the tasks based on the students need certain skills (Silver & Herbst, 2007). First, the teacher must be aware of the students’ perspectives on the task and guide their thoughts in accordance with the learning opportunities (Hallman-Thrasher, 2017; Sun & van Es, 2015). This criterion, which may be explained by the teachers’ understanding of the mathematical and pedagogical potentials of the tasks, is strongly associated with teacher task knowledge (Chapman, 2013; Liljedahl et al., 2007; Sullivan et al., 2013). Thus, it influences the selection of appropriate tasks and their effective implementation, questions to ask during the implementation, anticipation of misconceptions and difficulties, and implementation of the necessary instructional measures (Taylan, 2020). This competence is hard to gain for both in-service and pre-service teachers because it depends on their experience in selecting appropriate tasks, analyzing them according to the learning goals, and designing new tasks. In this study, we used the notion of “attention” (Mason, 1998) to investigate how pre-service mathematics teachers (PMTs) evaluate tasks’ mathematical and pedagogical affordances in analyzing and designing them. Hence, the purpose of this study is to investigate PMTs’ attention to tasks’ mathematical and pedagogical affordances while analyzing tasks and how their attention reflects their original task design.

Background and Rationale for Research

Teacher attention

The nature of awareness and the structure of attention are the essential ideas that underlie meaningful instruction (Mason, 1998). “Teaching is fundamentally about attention, producing shifts in the locus, focus, and structure of attention, and these can be enhanced for others by working on one’s own awareness” (Mason, 1998, p. 244). Teacher attention plays a critical role in educational settings since it shapes teachers’ practices in the classroom (Mason, 2008). Research on teachers’ attention to classroom situations has shown that novice teachers, especially, prioritize management of learning over other aspects, such as mathematical tasks (e.g., Jacobs et al., 2010; Sherin & van Es, 2005). They may overlook opportunities to expand on students' mathematical ideas to improve instruction (Mason, 1998).

Students can learn about a mathematical subject using mathematical tasks (Stein & Smith, 1998). To choose tasks that are appropriate for the learning objectives, teachers must be aware of the features of mathematical activities (Liljedahl et al., 2007). However, according to Stephens (2006) and Paparistodemou et al. (2014), pre-service teachers could not attend to the ways in which tasks could encourage mathematics learning. According to several studies (Jacobs et al., 2010; Van Es & Sherin, 2008), attention can be learned over time and encouraged by teacher education. Therefore, it is crucial to investigate what prospective teachers are considering when planning tasks for implementation and anticipating students' responses to tasks. This can help teacher educators produce the proper support, guidance, and support mechanisms (Ayalon & Hershkowitz, 2018). Professional development based on task analysis assists teachers in understanding the affordances and limits of tasks (Johnson et al., 2016; Son & Kim, 2015). Teacher professional development should help teachers grasp the dynamics of task-related decision-making in the classroom (Sullivan & Mousley, 2001).

Cognitive demand level (CDL)of tasks

Studies show that students’ understanding is enhanced by tasks (Thanheiser, 2015). In addition to this, to ensure that students remain motivated, it is essential that the tasks for mathematics lessons exhibit a particular level of difficulty (Rimma, 2016). The cognitive demand framework was proposed by Stein and Lane (1996) to categorize various mathematical activities according to the degree of mathematical reasoning they elicit. According to the Task Analysis Guide by Stein et al. (2000), four different levels of cognitive demand are identified within the framework (see Table 1).

Table 1. *The Definitions of CDLs of Mathematical Tasks (adapted from Stein et al., 2000)*

LEVEL OF DEMAND	DEFINITION
1.Memorization	Students retain previously taught information, rules, formulas, and definitions.
2.Procedures without connection	Students solve problems using previously demonstrated algorithms without tying them to the underlying concepts, meaning, or comprehension.
3.Procedures with connection	Students use previously established procedures to solve problems, while maintaining close ties to the underlying mathematical principles.
4.Doing mathematics	Students solve problems requiring complicated, non-algorithmic thought for which there is no fixed solution.

It is also challenging for teachers to administer tasks with high CDLs (Monarrez & Tchoshanov, 2020). They struggle with how to order student responses in discussions, especially when using open-ended and challenging tasks (Xu & Mesiti, 2022). Challenges with students' knowledge, teachers' knowledge, and curriculum are the barriers teachers face while attempting to comprehend and implement tasks with high CDLs. So, teachers require assistance to execute demanding tasks (Monarrez & Tchoshanov, 2020).

Significance of the study

Teachers must develop teaching-learning procedures based on challenging mathematics tasks and establish an appropriate learning setting in the classroom to effectively implement student-centered teaching. Then, teachers can understand how their students think about mathematics and make decisions about how to teach them that will help them learn (Ball & Forzani, 2011). Since teachers should have the ability to employ tasks that appropriately build the mathematical thinking of students (Lithner, 2017), mathematics teacher educators can figure out how to help prospective teachers learn to use what tasks afford mathematically and pedagogically and how to manage effective discussions in task implementation (Ayalon & Hershkowitz, 2018; Johnson et al., 2016; Liljedahl et al., 2007; Son & Kim, 2015; Sullivan & Mousley, 2001). When we understand what prospective teachers need, we can provide environments that support their competencies for analysis, adaptation, and the design of tasks. Our goal was to assist PMTs in being ready to instruct effectively in their upcoming professional careers. Besides, we can evaluate the effectiveness of the outputs of the teacher training program. Taking into consideration these concerns, the present study intends to contribute to revealing PMTs' attention to the pedagogical and mathematical affordances of the tasks they selected from textbooks when they analyze them and develop original tasks. Hence, the following are research questions:

1) How do PMTs attend to mathematical and pedagogical affordances offered by tasks as they analyze and revise their analysis?

2) How does their attention to task-specific elements reflect on the CDLs of their designed tasks?

METHOD

Qualitatively constructed research permits the analysis of a problem by addressing complicated and detailed understandings of the subject (Creswell, 2007). This study aims to examine PMTs' identification of mathematical and pedagogical affordances that tasks may have within the scope of task analysis and design in detail. Thus, a qualitative collective case study (Stake, 1995) was adapted. This research approach lets researchers examine individual cases to explain a situation, phenomenon, or experience. Individual studies explain the "why" and "how," and contrasting cases side by side helps explain the problem (Schoepf & Klimow, 2022).

Participants

This study was carried out with the participation of five PMTs (four female and one male) who are second graders in the four-year teacher training program at a state university. In this context, a convenient sampling method was used. In cases where random sampling is difficult, the researcher uses this method based on the time, place, and volunteerism of the participants (Fraenkel et al., 2012). The participants took the course

"Task-Design in Mathematics Education," and within the scope of this course, they studied the theoretical and philosophical foundations of the concept of a mathematical task, the significance of tasks in mathematics education, and the principles of creating mathematics tasks. The participants are called Nur, Asu, Ela, Can, and Ece (all names are pseudonyms).

Data Collection

Figure 1 shows the study's data collection process:

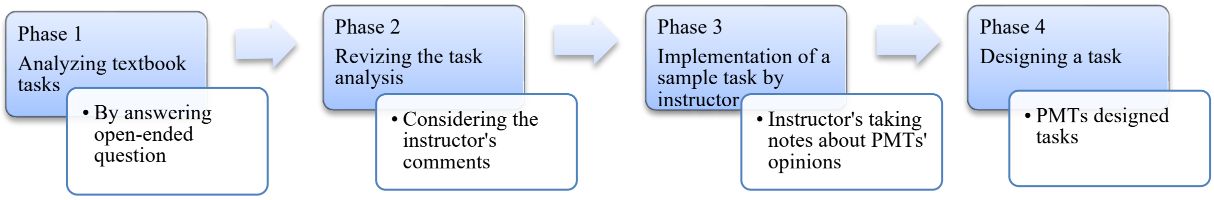


Figure 1. *Data collection process*

First, the PMTs were asked to select a task from the current textbooks (see Appendix) and to analyze this task. To guide the PMTs in analyzing the task, we prepared seven open-ended questions (see Table 2) based on the literature (Hughes, 2006; Smith et al., 2008; Stephan et al., 2017). The PMTs wrote a report consisting of the responses to these questions. Then, the researcher provided PMTs with specific comments (e.g., Which terms are included in this task? Please clarify them); Please be specific to mathematics and the content of the task: What will the student be able to do with these concepts, and how will they make connections between these concepts? Could you be more specific about those mistakes or misconceptions and how to fix them? For example, could we think of a cylinder as a prism? Why? If the cylinder is also a prism, how can we calculate its area?) on their work, following which the PMTs were requested to revise their initial analysis. With these modifications, a second written report was produced by the PMTs. In addition, the researcher implemented "the bag of marbles" task proposed by Smith et al. (2008) to serve as an example of task implementation. During the implementation phase, the researcher recorded the PMTs' opinions on both the task and the implementation. Last, PMTs were asked to design a unique task to target a learning area of their selection.

Data Analysis

Data were obtained through PMTs' two written reports and designed tasks. They answered the open-ended questions while analyzing textbook tasks (Phase1-Ph1) and revising the task analysis (Phase 2-Ph2), and they finally designed tasks (Phase 4-Ph4). In Phase 3 (Ph3), they only observed the task implementation process. Firstly, we identified PMTs' remarks that they attended to such as mathematical and pedagogical affordances supporting students' learning. We also further classified these two categories (see Table 2). We then used inductive content analysis to investigate to what extent mathematical elements (general, specific to task, and partially specific to task) caught PMTs' attention throughout the two phases (Ph1 and Ph2). Last, to analyze PMTs' designed tasks, we used the definitions of the cognitive demand levels (Stein et al., 2000). We classified PMTs' designed tasks into four levels and examined how PMTs' attention was reflected in each level of tasks.

Table 2. *The Categories and Sub-Categories of Attention to Task's Affordances*

ATTENTION TO…		RELATED OPEN-ENDED QUESTIONS
Mathematical affordances to support students' thinking	Identification of goal	What is the goal for the task?
	Identification of strategies	What would the strategies that students use to solve the task?
	Identification of prior concepts	What prior knowledge would students need to have to begin to work on the task?
Pedagogical affordances to support students' thinking	Anticipation of student thinking	What misconceptions and difficulties might students have while working on the task?
	Instructional questions for getting started on the task	

	Instructional questions for focusing on mathematical ideas	If you would maintain the task in class, what questions will you ask to help students get started on the task? If you would maintain the task in class, what questions would you ask to focus students' thinking on the key mathematical ideas in the task?
	Assessment students' understanding	What indicates that students understand the intended mathematical idea?

The researchers independently categorized each participant's responses to determine an inter-rater reliability of 92%. We examined score irregularities until we reached an almost unanimous conclusion. Also, the study followed a method and data was gathered at regular intervals strengthened the proper identification of patterns in the data. Together, the two researchers determined the cognitive level of the PMTs' designed tasks.

Ethic

All necessary permissions were obtained Süleyman Demirel University Social and Human Sciences Ethics Committee with the ethical permission dated 01.04.2023 and 131/28 certificate issue number.

FINDINGS

The findings of the study are organized in a way to understand how the PMTs attend to mathematical and pedagogical affordances presented by their selected tasks. Accordingly, the findings related to PMTs' attention to mathematical affordances, and then the findings related to PMTs' attention to pedagogical affordances are presented. In addition, the opinions of PMTs on the implementation of a sample task are presented. Last, the cognitive demand levels of PMTs' designed tasks are discussed along with their attention.

PMTs' Attention to Mathematical Affordances in Ph1 and Ph2

Table 3 displays the extent (to which general, specific to task, and partially specific to task) mathematical elements are present in PMTs' attention related mathematical affordances from Ph1 to Ph2.

Table 3. *PMTs' Attention to Mathematical Affordances*

ATTENTION TO...		ELA		NUR		ASU		CAN		ECE	
		Ph1	Ph2	Ph1	Ph2	Ph1	Ph2	Ph1	Ph2	Ph1	Ph2
Mathematical affordances to support students' thinking	Identification of goal	ST^p	ST	ST	ST	ST	ST	ST	ST	ST^p	ST
	Identification of strategies	G	ST	ST	ST	ST^p	ST	G	G	G	G
	Identification of prior concepts	ST	ST	ST^p	ST^p	ST	ST	ST	ST	ST	ST

G indicates general comments.
ST indicates comments specific to task.
ST^p indicates comments partially specific to task.

Table 3 shows that not all PMTs' attentions have changed from Ph1 to Ph2. However, it can be stated that some PMTs experienced a change from general comments to specific task comments and from partially specific comments to specific task comments. Nur and Asu stand out with better performances than other PMTs for attending mathematical affordances. Ela improved her comments with mathematical elements specific to the task when she was in Ph2. However, Can and Ece did not improve their comments on the identification of strategies as specific to the task

Identification of goal

The PMTs adequately described the related learning objectives from the mathematics curriculum for their selected tasks (see Appendix) in Ph1. However, the PMTs were also supposed to explain how their tasks might support the learning objectives specifically to task. Thus, Ela and Ece's explanations were determined as partially specific to task (see Table 3). For instance, Nur's explanation was specific to task and described the learning objective as *follows "Determines that the areas of the shapes are the number of unit squares that cover that area. In addition to the regular shapes, it is also possible to work with notched shapes such as leaves, and hands drawn on squared paper."* She also explained how the task assisted students to gain this learning objective as follows:

"With this task, students can initially distinguish the concepts of area and area measurement. They can use the (non-linear) relationship between length and area measurements, the concepts of ratio and scale in estimation and mapping. They can use strategies to complete the square unit numbers and non-perfect squares they will find by counting or using the area of the rectangle during the area measurement of the leaf. With the use of 1 cm and 0.5 cm squared papers, they may feel the need to use a standard unit as well as the proportional relationship between them (Nur-Ph1 and Ph2)."

Other PMTs improved and detailed their identified goals in Ph1 when they move into Ph2 (see Table 3). For example, Ela's comments included mathematical characteristics when compared to her comments in Ph1. She explained the goal of the task for the activity with general terms in Ph1 such as reasoning mathematically or discovering the connection between the concepts, while she specified almost all the mathematical characteristics which were specific to the task in Ph2 as follows:

"It will contribute positively to the student's mathematical reasoning skills. Because in the questions, the student is expected to think about certain things, relate them and answer them as such. At the end of this task, the student will discover the interconnection of the subjects in mathematics (Ela-Ph1)."

"The student is expected to think about the transition from the side length of the rectangle to the height of the cylinder, from the long side of the rectangle to the perimeter of the base of the cylinder (circular region), make these relations. At the end of this task, the student will discover the connection between the short-long side measure of rectangle, height-perimeter of right cylinder. It is also aimed to get the relation related to the surface area calculation of the right cylinder by utilizing the concrete materials (the unfolding and closed form of the right cylinder) (Ela-Ph2)."

Identification of strategies

The majority of PMTs (Ece, Ela and Can) provided very general comments on the strategies in Ph1 (see Table 3). For instance, Ela wrote, *"Each student will come up with a different solution by thinking, explaining their thoughts and making associations in the activity."* and Can wrote, *"The task requires the student to know the operation and graphic interpretation used to get these values (median, mode, mean) and then answer each question."* The comments were generic and not about the possible student strategies.

After the instructor's feedback, Ela and Asu were able to provide mathematical details in Ph2 by focusing on the strategies that students could use. For example, Ela commented as follows:

"When we make measurements from the opening of the cylinder, the students will recognize that the short side of the rectangle is equal to the height of the cylinder. They find the circumference of the circle, and they will notice that the value of circumference is equal to the long side of the rectangle. Thus, they need to find that the perimeter of the circle is equal to the side length of the rectangle and the short side of the rectangle is equal to the height of the right circular cylinder. Last, they need to find that the area of the cylinder is the sum of the base areas and the lateral area (Ela-Ph2)." However, Ece and Can's comments did not improve mathematically properly. For example, Ece stated "*Following all the steps given in the task is a path for students to use."* in Ph1. Then, she stated *"Trial-error method"* as a strategy for mental computation in Ph2. This is not a mental computation strategy, and it also could not be an effective strategy for reasoning

mathematically. Thus, she wrote a comment that did not contain any details about the mental computation strategies that students could use to complete the task.

Identification of prior concepts

In this category, unlike the other categories, the PMTs could completely or partially stated pre-mathematics concepts specific to tasks in Ph1 (see Table 3). Nur, who did it partially, specified the skills and preliminary concepts required partially. She emphasized many of the mathematical concepts, definitions, and ideas necessary for the task except rate and ratio concepts and multiplicative thinking to evaluate the use of 1 cm and 0.5 cm squared paper. She explained the prior concepts for carry out the task as follows:

"They should know that area is the space that covers, and measuring area is finding the size of that amount of space. They should know the units used in measurement. For example, they should be familiar with the idea of counting unit squares when measuring areas (Nur-Ph1 and Ph2)."

PMTs' Attention to Pedagogical Affordances in Ph1 and Ph2

Table 4 shows to what extent (general, specific to task, and partially specific to task) mathematical elements in PMTs' attention related pedagogical affordances of tasks is through Ph1 to Ph2.

Table 4. *PMTs' Attention to Pedagogical Affordances*

ATTENTION TO...		ELA		NUR		ASU		CAN		ECE	
		Ph1	Ph2	Ph1	Ph2	Ph1	Ph2	Ph1	Ph2	Ph1	Ph2
Pedagogical affordances to support students' thinking	Anticipation of student thinking	ST^p	ST	ST^p	ST	G	ST^p	ST^p	ST	G	ST^p
	Assessment students' understanding	G	ST	G	ST	G	ST	G	ST	G	G
	Instructional questions for getting started task	ST^p	ST	G	ST	ST	ST	ST^p	ST	G	ST
	Instructional questions for focusing on mathematical ideas	G	ST	G	ST	G	ST	G	ST	G	G

G indicates general comments.
ST indicates comments specific to task.
ST^p indicates comments partially specific to task.

Table 4 shows that all PMTs' attentions have changed from Ph1 to Ph2 except Ece, and all PMTs experienced a change from general comments to specific task comments and from partially specific comments to specific task comments. However, Ece could not improve her comments for assessing students' understanding or develop instructional questions for focusing on mathematical ideas specific to the task.

Anticipation of student thinking

In order to attend possible misconceptions and difficulties while working on tasks, either the PMTs (Ela, Nur, and Can) could address some of the students' possible misconceptions mathematically or the PMTs (Ece and Asu) commented generally about students' understanding in Ph1 (see Table 4). After the instructor's feedback, they added possible misconceptions specific to the mathematical idea of the task and difficulties that students may have.

Nur made comprehensive comments on students' thinking in Ph1 and Ph2 when compared to other PMTs. She added students' difficulty in understanding area and area measurement, detailed students' difficulty in understanding the ratio of 1 cm and 0.5 cm and added the misconception about comparing fractions in Ph2: *"Students may not be able to distinguish the concepts of area and area measurement. Area measurement is the number of units of measurement needed to cover a given area. Students may not be able to define this concept. They may interpret the area measurement as "...the area being the bounded region" ... When leaves are drawn on 1 cm squared paper and 0.5 cm squared paper and counted as complete squares, they may think* *that the number of squares counted is more on 1 cm paper because the number of squares is 1 cm larger. Fraction knowledge is important here. We divide two whole squares of equal size into a different number of unit squares. So, the denominators are different. What the student should know here; the smaller the fraction, the squarer the unit will be in the smaller fraction (Nur-Ph2)."*

On the other hand, while Asu and Ece made general comments on students' thinking, they also detailed and improved their comments specific to the mathematical idea of the task partially. For example, Asu stated a misconception in Ph1 as follows: *"Students may have the misconception on the area of algebra tiles given in the figure and they may add the length of edges instead of multiplying them (Asu-Ph1)."*

Then, she added other possible misconceptions to her explanations in Ph2 as follows:

"... They may think that the expressions with x^2 and the expressions x given on the tiles are of the same type. Thus, they may have misconception about similar terms in algebraic expressions. They can find $x^3=x+x^2$ when adding tiles incorrectly ... They can use the perimeter formula instead of the area formula (Asu-Ph2)."

She also would indicate the difficulty and misconception in using distributive property in multiplication of two algebraic expressions and this would be more comprehensive comment.

Assessment students' understanding

It is quite remarkable that all PMTs could not make comments specific to the task of assessing students' understanding in Ph1. They mostly made comments with generic issues about the tasks (see Table 4). The PMTs generally stated that they would evaluate whether all students could give the same answer (Ece), explain what they were doing (Ela, Asu, and Can), or get the right answers from the students (Nur). Thus, they made general comments and did not specify according to the mathematical idea of the task. The interpretations of the four PMTs in Ph2 included more task-specific features, except Ece (see Table 4). For example, Nur made evaluations parallel to the learning objective and goal that she determined as follows: *"I decide that the student has understood when students stated that area and area measurement are different concepts; realized that area was the laying of a plane in a region where measurement can be made, while area measurement was the number of units of measure needed to cover that region, and they effectively used the unit square paper in measuring the area of irregular shapes (Nur-Ph2)."*

Asu had other noteworthy mathematically detailed comments. She mentioned that the sum of the areas is the product of the multiplication of two algebraic expressions (as the edges of a rectangle). That was one of the ideas specific to the task, as follows: *"Since the sum of the algebra tiles in the given figure for the area and the multiplication of their edges express the area, I would expect them to come to a state where they can tell that both results can be written as a product of algebraic expressions (Asu-Ph2)."* However, Ece could not improve her comment mathematically; it was specific to the task, and she made the same comments in Ph2.

Instructional questions for getting started the task

The PMTs (Nur and Ece) usually devised general questions at Ph1 (see table 4). We observed that these PMTs did not address enough mathematical characteristics in their explanations. For example, Ece responded "*Make students think (Ph1)*" for getting students started. However, she did not generate any starting questions. She would connect the mathematical idea of the task with everyday life or experiences. After the instructor's feedback, Ece generated the following questions: *"What mental-computation strategy does a grocery use while giving remainder of money to a customer as quickly as possible? How can I make addition without paper and pencil in daily life? (Ece-Ph2)."*

On the other hand, Ela and Can added questions referring mathematical concepts in the tasks in Ph2. For example, Can generated following questions in Ph1 and Ph2: *"How do you compare and evaluate the scores you get from any exam? (Can-Ph1)" ".... Let's say we examine the average age in cities of Turkey. With this information, can we make comments on which province has the young population too low and which province has the elderly population too low? How do we do? Can you give an example? (Can-Ph2)"* Can added questions to make students think and discuss on the average concept. Similarly, Ela generated a question related real life experiences in Ph1. Then, in Ph2, she added questions that make students think on the mathematical concept (surface area of a right circular cylinder) of the task.

Instructional questions for focusing on mathematical ideas

It is rather interesting that all PMTs were unable to develop task-specific questions for focusing on mathematical ideas in Ph1 (see Table 4). The PMTs would clearly state the questions they would ask to focus on the mathematical ideas of the task to support students' understanding. However, they usually refer to the questions of the task. They did not develop any other questions that helped students understand the mathematical concepts and ideas of the task.

In Ph2, four PMTs developed questions that helped students focus on the mathematical concepts and ideas, except Ece (see Table 4). For example, Nur suggested task-specific discussion questions that would allow students to understand mathematical ideas and to make connections between the strategies used in 1 and 0.5 cm squared papers. Last, she developed a question that requires students to need a standard unit for measurement and learn the different strategies presented. Her generated questions were as follows: *"What happens when you do the work with 1 cm squared paper with 0.5 cm squared paper? Does the number of squares you count increase or decrease? Is it 1 cm paper or 0.5 cm paper that allows for more precise in measuring area? How your observations about the measurement using 1 cm and 0.5 cm squared papers are applied on the sample of the Turkey map? How can you standardize your measures? (Nur-Ph2)"*. On the other hand, Ece did not develop questions specific to task for focusing on mathematical ideas in two phases. She stated that "*I ask the students why the result of the task is like this, and I ask them to explain their answers with justifications.*" She only indicated what kind of questions she would ask and did not exemplify these questions.

PMTs' Opinions on the Implementation of a Sample Task in Ph3

In Ph3, the course instructor used a rich mathematics task (Smith, Bill, & Hughes, 2008) in a classroom setting where the PMTs acted as middle school students. They were encouraged to consider the open-ended questions and share their responses before the implementation. Some notes from the instructor's research journal were as follows: *"The PMTs appeared to be pleased with participating with the task presented to them. But prior to the sample application, they did not try for more than two or three alternative solutions that could come from the students. They couldn't, however, agree on how the students' answers should be handled in the classroom. Although some claimed they would prioritize incorrect thinking or misconceptions, others said that they would follow an order that proceeded from simple to complex. After the implementation, they said that they were able to obtain answers to many of the open-ended questions."*

The instructor also documented the PMTs' verbal responses to both the task and the implementation in her journal after the implementation. Ela's opinions on the task implementation process were as follows: "*The task was nice and had a high cognitive level. We would not have gotten good results if we had done only with the presented question of the task. But, since our instructor prepared by considering about the open-ended questions before, the task could be carried out efficiently. Consequently, this may create beneficial outcomes for students. I also want to carry out same task in the future"*.

Other notes from the instructor's journal according to the PMTs' reflections are as follows: *"The PMTs perceived the task as a simple question before working on the task, but they stated that the task was enriched by referring several points about student thinking during the implementation process."* From this point of view, PMTs believe that a worthwhile task always requires complex processes.

Cognitive Demand Level of PMTs' Designed Tasks

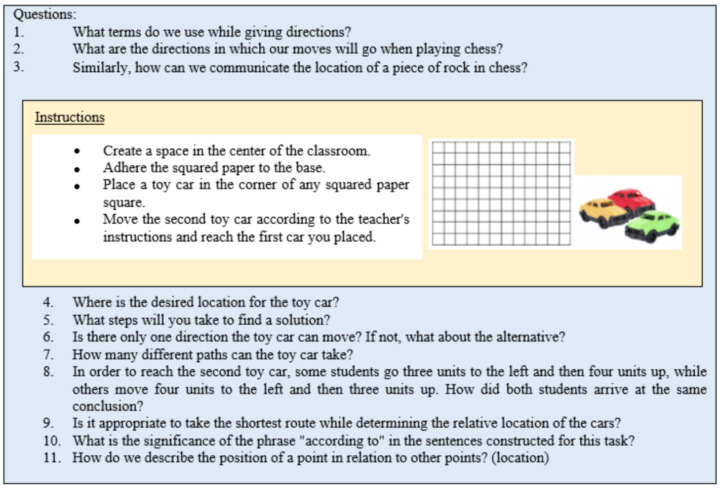
Table 5 shows the CDL of PMTs' designed tasks. According to the table, Nur and Asu designed higher level (Level 3) tasks than the other PMTs' tasks.

Table 5. *The Relationship Between PMTs' Designed Tasks' CDL and Generated Instructional Questions*

PMTs	CDLs of PMTs' SELECTED TASKS	CDLs of PMTs' DESIGNED TASKS
Ela	3	2
Nur	3	3
Asu	3	3
Can	2	1
Ece	1	1

Table 5 shows that the CDL levels of the tasks that the PMTs designed were either equal to or lower than the CDL levels of the tasks they selected from the textbooks. We also discovered that the CDLs of the PMTs' designed tasks were comparable to their performance on their attention to instructional questions in the context of pedagogical affordances. We may claim that the CDLs of the tasks designed by PMTs improve according to their attention to their generated instructional questions for getting started and focusing on mathematical ideas.

We discovered that the tasks in Level 2 and Level 3 employed instructional prompts to help the students' connecting procedures with mathematical concepts when evaluating the tasks from a pedagogical standpoint. For example, Nur designed a fifth-grade task aligned with the learning objective (see Figure 2). She begins with real-life questions designed to get students thinking while also connecting to the mathematical concepts (Q1 and Q2). Answering these questions, students may utilize terms like right-left and front-back. Students begin to consider the mathematical concepts they will learn with Q3. Also, the task's instructions are clear. Since the other questions proceed progressively, the task questions prepare students for the intended mathematical concept by provoking thinking. Students would use previously established procedures to learn the underlying mathematical concepts.


Questions:
1. What terms do we use while giving directions?
2. What are the directions in which our moves will go when playing chess?
3. Similarly, how can we communicate the location of a piece of rock in chess?

Instructions

- Create a space in the center of the classroom.
- Adhere the squared paper to the base.
- Place a toy car in the corner of any squared paper square.
- Move the second toy car according to the teacher's instructions and reach the first car you placed.

4. Where is the desired location for the toy car?
5. What steps will you take to find a solution?
6. Is there only one direction the toy car can move? If not, what about the alternative?
7. How many different paths can the toy car take?
8. In order to reach the second toy car, some students go three units to the left and then four units up, while others move four units to the left and then three units up. How did both students arrive at the same conclusion?
9. Is it appropriate to take the shortest route while determining the relative location of the cars?
10. What is the significance of the phrase "according to" in the sentences constructed for this task?
11. How do we describe the position of a point in relation to other points? (location)

Figure 2. *Nur's designed task*

However, we may assert that instructional prompts are inadequate, particularly for Can and Ece's tasks. Typically, they did not include proper prompting questions for discussion and instructions. Consequently, their tasks have low CDLs. For example, Ece has prepared a fifth-grade task with the objective "The student calculates the sum of the measurements of the interior angles of triangles and quadrilaterals and identifies the missing measure of angle" (see Figure 3). First, the instructions are insufficient for this task. She must clarify what she means by "center." In addition, she has already provided the total degree measurements of the triangle's interior angles that students will calculate in the question (Q1). Students have learned this information in previous years. Instead, she should have enabled the students to independently find this formula. She selects the rectangle as opposed to any other quadrilateral to support the students' generalization of the rule. However, it is not meaningful for the students to find the sum of the measurements of the interior angles of the rectangle because they already know that the degree measure of each inner angle of the rectangle is 900. We evaluated this task at Level 1 from a cognitive standpoint because the display of this information in the task's instructions requires students to recall only prior knowledge.

Instructions:
1. Cut the angle-indicating pieces from the corners of the triangle and rectangle (cardboard) provided.
2. Stick them around the center.

Questions:
1. Can you prove that the total of the triangle's interior angles is 180 degrees?
2. How do we perform for a rectangle?
3. How much does an angle of the rectangle measure in degrees?
4. What is the sum of the measures of the rectangle's inside angles?
5. Can the same rule be used to other polygons?
6. How else can the total of all angles be calculated?

Figure 3. *Ece's designed task*

In sum, in task-analysis processes, Ece and Can's attention was weak when we compared with other PMTs. Other PMTs recognized their general comments with the instructor's feedback and improved their attention specifically in Ph2. However, Ece and Can could not improve their attention specific to tasks as well as others. We may say that their weak attention reflected their designed tasks' low level. Similarly, other PMTs, particularly Nur and Asu, usually made comprehensive comments before or after feedback in task analysis. Thus, their strong attention reflected their designed tasks as high-level.

DISCUSSION, CONCLUSION, RECOMMENDATIONS

In this study, the PMTs' attention to mathematical and pedagogical affordances to support students' thinking was examined in the context of analyzing a mathematical task, revising the analysis, and finally designing an original task. The majority of PMTs demonstrated an improvement in their attending abilities in Ph2. As a result of being pushed to identify the mathematical elements of activities, PMTs described more mathematical aspects of the tasks. In a similar vein, Ulusoy (2020) reported that as prospective mathematics teachers concentrated on identifying mathematical elements, they shifted their attention to content-specific components of teaching rather than focusing on general aspects. Particularly, Ela, Nur, and Asu showed continual improvement in their ability to pay attention. In this regard, we may conclude that they have strong conceptual and pedagogical subject knowledge and can explain with more mathematical and pedagogical elements than others. Another significant observation was that Ece attended to pedagogical affordances less than other PMTs, although she performed well in attending to mathematical affordances, particularly identification of goals and prior concepts in Ph2. As this PMT just completed a lack of content knowledge with the feedback, it is possible to assert that she had a lack of knowledge of students and teaching. Attending to content-specific aspects of teaching necessitates a mathematical understanding of teaching to recognize mathematically relevant indicators of strong mathematics instruction (Schlesinger et al., 2018).

In Ph3, the researcher's journal presents crucial information addressing the PMTs' task-design and implementation aspects. According to the notes, the PMTs believe that all good tasks involve complex procedures. However, they missed crucial instructional phases of implementation. Prior to working on the task in Ph3, the PMTs viewed the problem as a simple or routine one, but they understood that the task was enhanced by the fact that several aspects of student thinking were addressed throughout the implementation. The implementation of a task is equally as important as the CDL of the task; even a task with a high degree of effectiveness may be executed poorly (Kaur, 2010).

In Ph4, PMTs with higher CDL designed tasks more effectively than their counterparts by attending instructional questions. These PMTs utilized instructional prompts to enhance students' thinking processes. Similarly, the growth of the PMTs' understanding is correlated with their growing attention to the mathematical and pedagogical aspects included in the tasks, and this development appears to influence the task (Lee et al., 2019). In contrast, it is interesting that two PMTs (Ece and Can) did not refer to instructional questions properly while designing assignments. To construct an effective task, it was necessary to consider all the mathematical and pedagogical elements (Paparistodemou et al., 2014).

Many of the tasks that PMTs selected from textbooks lack many of the features that should be present in a rich task and have low CDLs as exercises (Basyal et al., 2022; Özgeldi & Esen, 2010; Ubuz et al., 2010). In this sense, Lee et al. (2019) discovered that examining the affordances and limitations of textbook tasks in terms of students' inquiry and investigating alternatives to overcome the limitations appears to aid in the development of PMTs' ability to recognize opportunities for students' inquiry embedded in tasks. Thus, PMTs require help in terms of evaluating and creating original tasks concerning their mathematical elements. Even so, it can be extrapolated that PMTs require more expertise in the implementation of complex tasks as well as important abilities in assessing the tasks concerning mathematical elements and translating them into tasks with high CDL.

The most significant limitation of this study is that PMTs are not given the opportunity to carry out the tasks with actual students. However, student responses and outcomes to a task will vary based on the characteristics of the group (Healy et al., 2013). There was no actual task implementation for students, and PMTs were asked to reflect potential learning opportunities in a hypothetical classroom environment. By applying the tasks to students, the growth of PMTs' attention may be evaluated. Also, the change in the PMTs' attention can be recorded by focusing on a specific topic and certain task for all or can be investigated among the grade levels (senior, junior, etc.).

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APPENDIX: The PMTs' selected tasks (The tasks are from the textbooks by Turkish Ministry of Education. The authors translated them.)

Can's selected-task

11 students' math scores are given in the table.
To determine the success status of the class by using this table answer the following questions. Add up the scores obtained by the students.
- Divide the sum by the number of students.
- What is the relationship between the sum and the data?
- Arrange and rank the scores in descending order, regardless of the students' names.
- Mark the middle score.
- If there were 12 students, what would you say about the middle score?
- Determine how many of each score was received.
- Identify which score was the highest.
- Which stage works better in determining the success of the whole class?

Table: Students and their scores

Students	Scores
Büşra	90
Atahan	80
Buse	100
Özge	75
Berkay	80
Tuncay	95
Defne	70
Ali Emir	80
Ada	90
Kayra	80
Efe	75

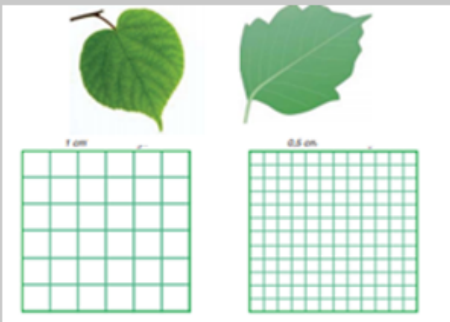
Nur's selected-task

Area of The Leaf
We can use fun mathematical methods to find the area of surfaces like leaves.
Your teacher will give you 1 cm and 0.5 cm squares of paper.
Find similar tree leaves and bring them to class.

Questions
1.Tree leaves are the parts of plants that get light from the sun and make their own food. How do you find out how much area a leaf has?
2.Can we find the approximate area of a leaf using gridded paper? Discuss in your group.
3.Put the leaf on 1 cm gridded paper and draw along the edges to form its shape. Count the number of perfect squares inside the figure. What are the incomplete squares on the sides? Discuss with your friends.
4. What happens if you do the above with a half-centimeter square of paper? The square you counted number increases or decreases compared to 1 cm gridded paper? Make a prediction. If it increases or decreases how many times does it increase or decrease? One times, two times, three times, four times? Discuss in your group.
5. Do the same operation as in the third step above, this time using 0.5 cm square paper. Repeat this i.e. find the area of the same leaves using 0.5 cm squares. How many squares did you count each time? Record your findings in a table.
6. Are your findings as you expected? If not, discuss with your friends why they are different.
7. Does 1 cm gridded paper provide a more precise measurement for finding the area or 0.5 one-inch squared paper? Why? Discuss with your friends.

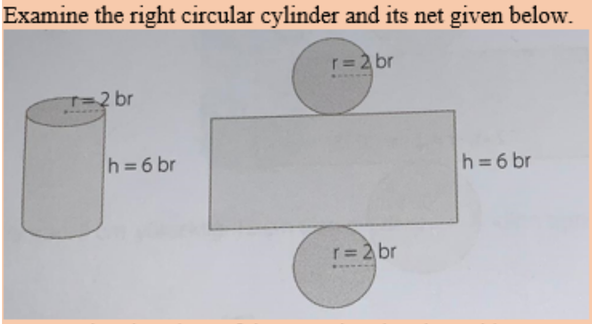
Using 1 cm square paper
First leaf (draw the approximate shape here)
Second leaf (draw the approximate shape here)
Number of perfect squares:
Number of perfect squares:
Sum of incomplete squares:
Sum:
Sum of incomplete squares:
Total:

Using 0.5 cm gridded paper
First leaf (draw the approximate shape here)
Second leaf (draw the approximate shape here)
Number of perfect squares:
Number of perfect squares:
Sum of incomplete squares:
Total:
Sum of incomplete squares:
Total:



Ela's selected-task

Examine the right circular cylinder and its net given below.

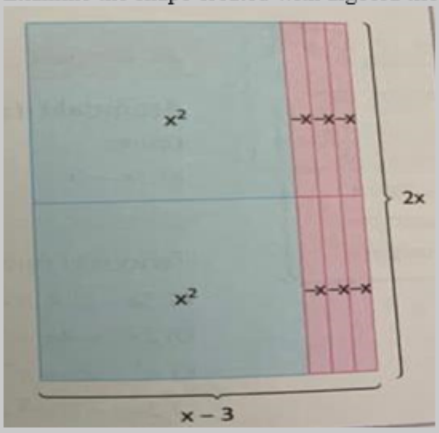


- Determine the edges of the net using the given sides.

Explain the relationship between the length of the short side of the rectangular face and the height of the right circular cylinder in the net.
What is the relationship between the length of the long side of the rectangular face in the net and the perimeter of the circular area that forms the base of the right circular cylinder?
How is the surface area of a right circular cylinder found?
✓ What is the relationship between the face area of a right circular cylinder and the areas of the plain areas that make up its net? Explain.

Asu’s selected-task

Examine the shape created with algebra tiles on the side.



Write the area of the figure as the sum of algebra tiles.
Write the area of the rectangular as the multiplication of the sides.
Write the relationship between the expression for the sum of algebra tiles and expression for the product of side lengths mathematically.
✓ Which side of the equation you have set is equal to the product of two algebraic expressions?
Which algebraic expressions are these factors?

Ece’s selected-task

Which is Easy?
Tools and Materials: paper, pen, calculator, clock
-Ask a friend to say two two-digit numbers and try to sum these numbers with different methods as soon as possible mentally.
-Write down your process and result on a piece of paper according to the methods you used.
-Ask your friend to check your result with a calculator.
-Ask your friend to say two two-digit numbers and try to subtract the smaller one from the other number with different methods mentally.
-Write down your process time and result on a piece of paper according to the methods you used.
-Ask your friend to check your result with a calculator.
-Tell your friends about the methods you use. Which method did you use in a shorter time?
-What do you think is the strategy that allows you to do mental addition and subtraction in the shortest and easiest way?

GENİŞ ÖZET

Giriş: Öğretmenler, öğrenci merkezli öğretim vizyonunu gerçekleştirmek için, zorlu matematiksel etkinliklere dayalı öğretme-öğrenme süreçleri oluşturmalı ve anlamlı matematiksel tartışmaları teşvik eden bir sınıf ortamı yaratmalıdır (Ayalon vd., 2021). Bu nedenle, matematikte çeşitli düzeylerde çeşitli amaçlara yönelik etkinliklerin seçilmesi ve etkinliklerin öğrencilere göre uyarlanması ve uygulanması belirli beceriler gerektirir (Silver ve Herbst, 2007). Öncelikle öğretmen, öğrencilerin etkinliğe yönelik bakış açılarının farkında olmalı ve onların düşüncelerini öğrenme fırsatları doğrultusunda yönlendirmelidir (Hallman-Thrasher, 2017; Sun ve van Es, 2015). Öğretmenlerin etkinliklerin matematiksel ve pedagojik potansiyellerini anlamaları ile açıklanabilecek bu kriter, öğretmen etkinlik bilgisi ile güçlü bir şekilde ilişkilidir (Chapman, 2013; Liljedahl vd., 2007; Sullivan vd., 2013). Öğretmen etkinlik bilgisi, uygun etkinliklerin seçimini ve etkili bir şekilde uygulanmasını, uygulama sırasında sorulacak soruları, kavram yanılgılarının ve zorlukların öngörülmesini ve gerekli öğretim önlemlerinin uygulanmasını etkiler (Taylan, 2020). Bu yetkinliğin hem öğretmen adayları hem de öğretmenler için kazanılması zordur çünkü uygun etkinlikleri seçme, öğrenme hedeflerine göre analiz etme ve yeni etkinlikler tasarlama konusundaki deneyimlerine bağlıdır. Bu çalışmada, matematik öğretmen adaylarının (MÖA) etkinlikleri analiz ederken ve tasarlarken etkinliklerin sunduğu matematiksel ve pedagojik olanakları nasıl değerlendirdiklerini araştırmak için "dikkat" (Mason, 1998) kavramı kullanılmıştır. Dolayısıyla, bu çalışmanın amacı, öğretmen adaylarının etkinlikleri analiz ederken etkinliklerin matematiksel ve pedagojik olanaklarına nasıl dikkat ettiklerini ve bu dikkatlerinin orijinal etkinlik tasarımlarına nasıl yansıdığını araştırmaktır.

Yöntem: Bu çalışmada nitel araştırma yöntemlerinden kolektif durum çalışması kullanılmıştır (Stake, 1995). Mevcut çalışma, bir devlet üniversitesinde dört yıllık öğretmen yetiştirme programında ikinci sınıf öğrencisi olan beş MÖA’nın katılımıyla gerçekleştirilmiştir. Bu bağlamda uygun örnekleme yöntemi kullanılmıştır. Katılımcılar "Matematik Eğitiminde Etkinlik Tasarımı" dersini almışlar ve bu ders kapsamında matematiksel etkinlik kavramının teorik ve felsefi temelleri, matematik eğitiminde etkinliklerin önemi ve matematik etkinlik oluşturma ilkeleri üzerine çalışmışlardır. Katılımcılar Nur, Asu, Ela, Can ve Ece olarak adlandırılmıştır. İlk olarak, MÖA’lardan mevcut ders kitaplarından bir etkinlik seçmeleri ve bu etkinliği analiz etmeleri istenmiştir (1.aşama). Ardından MÖA’lardan ilk analizlerini gözden geçirmeleri istenmiştir. Bu değişikliklerle birlikte, MÖA’lar tarafından bir yazılı rapor hazırlanmıştır (2.aşama). Buna ek olarak araştırmacı, örnek bir etkinlik uygulaması gerçekleştirerek, MÖA’ların hem etkinliğe hem de uygulamaya ilişkin görüşlerini kaydetmiştir (3.aşama). Son olarak, MÖA’lardan kendi seçtikleri bir öğrenme alanını hedefleyen özgün bir etkinlik tasarlamaları istenmiştir (4.aşama). Elde edilen verilerin analizinde içerik analizi kullanılmıştır. İlk olarak, matematiksel ve pedagojik olanaklar olarak iki kategori belirlenmiştir. Daha sonra, matematiksel unsurların MÖA’ların açıklamalarında ne ölçüde (genel, etkinliğe özgü ve kısmen etkinliğe özgü) olduğu analiz edilmiştir. Son olarak, adayların tasarladıkları etkinlikler bilişsel istem düzeylerine göre sınıflandırılmıştır.

Bulgular: Matematiksel açıdan, Nil ve Asu matematiksel olanakları dikkate alma konusunda diğer MÖA’lara göre daha iyi performanslarıyla öne çıkmıştır. Ela, 2.aşamada etkinliğe özgü matematiksel unsurlarla yorumlarını geliştirmiştir. Ancak, Can ve Ece etkinliğe özgü stratejilerin tanımlanmasına yönelik yorumlarını geliştirememiştir. Pedagojik açıdan, MÖA’lar 1.aşamada pedagojik olanakları dikkate alma konusunda genel yorumlar yapmış ve genel sorular geliştirmişlerdir. Daha sonra, 2.aşamada etkinliğe özgü matematiksel unsurlarla yorumlarını ve sorularını geliştirmişlerdir. Ancak, Ece öğrencilerin anlamalarını değerlendirmek için yorumlarını geliştirememiş ve etkinliğe özgü matematiksel fikirlere odaklanmak için öğretimsel sorular geliştirememiştir. MÖA’ların tasarladıkları etkinliklerin bilişsel istem seviyelerinin, ders kitaplarından seçtikleri etkinliklerin bilişsel istem seviyelerine eşit ya da daha düşük olduğu görülmüştür. Ayrıca, MÖA’ların tasarladıkları etkinliklerin bilişsel istem seviyelerinin, pedagojik olanaklar bağlamında öğretimsel soruları dikkate alma performanslarıyla karşılaştırılabilir olduğu söylenebilir. MÖA’lar tarafından tasarlanan etkinliklerin seviyelerinin, başlangıç ve matematiksel fikirlere odaklanma için oluşturdukları öğretim sorularına gösterdikleri dikkate göre arttığı söylenebilir. Genel olarak, etkinlik analiz süreçlerinde Ece ve Can'ın dikkati diğer MÖA’lara kıyaslandığında zayıf kalmıştır. Diğer MÖA’lar genel yorumlarını eğitmenin geribildirimi ile fark etmiş ve 2.aşamada dikkatlerini geliştirmişlerdir. Ancak Ece ve Can etkinliğe özgü dikkatlerini diğerleri kadar geliştirememiştir. Zayıf dikkatlerinin, tasarladıkları etkinliklerin düşük seviyeli olmasını da etkilediği söylenebilir. Benzer şekilde, diğer MÖA’lar, özellikle Nur ve Asu, etkinlik analizinde geri bildirimden önce veya sonra genellikle kapsamlı yorumlar yapmışlardır. Dolayısıyla, güçlü dikkatleri tasarladıkları etkinliklerin yüksek seviyeli olmasına da yansımıştır.

Tartışma: MÖA’ların çoğunluğu 2.aşamada dikkate alma becerilerinde bir gelişme göstermiştir. Etkinliklerin matematiksel unsurlarını belirlemeye zorlanmanın bir sonucu olarak, adaylar etkinliklerin daha çok matematiksel yönlerini tanımlamışlardır. Benzer bir şekilde, Ulusoy (2020) matematik öğretmen adaylarının matematiksel unsurları belirlemeye odaklandıkça, dikkatlerini genel yönlere odaklanmak yerine öğretimin içeriğe özgü bileşenlerine kaydırdıklarını bildirmiştir. 3.aşamada, araştırmacının günlüğü MÖA’ların etkinlik tasarımı ve uygulama yönlerini ele alan önemli bilgiler sunmaktadır. Notlara göre, MÖA’lar tüm iyi etkinliklerin karmaşık prosedürler içerdiğine inanmaktadır. Ancak, uygulamanın önemli öğretim aşamalarını gözden kaçırmışlardır. 3.aşamadaki etkinlik üzerinde çalışmadan önce, MÖA’lar problemi basit ya da rutin bir problem olarak görmüşlerdir, ancak etkinliğin uygulama boyunca öğrenci düşüncesinin çeşitli yönlerinin ele alınmasıyla geliştirildiğini anlamışlardır. Bir etkinliğin uygulanması, etkinliğin bilişsel seviyesi kadar önemlidir (Kaur, 2010). Daha iyi performans gösteren MÖA’lar, 4.aşamada öğretim sorularının önemli özelliklerini dikkate alarak etkinlikleri daha etkili bir şekilde tasarlamıştır. Bu MÖA’lar, öğrencilerin düşünme süreçlerini geliştirmek için öğretimsel ipuçlarından yararlanmıştır. Benzer şekilde, MÖA’ların anlayışlarının gelişimi, etkinliklerde yer alan matematiksel ve pedagojik yönlere artan dikkatleri ile ilişkilidir ve bu gelişimin etkinliği etkilediği görülmektedir (Lee vd., 2019). Etkili bir etkinlik oluşturmak için tüm matematiksel ve pedagojik unsurların dikkate alınması gerekmektedir (Paparistodemou vd., 2014).

Sonuç ve Öneriler:

- Genel olarak MÖA’lar matematiksel unsurlarına ilişkin özgün etkinlikleri değerlendirme ve oluşturma konusunda yardıma ihtiyaç duymaktadır.
- Gelecek çalışmalara öneri olarak, etkinlikler öğrencilere uygulanarak, öğretmen adaylarının dikkatlerinin gelişimi değerlendirilebilir.
- Ayrıca, öğretmen adaylarının dikkatindeki değişim, herkes için belirli bir konuya ve belirli bir etkinliğe odaklanarak incelenebilir veya sınıf seviyeleri (son sınıf, üçüncü sınıf vb.) arasında araştırılabilir.