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# AN EXAMINATION OF $\boldsymbol{7}^{\text {TH }}$ GRADE STUDENTS' MISTAKES IN ALGEBRAIC EXPRESSIONS 

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#### Abstract

The aim of this study is to determine students' achievement rates in algebraic expressions and the mistakes they made. $1007^{\text {th }}$ grade students from 4 different schools in the center of Burdur attended to the research. The study is a qualitative research, which was carried out in the survey design, and the data was analyzed using descriptive analysis. A test that consisted of 6 open-ended questions about algebraic expressions was used as the measurement tool. During the development of the questions, algebra learning area of the $7^{\text {th }}$ grade was considered as the scope. A 9-question test was developed by consulting expert and teacher opinions in the first place. This test was applied in a $7^{\text {th }}$ grade class and 2 questions were excluded as they were not appropriate in terms of difficulty levels, and also, some questions which caused problems in understanding were revised. The students' achievement rate is $68 \%$ in addition operations in the algebraic expressions, $43 \%$ in expressing a verbal expression algebraically and $26 \%$ in expressing a geometric representation as an algebraic expression. Their overall average is $69 \%$. Looking at the reasons behind the mistakes from a broader perspective, it was determined that the main reasons were failure to attribute a meaning to the unknown and therefore doing the operations by assigning a value to the unknown. In other words, it could be asserted that failure to understand the main idea of algebraic expressions (the concept of variable and the concept of algebraic expression involving variables) as a concept properly is the major reason.


Key words: Algebraic expressions, conceptual learning, $7^{\text {th }}$ grade students

## INTRODUCTION

Algebra is a branch of mathematics, which focuses on symbolization of general numerical relations and operations on mathematical structures (Kieran, 1992). Usiskin (1988) described four conceptions of algebra as generalized arithmetic, problem solving methods, the relation among quantities and the study of structures. Lacampagne (1995) stated that algebra was the language of mathematics and a prerequisite of advanced mathematics. According to Kaf (2007), algebraic thinking is a way of thinking which includes such skills as reasoning, using representations, understanding variables, explaining the meaning of symbolic representations, working with models to develop mathematical opinions and carrying out transformations among representations.

The reasons why students have difficulty in understanding algebra are as following: "Not being able to simplify algebraic expressions" (Dede, 2005), "Difficulties they experience in transition from arithmetic to algebra" (Dooren-Verschaffel-Ongehena, 2003; Van Ameron, 2003), "Misinterpreting equations" (Real, 1996), "Difficulty in writing algebraic verbal problems as equations" (Dede 2004; Herscovics-Kieran, 1980; MacGregor-Stacey, 1996; Real, 1996; Stacey-MacGregor, 2000), "Perceiving equations as if they were different phenomena from reality" (Pope, 1994).

One of the major causes of incompetency in algebra teaching is the curriculum while another one is the teacher. The teacher is expected to be aware of the facts their students may face while learning algebra. The facts that students may encounter can be examined under six topics as following (Altun, 2007): "A letter represents a single numeric value", "Cases when the letter value has no importance", "A letter is used as an object", "Cases when a letter is used as a special variable", "Cases when a letter represents a number within a range", "Cases when a letter is perceived as a variable".

Bednarz and Janvier (1996) classified school algebra under four conceptions: generalization, problem solving, modelling and functions. In many of the activities that form the concepts in school algebra subjects, algebraic

[^0]thinking and algebraic symbolizing aspects are involved. Students must definitely gain these two abilities which complement algebraic understanding. Activities to be carried out at schools in order to improve students' algebraic thinking can be supported with those four different study types (NCTM, 2000): Activities to understand patterns, relations and functions (1), Activities to show and analyze mathematical situations and structures using symbols of algebra (2), Activities to use mathematical models to show and understand quantitative relations (3), Activities to analyze changes in different contexts by drawing graphs, and using technology-aided programs or worksheets (4).

## METHOD

The aim of this study is to determine students' achievement rates in algebraic expressions and the mistakes they made. $1007^{\text {th }}$ grade students from 4 different schools in the center of Burdur attended to the research. The study is a qualitative research, which was carried out in the survey design, and the data was analyzed using descriptive analysis. A test that consisted of 6 open ended questions about algebraic expressions was used as the measurement tool. During the development of the questions, algebra learning area of the $7^{\text {th }}$ grade was considered as the scope. A 9-question test was developed by consulting expert and teacher opinions in the first place. This test was applied in a $7^{\text {th }}$ grade class and 2 questions were excluded as they were not appropriate in terms of difficulty levels, and also, some questions which caused problems in understanding were revised.

## FINDINGS

This section includes the rates of students' answering the questions correctly and among the factors that cause failure, only those included in algebraic expressions are presented.

## Findings Related to the First Problem

The students were asked to write the most simplified equivalent for the algebraic expression of " $2 x-3+4 x-5$ ". $68 \%$ of the students answered this question correctly. Incorrect solutions appeared as follows:

- After variables and fixed terms were put in operation separately, equalization of the results. $2 x+4 x=6 x$, -$3-5=-8,6 x=-8$.
- Doing operations just for coefficients ignoring the variable: $2-3+4-5=-2$
- Assigning any value to x and finding a solution.


## Findings Related to the Second Problem

The students were asked to write "the algebraic expression that shows $3 / 5$ of an $x$ plus 4 ". $36 \%$ of the students answered this question correctly. The causes for mistakes appeared to be as follows:

- Taking a number plus 4 as ( 4 x ).
- Not using parenthesis.
- Assigning random values to x .
- Writing 4.(3/5) ignoring the unknown variable.


## Findings Related to the Third Problem

The students were given the problem "In the algebraic expression $-\mathrm{a}+2 \mathrm{~b}+3 \mathrm{c}=18$, what is c while $\mathrm{a}=1 \mathrm{and} \mathrm{b}=2$ ?" $78 \%$ of the students solved this problem correctly. The causes for mistakes appeared to be as follows:

- Failing to understand the question.
- In 2 b expression when $\mathrm{b}=2$, thinking that the expression was 22 .
- Considering that $c$ would be 3 if $a=1, b=2$.


## Findings Related to the Fourth Problem



In the figure, as ABC is an equilateral triangle and BCDE is a rectangle, write the algebraic expression that gives the perimeter length of the shape in x and $\mathrm{y} .35 \%$ of the students answered this question correctly. The causes for mistakes appeared to be as follows:
In the algebraic expression giving the perimeter, finding a numeric result by assigning random values to the variables
Finding a perimeter length by assigning values to variables

## Findings Related to the Fifth Problem

"Selin's age is $3 x$ and Yiğit's age is $4 x$. How old will Yiğit be when Selin reaches Yiğit's age?"
$71 \%$ of the students answered this question correctly. The causes for mistakes appeared to be as follows:

- Failing to understand the question.
- Finding a numerical result by assigning a value to x in the algebraic expression they obtained.
- Solving by assigning arbitrary numbers for x .


## Findings Related to the Sixth Problem

In this question, the students were asked to find algebraic expression giving the area of the black part. $16 \%$ of the students answered this question correctly. The reasons for mistakes were as follows:


Failing to understand the rectangle whose edge lengths were given as x .
Making errors in arithmetic operations in the algebraic expressions such as writing $x . x=2 x, x^{2}+x^{2}=x^{4}, x+x=x^{2}$.
Finding a numerical result by assigning arbitrary values to $x$ in the algebraic expression they obtained.

## Findings Related to the Seventh Problem

The question given to the students was as follows: " 3 trees of x meter height are planted in a garden. Each year these trees grow y meter. In this respect, write the total height of the 3 trees after 2 years as an algebraic expression". $32 \%$ of the students answered this question correctly. The causes of mistakes appeared as follows:

- Finding a numerical solution by assigning arbitrary values to variables in the expression they obtained.
- Initially by assigning arbitrary values to $x$ and $y$, doing numerical operations.
- Thinking wrongly such as $2 x+3 y, 3.2 x=6 x, 3 x+2 y, 3 x=6 y$.
- Errors in arithmetic operations in algebraic expressions such as $y+y=y^{2}, y^{2}+y^{2}+y^{2}=3^{y^{2}}$.


## DISCUSSION AND CONCLUSION

The students' achievement rate is $68 \%$ in addition operations in the algebraic expressions, $43 \%$ in expressing a verbal expression algebraically and $26 \%$ in expressing a geometric representation as an algebraic expression. Their overall average is $69 \%$. In studies carried out in the USA, Thailand, Japan and Korea, $43 \%, 46 \%, 57 \%$ and $64 \%$ of the students respectively could write a verbally stated expression algebraically (Kaş, 2010).

In the present study, the causes of mistakes within the scope of algebraic expression are as follows:

- Putting variable and fixed terms into operation separately and equalizing the results.
- Doing operations for coefficients only by ignoring the variable. In the study carried out by Küchemann (1981), it was also found that students ignored algebraic letters.
- Getting a solution by assigning a value to the variable.
- Digitizing the algebraic expression by assigning a value to the variable in the solution obtained.
- Failure to pay attention to the use of parenthesis.
- Failure to attribute a meaning to $x$ in a geometrical shape. In Dindyal's (2003) study, it was found that students had various conceptual difficulties in understanding the nature of the concept of variable and forming algebraic expressions while solving geometry problems.
- Errors in arithmetic operations in the algebraic expressions. Bağdat (2013) found that $8^{\text {th }}$ grade students had difficulty using and doing operations with the symbols in algebraic expressions.

Looking at the reasons behind the mistakes from a broader perspective, it was determined that the main reasons were failure to attribute a meaning to the unknown and therefore doing the operations by assigning a value to the unknown. In other words, it could be asserted that failure to understand the main idea of algebraic expressions (the concept of variable and the concept of algebraic expression involving variables) as a concept properly is the major reason. Similarly, in the study carried out by Dede, Yalın and Argün (2002), it was found that $8^{\text {th }}$ grade students could not attribute a meaning to the concept of variable and were incompetent in doing operations with algebraic expressions involving variables. Akgün (2007) stated that students had several difficulties and misconceptions in understanding the concept of variable and distinguishing between different uses of this concept. Moreover, it was determined that students found it difficult to carry out operations with expressions including variables or letters and to build relations between variables and word problems.

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