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Ranking with Statistical Variance Procedure based Analytic Hierarchy Process

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ÖZET

Bu çalışmada çok kriterli karar analizi yöntemlerinden Analitik Hiyerarşi Prosesi (AHP) ile sıralama yöntemini temel alan çok kriterli bir nesnel sıralama yöntemi sunulmaktadır. Çok kriterli karar analizi yöntemleri, matematiksel altyapılarındaki farklılıklar nedeniyle, aynı sıralama problemi için farklı sıralama çözümleri üretebilmektedir. AHP ile sıralama yönteminde karar vericilerin 1-9 ölçeğinde belirttiği tercihler ile pozitif karşılaştırmalar matrisleri oluşturulmaktadır. Ancak karar vericiler ufak çaplı bir sıralama problemi için bile çok sayıda karşılaştırma yaparken öznel yargılar tutarsız sıralamalara neden olabilmektedir. Bu çalışmada sunulan AHP'nin sadeleştirilmiş hali olan İstatistiksel Varyans Prosedürü (İVP) temelli AHP (İVP-AHP), çok kriterli bir veri setindeki alternatiflerin sıralamasını maliyetli anket süreçlerine başvurmadan kriter değerlerine göre belirlemektedir. Nesnel bir sıralama için İVP ve vektörel normalizasyonu AHP ile bütünleştiren İVP-AHP yönteminde kriter ağırlıkları İVP ile belirlenirken alternatiflerin karşılaştırmalar matrisleri normalize edilmiş gözlem değerlerinden oluşmaktadır. İVP-AHP ile sıralama yöntemi, AHP ile sıralama yönteminin güçlü özelliği olan karşılaştırmalar matrislerini kullanırken tutarlılık ölçümlerine ihtiyaç duymamaktadır. İVP-AHP yönteminde sadece sıralanması istenen alternatifler, seçimi etkileyen kriterler ve alternatiflerin kriter değerlerinin bilinmesi yeterli olup bu parametreler için –AHP yönteminde olduğu gibi– karar verici yargılarına ihtiyaç bulunmamaktadır. Bu çalışmada örnek bir veri setinden AHP ve İVP-AHP yöntemleri ile elde edilen karşılaştırmalı bulgular, işlem kolaylığı ve AHP yöntemindeki öznelliği gidermesi açısından İVP-AHP sıralama yönteminin etkin ve nesnel bir sıralama yöntemi olduğuna işaret etmektedir.

Anahtar Kelimeler: Analitik Hiyerarşi Prosesi, Çok Kriterli Karar Verme, Karşılaştırmalar Matrisi, Vektörel Normalizasyon

ABSTRACT

This study introduces an objective multicriteria ranking method based on the Analytic Hierarchy Process (AHP). Different multicriteria decision analysis methods generate different solutions for the same ranking problem because of their varying mathematical models. In AHP, decision makers construct positive comparison matrices from their preferences by using a scale of 1-9. However, even a simple ranking problem requires numerous comparison matrices while subjective judgments lead to inconsistent rankings. As a simplified version of the AHP, the Statistical Variance Procedure (SVP) based AHP (SVP-AHP) extracts the ranking of alternatives from a multicriteria dataset without referring to costly survey processes. SVP-AHP uses pairwise comparison matrices, the powerful tool of AHP, and it does not need to measure consistency. For an objective ranking of alternatives, SVP-AHP embeds vector normalization and SVP into the AHP. SVP determines criteria weights while pairwise comparison matrices for alternatives are constructed using the normalized observations. In SVP-AHP, it is sufficient to know only criteria and alternative values, unlike AHP, where the model requires decision makers' judgments. Results of the AHP and SVP-AHP for the example in this study point out that SVP-AHP is an efficient ranking method because of its computational efficieny and objectivity.

Keywords: Analytic Hierarchy Process, Multicriteria Decision Making, Pairwise Comparison Matrix, Vector Normalization

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1. INTRODUCTION

In management science, many problems from facility location to production scheduling involve selection by ranking alternatives. Research from mathematics, informatics, and decision sciences devoted to the multicriteria decision theory have extensive applications in business analytics, economics, and related fields for determining the optimal choice by classifying or ranking multiple alternatives. Traveling salesman problem, for example, aims to find the minimum traveling distance of a salesperson by ranking various demand points in their order of visit. Internet search engines rank the listed web sites according to various criteria –i. e. content quality. Routing problems also aim to find the order of jobs in a specific time window [1]. The field of multicriteria decision analysis has been developing since the second half of the 20th century [2].

Main advantage of Multicriteria Decision-Making (MCDM) models stems from the fact that criteria values of alternatives melt in the same pot for a holistic evaluation, and typically, these models do not require criteria selection or statistical significance tests. Analytic Hierarchy Process (AHP) is an MCDM and ranking model that converts verbal judgments of the decision makers (DM) into quantitative expressions. However, AHP includes a difficult surveying process for evaluating alternatives, and is limited to subjective judgments. Besides, the model requires repetitive pairwise comparisons that confuse DM and bewilder their judgmental abilities.

This study presents an objective ranking method [3], which embeds vector normalization and Statistical Variance Procedure (SVP) into the AHP. As a simplified version of the AHP, the method of SVP-AHP computes criteria weights from the data itself, eliminates the issue of consistency due to subjective judgments, and yet continues to benefit from the strength of pairwise comparison matrices. Organization of the rest of the paper is as follows: Section 2 reviews the AHP, Section 3 explains vector normalization, and Section 4 describes the SVP as a weighting method in the MCDM models. Section 5 introduces the methodology for integrating vector normalization, SVP and AHP into an objective ranking method and the proof of the benefit of SVP-AHP on the consistency issue of the AHP model. Section 6 compares the empirical results of a flat selection problem obtained with the AHP and the SVP-AHP models while discussing how the SVP-AHP compensates for the drawbacks of the AHP. Finally, Section 7 concludes with closing remarks.

2. ANALYTIC HIERARCHY PROCESS

There exist various MCDM models and each has a different capability for determining the best alternative of a set of possible solutions. MCDM models are applicable to business, economics and related fields, from production to finance [4][5]. A literature review on the wide-ranging MCDM models for decision support systems with applications, model development approaches, and software implementations are available in [6]. Tsoukiàs [7] states that compared to the structural precision of the multiobjective programming models, the configuration of an MCDM model for a decision problem is based upon an abstract language where subjective judgment, intuition, experience, and preferences are at the forefront. AHP, ELECTRE, and TOPSIS are among the well-known ranking methods that aim to generate a solution for an MCDM problem.

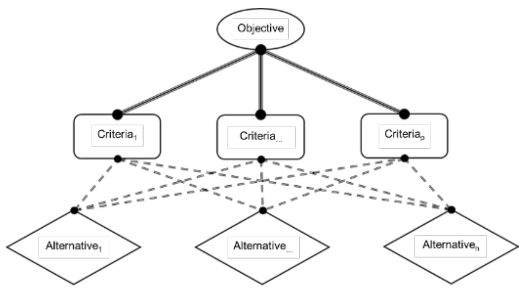

Figure 1. Multilevel AHP structure

Saaty [8] has developed the quantitative MCDM method of AHP to determine the relative rankings of alternatives by using the principal eigenvector of a symmetrically reciprocal positive pairwise comparison matrix. The AHP has found a global application area [9]. Figure 1 shows the structure of the AHP model that includes the interacting goal, criteria, and alternatives.

AHP has a three-phase calculation process for ranking a set of alternatives at an MCDM setting. All DM express their comparative judgments for each criterion with respect to each criterion, and the AHP determines criteria weights from these judgments in the 1st phase. For each criterion, DM compare alternatives to each other for the weights of alternatives in the 2nd phase. The last phase computes the weighted average of weights of criteria and weights of alternatives where the corresponding alternative with the maximum weighted average is the optimum solution of the AHP model. A summary of the computations of the AHP model:

- Phase 1: Compute weights for all criteria
- Phase 2: Compute weights for all alternatives with respect to each criterion
- Phase 3: Compute the weighted average of the weights of criteria and alternatives

In the first two phases, DM state their judgments in the form of pairwise priorities on a 1-9 scale [8]. The input of the AHP model is a symmetrically reciprocal positive pairwise comparison matrix. If DM compare that the evaluated criteria or alternatives seem to be equal to each other, then the pairwise comparison equals to 1. If DM express that a criterion or an alternative is definitely important compared to the other criterion or alternative, then the resulting pairwise comparison is 9. Other values of the 1-9 scale represent the pairwise priorities in-between. Diagonal values of the symmetrically reciprocal positive pairwise comparison matrices are always 1 because within the AHP model they represent the pairwise comparison of a criterion or an alternative to itself.

Consistency [10] in the AHP model controls whether the logic of the pairwise priorities is free from contradiction. The symmetrically reciprocal positive pairwise comparison matrix $[A]_{p\times p}$ is consistent or transitive, if $a_{ij}.a_{jk} = a_{ik}$ $(\forall\, i,j,k)$ [11][12]. This property also states that all rows and columns of A are linearly dependent and the determinant of such a matrix is zero. A is also consistent if $A\vec{w} = p\vec{w}$ where $\vec{w}$ $(w_i\,, i = 1,2,\ldots,p)$ is the resulting weight vector of criteria or alternatives [13].

AHP does not expect the priorities of the DM to be always consistent. Therefore, the AHP allows the pairwise comparison matrices to be inconsistent up to a certain level. As a heuristic normalization method for determining the relative importance of criteria and alternatives, the mathematical structure of the AHP stems from eigenanalysis, which provides an opportunity to calculate the inconsistency of the DM priorities [14][15]. Inconsistency in the AHP framework measures the approximation error between the real principal eigenvalue of the pairwise comparison matrix and λ_{max} of the AHP model by using the random index methodology that associates the approximation error with a possible inconsistency within DM priorities. λmax is the maximum eigenvalue that a pairwise comparison matrix can obtain. This heuristic process of normalization approximates the eigenvalue of the principal eigenvector of the pairwise comparison matrix. If the pairwise comparison matrix is consistent then $\lambda_{max} = p$; its rows and columns are linearly dependent, and thus its determinant is zero [11][16].

3. VECTOR NORMALIZATION

An MCDM method typically hosts a normalization process for removing the effect of different measuring units. Opricovic and Tzeng [17] express that normalization eliminates the units of criteria and makes criteria dimensionless. Thus, normalization allows for the evaluation of conflicting criteria within the same decision framework. The analyses of the effects of normalization on several MCDM methods are available in [18] and [19]. SVP-AHP implements vector normalization –as in TOPSIS– for an arbitrary ranking problem.

Let $[A]_{n\times p} = \begin{bmatrix} a_{11} & \cdots & a_{1j} \\ \vdots & \ddots & \vdots \\ a_{i1} & \cdots & a_{ij} \end{bmatrix}$ be the input of an MCDM problem with n alternatives to be evaluated by p criteria, where a_{ij} is the i^{th} observation value with respect to the j^{th} criterion $(i = \{1,2,\ldots,n\};\ j = \{1,2,\ldots,p\})$. The following equations normalize $[A]$ vectorially and convert it to $[T]_{n\times p}$:

- If the evaluated criterion of A is a benefit criterion, then,

$$t_{ij} = \frac{a_{ij}}{\sqrt{\left(\sum_{i=1}^{n}\sum_{j=1}^{p}\left|a_{ij}\right|^{2}\right)}} \tag{1}$$

- If the evaluated criterion of A is a cost criterion, then,

$$t_{ij} = \frac{(1/a_{ij})}{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{p}\left|(1/a_{ij})\right|^2}} \qquad (2)$$

where t_{ij} are elements of T

4. STATISTICAL VARIANCE PROCEDURE

Weight extraction is one of the most difficult processes in MCDM models [20] and the most important part of the AHP is to determine criteria weights [13]. A classification and a detailed discussion of the MCDM models are available in [21] along with their embedded weighting methods and their categorization. SVP is an objective weighting method that assigns an objective weight to each criterion using variances [22]. Variance of a dataset carries an important information, and it is suitable for comparing the criteria weights after normalization [23]. SVP-AHP embeds the SVP into the AHP, after vectorially normalizing the MCDM dataset.

If $[T]_{n\times p} = \begin{bmatrix} t_{11} & \dots & t_{1j} \\ \vdots & \ddots & \vdots \\ t_{i1} & \dots & t_{ij} \end{bmatrix}$ is the normalized dataset where t_{ij} is the i^{th} observation with respect to the j^{th} criterion, then the sample variance of the j^{th} criterion of T:

$$\mathrm{s_j} = \sum_{i=1}^{n}\sum_{j=1}^{p}(\mathrm{t_{ij}} - \bar{\mathrm{t}}_{\mathrm{ij}})^2 \Big/ (\mathrm{n}-1) \qquad (3)$$

The ratio of $\mathrm{s_j}$ to the total variance of T:

$$\mathrm{c_j} = \mathrm{s_j} \Big/ \textstyle\sum_{j=1}^{p} \mathrm{s_j} \qquad (4)$$

where $\mathrm{c_j}$ is the objective weight of the j^{th} criterion that the SVP assigns.

5. METHODOLOGY

SVP-AHP computes the weight parameter via SVP by dividing the variance of each criterion to the total variance of the normalized dataset. Using this method –instead of 1-9 scale– allows for the construction of consistent pairwise comparions matrices when determining criteria and alternative weights at the first two stages of AHP.

Let $[T]_{n\times p}$ be the vectorially normalized dataset of an MCDM problem obtained after applying Equation 1 and Equation 2. To extract weights from criteria or alternatives, instead of asking DM their judgments on the priorities, let $[C]_{p\times p}$ be the symmetrically reciprocal positive and transitive pairwise comparison matrix extracted from T using Equations 3 and 4:

$$[C]_{p\times p} = \begin{bmatrix} c_{j_1}/c_{j_1} & c_{j_1}/c_{j_2} & \dots & c_{j_1}/c_{j_p} \\ c_{j_2}/c_{j_1} & c_{j_2}/c_{j_2} & \dots & c_{j_2}/c_{j_p} \\ \vdots & \vdots & \dots & \vdots \\ c_{j_p}/c_{j_1} & c_{j_p}/c_{j_2} & \dots & c_{j_p}/c_{j_p} \end{bmatrix}$$

For simplicity, denote c_j with e and note that $\sum e = 1$ since $\sum \mathrm{c_j} = 1$ as stated in Equation 4.

$$[E]_{p\times p} = \begin{bmatrix} e_1/e_1 & e_1/e_2 & \dots & e_1/e_p \\ e_2/e_1 & e_2/e_2 & \dots & e_2/e_p \\ \vdots & \vdots & \dots & \vdots \\ e_p/e_1 & e_p/e_2 & \dots & e_p/e_p \end{bmatrix}$$

For the weight of the first criterion, AHP computes the sum of the first column:

$$\mathrm{b_1} = \frac{(e_1/e_1) + (e_2/e_1) + \dots + (e_p/e_1)}{e_1} = \frac{1}{e_1} \quad (5)$$

The generalization of Equation 5 leads to:

$$\mathrm{b_j} = \frac{1}{e_j} \qquad (6)$$

Continuing with the normalization of the AHP:

$$[N] = \begin{bmatrix} 1/b_1 & e_1/e_2b_2 & \dots & e_1/e_pb_p \\ e_2/e_1b_1 & 1/b_2 & \dots & e_2/e_3b_3 \\ \vdots & \vdots & \dots & \vdots \\ e_p/e_1b_1 & e_p/e_2b_2 & \dots & 1/b_p \end{bmatrix}$$

The average of the first row of N at Equation 7 gives the weight of the first criterion:

$$w_{p_1} = \frac{(1/b_1) + (e_1/e_2b_2) + \dots + (e_1/e_pb_p)}{p} = e_1 \quad (7)$$

Generalizing Equation 7:

$$w_p = e \qquad (8)$$

Pairwise comparison matrix of weights is consistent, $\lambda_{max_C} = p$, and $|C| = 0$.

6. EMPIRICAL RESULTS

An MCDM dataset of the flat selection problem in [24] consists of 4 alternatives and 9 criteria as given in the following decision matrix, $[D]$:

$$D_{4\times 9} = \begin{bmatrix} 375{,}000 & 110 & 4 & 2 & 1 & 1 & 1 & 1 & 4 \\ 310{,}000 & 120 & 4 & 4 & 2 & 2 & 2 & 2 & 1 \\ 330{,}000 & 130 & 5 & 2 & 3 & 3 & 3 & 2 & 3 \\ 350{,}000 & 140 & 3 & 1 & 4 & 4 & 4 & 3 & 2 \end{bmatrix}$$

4 alternative flats of X, W, Y and Z in the rows of D are to be evaluated according to 9 criteria at the columns of D. These criteria respectively measure the flat's price, the flat's area, the number of rooms of the flat, the direction of the facade of the flat, quality of materials used at the flat, quality of building in general, earthquake resistance of the building, availability of car park, and distance to the city center. All criteria other than price, area, and number of rooms use an ordinal scale from 1 to 4; where 4 represents the best preference, and all criteria other than price are benefit criteria. The weights of criteria and alternatives of the AHP problem are as follows:

$$AHP_{weights\ of\ criteria} = [.26 \quad .10 \quad .07 \quad .04 \quad .06 \quad .03 \quad .22 \quad .09 \quad .12]$$

$$AHP_{weights\ of\ alternatives} = \begin{bmatrix} .07 & .07 & .14 & .16 & .04 & .04 & .04 & .04 & .62 \\ .47 & .11 & .14 & .64 & .10 & .10 & .08 & .13 & .04 \\ .30 & .29 & .57 & .04 & .28 & .25 & .26 & .13 & .23 \\ .17 & .52 & .14 & .16 & .58 & .61 & .62 & .69 & .11 \end{bmatrix}$$

The weighted average of criteria and alternative weights for the alternatives of X, W, Y and Z:

$$AHP \rightarrow \begin{bmatrix} X \\ W \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.13 \\ 0.23 \\ 0.28 \\ 0.36 \end{bmatrix} \therefore\ Z > Y > W > X$$

The critical ratios, in other words, the measures of inconsistency respectively for criteria and alternative weights are .03, .07, .01, .00, .00, .05, .07, .09, .09, .09, and, .07.

Note that the evaluation of this ranking problem with the AHP requires 80 comparison matrices of which 36 are pairwise comparisons for determining the weights of criteria, and 54 are pairwise comparisons for determining the weights of alternatives.

6.1. Normalizing the Decision Matrix

Application of the methodology in Section 5 starts with vector normalization as given in Equation 1, and Equation 2 at Section 3. The normalization of $D_{4\times 9}$ leads to $[T]_{4\times 9}$:

$$T = \begin{bmatrix} .45 & .44 & .49 & .40 & .18 & .18 & .18 & .24 & .73 \\ .55 & .48 & .49 & .80 & .37 & .37 & .37 & .47 & .18 \\ .51 & .52 & .62 & .40 & .55 & .55 & .55 & .47 & .55 \\ .48 & .56 & .37 & .20 & .73 & .73 & .73 & .71 & .37 \end{bmatrix}$$

6.2. Weights of Criteria with the Statistical Variance Procedure

The sample variances of each criterion of T using Equation 3 in Section 4:

$$S = [.00 \quad .00 \quad .01 \quad .06 \quad .06 \quad .06 \quad .06 \quad .04 \quad .06]$$

The consistent weights –as shown in Section 5– of the SVP-AHP, obtained using Equation 4 in Section 4:

$$C = [.00 \quad .01 \quad .03 \quad .19 \quad .16 \quad .16 \quad .16 \quad .11 \quad .16]$$

6.3. Weights of Alternatives with the Analytic Hierarchy Process

In this phase, SVP-AHP computes the weights of alternatives exactly the same way as the AHP does. The only difference from AHP is that SVP-AHP uses criteria values of alternatives as the input, instead of DM priorities. For each criterion, SVP-AHP compares alternatives to each other using the values of T.

The construction of comparions matrices of alternatives are directly from criteria values. For example, a symmetrically reciprocal and consistent pairwise comparison matrix of alternatives constructed for the last column of T, which is the normalized distance criterion ($[.73 \quad .18 \quad .55 \quad .37]^{\mathrm{T}},$):

$$\begin{bmatrix} X \\ W \\ Y \\ Z \end{bmatrix}_{Distance} = \begin{bmatrix} 1 & 4.00 & 1.33 & 2.00 \\ 0.25 & 1 & 0.33 & 0.50 \\ 0.75 & 3.00 & 1 & 1.50 \\ 0.50 & 2.00 & 0.67 & 1 \end{bmatrix}$$

Continuing with the same computations of the AHP model, dividing each column's elements by the sum of each column's elements determines each alternative's weight with respect to distance. The consistent weights of alternatives with respect to distance criterion: $[X \quad W \quad Y \quad Z]_{\text{weight\_distance}}{}^{\mathrm{T}} = [.40 \quad .10 \quad .30 \quad .20]^{\mathrm{T}}$

Repeating the same procedure for each criterion constructs the matrix of weights of alternatives:

$$W_{alternatives} = \begin{bmatrix} .23 & .22 & .25 & .22 & .10 & .10 & .10 & .13 & .40 \\ .27 & .24 & .25 & .44 & .20 & .20 & .20 & .25 & .10 \\ .26 & .26 & .31 & .22 & .30 & .30 & .30 & .25 & .30 \\ .24 & .28 & .19 & .11 & .40 & .40 & .40 & .38 & .20 \end{bmatrix}$$

6.4. Ranking Expected Values

The expected values of criteria and alternative weights for the alternatives of X, W, Y and Z:

$$SVP - AHP \rightarrow \begin{bmatrix} X \\ W \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.18 \\ 0.24 \\ 0.28 \\ 0.30 \end{bmatrix}$$

Thus, the ranking of alternatives via SVP-AHP yields the same ranking results obtained with AHP:

$$Z > Y > W > X$$

6.5. Discussion

In the AHP results of the problem in [24], the DM gave the top three weights to the criteria of price of the flat, earthquake resistance of the building, and the distance to the city center, with approximate weights of 26%, 22%, and 12% respectively as given in the beginning of this Section 6. On the other hand, the method of SVP-AHP determined the top priorities to be the direction of the facade, quality of materials used at the flat, quality of building, earthquake resistance of the building, and the distance to the city center, where the approximate weights are 19% for the direction of the facade and 16% for the other criteria. SVP-AHP denoted very little weight to price, approximately 0.5%, since flat prices are not widely spread out.

Hybrid MCDM models can include various MCDM and weighting methods in different stages of the problem. For example, [25] compares alternative locations for solar power plants using geographical information systems and MCDM tools where AHP determines criteria weights and TOPSIS ranks alternatives. As a hybrid MCDM ranking method, SVP-AHP uses SVP to determine criteria weights. After the vector normalization of the dataset to add objectivity into the process, the ranking proceeds with the methodology of AHP.

Managerial data is mainly composed of quantitative observations of DM preferences and behavior of the market; and SVP-AHP provides the capacity for objective ranking because it does not inject any further DM subjectivity into the ranking process. It is noteworthy to add that AHP seeks the independence of criteria, yet many MCDM problems that use the AHP ignores the independency assumption, since it is nearly impossible to show a theoretical independence among variables and observations. In any case, SVP-AHP presents a pragmatic approach, as well as other critical points of the AHP model that arise in large-scale ranking problems –i. e. rank reversal or consistency. Objectivity of the experimental results of SVP-AHP is clear as they compare efficiently to those of the mainstream MCDM methods [3].

7. CONCLUSION

Different MCDM methods generate different solutions for the same ranking problem because of the varying mathematical models. SVP-AHP is an objective ranking method that takes advantage of the methodology of AHP and pairwise comparison matrices along with the vector normalization, and the SVP. As the simplified version of the AHP, the method of SVP-AHP ranks alternatives with their factual criteria values.

In SVP-AHP, DM only need to specify the related criteria of the decision problem to rank alternatives. Therefore, SVP-AHP is free of time-consuming evaluations and consistency measurements. Future research directions are towards building a stochastic version of the SVP-AHP and analyzing its ranking results in various applications with a comparison to other related methods.

REFERENCES

[1] Esen, H. Ö., 2008, "Applied Operational Research" ("*Uygulamalı Yöneylem Araştırması*"), (S. Tolun, Ed.), Çağlayan Kitabevi.

[2] Roy, B., & Vanderpooten, D., 1997, "An overview on "The European school of MCDA: Emergence, basic features and current works", European Journal of Operational Research, 99, 26–27.

[3] Tayalı, H. A., 2016, "Statistical variance procedure based analytic hierarcy process: An application on multicriteria facility location selection", Retrieved from https://tez.yok.gov.tr/UlusalTezMerkezi/.

[4] Ömürbek, N., & Mercan, Y., 2014, "Performance Evaluation of Sub-manufacturing Sectors Using TOPSIS and ELECTRE Methods", Cankiri Karatekin University Journal of the Faculty of Economics and Administrative Sciences, 4(1), 237–266.

[5] Xidonas, P., Mavrotas, G., & Psarras, J., 2009, "A multicriteria methodology for equity selection using financial analysis", Computers and Operations Research, 36(12), 3187–3203.

[6] Zopounidis, C., & Doumpos, M., 2002, "Multicriteria classification and sorting methods: A literature review", European Journal of Operational Research, 138(2), 229–246.

[7] Tsoukiàs, A., 2008, "From decision theory to decision aiding methodology", European Journal of Operational Research, 187(1), 138–161.

[8] Saaty, T. L., 1977, "A scaling method for priorities in hierarchical structures", Journal of Mathematical Psychology, 15(3), 234–281.

[9] Sipahi, S., & Timor, M., 2010, "The analytic hierarchy process and analytic network process: An overview of applications", Management Decision, 48(5), 775–808.

[10] Nelson, D., 2008, "The Penguin Dictionary of Mathematics", Penguin UK.

[11] Alonso, J. A., & Lamata, M. T., 2006, "Consistency in the analytic hierarchy process: a new approach", International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 14(4), 445–459.

[12] Farkas, A., 2007, "The analysis of the principal eigenvector of pairwise comparison matrices", Acta Polytechnica Hungarica, 4(2), 99–115.

[13] Taha, H. A., 2007, "Operations Research: An Introduction", Pearson Education International.

[14] Saaty, T. L., 2003, "Decision-making with the AHP: Why is the principal eigenvector necessary", European Journal of Operational Research, 145(1), 85–91.

[15] Saaty, T. L., 2008, "Relative measurement and its generalization in decision making; why pairwise comparisons are central in mathematics for the measurement of intangible factors", Revista de La Real Academia de Ciencias Exactas, Fisicas Y Naturales. Serie A. Matematicas, 102(2), 251–318.

[16] Peláez, J. I., & Lamata, M. T., 2003, "A new measure of consistency for positive reciprocal matrices", Computers and Mathematics with Applications, 46(12), 1839–1845.

[17] Opricovic, S., & Tzeng, G. H., 2004, "Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS", European Journal of Operational Research, 156(2), 445–455.

[18] Özdağoğlu, A., 2013, "The effects of different normalization methods to decision making process in TOPSIS" ("*Farklı normalizasyon yöntemlerinin TOPSIS'te karar verme sürecine etkisi*"), Ege Academic Review, 13(2), 245–257.

[19] Pavlicic, D. M., 2001, "Normalisation affects the results of MADM methods", Yugoslav Journal of Operations Research, 11(2), 251–265.

[20] Tervonen, T., Figueira, J., Lahdelma, R., &

Dias, J., 2009, "A stochastic method for robustness analysis in sorting problems", European Journal of Operational Research, 192(1), 236–242.

[21] Zardari, N. H., Ahmed, K., Shirazi, S. M., & Yusop, Z. Bin., 2015, "Weighting Methods and their Effects on Multi-Criteria Decision Making Model Outcomes in Water Resources Management", Springer International Publishing.

[22] Rao, R., & Patel, B., 2010, "A subjective and objective integrated multiple attribute decision making method for material selection", Materials & Design, 31(10), 4738–4747.

[23] Charilas, D. E., Panagopoulos, A. D., & Ourania, M. I., 2014, "A Unified Network Selection Framework Using Principal Component Analysis and Multi Attribute Decision Making", Wireless Personal Communications, 74(1), 147–165.

[24] Timor, M., 2011, "Analytic Hierarchy Process" ("*Analitik Hiyerarşi Prosesi*") Istanbul, Türkmen Kitabevi, 102.

[25] Sánchez-Lozano, J., & Teruel-Solano, J., 2013, "Geographical Information Systems (GIS) and Multi-Criteria Decision Making (MCDM) methods for the evaluation of solar farms locations: Case study in south-eastern Spain", Renewable and Sustainable Energy Reviews, 24, 544–556.