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TEACHING FRACTIONAL ORDER CONTROL SYSTEMS USING INTERACTIVE TOOLS

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ABSTRACT: Much of subjects being taught in a first course on control theory in Electrical and Electronics Engineering appears to have changed little. Although the basic theories, methods and applications on classical control in textbooks remain unchanged, there have been many new developments in the field of control theory in recent years. One of such topic is fractional order control systems which is based on fractional order calculus and can be used to model physical system more exactly than integer order systems. The purpose of this paper is to show how fractional order control methods can be introduced into a first course on classical control using interactive tools such as Matlab and LabView.

Key words: Education, control theory, fractional order system, interactivity, labview

INTRODUCTION

The subjects being taught to undergraduate students on automatic control, despite advances in the field of control theory, seem to have changed little over last many years. However, developments in computer technology now allow us to use new and high-quality educational methods such as interactive tools, virtual and remote laboratories to make use of World Wide Web etc. (Dormido, 2002). Today's computer software programs such as Matlab and LabVIEW can be used effectively to teach some advanced subjects to the students without going into details of mathematical derivations. Although the fundamental control theory concepts known as classical control theory are necessary for control theory education and because of its many advantages we think this must continue in the future, it will be important that any developments in control theory which can be linked to classical control are good candidates for consideration in basic feedback control education by using interactive tools. The interactive tools are extremely useful and they enable students to explore changes in system performance as parameters are varied, and to do so looking at several diagrams simultaneously. This can be done without any programming by just using a mouse to adjust any parameters and the effects can be immediately seen. Matlab, LabVIEW and Simulink provide an excellent environment for such studies to teach additional topics besides subjects of classical control theory.

One such topic is the recent development in methods to analyse systems using fractional order calculus (Das, 2008; Podlubny, 1999a). This is an important topic since most real physical system can be modeled more adequately by fractional order differential equations. The applications of fractional order differential equations to the problems in the control theory have been increased in recent years and promising results have been obtained (Xue et al., 2007; Monje et al., 2010). It has been shown that there is a strong link between classical control methods and fractional order approaches. Therefore, from educational point of view, it is important to teach developed results based on fractional order calculus to see the effects of fractional order integrator and derivative on control system performance. The purpose of this paper is to show how the results based on fractional order concepts can be introduced into a basic classical feedback control theory course given for undergraduate students and the teaching can be supported by software tools.

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The layout of the paper is as follows. In the next section the concept of fractional order system is introduced and examples are given. Section 3 discusses the teaching of fractional order control systems using Matlab and LabVIEW with application examples. Finally some conclusions are given in Section 4.

FRACTIONAL ORDER CONTROL SYSTEMS

Fractional order derivative and integrator can be considered as an extension of integer order derivative and integrator operators to the case of non-integer orders and it is defined in general form as the following (Chen et al., 2009),

$${}_a D_t^\alpha = \begin{cases} \frac{d^\alpha}{dt^\alpha} & \alpha > 0 \\ 1 & \alpha = 0 \\ \int_a^t (d\tau)^{(-\alpha)} & \alpha < 0 \end{cases} \quad (1)$$

where, ${}_a D_t^\alpha$ represents fundamental non-integer order operator of fractional calculus. Parameters a and t are the lower and upper bounds of integration, and $\alpha \in R$ denotes the fractional-order (Oustaloup et al., 2000). Two definitions used for the general fractional derivative and integrator are the Riemann-Liouville definition and the Caputo definition (Chen et al., 2009). The Riemann-Liouville definition for the fractional-order derivative of order $\alpha \in R$ has the following form

$${}_a D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau) d\tau}{(t-\tau)^{\alpha-n+1}} \quad (2)$$

where $\Gamma(\cdot)$ is Euler's gamma function and $n-1 < \alpha < n$, $n \in \mathbb{Z}$. An alternative definition for the fractional-order derivative was given by Caputo as

$${}_a D_t^\alpha = \frac{1}{\Gamma(n-\alpha)} \int_a^t \frac{f^n(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \quad (3)$$

where $n-1 < \alpha < n$, $n \in \mathbb{Z}$. The Laplace transform of the Caputo fractional order derivative has the following result that is particularly significant for fractional order system modeling (Monje et al, 2010):

$$\int_0^\infty e^{-st} {}_0 D_t^\alpha f(t) dt = s^\alpha F(s) - \sum_{k=0}^{n-1} s^{\alpha-k-1} f^{(k)}(0) \quad (4)$$

When $f(0) = f^{(1)}(0) = f^{(2)}(0) = f^{(3)}(0) = \dots, f^{(n-1)}(0) = 0$ is considered, a basic property facilitating for design and analysis of fractional order systems is expressed for Laplace transform of fractional order derivative as $L\{D^\alpha f(t)\} = s^\alpha F(s)$.

In general, fractional order LTI systems were described by the following fractional order differential equation form as (Monje et al, 2010):

$$v_n D_n^{\alpha_q} y(t) + v_{n-1} D_n^{\alpha_{q-1}} y(t) + \dots + v_1 D_n^{\alpha_1} y(t) + v_0 y(t) = u_p D_p^{\beta_p} r(t) + u_{p-1} D_p^{\beta_{p-1}} r(t) + \dots + u_1 D_p^{\beta_1} r(t) + u_0 r(t) \quad (5)$$

By applying $L\{D^\alpha f(t)\} = s^\alpha F(s)$, a general form of transfer function of fractional order LTI systems were expressed as,

$$G(s) = \frac{Y(s)}{R(s)} = \frac{\sum_{j=0}^p u_j s^{\beta_j}}{\sum_{i=0}^q v_i s^{\alpha_i}} \quad (6)$$

where, denominator polynomial coefficients v_i and numerator polynomial coefficients u_j are polynomial coefficients and fractional orders of the LTI system are denoted by $\alpha_i \in R$ ($i = 0, 1, 2, 3, \dots, q$) and $\beta_j \in R$ ($j = 0, 1, 2, 3, \dots, p$).

Fractional Order PID Controller

Fractional order PID controller ($PI^\lambda D^\mu$) which has five tuning controller parameters including an integrator of order λ and an differentiator of order μ provides a better response than the integer order PID controller when used both for the integer-order systems and fractional-order systems (Chen et al., 2009). $PI^\lambda D^\mu$ is described in time domain in (Podlubny, 1999b) as follows;

$$u(t) = k_p e(t) + T_i D^{-\lambda} e(t) + T_d D^{\mu} e(t) \tag{7}$$

where k_p is the proportional gain, T_i the integration constant, T_d the differentiation constant; λ and μ are positive real numbers. The frequency domain formula is given in (Petras, 1999).

$$C(s) = \frac{U(s)}{E(s)} = k_p + \frac{k_i}{s^2} + k_d s^{\mu} \tag{8}$$

In Equation (8), classical PID controller can be obtained for $\lambda = 1$ and $\mu = 1$.

Integer Order Approximation Methods for Fractional Order Systems

Fractional order functions are infinite dimensional functions and hence it is very difficult to implement them practically or simulate them numerically (Vinagre et al., 2000). Therefore, several integer order approximation methods were proposed for realization of fractional order systems by replacing them with integer order approximations. Oustaloup presented an approximation method based on recursive distribution of poles and zeros in a limited frequency (Oustaloup et al., 2000). Another approximation method using the gain of the fractional order transfer functions at certain frequencies was suggested by Matsuda, (Matsuda et al., 1993). Recently, SBL fitting approximation method has been presented for fractional order derivative and integrator operators (Deniz et al., 2016). The integer order approximations model obtained by SBL fitting method is given in Table 1 (Deniz et al., 2016).

Table 1. Lists of fractional order derivative operator approximations determined using proposed method for $\omega \in [0.01, 1.2]$.

s^{α}	Fractional Order Derivative Approximations by SBL fitting
$s^{0.1}$	$\frac{5532s^4 + 1.338 \times 10^4 s^3 + 4147s^2 + 191.5s + 1}{4392s^4 + 1.353 \times 10^4 s^3 + 5063s^2 + 284.5s + 1.894}$
$s^{0.2}$	$\frac{7718s^4 + 1.714 \times 10^4 s^3 + 4974s^2 + 214.2s + 1}{4810s^4 + 1.743 \times 10^4 s^3 + 7372s^2 + 470.2s + 3.587}$
$s^{0.3}$	$\frac{1.087 \times 10^4 s^4 + 2.227 \times 10^4 s^3 + 6052s^2 + 242.6s + 1}{5224s^4 + 2.263 \times 10^4 s^3 + 1.083 \times 10^4 s^2 + 783.7s + 6.829}$
$s^{0.4}$	$\frac{1.553 \times 10^4 s^4 + 2.949 \times 10^4 s^3 + 7507s^2 + 279.8s + 1}{5622s^4 + 2.972 \times 10^4 s^3 + 1.615 \times 10^4 s^2 + 1324s + 13.16}$
$s^{0.5}$	$\frac{2.271 \times 10^4 s^4 + 4.012 \times 10^4 s^3 + 9566s^2 + 330.7s + 1}{5987s^4 + 3.978 \times 10^4 s^3 + 2.46 \times 10^4 s^2 + 2283s + 25.87}$
$s^{0.6}$	$\frac{3.447 \times 10^4 s^4 + 5.677 \times 10^4 s^3 + 1.268 \times 10^4 s^2 + 405.6s + 1}{6300s^4 + 5.493 \times 10^4 s^3 + 3.88 \times 10^4 s^2 + 4074s + 52.66}$
$s^{0.7}$	$\frac{5.564 \times 10^4 s^4 + 8.562 \times 10^4 s^3 + 1.791 \times 10^4 s^2 + 528.2s + 1}{6537s^4 + 8.007 \times 10^4 s^3 + 6.487 \times 10^4 s^2 + 7704s + 113.8}$
$s^{0.8}$	$\frac{1.008 \times 10^5 s^4 + 1.452 \times 10^5 s^3 + 2.844 \times 10^4 s^2 + 770.1s + 1}{6666s^4 + 1.298 \times 10^5 s^3 + 1.213 \times 10^5 s^2 + 1.628 \times 10^4 s + 275.5}$
$s^{0.9}$	$\frac{2.428 \times 10^5 s^4 + 3.28 \times 10^5 s^3 + 6.012 \times 10^4 s^2 + 1488s + 1}{6648s^4 + 2.773 \times 10^5 s^3 + 3.006 \times 10^5 s^2 + 4.561 \times 10^4 s + 887.3}$

An example is provided below relating to SBL fitting method. For this case, the values in Table 1 are used to obtain integer order approximation model of fractional order derivative operator. Also, these values can be used as $1/s^{\alpha}$ for integer order approximation model of fractional order integrator operator.

Example 1: Consider the fractional order transfer function

$$G(s) = \frac{1}{s^{1.2} + 1} \tag{9}$$

For the fractional order derivative $s^{0.2}$, SBL fitting method, Matsuda's method and Oustaloup's method provide the integer order approximation model in Table 2. Also, equivalent integer order transfer function $G_T(s)$ of fractional order transfer function $G(s)$ can be seen in Table 2.

Table 2. Equivalent integer order transfer function $G_T(s)$ of fractional order transfer function $G(s)$

Method	Integer order approximation models for $G(s) = \frac{1}{s^{1.2} + 1}$
SBL	$s^{0.2} = \frac{7718s^4 + 1.714 \times 10^4 s^3 + 4974s^2 + 214.2s + 1}{4810s^4 + 1.743 \times 10^4 s^3 + 7372s^2 + 470.2s + 3.587}$ $G_T(s) = \frac{4810s^4 + 1.743 \times 10^4 s^3 + 7372s^2 + 470.2s + 3.587}{7718s^5 + 2.195 \times 10^4 s^4 + 2.241 \times 10^4 s^3 + 7586s^2 + 471.2s + 3.587}$
Matsuda	$s^{0.2} = \frac{3.357s^4 + 161s^3 + 453.9s^2 + 95s + 1}{s^4 + 95s^3 + 453.9s^2 + 161s + 3.357}$ $G_T(s) = \frac{s^4 + 95s^3 + 453.9s^2 + 161s + 3.357}{3.357s^5 + 162s^4 + 548.9s^3 + 548.9s^2 + 162s + 3.357}$
Oustaloup	$s^{0.2} = \frac{2.512s^5 + 98.83s^4 + 531.7s^3 + 442.3s^2 + 56.87s + 1}{s^5 + 56.87s^4 + 442.3s^3 + 531.7s^2 + 98.83s + 2.512}$ $G_T(s) = \frac{s^5 + 56.87s^4 + 442.3s^3 + 531.7s^2 + 98.83s + 2.512}{2.512s^6 + 99.83s^5 + 588.6s^4 + 884.5s^3 + 588.6s^2 + 99.83s + 2.512}$

Figure 5 shows a comparison of the amplitude and phase responses. Here, the exact solution for $G(s) = \frac{1}{s^{1.2} + 1}$ was obtained by calculating phase and amplitude of $\frac{1}{(j\omega)^{1.2} + 1}$ considering $(j\omega)^\alpha = \omega^\alpha (\cos(\frac{\pi}{2}\alpha) + j \sin(\frac{\pi}{2}\alpha))$.

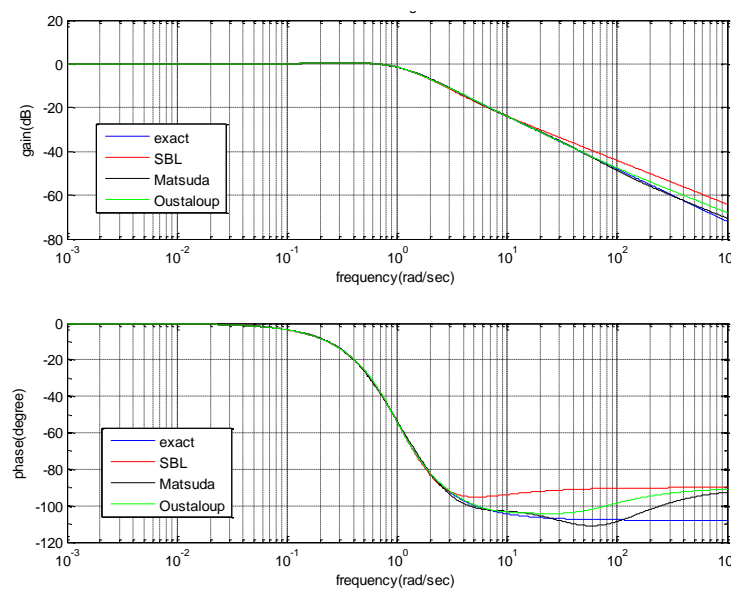


Figure 1. Amplitude and phase responses of $G(s)$ and approximate integer order models, SBL fitting method, Matsuda's method and Oustaloup's method

Example 2: In this example, SBL fitting method is used to simulate the closed loop control system with PI^λ controller as shown in Figure 2 in MATLAB. Consider the fractional order PI controller

$$C(s) = k_p + \frac{k_i}{s^\lambda} \tag{10}$$

To use fractional order controller $C(s)$ for simulation of the closed loop control system in MATLAB, an integer order approximation method has to be applied for fractional operator s^λ . When an equivalent integer order

approximation model of $C(s)$ is determined using an integer order approximation method, the closed loop control system as shown in Figure 2 can be simulated to obtain the closed loop response. For this, fractional order operator is replaced with its equivalent integer order model in closed loop control system.

An illustrative example which contains SBL fitting approximation method is given to clarify this strategy. Assuming that the plant transfer function is given by $G(s) = \frac{1}{s^2 + 3s + 1}$ and PI^λ controller is formed as $C(s) = k_p + \frac{k_i}{s^{0.9}}$ in Figure 2, one can find stability region of the closed loop control system using SBL analysis (Hamamci, 2008). Then, the controller parameters k_p and k_i can be selected in stability region to obtain the step response of configuration in Figure 2.

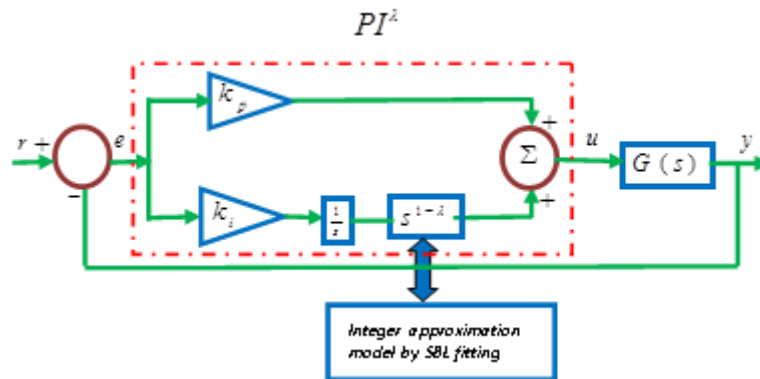


Figure 2. The closed loop control system with PI^λ simulated in MATLAB

Figure 3 shows different step responses obtained for different controller parameters (k_p, k_i) with $\lambda = 0.9$ selected from stability region as $(0.2604, 2.2281)$, $(0.6843, 4.7544)$ and $(0.1129, 1.2807)$. Better closed loop step responses can be obtained for different controller parameters (k_p, k_i, λ) selected from stability region.

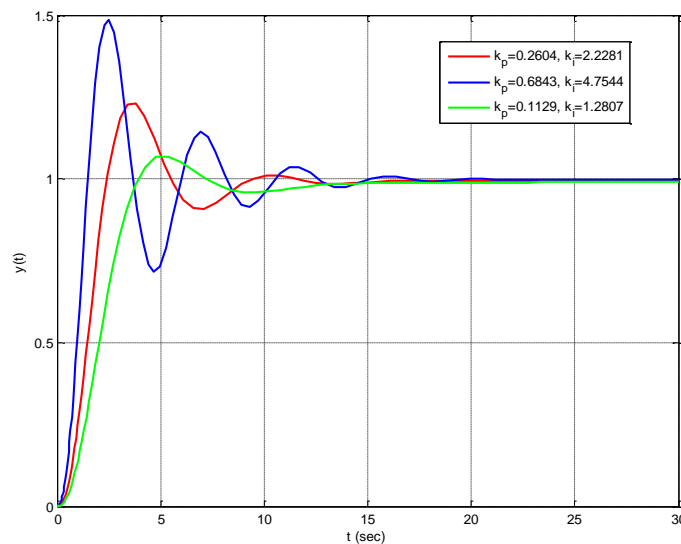


Figure 3. Step responses of the closed loop control system with PI^λ for different values k_p and k_i

Teaching Fractional Order Control Systems Using Matlab And Labview

MATLAB is a high-performance language for technical computing. It integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation. Simulink can easily analyze model and simulate control systems (MathWorks, <http://www.mathworks.com/>). Simulink can work with the MathWorks Real-Time Workshop. Real-Time Workshop generates and executes stand-alone C code for developing and testing algorithms modeled in Simulink. The resulting code can be used for many real-time and non-real-time applications, including simulation acceleration, rapid prototyping, and hardware-in-the-loop testing (Rodriguez, et al. 2005). So

Simulink is also widely used in simulation of all kind of systems. However, Simulink lack the imitation of physical instruments or equipment in appearance and operation. That's why, it is a good idea to combine LabVIEW and MATLAB in simulation of control system (Xuejun et al, 2007).

The LabVIEW (Laboratory Virtual Instrument Engineering Workbench) software is a graphical programming language and used to develop a virtual instrument (vi) that includes a front panel and a functional block diagram. User enters input from the front panel of the vi. LabVIEW has become a vital tool for engineers and scientists in research throughout academia, industry, and government labs. LabVIEW is taught at many universities in Europe and USA. LabVIEW has been used in the classroom for teaching of difficult subjects. For example, the graphical programming approach of LabVIEW has been used to teach computer science concepts. The graphics make the concepts more intuitive and easier to understand.

LabVIEW is used to teach control systems. The graphical programming approach is based on dataflow theory which is an ideal platform for learning how signals flow from one function to another such as from an acquisition function through a filter to a spectrum analysis and finally to a graph. Each function is an icon that is wired to other icons as in the example given in Figure 4. The wires are the signals flowing from one icon to the next. Research and industry use LabVIEW for automated test (ATE), medicine, chemistry, process control, simulation, calibration, and general-purpose data acquisition and analysis (Vento, 1988).

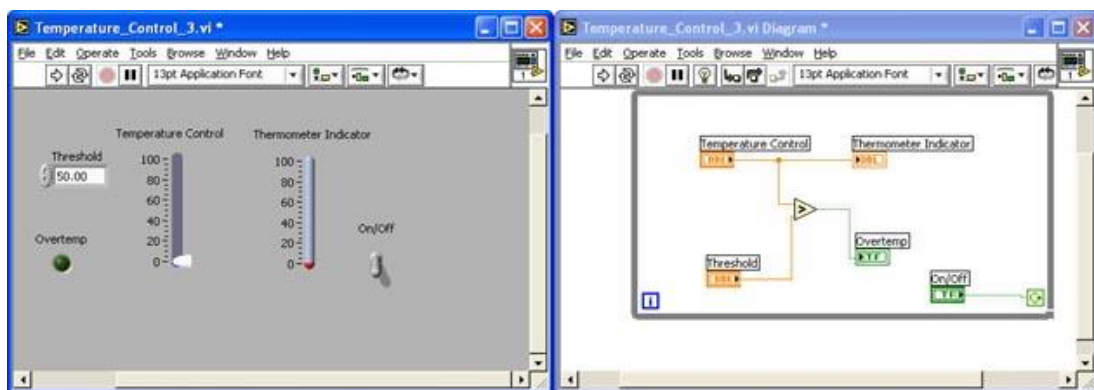


Figure 4. The Front and Block Diagram Panel Image of LabVIEW

The temperature control system given in Figure 4 performs on LabVIEW environment. The application includes two windows. The first window is front panel and second is block diagram panel of the program. The front panel is developed as a graphical interface and it is performed interactively. The block diagram panel is that provided wire connection and data flow.

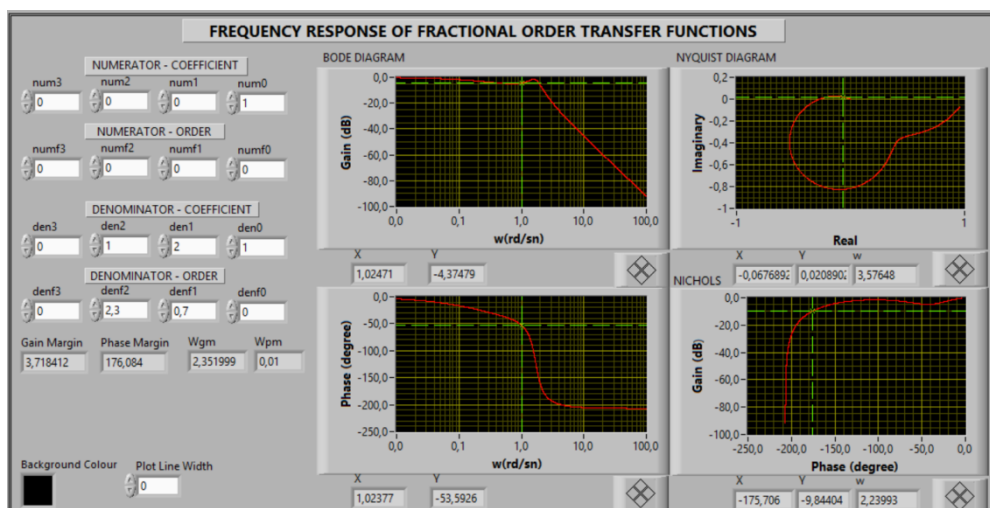


Figure 5. The Front Panel Image of Frequency Response Application of FOTF using LabVIEW

The application given in Figure 5 has been developed in the LabVIEW environment for the frequency domain analysis of fractional order control systems. The Bode, Nyquist and Nichols diagrams for any fractional order transfer function can be plotted by using the program. The program with this properties helps to design suitable controller for a given control system. The program is especially suitable for use in the field of education since it has interactive features.

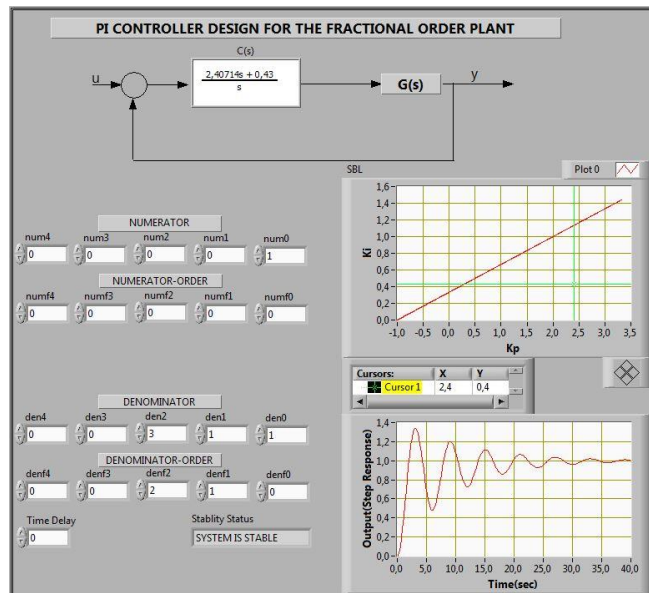


Figure 6. The Front Panel Image of PI Controller Design Application for the Fractional Order Plant using LabVIEW

The application given in Figure 6 runs the stability boundary locus (SBL) method to find all stabilizing PI controllers in a stability region for closed loop control systems with fractional order plant transfer function using LabVIEW application. Effect of the parameters of controllers selected from the stability region can be immediately observed and step response of the system can be immediately plotted. Thus, the controller which gives the best results can be designed based on proposed interactive approach.

Example 3: Consider the transfer function as below,

$$G(s) = \frac{5}{s^{2.2} + 7s^{0.9} + 2} \quad (11)$$

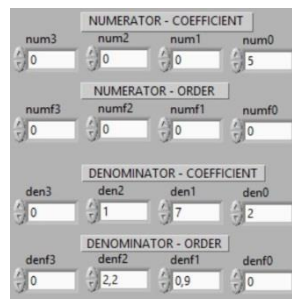


Figure 7. Transfer Function Input Panel for Frequency Response Application

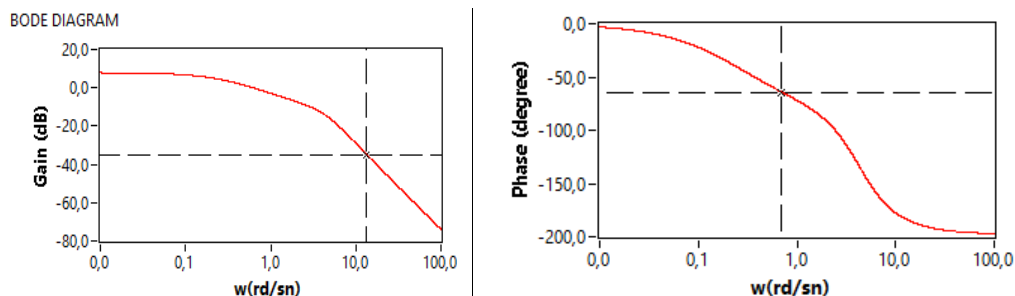


Figure 8. Bode Diagram

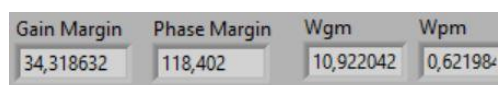


Figure 9. Gain and Phase Margin Panel

In this example, frequency response application of fractional order transfer functions is examined. Transfer function given in Eq. (11) is entered to panel as shown in Figure 7. Bode, Nyquist Nichols diagrams are

simultaneously plotted as shown in Figure 8 and 9. Also, the application computes gain and phase margin values which are shown in Figure 9. The application has interactive features, thus user can try different values, while the application is performing.

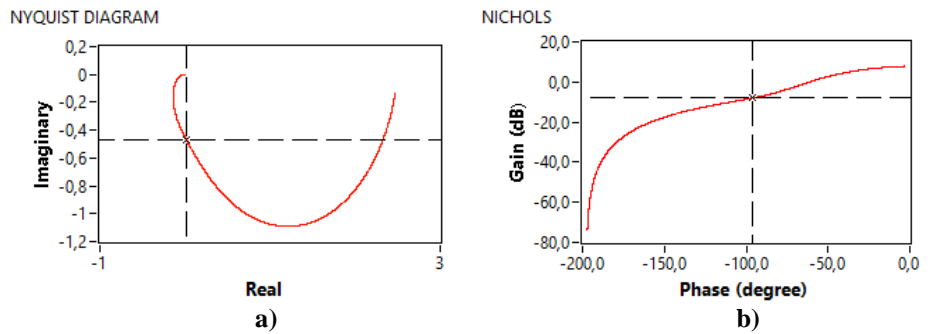


Figure 10. a) Nyquist Diagram b) Nichols Diagram

Example 4: Consider the transfer function with time delay as below,

$$G(s) = \frac{2}{s^{2.4} + 2.5s^{1.2} + s^{0.4}} e^{-2s} \tag{12}$$

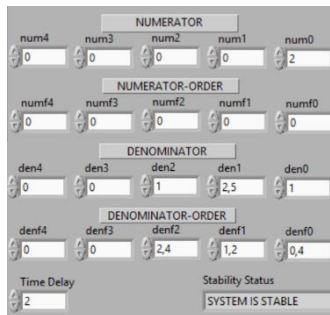


Figure 11. Transfer Function Input Panel for PI Controller Design Application

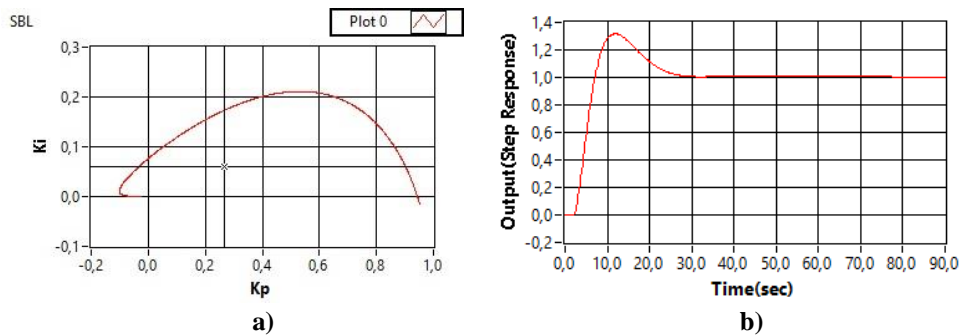


Figure 12. a) Selected Point in Stability Region b) Stable Step Response for Closed Loop System

In this example, PI controller design for fractional order transfer functions is examined. Transfer function given in Eq. (12) is entered to panel as shown in Figure 11. SBL graph is first plotted when the program is run. Then, user can select any points in the controller parameter plane which is shown in Figure 12 (a) by moving mouse on the opened SBL plot window. As seen in Figure 12 (a), selected point is $K_p = 0.262545$, $K_i = 0.06$ and this point is in stability region. Thus, the closed loop system is stable and step response of the system is plotted for the selected point and the result is given in Figure 12 (b). According to selected point, the PI controller is designed as shown in Figure 13.

$$C(s) = \frac{0,262545s + 0,06}{s}$$

Figure 13. Designed PI Controller According to Selected Point

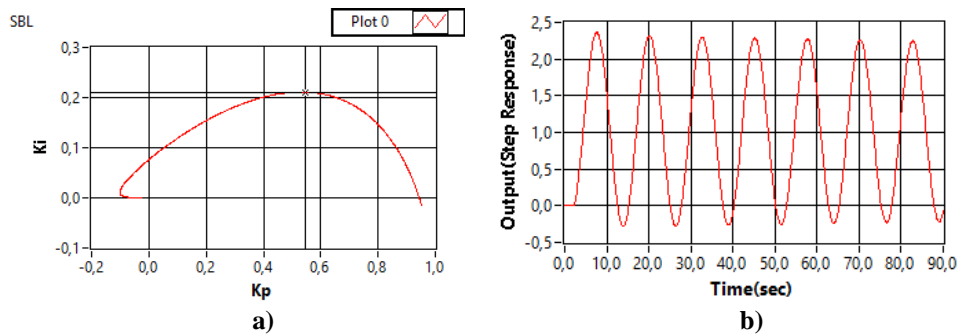


Figure 14. a) Selected Point on Stability Curve b) Critical Step Response for Closed Loop System

Now, consider another point as seen in Figure 14 (a), where selected point is $K_p = 0.546182$, $K_i = 0.21$ and this point is exactly on SBL curve. Thus, the closed loop system is critically stable and step response of the system is plotted for the selected point and the result is given in Figure 12 (b). According to selected point, the PI controller is designed as shown in Figure 15.

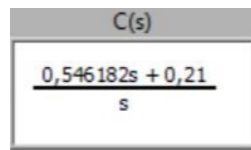


Figure 15. Designed PI Controller According to Selected Point

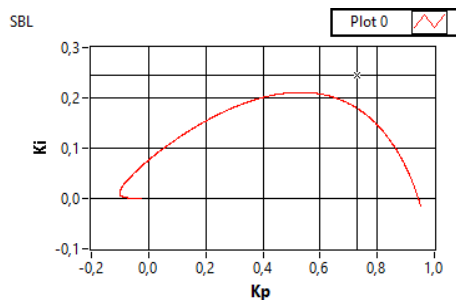


Figure 16. Selected Point Outside Stability Region

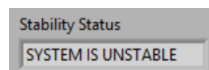


Figure 17. Stability Status Panel

Consider another point as seen in Figure 16, where selected point is outside the SBL curve. Thus, the closed loop system is unstable and step response is not plotted, because the application uses Inverse Fourier Transform Method (IFTM) for plotting the time response and IFTM method is defined in stable condition (Atherton et al, 2015). Thus, the application writes 'SYSTEM IS UNSTABLE' on stability status panel.

CONCLUSION

The objective of this paper has been to draw attention to the new developments in the field of control systems with fractional order derivative and integrator and to show how these ideas can be introduced into a first course on classical control theory. The methods are based on fractional order calculus and allow students to think more practically in terms of representation of real systems with fractional order differential equations. It has also been shown that with suitable software, the theoretical results obtained in the field of fractional order control can be used for the analysis and design of systems.

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