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Abstract

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Research Article

Cognitive and Metacognitive Strategies in Problem-Posing Tasks in the Context of Science*

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Abstract

This study aims to determine the cognitive and metacognitive strategies used by pre-service mathematics teachers for activating mental processes in a semi-structured problem-posing task. A holistic multiple-case design was used to in this study. For the case study, five voluntary pre-service mathematics teachers participated in this study. This task involves problem-posing in the context of science appropriate for different mathematical expressions. A think-aloud protocol, a semi-structured interview, observation and the pieces of papers for each question were used in this study. Open coding was performed using the continuous comparative analysis technique. The main results are that (a) they used various cognitive and metacognitive strategies to activate mental processes in problem-posing, (b) these strategies differed both in diversity and the usage of frequency of them and some strategies are either domain-specific or general-specific and (c) the use of metacognitive strategies is more common than cognitive strategies.

Keywords: Cognitive strategies, mathematical expressions, metacognitive strategies, semi-structured problem-posing, the context of science

1. INTRODUCTION

Studies on science and mathematics education have focused on the need to integrate the disciplines of science and mathematics since the beginning of the 20th century (McBride & Silverman, 1991). This is because while science uses mathematics to calculate and explain relationships between concepts (McBride & Silverman, 1991), science is used as a context to pose mathematical problems (Davison, Miller & Metheny, 1995). In the 21st century, the importance of interdisciplinary education started to increase (Kim & Cho, 2015) with the prevalence of the idea that using concepts from different disciplines can help learners solve real-world problems (Burrows & Slater, 2015). The integration of science and mathematics education has paved the way for STEM education. STEM education emerged with the combination of science, technology, engineering and mathematics disciplines (Dugger, 2010). It is aimed to develop 21st century skills in students through the integration of science and mathematics, as well as technology and engineering (Blackley & Howell, 2019). STEM education is an approach that enhancing critical thinking, problem solving and high-level thinking skills (Gül, 2019). The importance of research on STEM education is increasing internationally (Li, Wang, Xiao & Froyd, 2020; Öztürk & Özdemir, 2020). This study focused on the integration of the disciplines of science and mathematics within the scope of problem-posing tasks.

1.1. Problem-Posing in the Context of Science in Mathematics Education

This study focused on "problem-posing" as a step of problem-solving. Problem-posing was added to Polya's problem-solving methodology as the fifth step by Gonzales (1994). Silver (1994) defined problem-posing as either creating new problems or questions or reforming a given problem to

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investigate a given situation. Stoyanova and Ellerton (1996) classified problem-posing situations according to their structures: (a) problem-posing tasks based on a specific problem as a structured form, (b) problem-posing tasks by giving the student an open-ended situation, by using the individual's own knowledge, skills, and experience as a semi-structured form, and (c) problem-posing tasks by giving a situation related to daily life as free problem-posing.

In problem-posing, informal contexts such as personal experiences, visual expressions such as pictures, graphs, or tables, and symbolic expressions such as equations are used. Additionally, mathematical expressions can be classified as internal and external expressions. Internal expressions address to abstractions of mathematical ideas or cognitive schemata developed through personal experiences, whereas external expressions address the physical depictions of a concept using elements such as numerals, algebraic equations, graphs, and tables (Pape & Tchoshanov, 2001). These expressions are used to improve the conceptual understanding and mathematics achievement of students (Kopparla et al., 2019). The semi-structured problem-posing context was used in this study, as it focused on the task of posing a problem in the context of science appropriate for three different mathematical expressions, which are tabular, graphical, and algebraic expression. Tabular, graphical and algebraic expressions are common expressions in both science and mathematics contexts. These expressions are more suitable for the nature of the semi-structured problem-posing context. Additionally, it was thought that it would be better to use more general expressions to reveal the metacognitive activities of the participants during the implementation.

1.2. Strategies Used in the Problem-Posing Process

In order to be successful in problem-posing, students always ask themselves questions like "What… changed?", "What if…?" and "What if ... not?" when they face a math problem, problem situation, or the answer to a problem (Ghasempour, Bakar & Jahanshahloo, 2013). They also resort to a number of strategies (Ghasempour et al., 2013) such as the "What if" or "What if not" strategy (Brown & Walter, 2005), the imitation strategy (Kojima, Miwa & Matsui, 2009), and the effective questioning strategy (English, 1997). Considering that problem-solving involves the usage of cognitive and metacognitive strategies (Schoenfeld, 1992), it is necessary to use both cognitive and metacognitive strategies in the process of problem-posing.

Performing a task is related to the formation of various mental processes. These processes include cognitive and metacognitive activities (Hıdıroğlu, 2018). For instance, learning is a cognitive process that requires the use of cognitive strategies (Yerdelen-Damar & Eryılmaz, 2021). Cognitive strategies are used to perform a task, while metacognitive strategies are used to understand how the task is performed (Garner, 1987). Cognitive strategies are needed to activate mental processes such as a better understanding of basic concepts and learning and remembering concepts (Leutwyler, 2009). Metacognitive strategies are needed to activate mental process such as becoming aware of one's mental activities, monitoring and evaluating them (Gunstone & Mitchell, 1998), and ensuring one's pre-regulation, directing one's attention, and selective attention and self-management while performing a task (O'Malley & Chamot, 1990). Questions such as "Why do I do something?" or "How is it done?" often trigger metacognition (Larkin, 2009). Cognitive and metacognitive processes can set the stage for each other. For example, cognitive processes can help students become aware of what they do and do not know, which can trigger monitoring and regulation processes. Monitoring and regulation can also improve the quality of the following cognitive processes (Roelle, Nowitzki & Berthold, 2017).

Students actively plan and monitor problem posing process and carry out self-evaluations. While the students use metacognitive strategies in this process, they are constantly engaged in how to understand information, improve a problem posing plan, formulate problems, solve problems and examine solutions (Taufik, Pagiling, Mayasari, Munfarikhatin, Natsir & Dadi, 2019). Although metacognitive strategies play an important role in problem-posing (Ghasempour et al., 2013), the literature on metacognition and problem-posing is very limited (Karnain, Bakar, Siamakani, Mohammadikia & Candra, 2014). Aydoğdu and Türnüklü (2023), examined the problem posing strategies of secondary school students on geometry learning. The study was carried out with 160 middle school students. In line with the findings obtained in the study, it was determined that students used thirteen different problem posing strategies while posing problems. The most frequently used problem posing strategies by the students were the Strategy of Adapting to Daily Life and the Strategy of Drawing a Figure and Posing the Problem According to this Figure Strategy. The problem posing strategies that the students used the least were the Backward Checkout Strategy and the Emotional Approach Strategy. Altun and Yeşilpınar-Uyar (2023) investigated the predictive relations between reading strategies metacognitive awareness and problem posing skills of seventh grade students. 373 seventh grade students participated in the research. As a result of the research, it was determined that reading strategies metacognitive awareness levels of seventh grade students significantly predicted problem posing skills and explained 42% of the change in problem posing skills. The authors recommend that teachers plan and implement interdisciplinary problem posing activities that include the use of different reading and problem posing strategies for the development of reading comprehension and cognitive awareness. It is also unclear which strategies are most effective for teaching problem-posing. In addition, general information about learners' problem posing processes is limited (Cai & Leikin, 2020). There is also limited information about both what these strategies are and the effectiveness of including these strategies in different problem-posing processes of students (Cai, Hwang, Jiang & Silber, 2015).

1.3. Objective and Significance

Problem-posing is an important part of research and practice in school mathematics, and it has been regarded as a critical intellectual activity in scientific research for a long time (Cai et al., 2015). The usage of various strategies in the development of this skill in teacher education is an important goal (Osana & Pelczer, 2015). Considering that metacognition has a critical role in the development of problem-posing skills (Osana & Pelczer, 2015), it is important for future teachers to be aware of the usage of metacognitive strategies in problem-posing. The aim of this study was to determine the cognitive and metacognitive strategies used by pre-service mathematics teachers for activating mental processes in a semi-structured problem-posing task in detail. For this purpose, answers were sought to the following questions in the context of science appropriate for different mathematical expressions: (a) What are cognitive strategies? (b) What are the metacognitive strategies used by pre-service mathematics teachers?

Much more research is needed to develop a widely applicable understanding of basic processes and strategies for problem-posing (Cai et al., 2015). It is thought that this study can contribute in terms of presenting recommendations for (a) developing a taxonomy of cognitive and metacognitive activities in problem-posing and (b) developing a scale for strategies used in problem-posing in the context of science.

2. METHOD

2.1. Research Design

A holistic multiple-case design was used to determine the strategies used by pre-service mathematics teachers to pose problems in the context of science appropriate for different mathematical expressions. In this design, there are multiple situations, and each situation is examined in a holistic way and compared to each other (Yin, 2018). In this study, three different cases -tabular, graphical and algebraic expressions- were investigated. Each case was studied and compared in terms of strategies used in problem-posing.

2.2. Participants

A semi-structured problem-posing test was implemented with 17 pre-service mathematics teachers who were studying at the education faculty of a university in a small city in the northern region of Turkey. For the case study, the criterion sampling method was used because the participants were selected based on the inclusion criterion requiring them to solve at least four items in the problem-posing test. Then, convenience sampling was used, and five voluntary pre-service mathematics teachers, one who had a low score, two who had medium scores, and two who had high scores in the test, were selected. The rate of the participants completing the problem-posing test varied between 40% and 90%. The rate of completion was 90% for tabular expressions, 65% for graphical expressions, and 65% for algebraic expressions. In line with this information, the sample was selected by maximum diversity sampling.

2.3. Data Collection Tools

A problem-posing test consisting of 10 questions and different mathematical expressions - tables, graphs, and algebraic expressions- was prepared by the researchers. In the preparation of the test, the questions including tabular, graphical and algebraic expressions in the science sections of national examinations such as the Examination for Transition from Elementary Education to Secondary Education (ETEESE), the Undergraduate Placement Examination (UPE), and the Field Proficiency Test (FPT) and international examinations such as PISA and TIMMS were examined. Attention was paid to the variety of data in the questions (one-digit positive numbers, two-digit positive numbers, decimal number) and the selection of the questions from basic science subjects on the secondary school level. The participants were asked to pose problems in the context of science related to daily life in accordance with the data in the given expression. Examples of questions in the problem-posing test are given in Figures 1, 7, and 9.

The problem-posing task rubric developed by Rosli, Capraro, Goldsby, y Gonzalez, Onwuegbuzie and Capraro (2015) was used to examine the participants' problem-posing cases. While deciding on the use of this rubric for this study, it was considered that this rubric had the criteria for problem-posing, it was current and comprehensive, and it was developed for pre-service secondary school teachers. This rubric is based on a 4-point scale ranging from 1 to 4 (1: Unsatisfactory and 4: Extended) in ascending order of proficiency. The problems posed by the participants were examined and scored in terms of appropriateness based on their structure/context, scientific concepts, mathematical expressions, and problem-posing design.

In this study, a think-aloud protocol and a semi-structured interview form consisting of eight main questions and some side questions developed by the researchers were used. A think-aloud protocol is when individuals perform a task and verbally express everything that crosses their minds during task performance (Jääskeläinen, 2010). It is used to assess metacognitive activities in educational research (e.g., Bannert & Mengelkamp, 2008). Therefore, in this study, the participants were asked to think aloud while posing problems to define the strategies they used cognitively or metacognitively and determine their purposes for using strategies. While preparing the semi-structured interview form, which was used to determine the cognitive and metacognitive strategies used in the problem-posing process and the purposes of the participants for using them, studies examining strategies used in problem-solving, problem-posing, and reading processes were investigated (e.g., Karnain et al., 2014; Kumlu, 2012; Mishra & Iyer, 2015). This form consisted of fifteen main items and some sub-items. Two of these items related to the planning phase of problem-posing, such as "What did you think when you encountered the tabular/graphical/algebraic expression?", were asked in the form. Six items were related to the stage of organizing the problem. For instance, questions such as "How did you go about posing the problem? Why?", "Did you have any difficulties while posing the problem? In which parts?", "When you became aware of that you were having difficulties, how did you go about solving this problem?" were asked in the form. Two of these items were related to the stage of solving the problem. To exemplify, there were questions such as "Can your problem be solved? Why?", "Can you solve the problem?" Five of these items were related to the phase of correcting the error and completing the problem. For instance, questions such as "Is there something missing or an error in the problem sentence or the solution of the problem you have posed? Why?" were asked in the form. This interview form was administered after the participants had written down and solved each problem-posing question to not affect their problem-posing process.

The problem-posing test and problem-posing process semi-structured interview form were submitted to two experts for their assessments. The domain of one of these experts is problem-solving in mathematics education, and the domain of the other is scientific reasoning skills in science education. After the feedback from the experts, these data collection tools were finalized.

This study used by the participants in posing problems were analyzed as data collection documents.

2.4. Data Collection Process

An interview calendar was created by determining the appropriate day and time for the participants and researchers. Before the interview, information was given about the purpose of the study and the process of thinking aloud. The interviews were recorded after the participants had been informed about recording. The questions in the problem-posing test were given to the participants. Participants were asked to think aloud while posing and solving problems. A piece of paper for each question was given to the participants to write down the problems they planned. These pieces of paper were analyzed as documents. One of the researchers also took notes on the ways the participants used in the problem-posing process and their thoughts that were expressed aloud, and the researcher took the role of an "external observer" by not interfering with the problem-posing process. After the completion of the problem-posing process for each question, the researcher interviewed the participants. The observations and interviews lasted about two hours. These observations and interviews were transcribed.

2.5. Data Analysis

To determine the problem-posing levels of the participants, the problem-posing task rubric developed by Rosli et al. (2015) was used, and a descriptive analysis was performed. Inductive content analysis, which is used to create concepts, categories, and themes from data (Kyngäs, 2020), was utilized to determine in detail the strategies used by the participants in the problem-posing process and their purposes for using these strategies. In the data analysis part of this study, the transcripts of the interviews with the participants, the observation notes of the researcher, and the pieces of paper used by the participants to pose problems were examined.

In this study, open coding was performed using the continuous comparative analysis technique (Straus & Corbin, 1998). In other words, the researchers read the transcript of the participants' problem-posing process line by line, took notes and defined the strategies they used and their purposes for using these strategies. These strategies were coded. In this study, the concept of strategy was discussed as a "general way to fulfill the problem-posing task" (Pelczer, Voica & Gamboa, 2008, p. 98). While naming the strategies, information in the literature on problem-posing, problem-solving, and reading strategies (e.g., Brown & Walter, 2005; Ekici, 2016; Gonzales, 1998; Silver, Mamona-Downs, Leung & Ann-Kenney, 1996) was used. A similar coding process was carried out for the strategy usage purposes of the participants. In the process of coding the purposes of the participants for using the strategies, tasks in the problem-posing steps (Polya, 1957), sub-dimensions of the Problem-Posing Skills Scale proposed by Pilten, Isik and Serin (2017), planning, monitoring, and evaluation processes of metacognitive regulation (Karnain et al., 2014), metacognitive awareness elements (Schraw & Dennison, 1994), cognitive prompts such as organization and elaboration, and metacognitive prompts such as monitoring, self-diagnosis, and planning remedial processes that encourage the use of cognitive and metacognitive strategies (Roelle et al., 2017) were used. Attention was paid to the conceptual nature of the names of the strategies and their intended use. Additionally, the coding of whether the strategies were cognitive or metacognitive was carried out according to the purposes of strategies. If an individual used a strategy to understand, learn, and remember a task, it was cognitive (Leutwyler, 2009), and if they used a strategy for being aware of, monitoring, and evaluating mental activities (Gunstone & Mitchell, 1998), it was metacognitive. For example, if the strategy of questioning was used to make sense of the components of the problem, this strategy was coded as a cognitive strategy. If this strategy was used to decide on the components of the problem or overcome difficulties in deciding on the components of the problem, this strategy was coded as a metacognitive strategy. It was determined that 34 different cognitive strategies were used to perform five different cognitive activities, and 53 different metacognitive strategies were used to perform ten different metacognitive activities.

Performing a task is related to the formation of various mental processes. These processes include cognitive and metacognitive activities (Hıdıroğlu, 2018). In fact, the purpose of using a strategy can be defined operationally as using it to perform a cognitive activity or a metacognitive activity. In this study, the usage purposes of the strategies were addressed based on mental activities that were activated in problem-posing in the results section. Based on this information, the strategies used to perform cognitive activities such as understanding the information that is given, activating prior knowledge about the given information, making sense of the components of the problem, organizing the problem, and realizing the problem's solution from the tasks to be completed regarding the problem-posing steps were coded as cognitive strategies. Deciding on the components of the problem, being aware of the difficulties encountered in deciding the components of the problem and overcoming this difficulty, paying attention to the important elements while organizing the problem, being aware of the difficulty encountered while organizing the problem and overcoming this difficulty, evaluating the correctness/plausibility of the problem's solution, being aware of the errors in the organized problem and monitoring errors (debugging), and evaluating the correctness/plausibility of the organized problem were coded as metacognitive activities The coding sets consisting of sample data segments for each mental activity and each strategy were coded by one of the researchers and an expert of metacognitive strategy in science education. The coders discussed until they reached a satisfactory agreement on the inconsistent coding sets, and the final taxonomy was reached by reviewing the literature. The problem-posing question regarding the context of graphical expression in the problem-posing test is given in Figure 1.

3. Using the data in the graphic on the side, pose a verbal problem about science subjects.

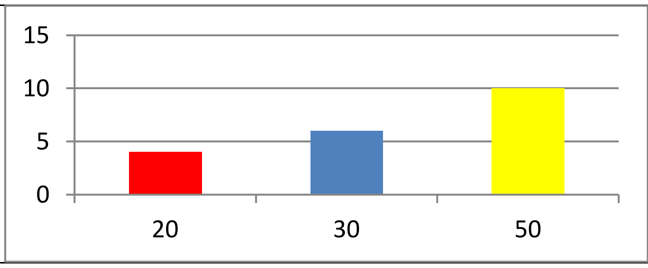
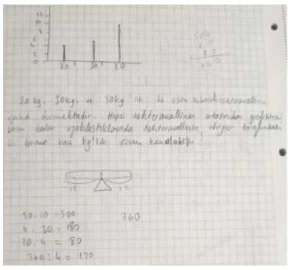


Figure 1. Graphical expression example

The paper of the problem posed by P1 was shown in Figure 2.



(Three objects of 20kg, 30kg, and 50kg put on a 24-unit wooden board. How much do objects put at a 4-unit on the other side of the seesaw must weigh when all present objects are moved away from the center of the seesaw by the unit on the graph?)

Figure 2. An example of p5's paper about the problem-posing in the context of graphical expression given

The sample coding section regarding the various strategies used to perform different activities by P1 who had a medium score from this test and their usage purposes in this process are given below:

P1: The column chart has 20, 30, and 50 at the bottom. 20 intersects with 4 on the y-axis, 30 intersects with 6 on the y-axis, and 50 intersects with 10 on the y-axis. There are 3 columns [Understanding the information given-Cognitive Activity (CA) /Examining the information given-Cognitive Strategy (CS)]. I can also consider the balance board. 20, 30, and 50 can be placed on a balance board. The numbers on the y-axis can also be the units where they stand [Making sense of the components of the problem-Cognitive Activity (CA) / Associating with prior knowledge and experiences-Cognitive Strategy (CS)].

Researcher's observation: The participant writes the problem [Organizing the problem-Cognitive Activity (CA) / Writing the problem designed in one's mind-Cognitive Strategy (CS)]

Researcher: How did you associate the problem with the graph?

P1: I did it by multiplying the intersections of the x-axis and the y-axis, which are marked directly. So, I came up with such a question from there [Making sense of the components of the problem -Cognitive Activity (CA) / Finding correlation -Cognitive Strategy (CS)].

Researcher: So, what did you think while writing? How did you decide on the balance problem?

P1: If there are small numbers and large numbers, my mind will go to balance questions, if there are numbers close to each other, I guess I will think of speed questions. Here, I thought of posing a balance problem since the y-axis consists of small numbers and the x-axis consists of large numbers [Deciding on the components of the problem-Metacognitive Activity (MCA) / Choosing the concepts according to the mathematical characteristics of the information given-Metacognitive Strategy (MCS)].

The cognitive and metacognitive activities of the participants and the strategies they used to perform these mental activities that were identified as a result of the analysis of the data are presented in the results section as a figure. Moreover, the frequencies of using cognitive and metacognitive strategies are presented as a word cloud. In the word cloud, the higher the frequency of using the strategy, the larger the font. While calculating these frequencies, the numbers of strategies used by five participants in the problem-posing test that included ten questions were added.

2.6. Validity and Reliability of the Study and Ethics

Triangulation, maximum variation, adequate engagement in data collection, rich and thick descriptions, and audit trail, which are strategies for promoting validity and reliability (Merriam & Tisdell, 2016), were used in this study. The triangulation method was employed by using multiple data collection tools which were semi-structured interviews, observations, and problem-posing sheets, and the think-aloud protocol was used to verify obtaining findings. Through data triangulation, the cognitive/metacognitive strategies and mental activities that become activated were coded, and reliable and common evidence was achieved.

The maximum variation strategy was used in sample selection. In this study, the participants had different problem-posing levels and their rates of completing the problem-posing test varied. One of the researchers spent two hours for each participant while interviewing them in line with the adequate engagement in data collection strategy. An 85-page observation and interview transcript were obtained in the study, indicating highly rich and dense descriptive data. The information about the usage of the audit trail strategy is available in the data collection process and data analysis section.

With regard to ethics, first of all, the necessary permissions were obtained from the Human Research Ethics Committee (letter dated 18.12.2020 and numbered 2020-135). Moreover, pre-service teachers who voluntarily agreed to participate in the study were included, and the identifying information of the participants was kept confidential by assigning them codes from P1 to P5.

3. FINDINGS

The mental processes that became activated and strategies that the participants used in problem-posing in the context of science appropriate for different mathematical expressions were examined within the scope of this study. The mental processes that were examined included cognitive activities and metacognitive activities. Information about the cognitive activities and the frequency of using of the cognitive strategies while posing problems in the context of different mathematical expressions are given in Table 1.

Table 1. The cognitive activities and the frequency of using of the cognitive strategies in problem posing

Cognitive activities	Cognitive strategies	Table		Graphic		Algebraic expression		Total	
		f	%	f	%	f	%	f	%
Understanding the information given	Estimating	1	0.90	0	0.00	0	0.00	6	1.47
	Examining the information given	8	7.21*	19	11.59*	19	14.96*	46	11.30*
	Finding correlations	8	7.21*	14	8.54*	15	11.81*	37	9.09
	Total	17	15.32	33	20.12	34	26.77	89	21.87
Activating his/her prior knowledge about the information given	Associating with previous problems	2	1.80*	3	1.83*	0	0.00	5	1.23*
	Associating with prior knowledge and experience	2	1.80*	4	2.44*	1	0.79*	7	1.72*
	Relating with formulas	1	0.90	1	0.61	1	0.79*	3	0.74
	Questioning	0	0.00	1	0.61	0	0.00	1	0.25
	Total	5	4.50	9	5.49	2	1.57	16	3.93
Making sense of the components of the problem	Associating with daily life	0	0.00	4	2.44	0	0.00	4	0.98
	Associating with previous problems	1	0.90	1	0.61	1	0.79	3	0.74
	Associating with prior knowledge and experience	6	5.41*	9	5.49*	4	3.15	19	4.67
	Envisioning the problem	7	6.31*	9	5.49*	6	4.72	22	5.41
	Estimating appropriate concepts to the given information	8	7.21*	13	7.93*	14	11.02*	35	8.60
	Finding correlations	2	1.80	5	3.05	3	2.36	10	2.46
	Making sense of the information given	10	9.01*	21	12.80*	8	6.30*	39	9.58
	Relating with different discipline issues	3	2.70	2	1.22	0	0.00	5	1.23
	Relating with formulas	4	3.60	3	1.83	3	2.36	10	2.46
	Questioning	1	0.90	5	3.05	0	0.00	6	1.47
	Total	42	37.84	72	43.90	39	30.71	153	37.59
Organizing the problem	Assigning symbolic representations to concepts in the problem	1	0.90	2	1.22	0	0.00	3	0.74
	Associating with daily life	1	0.90	1	0.61	1	0.79	3	0.74
	Finding correlations	1	0.90	0	0.00	1	0.79	2	0.49
	Making sense of the components of the problem	7	6.31*	4	2.44	6	4.72*	17	4.18
	Making sense of the information given	4	3.60*	5	3.05*	5	3.94*	14	3.44
	Making the operation easier	2	1.80	1	0.61	1	0.79	4	0.98
	Relating information given with the other components of the problem	0	0.00	3	1.83	5	3.94*	8	1.97
	Relating with different discipline issues	0	0.00	1	0.61	2	1.57	3	0.74
	Relating with formulas	1	0.90	0	0.00	1	0.79	2	0.49
	Sampling	0	0.00	1	0.61	0	0.00	1	0.25
	Sampling the information given	1	0.90	1	0.61	0	0.00	2	0.49
	Using mathematical structure in the solution	1	0.90	2	1.22	0	0.00	3	0.74
	Using scientific expressions	2	1.80	0	0.00	0	0.00	2	0.49
	Using scientific notation	0	0.00	0	0.00	2	1.57	2	0.49

Table 1. (Continued)

Cognitive activities	Cognitive strategies	Table		Graphic		Algebraic expression		Total	
		f	%	f	%	f	%	f	%
Organizing the problem	Using symbols	1	0.90	0	0.00	2	1.57	3	0.74
	Using the mathematical structure of the information given	0	0.00	0	0.00	1	0.79	1	0.25
	Using unit	0	0.00	1	0.61	1	0.79	2	0.49
	Writing the problem conceived in the mind	6	5.41*	11	6.71*	4	3.15	21	5.16
	Writing without thinking	2	1.80	3	1.83	7	5.51*	12	2.95
	Total	30	27.03	36	21,95	39	30.71	105	25.80
Realizing the problem solution	Associating with prior knowledge and experience	2	1.80	1	0.61	0	0.00	3	0.74
	Converting the unit	1	0.90	0	0.00	0	0.00	1	0.25
	Establishing equality	1	0.90	0	0.00	0	0.00	1	0.25
	Expressing the steps of the process in one's own sentences	1	0.90	1	0.61	0	0.00	2	0.49
	Finding correlations	3	2.70*	2	1.22*	0	0.00	5	1.23
	Interpreting the information given with concepts	1	0.90	1	0.61	0	0.00	2	0.49
	Percentage calculation	1	0.90	0	0.00	0	0.00	1	0.25
	Proportioning	1	0.90	3	1.83*	4	3.15*	8	1.97
	Using formulas	4	3.60*	6	3.66*	9	7.09*	19	4.67
	Using four operations	2	1.80	0	0.00	0	0.00	2	0.49
	Total	17	15.32	14	8.54	13	10.24	44	10.81
	General Total	111	100	164	100	127	100	407	100

** It shows that the cognitive strategies frequently used in each cognitive activity category for each representation.*

As seen Table 1, the participants used various cognitive strategies to perform cognitive activities which included understanding the information that is given, activating one's prior knowledge about the information given, making sense of the components of the problem, organizing the problem, and realizing the problem's solution in the problem-posing process. The number of cognitive strategies used to perform each cognitive activity varied. For example, three different strategies for understanding the information given and nineteen different strategies for organizing the problem were defined. It was found that some of the cognitive strategies were used to activate only one cognitive activity, and some were used to activate multiple cognitive activities. For example, the strategy of examining the information that is given was used only to understand the information given, and the strategy of relating with the formulae was used to activate one's prior knowledge about the information given, make sense of the components of the problem, and organize the problem. Furthermore, it was determined that the frequency of using of some strategies such as making sense of the information given and estimating the appropriate concepts to the information given in order to make sense of the components of the problem was high in performing some activities in all three mathematical representations. The finding correlations strategy was the most frequently used strategy. This was followed by making sense of the information that is given, examining the information given, estimating the appropriate concepts, associating with prior knowledge and experience, visual imagery of the problem, writing the problem conceived in the mind, using formulae, and making sense of the components of the problem. The frequencies of using these strategies while posing problems were higher than those of other cognitive strategies.

The number of different cognitive strategies used in problem posing in the context of table representation given was the highest in terms of diversity (f=29 and 85% for table, f=25 and 74% for graphic, f=21 and 62% for algebraic expression). The frequency of using cognitive strategies in problem posing in the context of graphic representation given was the highest (f=111 and 27% for table, f=164 and 40% for graphic and f=127 and 31% for algebraic expression). The use of strategies for examining information given, finding correlations, estimating appropriate concepts, making sense of the components of the problem, making sense of information given, and using formulas were higher than other cognitive strategies in all three representations. There were also strategies that were commonly used in all three mathematical representations, but were more frequently used in one or both. For example, the strategy of associating with previous problems was more frequently used in the context of the table and the graph, writing without thinking was more frequently used in the context of the algebraic expression in problem posing. It was determined that the participants carried out metacognitive activities to decide on the components of the problem, be aware of the difficulties encountered in deciding the components of the problem and organizing the problem and overcome these difficulties, pay attention to the important elements while organizing the problem, be aware of the errors in the organized problem, monitor the errors in the organized problem, and assess the accuracy/plausibility of the solution and of the organized problem. The strategies used to perform these activities are defined as metacognitive strategies. The usage frequencies of the metacognitive strategies used to perform these metacognitive activities while posing problems in the context of different mathematical expressions are given in Table 2.

Table 2. The metacognitive activities and the frequency of using of the metacognitive strategies in problem posing

Metacognitive activities	Metacognitive Strategies	Table		Graphic		Algebraic expression		Total	
		f	%	f	%	f	%	f	%
Deciding on the components of the problem	Choosing the concepts according to the mathematical characteristics of the information given	5	4.39*	14	8.28*	10	9.01*	29	7.36
	Interpreting the information given with concepts	5	4.39*	8	4.73*	4	3.60	17	4.31
	Questioning	1	0.88	4	15.38	9	8.11*	14	3.55
	Total	11	9.65	26	1.18	23	20.72	60	15.23
Being aware of the difficulties encountered in deciding the components of the problem	Comparing the consistency of the information given with the concepts	1	0.88	2	15.38	0	0.00	3	0.76
	Questioning	7	6.14*	23	13.61*	12	10.81*	42	10.66
	Self-questioning	1	0.88	2	1.18	3	2.70	6	1.52
	Total	9	7.89	27	15.98	15	13.51	51	12.94
Overcoming difficulties in deciding on the components of the problem	Arranging the root of the question in accordance with the information given	0	0.00	1	0.59	0	0.00	1	0.25
	Associating with previous situations	1	0.88	1	0.59	0	0.00	2	0.51
	Changing the concepts associated with the information given	5	4.39*	9	5.33*	4	3.60*	18	4.57
	Changing the question root	1	0.88	0	0.00	0	0.00	1	0.25
	Choosing the concepts according to the mathematical characteristics of the information given	0	0.00	1	0.59	3	2.70	4	1.02
	Interpreting the information given with concepts	0	0.00	1	0.59	0	0.00	1	0.25
	Limiting the concepts with which the information given is associated	0	0.00	1	0.59	0	0.00	1	0.25
	Making additions	0	0.00	2	1.18	0	0.00	2	0.51
	Questioning	1	0.88	3	1.78	2	1.80	6	1.52
	Quitting	1	0.88	6	3.55*	7	6.31*	14	3.55
	Relating with different discipline issues	1	0.88	0	0.00	0	0.00	1	0.25
	Review	2	1.75*	4	2.37*	0	0.00	6	1.52
	Writing the problem conceived in the mind	1	0.88	0	0.00	0	0.00	1	0.25
	Total	13	11.40	29	17.16	16	14.41	58	14.72

Table 2. (Continued)

Metacognitive activities	Metacognitive Strategies	Table		Graphic		Algebraic expression		Total	
		f	%	f	%	f	%	f	%
Paying attention to the important elements while organizing the problem	Associating with daily life	0	0.00	1	0.59	0	0.00	1	0.25
	Choosing a solution according to the mathematical characteristics of the information given	0	0.00	1	0.59	0	0.00	1	0.25
	Comparing the consistency of the information given with the context of the problem	1	0.88	1	0.59	0	0.00	2	0.51
	Differentiating the question pattern	4	3.51*	0	0.00	0	0.00	4	1.02
	Distinguishing necessary /unnecessary information	1	0.88	1	0.59	0	0.00	2	0.51
	Highlighting key elements related to the problem	1	0.88	0	0.00	0	0.00	1	0.25
	Increasing the number of operation steps	0	0.00	1	0.59	1	0.90	2	0.51
	Interpreting key elements related to the problem	7	6.14*	11	6.51*	3	2.70*	21	5.33
	Interpreting the difficulty of the problem	1	0.88	0	0.00	0	0.00	1	0.25
	Questioning	0	0.00	1	0.59	0	0.00	1	0.25
	Relating the context of the problem to its solution	0	0.00	3	1.78	2	1.80	5	1.27
	Trying different solutions	0	0.00	1	0.59	0	0.00	1	0.25
	Using explanatory statements	4	3.51*	6	3.55*	6	5.41*	16	4.06
	Visualization	1	0.88	0	0.00	0	0.00	1	0.25
	Total	20	17.54	27	15.98	12	10.81	59	14.97
Being aware of the difficulties encountered in organizing the problem	Questioning	5	4.39*	2	1.18	0	0.00	7	1.78
	Self-questioning	2	1.75	3	1.78*	0	0.00	5	1.27
	Total	7	6.14	5	2.96	0	0.00	12	3.05
Overcoming difficulties in organizing the problem	Adding variables	1	0.88	0	0.00	0	0.00	1	0.25
	Assigning symbolic representations to concepts in the problem	2	1.75*	0	0.00	0	0.00	2	0.51
	Associating with previous situations	1	0.88	0	0.00	0	0.00	1	0.25
	Changing the context of the problem	1	0.88	0	0.00	0	0.00	1	0.25
	Changing the question root	2	1.75*	1	0.59*	0	0.00	3	0.76
	Limiting the concepts with which the information given is associated	0	0.00	1	0.59*	0	0.00	1	0.25
	Limiting the number of conditions added to the problem	1	0.88	0	0.00	0	0.00	1	0.25
	Self-questioning	1	0.88	0	0.00	0	0.00	1	0.25
	Total	9	7.89	2	1.18	0	0.00	11	2.79
Assessing the accuracy/ plausibility of the solution	Comparing the consistency of the answer with the expected answer	2	1.75	0	0.00	0	0.00	2	0.51
	Comparing the consistency of the answer with the root of the problem	2	1.75	0	0.00	0	0.00	2	0.51
	Crosschecking	1	0.88	0	0.00	3	2.70*	4	1.02
	Free from calculation errors	1	0.88	2	1.18	3	2.70*	6	1.52
	Interpreting the answer in real life context	0	0.00	1	0.59	2	1.80	3	0.76
	Interpreting the answer with concepts	0	0.00	0	0.00	2	1.80	2	0.51
	Interpreting the solution process	4	3.51*	7	4.14*	3	2.70*	14	3.55
	Quitting	0	0.00	1	0.59	0	0.00	1	0.25
	Self-questioning	4	3.51*	5	2.96*	3	2.70*	12	3.05
	Trying different solutions	2	1.75	2	1.18	1	0.90	5	1.27
	Total	16	14.04	18	10.65	17	15.32	51	12.94

Table 2. (Continued)

Metacognitive activities	Metacognitive Strategies	Table		Graphic		Algebraic expression		Total	
		f	%	f	%	f	%	f	%
Being aware of the errors in the organized problem	Comparing question root with solution consistency	1	0.88*	0	0.00	1	0.90	2	0.51
	Comparing the consistency of the information given with the context of the problem	1	0.88*	0	0.00	3	2.70*	4	1.02
	Comparing the consistency of the solution and the context of the problem	1	0.88*	0	0.00	0	0.00	1	0.25
	Questioning	0	0.00	1	0.59*	0	0.00	1	0.25
	Review	0	0.00	1	0.59*	0	0.00	1	0.25
	Total	3	2.63	2	1.18	4	3.60	9	2.28
Monitoring the errors in the organized problem / Debugging	Adding conditions to the problem	0	0.00	1	0.59*	0	0.00	1	0.25
	Changing the context of the problem	1	0.88	0	0.00	0	0.00	1	0.25
	Making inferences	1	0.88	0	0.00	0	0.00	1	0.25
	Organizing the components of the problem	2	1.75*	1	0.59*	3	2.70*	6	1.52
	Review	1	0.88	0	0.00	0	0.00	1	0.25
	Total	5	4.39	2	1.18	3	2.70	10	2.54
Assessing the accuracy/ plausibility of the organized problem	Adding different conditions to the problem	0	0.00	2	1.18	0	0.00	2	0.51
	Adding thought-provoking expressions	0	0.00	2	1.18	1	0.90	3	0.76
	Associating with daily life	0	0.00	0	0.00	2	1.80	2	0.51
	Changing the context of the problem	0	0.00	1	0.59	0	0.00	1	0.25
	Changing the question root	1	0.88	0	0.00	1	0.90	2	0.51
	Comparing the consistency of the information given with daily life	2	1.75	0	0.00	0	0.00	2	0.51
	Comparing the consistency of the information given with the context of the problem	1	0.88	2	1.18	3	2.70*	6	1.52
	Converting the unit	0	0.00	1	0.59	0	0.00	1	0.25
	Distinguishing necessary/unnecessary information	2	1.75	1	0.59	0	0.00	3	0.76
	Highlighting key elements related to the problem	0	0.00	1	0.59	0	0.00	1	0.25
	Increasing the number of operation steps	0	0.00	1	0.59	0	0.00	1	0.25
	Interpreting key elements related to the problem	0	0.00	1	0.59	5	4.50*	6	1.52
	Making additions	4	3.51*	11	6.51*	4	3.60*	19	4.82
	Making changes in the expressions in the problem	2	1.75	3	1.78*	0	0.00	5	1.27
	Making the operation difficult	0	0.00	1	0.59	0	0.00	1	0.25
	Questioning	2	1.75	2	1.18	2	1.80	6	1.52
	Rereading	2	1.75	0	0.00	0	0.00	2	0.51
	Review	2	1.75	0	0.00	2	1.80	4	1.02
	Self-questioning	3	2.63*	2	1.18	0	0.00	5	1.27
	Visualization	0	0.00	0	0.00	1	0.90	1	0.25
	Total	21	18.42	31	18.34	21	18.92	73	18.53
	General Total	114	100	169	100	111	100	394	100

** It shows that the metacognitive strategies frequently used in each metacognitive activity category for each representation.*

In Table 2, it is seen that the number of metacognitive strategies used to perform each metacognitive activity differed. For example, two different strategies for being aware of the difficulties encountered in organizing the problem, and twenty different strategies to assess the accuracy/plausibility of the organized problem were identified. Furthermore, it was determined that some of the metacognitive strategies were used to perform only one metacognitive activity, and some to perform more than one metacognitive activity. For example, the strategy of adding variables was used only to overcome the difficulties encountered in the problem posing, and the strategy of choosing concepts according to the mathematical characteristics of the information given was used both in deciding the components of the problem and overcoming the difficulties in deciding on the components of the problem. Moreover, it was observed that the frequency of using of some strategies, such as the strategy of choosing concepts according to the mathematical characteristics of the information given in order to decide on the components of the problem was high in performing in all three mathematical representations. The variety of metacognitive strategies used in problem posing in table representation given was the most and the least for algebraic expression (*f*=39 and 73.5% table, *f*=34 and 64% for graphic, *f*=25 and 47% for algebraic expression). The frequency of using metacognitive strategies for graphic representation given was highest, and it was least for algebraic expression (*f*=114 and 29% for table, *f*=169 and 43% for graphic and *f*=111 and 28% for algebraic expression). The frequency of using *choosing the concepts according to the mathematical characteristics of the information given, questioning, changing the concepts associated with the information given, interpreting the key elements of the problem, using explanatory statements, interpreting the solution process, self-questioning, organizing the components of the problem and making additions,* was higher than the other metacognitive strategies in all three representations. It was determined that a strategy was specific to only one or two mathematical representations. For instance, visualization was specific to only in the context of table, crosschecking strategy was used in the context of both table and algebraic expression.

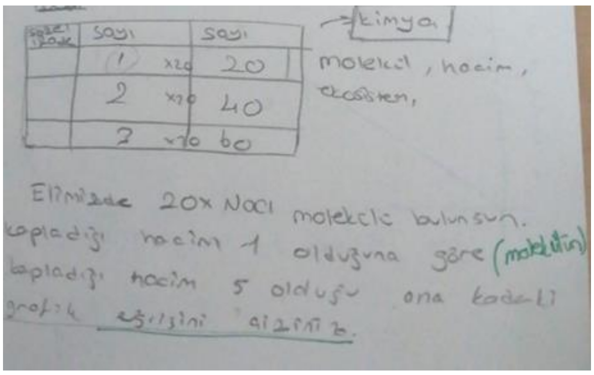
The problem-posing question regarding the context of tabular expressions in the problem-posing test is given in Figure 3.

Verbal expression	Number	Number
	1	20
	2	40
	3	60

2. Using the data in the table on the side, pose a verbal problem about science subjects.

Figure 3. Tabular expressions example

The paper of the problem posed by P5 was shown in Figure 4.



(Suppose we have 20X NaCl molecules. Since the volume occupied by this molecule is 1, draw the graph curve until the volume occupied by the molecule is 5.)

Figure 4. An example of p5's paper about the problem-posing in the context of table given

The sample section regarding the mental activities and various cognitive and metacognitive strategies used by P5 who had a low score from this test is given below:

P5: *... again, there are 4 rows and 3 columns [[here]]. [[There is]] a ratio, a proportion. For instance, from 20 to 40, the constant increases by 20, that is, there is an increase. 1, 2, and 3 increase by [[/in increments of]] 1.* [Understanding the information that is given - Finding Correlations (CS)] *How can we pose this problem? I thought of an increase in diversity as the number of living beings increases. When [[the numbers of]] plants and animals increase, the number of living animals increases, our ecosystem is [[/becomes]] broader, [[it is described that]] [[it]] becomes the number 60. Can it be like this? You know, they say there are more animals in rainforests but fewer animals in deserts.* [Making sense of the components of the problem - Associating with prior knowledge and experiences, Questioning, Making sense of the information that is given (CS)] (Observation – Thinking aloud protocol)
R: *What way did you go about when writing the problem down? Why did you include the concept of molecules in chemistry?*
P5: *...because I thought I could associate the numbers with chemistry. There was a ratio between the numbers, a regular proportion. You see, 1, 2, 3… it was increasing by 20 on the other side. This is why I thought [[about it]] like this.* [Overcoming the difficulties encountered in deciding on the components of the problem - Choosing the concepts according to the mathematical characteristics of the information that is given (MCS)] (Interview)

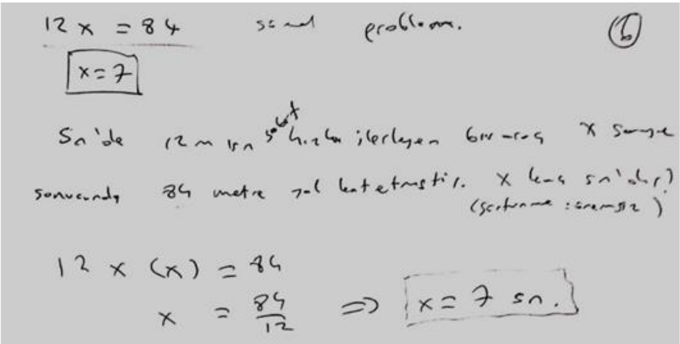
It may be stated that P5 had difficulty in defining the root of the question and changed the concepts to overcome this, and they chose the concept according to the mathematical characteristics of the information that was given. The task of posing a problem in the context of science and algebraic expressions is given in Figure 5.

7. 12.x = 84

Using the data in the equation above, pose a verbal problem about science subjects.

Figure 5. Algebraic expressions example

The paper of the problem posed by P2 was shown in Figure 6.



(A vehicle moving at a constant speed of 12 m/s covered a distance of 84 meters in x seconds. What is x? (Friction is negligible)

Figure 6. An example of p4's paper about the problem-posing in the context of algebraic expression given

The sample section regarding the mental activities and various cognitive and metacognitive strategies used by P2 who had a high score from this test is given below:

R: *Why did you choose the subject of speed?*
P2: *Because there were two multiplication operations side by side in the equation. From the simplest, the speed formulas came to my mind directly. The distance traveled is equal to time times speed. X=V.t. Since there are 2 multiplication operations here, I said that this question can be written directly.* [Deciding on the components of the problem - Choosing the concepts according to the mathematical characteristics of the information given (MCS)].
R: *Well, when you look at your problem, are there parts that you would like to make corrections or missing parts?*

P2: *There is no problem with the problem, but for example, it could have been asked graphic… just changing the form of my problem would have been better. I also verbally said that the thing is moving at a speed of 12 m/s, but I should have stated that it was moving steadily there. For example, I did not write. Friction is negligible. It had to be mentioned as well.* [Assessing the correctness / plausibility of the organized problem - Changing the question root, Adding different conditions to the problem (MCS)]

It may be stated that P2 used the strategy of choosing the concepts according to the mathematical characteristics of the information that is given to perform the activity of deciding on the components of the problem. It may be concluded that they used the strategies of changing the question's root and adding different conditions to the problem to assess the correctness/plausibility of the organized problem.

4. DISCUSSION and CONCLUSION

In this study, three main results were reached. The first of these was that the participants used various cognitive and metacognitive strategies to activate their mental activities. This situation can be explained by the possibility that the participants had the strategy repertoire (Hartman, 2001a) and prior knowledge and experience about learning the usage of strategies. Learning to use strategies can take place through teacher modeling (Van Keer, 2004) or interaction with a teacher, peer, family member, or an older person (Paris & Hamilton, 2009).

The second result of the study was that cognitive strategies such as finding correlations, examining the information that is given, estimating the appropriate concepts for the information that is given and metacognitive strategies such as questioning, choosing the concepts according to the mathematical characteristics of the information given, self-questioning, and interpreting the key elements of the problem were used more than other strategies. Some other studies in the literature on reading, problem-solving, and problem-posing also reported that students use these strategies (e.g., Gonzales, 1998; Meijer, Veenman & van Hout-Wolters, 2006; Silver et al., 1996). The high usage frequencies of some strategies can be explained by that these strategies are both domain-specific and general-specific (Hartman, 2001b). Learners can adapt the strategies they use to complete other tasks to the problem-posing task. Moreover, some strategies such as questioning and associating with previous situations were used to perform both cognitive and metacognitive activities. There are studies showing that strategies can be both cognitive and metacognitive depending on their purposes of usage (e.g., Kumlu, 2012). Furthermore, the variety and frequency of using metacognitive strategies are higher than the variety and frequency of using cognitive strategies. This is because problem-posing is not just about completing the mathematical activity. It focuses more on the relationships between mathematical ideas, and as a result, it triggers high-level thinking, different thinking, and metacognitive skills (Ghasempour et al., 2013).

The third result of this study was that the strategies used to perform each mental activity differed in diversity. This may be because there are many potential processes in problem-posing, and these vary depending on the type of problem that is being addressed (Cai et al., 2015). Moreover, the nature of the task (Duncan & McKeachie, 2005), the learner's perceptions of the difficulty of the task (Meijer et al., 2006; Oxford, 1990), and the learner's need to complete the task (Alavi & Kaıvanpanah, 2006) affect their usage of strategies.

To summarize the results of the study in general, the pre-service mathematics teachers used both cognitive and metacognitive strategies to perform various mental activities during the problem-posing process. Since problem-posing is a high-level skill, the use of metacognitive strategies is more common. Using a large number of strategies also indicates that individuals have a repertoire of strategies for problem-posing. While some of these strategies are general strategies such as those used in reading and problem-solving, some are specific to problem-posing.

This study was limited to semi-structured problem-posing, pre-service mathematics teachers, and cognitive and metacognitive activities and strategies in this context. Moreover, three different mathematical expressions, tabular, graphical, and algebraic expressions, were focused on in the study. Similar studies can be performed with different types of mathematical expressions, e.g., symbols, concrete objects, and pictures. This study can be implemented with primary and secondary school students and pre-service science teachers. By making use of the findings of this study, a metacognitive awareness scale for problem-posing can be developed.

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Ethics Committee Decision

Ethical approval and written permission for this study were obtained from the Social and Human Research Ethics Committee of Sinop University with the decision dated 18/12/2020 and numbered 2020/135.

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