



## TESTING FOR SLOPE HOMOGENEITY IN DYNAMIC PANELS USING THE WILD BOOTSTRAP $\tilde{\Delta}_{adj}$ TEST

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### Abstract

This paper considers testing for slope homogeneity in dynamic panel data models with heteroskedastic disturbances. We investigate the use of a wild bootstrap framework is suggested to approximate the distribution of the  $\tilde{\Delta}_{adj}$  statistic, proposed by Pesaran and Yamagata (Pesaran, M.H., Yamagata, T., Testing slope homogeneity in large panels, Journal of Econometrics 142, 50–93, 2008). Simulation experiments show that the wild bootstrap  $\tilde{\Delta}_{adj}$  test performs well in this context.

**Keywords:** Testing slope homogeneity,  $\tilde{\Delta}_{adj}$  test, dynamic panels, wild bootstrap  
**Jel Classification:** C12, C15, C33

### 1. INTRODUCTION

This study proposes tests for slope parameter homogeneity in the following stationary AR(1) panel data model:

$$y_{it} = \alpha_i(1 - \beta_i) + \beta_i y_{i,t-1} + \varepsilon_{it} \quad i = 1, \dots, N; t = 1, \dots, T \quad (1)$$

where  $\alpha_i$  the individual effect and  $\varepsilon_{it}$  is the error term. Testing slope homogeneity is

important because the assumption of the unknown slope coefficients  $\beta_i$  is to be constant across cross-sections and over time does not always hold. Various statistics for testing slope homogeneity in panel data models have been proposed, including Robertson and Symons (1992), Pesaran and Smith (1995), Pesaran et al. (1996), Phillips and Sul (2003), Pesaran and

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Yamagata (2008). The testing procedure of Pesaran and Yamagata (2008) is based on assumption of serially uncorrelated errors in dynamic models. In this approach regression errors that are heteroskedastic is permitted. As yet, however, there have been small sample bias. A technique used to overcome the unknown form of heteroskedastic errors is the so-called wild bootstrap, developed by Liu (1988), Wu (1986) and Beran (1986). Mammen (1993) provides further evidence of this approach. Liu and Mammen also propose that the asymptotic distribution of various statistics is the same as the asymptotic distribution of their wild bootstrap counterparts. Thus, this paper focuses on the heteroskedasticity-robust  $\tilde{\Delta}_{adj}$  test in autoregressive panels.

In this work, via Monte Carlo study, we want to see if a bootstrap approach can improve the finite sample properties of  $\tilde{\Delta}_{adj}$  test in dynamic models. The plan of the paper is the following. In Section 2 we introduce the model and test statistic. In Section 3 we deal with the problem of bootstrapping when the data generating process has heteroskedasticity. Section 4 presents the Monte Carlo study and concludes.

## 2. MODEL AND TEST STATISTIC

Stacking the time series observations in model (1) for  $i$  :

$$\mathbf{y}_i = \alpha_i(1 - \beta_i)\boldsymbol{\tau}_T + \beta_i\mathbf{y}_{i-1} + \boldsymbol{\varepsilon}_{it} \quad i = 1, \dots, N; \quad (2)$$

where  $\mathbf{y}_{i-1} = (y_{i0}, y_{i1}, \dots, y_{iT-1})'$  and  $\boldsymbol{\tau}_T$  is a  $T \times 1$  vector of ones.

In this set up the null hypothesis of interest is

$$H_0: \beta_i = \beta \quad \text{for all } i = 1, \dots, N \quad (3)$$

And the alternative hypothesis is

$$H_1: \beta_i \neq \beta_j \quad \text{for some } i \neq j \tag{4}$$

In this dynamic case the modified version of the Swamy test statistic is given by

$$\tilde{S} = \sum_{i=1}^N \tilde{\sigma}_i^{-2} (\hat{\beta}_i - \tilde{\beta}_{WFE})^2 (\mathbf{y}'_{i-1} \mathbf{M}_\tau \mathbf{y}_{i-1}) \tag{5}$$

where the individual slope estimator is written as

$$\hat{\beta}_i = (\mathbf{y}'_{i-1} \mathbf{M}_\tau \mathbf{y}_{i-1})^{-1} \mathbf{y}'_{i-1} \mathbf{M}_\tau \mathbf{y}_{i-1} \tag{6}$$

and

$$\tilde{\beta}_{WFE} = \left( \sum_{i=1}^N \frac{\mathbf{y}'_{i-1} \mathbf{M}_\tau \mathbf{y}_{i-1}}{\tilde{\sigma}_i^2} \right)^{-1} \sum_{i=1}^N \frac{\mathbf{y}'_{i-1} \mathbf{M}_\tau \mathbf{y}_{i-1}}{\tilde{\sigma}_i^2} \tag{7}$$

$$\tilde{\sigma}_i^2 = \frac{(\mathbf{y}_i - \tilde{\beta}_{FE} \mathbf{y}_{i-1})' \mathbf{M}_\tau (\mathbf{y}_i - \tilde{\beta}_{FE} \mathbf{y}_{i-1})}{T - 1} \tag{8}$$

where  $\tilde{\beta}_{FE} = (\sum_{i=1}^N \mathbf{y}'_{i-1} \mathbf{M}_\tau \mathbf{y}_{i-1})^{-1} \sum_{i=1}^N \mathbf{y}'_{i-1} \mathbf{M}_\tau \mathbf{y}_{i-1}$  with  $\mathbf{M}_\tau = \mathbf{I}_T - \boldsymbol{\tau}_T (\boldsymbol{\tau}'_T \boldsymbol{\tau}_T)^{-1} \boldsymbol{\tau}'_T$ ,  $\mathbf{I}_T$  is an identity matrix of order  $T$ .

The  $\tilde{\Delta}_{adj}$  testing approach of Pesaran and Yamagata (2008) is constructed as<sup>1</sup>

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<sup>1</sup>  $E(\tilde{z}_{it}) = k$  and  $Var(\tilde{z}_{it}) = \frac{2k(\tau-k-1)}{\tau+1}$ , defined by Pesaran and Yamagata (2008).



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$$\tilde{\Delta}_{adj} = \sqrt{N} \left( \frac{N^{-1} \tilde{S} - E(\tilde{z}_{iT})}{\sqrt{\text{Var}(\tilde{z}_{iT})}} \right) \quad (9)$$

### 3. BOOTSTRAP $\tilde{\Delta}_{adj}$ TEST

Our suggestion is to use the wild bootstrap to mimic heteroskedasticity of unknown form in the bootstrap distribution. The algorithms of the bootstrap data generating process (DGP) are as follows.

Step 1: Under the null hypothesis we obtain the residuals as

$$\hat{\varepsilon}_{it} = y_{it} - \hat{\alpha}_i - \tilde{\beta}_{WFE} y_{it-1} \quad (10)$$

Step 2: We generate a random variable suggested by Mammen (1993),

$$v_{it}^* \begin{cases} \frac{(1 - \sqrt{5})}{2} & \text{with probability } p = (1 + \sqrt{5})/2\sqrt{5} \\ \frac{(1 + \sqrt{5})}{2} & \text{with probability } 1 - p \end{cases} \quad (11)$$

Using the  $\hat{\varepsilon}_{it}$  and  $v_{it}^*$ , we generate an independent and identical sample in the following way:

$$(\varepsilon_{it}^* = \hat{\varepsilon}_{it} \times v_{it}^*)$$

Step 3: Obtain the wild bootstrap version of the  $\tilde{\Delta}_{adj}$  test,  $\tilde{\Delta}_{adj}^{WB}$ .

Then we repeats Steps 2-3  $B$  times, yielding an empirical distribution for  $\tilde{\Delta}_{adj}^{WB}$ .

### 4.SIMULATIONS RESULTS

In this section, we investigate the finite sample behaviour of the  $\tilde{\Delta}_{adj}$  and  $\tilde{\Delta}_{adj}^{WB}$  tests by using the Monte Carlo experiment. The following DGP is derived from (1)

$$y_{it} = \alpha_i(1 - \beta_i) + \beta_i y_{i,t-1} + \varepsilon_{it} \quad i = 1, \dots, N; t = 1, \dots, T \tag{12}$$

where  $\alpha_i \sim N(1,1)$  and  $\varepsilon_{it}$  is generated as similar to the Pesaran and Yamagata (2008)

$$\varepsilon_{it} \sim IIDN(0, \sigma_i^2) \tag{13}$$

with  $\sigma_i^2 \sim IID \chi^2(2)/2$ .  $\alpha_i$  and  $\sigma_i^2$  are fixed across replications with  $y_{i,-100} = 0$ . The first 100 observations are discarded. The null hypothesis is defined by  $\beta_i = \beta = 0.5$ , against the alternative  $\beta_i \sim IIDU[\beta - 0.1, \beta + 0.1]$ . Our results are based on 2000 independent replications with  $(N, T) = \{20, 30, 50, 100, 200\}$ . Table 1 presents empirical size and power of the  $\tilde{\Delta}_{adj}$   $\tilde{\Delta}_{adj}^{WB}$  tests for the nominal level of 5%. In the all experiments, we use the Warp-speed bootstrap algorithm proposed by Giacomini et al 2013).

**Table 1: Size and size-adjusted power of the  $\tilde{\Delta}_{adj}$  and  $\tilde{\Delta}_{adj}^{WB}$  tests**

N/T	Size					Power				
	20	30	50	100	200	20	30	50	100	200
$\tilde{\Delta}_{adj}$ test										
20	3.35	4.40	5.40	3.90	4.90	5.70	9.60	18.05	39.40	81.10
30	2.90	4.85	5.15	4.85	4.25	5.35	11.50	20.95	47.55	88.75
50	3.25	3.60	5.00	4.40	4.75	6.10	12.25	24.95	64.25	97.65
100	2.05	3.45	4.15	5.15	5.45	6.50	14.35	36.60	83.60	99.85
200	1.10	3.20	3.95	4.25	4.70	7.50	15.00	40.25	85.55	100
$\tilde{\Delta}_{adj}^{WB}$ test										
20	4.25	6.65	5.60	4.15	6.25	6.45	10.15	15.05	35.65	84.75
30	4.80	5.75	6.25	4.90	5.25	6.90	10.65	20.40	40.45	94.85
50	4.30	4.55	4.75	4.20	4.15	7.25	16.00	25.45	73.15	100
100	3.15	4.60	5.20	5.00	5.40	9.00	22.20	35.75	85.00	100
200	3.00	4.75	5.25	4.85	4.90	16.50	35.00	75.00	100	100



We have the following findings. First, the  $\tilde{\Delta}_{adj}$  test becomes slightly undersized especially in large  $N$  and small  $T$  panels when the errors are heteroskedastic. In contrast, the  $\tilde{\Delta}_{adj}^{WB}$  test has well sized almost always within reasonable margins. The power of the two tests increases the sample size according to the theoretical results of Pesaran and Yamagata (2008).

To summarize, this paper considers corrections for the asymptotic  $\tilde{\Delta}_{adj}$  test in large panels with heteroskedastic disturbances and an unknown pattern of heteroskedasticity. We have shown that the wild bootstrapped version of  $\tilde{\Delta}_{adj}$  test is never any worse behaved than the  $\tilde{\Delta}_{adj}$  and is usually markedly better by Monte Carlo experiments. This version of the wild bootstrap should be used in empirical studies that involve in autoregressive models.

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