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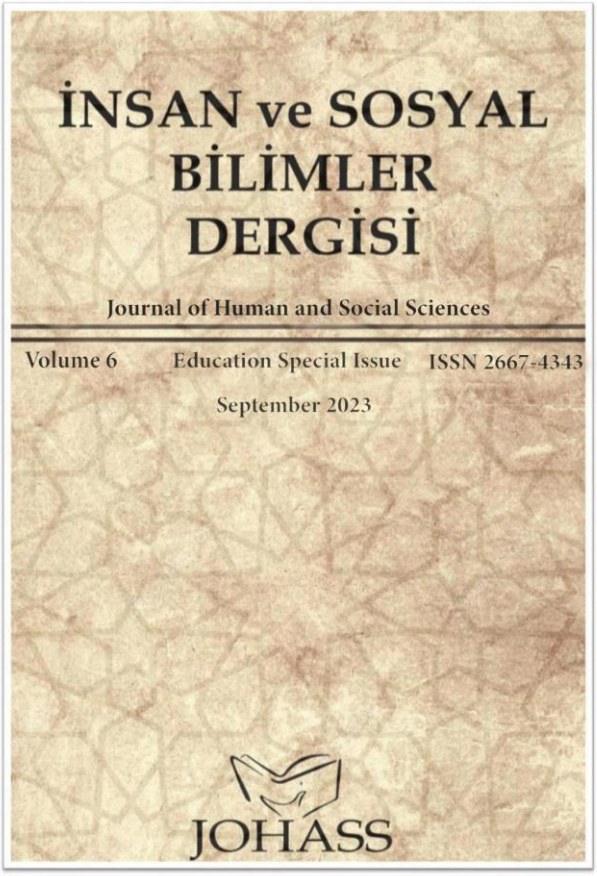

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Pre-Service Science Teachers' Understanding of Infinity Concept: The Case of Image on Convex Lens

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Pre-Service Science Teachers' Understanding of Infinity Concept: The Case of Image on Convex Lens

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Abstract **Research Article**

In this study, pre-service science teachers' definitions of the concept of infinity were tried to be revealed and their understanding of the concept was tried to be examined through the image formation experiment in a convex lens. The study was conducted with 31 pre-service science teachers studying in the 3rd year of science teaching at a state university. Two separate scales were used as data collection tools in the research. The scales consist of open-ended questions designed to reveal pre-service teachers' definitions of the concept of infinity and to evaluate their knowledge about image formation and image properties in convex lenses in this context. One of the scales (Scale-1) was applied before the image formation demonstration experiment in the convex lens, and the other (Scale-2) was applied after the application. The data obtained from the scales were analyzed using the descriptive analysis technique. Pre-service science teachers' definitions of infinity were grouped under three themes: general, mathematical and physical. Pre-service teachers mostly define and understand infinity with codes such as "unknown, undetectable, unmeasurable, undetectable, inexplicable, incomprehensible "under the "general" theme which includes the sub-themes of "unknown" and "having no end". This understanding of the prospective teachers leads them to the misunderstanding that the image of the object at infinity in the focal point of the convex lens is virtual and cannot be seen.

Keywords: Science education, infinity, image in optics

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Introduction

The concept of infinity has an important place in different disciplines and fields of education, and many studies have been conducted on students' understanding of infinity. These studies, conducted in different disciplines such as mathematics, philosophy, psychology and cognitive sciences, aim to investigate how students perceive, understand and learn the concept of infinity (Brieger, 2022; İşleyen, 2013; Singer & Voica, 2010; Howes & Rosenthal, 2001; Monaghan, 2001). Research examining issues such as comprehension strategies, mental processes, misconceptions and learning difficulties related to the concept of infinity is important to assist learning-teaching processes. It is necessary for students to develop their mental models in the process of learning the concept of infinity and to express these models with their own perception and understanding in order for them to learn meaningfully and fully. For example, in a study conducted with children aged 8-12, Smith, Solomon, and Carey (2005) found a high level of consistency between children's thoughts about the infinite divisibility of weight and the infinite divisibility of numbers. There are multiple, two-way interactions between number and matter domains as children develop their understanding of rational number and the continuity of matter, space, and weight (Singer & Voica, 2010). On the other hand, there are also studies conducted with teachers. Palamioti and Zachariades (2022), examine how different contexts affect secondary school mathematics teachers' understanding of infinity through participation in tasks in different contexts. It is stated that the context affects secondary school mathematics teachers' concepts of infinity and allows teachers' misconceptions to emerge.

Studies on students' understanding of infinity play an important role in improving mathematics education and developing students' mathematical thinking skills (Yıldız & Körpeoğlu, 2018). How students solve a mathematical problem and how they take the intellectual steps in the solution process has also been the subject of research. The concept of infinity is a subject addressed in disciplines such as mathematics and philosophy, and these disciplines also have an important place in the field of education. Some studies on the concept of infinity in different fields examine how the concept is perceived and the effects of this concept on the human mind:

1. Infinity in mathematics education: In mathematics education, the concept of infinity is taught especially in advanced mathematics courses such as number theory, analysis and geometry. In these lessons, the concept of infinity is

associated with topics such as the classification of numbers, infinite series and limits (Oflaz & Polat, 2022; Krátká, Eisenmann & Cihlář, 2021; Fernández-Plaza & Simpson, 2016; Merand, 2014; Fischbein, Tirosh, & Hess 1979).

2. In the field of philosophy , the concept of infinity is discussed in disciplines such as philosophy of religion and philosophy of existence, as well as subjects such as metaphysics, epistemology and logic (Başaran, 2016; Türkmen, 2014). Students can improve their intellectual skills by examining philosophical discussions on the concept of infinity.
3. Infinity in psychology education: The concept of infinity in the field of psychology is especially researched in fields such as cognitive psychology and developmental psychology (Oflaz & Polat, 2022) . These studies are carried out to understand how students perceive, understand and learn the concept of infinity.
4. Infinity in computer science education: In computer science education, the concept of infinity is important, especially in fields such as algorithms, computational theory, and artificial intelligence. In these courses, students can improve their problem-solving skills by using the concept of infinity (Oflaz & Polat, 2022).

In mathematics courses, the concept of infinity is especially researched by mathematicians in different topics such as number theory, analysis and topology (Savuran & Isiksal- Bostan, 2022; Çelik & Akşan, 2013; Kolar & Cadez, 2012; Dede & Soybaş, 2011; Tsamir, 2002; Tirosh, 1999).

Studies on the concept of infinity are also carried out in physics and science courses (Suárez et al . 2021; Arabacıoğlu, Oğuz-Unver & Unver, 2014). Scientists such as Albert Einstein, Stephen Hawking, Richard Feynman, Steven Weinberg, Brian Greene, Lisa Randall, Roger Penrose should be mentioned as some of the leading names working on the concept of infinity in the field of physics. Especially in fields such as cosmology, astrophysics, thermodynamics, mechanics and electricity-magnetism, the concept of infinity has an important place and helps students understand these subjects. In physics and science courses, the concept of infinity plays an important role in the specific topics of the disciplines and helps students understand these topics.

In physics and science courses, particularly theoretical physicists, astrophysicists, thermodynamicists, and cosmologists are interested in researching and teaching students the concept of infinity. For example, in cosmology and astrophysics courses, students learn about topics such as whether the universe is infinite or has a finite size, and the rate of expansion of the universe. These topics provide important information about the structure and nature of the universe and develop students' cosmological modeling skills. In thermodynamic courses, the concept of infinity is used to analyze the behavior of thermodynamic systems. Students learn topics such as infinite thermodynamic systems, near-infinite systems, and zero point energy. These topics are important for understanding the laws of thermodynamics. The concept of infinity is also used in mechanics and electricity-magnetism courses. For example, students learn how the concept of infinity is used in topics such as quantum mechanics and special relativity. These courses develop students' mathematical modeling and calculation skills.

In physics courses in electircity at the undergraduate level of science, when calculating the electric field intensity occurring at a very close or a far away point to a charged region in some cases, length, surface or volume regions such as an infinitely long charged rod, an infinitely large thin plate or an infinitely large sphere are defined. In some cases infinitely small length, surface or volume elements are defined. Total intensity calculations for the entire line, surface or volume by defining the unit intensity on the volume element are included in the course content. In this or similar cases, if the student does not understand or cannot understand the infinite definitions or cases in optics, mechanics, electricity and magnetism, it will be difficult for them to understand and comprehend the problem situation. Some students may be at lower levels in terms of the ability to carry out algebraic or geometric operations related to the solution of a physics problem, but understanding the problem situation and visualizing it in the mind with mental design is a behavior expected from students who have successfully taken the general mathematics and physics courses. A mathematically literate student who has taken general mathematics I-II and general physics I-II courses should be able to understand the typical and characteristic problem situations of physics course subjects and put forward the necessary analyses. However, it is a known fact that students' success in science and mathematics courses, from primary school to university, is generally not at the desired level. Not being able to understand problem situations in science and mathematics classes, not being able to analyze them, not being able to make the necessary analyzes is an undesirable situation from the perspective of meaningful learning and constructivist perspectives. Therefore, in order to facilitate and deepen students' understanding of science subjects, it is necessary to make concrete determinations about the effectiveness of factors affecting perception, understanding and success in learning-teaching activities.

Although the 5th unit of the 7th grade of the science curriculum (MEB, 2018a) includes achievements regarding image formation and properties in the subjects of Mirrors and Refraction of Light and Lenses, it is seen that concepts such as infinity, virtual and real images are not mentioned. Again, while the subjects related to mirrors and lenses of the 10th grade optics unit of the physics curriculum (MEB, 2018b) include the achievements related to image formation and image properties, it is seen that concepts such as infinity, virtual and real images are not mentioned. There is a relationship between image formation in a convex lens and the concept of infinity. Convex lenses are an important tool used in optics, and the focusing and image-forming properties of lenses are related to the concept of infinity.

Although the concept of infinity is not included in the primary and secondary education curriculum, it is necessary to pay attention to the information and classroom activities that will affect the development of students' understanding of this concept. Because it is accepted that this concept develops from the end of primary school and the first years of secondary school (Fischbein et al. 1979). In addition, with the development of this concept, the diversity of factors affect the conceptualisation is also emphasized (Date et al. 2017). These factors include the impact of teachers' understanding of infinity and students' previous understanding of this complex concept (Monaghan 2001). As a matter of fact, according to Montes, Carrillo and Ribeiro (2014), in terms of teacher knowledge and training, teachers should have a good level of knowledge about the stages that students need to go through to reach the understanding of infinity, because this will enable them to respond appropriately to students' questions, to better choose and organize classroom activities, will provide their analysis and ranking. In addition to understanding the developmental aspects of understanding the concept of infinity; teachers must also know how to introduce the concept to their classrooms in a way that does not restrict their students' development (Montes et al., 2014).

In geometric optics, the concept of infinity is associated with certain situations related to image formation in a convex lens, stating that the image is infinitely distant. A convex lens has the feature of focusing parallel rays by combining them at a point. The light rays come from the infinite object, that is, the rays coming from a very distant object on the principal axis of a convex lens, are considered almost parallel to the principal axis, and the image is formed after the rays are refracted in the lens. This point is the focal point of the lens. If an object is infinitely far away, the incoming parallel rays pass through the lens and converge at the focal point of the lens. In the opposite case, that is, if the object is at the focal point, the image is formed at a distant point on the side where the rays are refracted. The image formed in this case is called the image formed at infinity. It is stated that the focal objects' image will form at infinity. Therefore, this concept is an important phenomenon used in optics and plays an important role in explaining the optical phenomena of reflection and refraction occurring in mirrors and lenses, and in the design and applications of optical systems.

In geometric optics, the concept of infinity and image formation, image properties in mirrors and lenses are important topics in the learning and teaching processes and helps students understand optical systems.

The problem statement of this study is "What is the understanding of pre-service science teachers about the concept of infinity?" In order to answer this question, answers to the following sub-problems were sought:

1. How do pre-service science teachers explain image formation at infinity and image properties (real, virtual) in a convex lens?
2. How do pre-service science teachers define the concept of infinity?

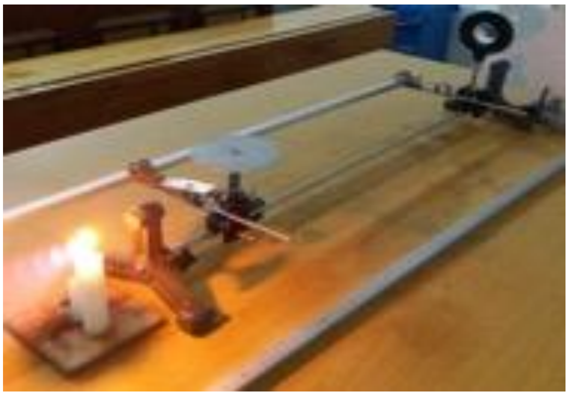
Method

Research Design

In this research, it was tried to reveal pre-service science teachers' understanding of infinity. The research was designed as a qualitative study. Pre-service science teachers were informed about the purpose and scope of the research and participation in the research was voluntary. In optics, some image states resulting from refraction and reflection are expressed with the concept of infinity. It is stated that in image formation in a convex lens, the rays coming from the focal point will proceed parallel to the principal axis after being refracted in the lens. It is also stated that the image of an object placed at the focal point on the principal axis of the convex lens will be at infinity. It is important here how the expression of infinity is understood by prospective teachers. Because correct understanding of optical events is effective in the realization of meaningful learning and the construction of knowledge. Therefore, the purpose of this research is to reveal how pre-service science teachers understand the concept of infinity. For this purpose, the image formation situation in a convex lens was used as a case study in this research. The experiment was not conducted for teaching purposes, no instructive or explanatory explanation was given, only image distances based on object distance were measured and the two measuring scale was applied to the pre-service teachers before and after the experiment. In the experimental setup, a convex lens with a focal length of 10 cm placed on the optical rail, a stand on which to place the candle used as an object, a screen on which to project the image of the candle, and a ruler were used. The burning candle was placed at certain object distances, starting from a point outside the center distance of the lens, up to the focal length, and a clear image was obtained on the screen. The object (burning candle) was placed starting from the off-center distances to the center distance, the distances between the center and the focal point, and the focal point and the point of real images formed on the screen are determined on the other side of the lens. Image distances were measured for each object distance. While the image distance was measuring when the candle (object) was at the focal point the larger, reversed and real image of the candle flame were shown. The real images on the screen on the other side of the lens were obtained at all position further a certain position when the clear image flame was begun to seen. During all these processes, no explanatory and instructive explanation of the event was included, only object and image distances were measured and data were recorded. With the help of a ruler, object distances and image distances were recorded in the table created on the board. The ruler was fixed to the optical rail, a laser (pointer) marker was used while reading the object and image distances on the ruler, and the alignments were made to coincide with the vertical center line of the lens and candle flame. The ruler used in the experiment is a standard aluminum ruler with 1 mm uncertainty. Some photos from the sample application demonstration experiment are given below.

Picture 1

The Image Position Close to the Focal Point When Object (Candle) Located Further from the Center Distance



Picture 2

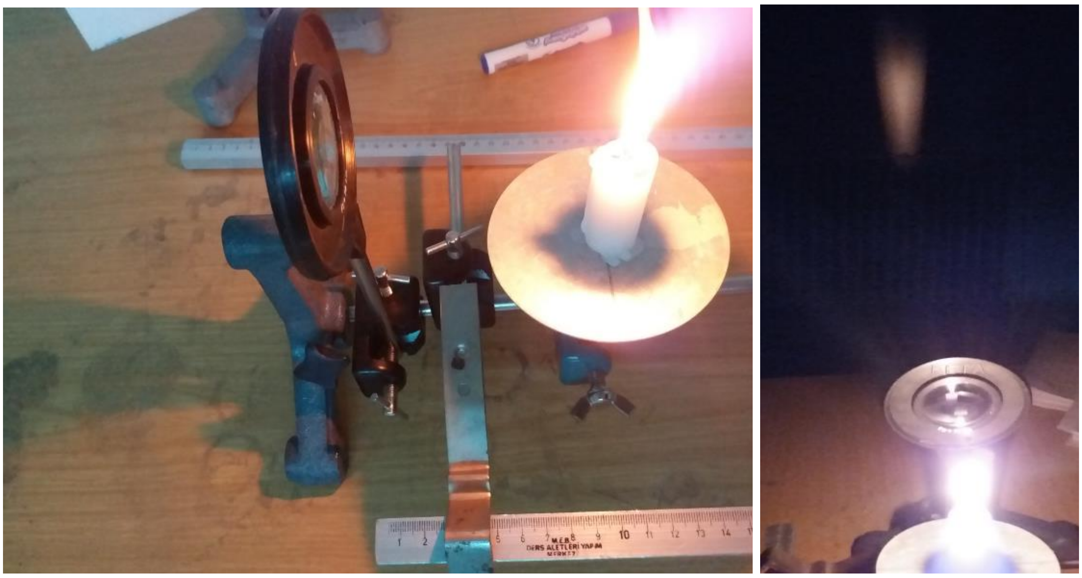
The Image Position at the Center Distance When Object Located on the Center Distance

Picture 3

The Real Image Formed When the Object is Between the Focal and The Center Point

Picture 4

The Object (Candle) Placed at The Exact Focal Point and the Real Image Reflected on the Screen at A Distance of Approximately 3 Meters (Real Images Are Available at All Position Except A Certain Distance from the Lens)



The experiment of image formation in a convex lens is included in both the secondary education curriculum and undergraduate programs. The concept of infinity, which is the subject of this research, is a concept that prospective teachers have encountered in their previous education and training processes. Two separate scales consisting of open-ended questions were prepared by the researcher in order to reveal the knowledge and understanding of prospective teachers about image formation in a convex lens and the concept of infinity. The questions in these two scales basically serve this purpose. In practice, image formation experiments were conducted on convex lenses for prospective teachers between two scales.

Study Group

In this research, participants were determined using the convenience sampling method and purposeful sampling. Convenience sampling in qualitative research is sampling conducted on volunteer individuals who are easy to reach, available, and willing to participate in the research (Christensen, Johnson, & Turner, 2015). Convenience sampling method means doing what is easy to save time, money and effort (Patton, 2014: 244). In purposeful sampling, participants are selected by the researcher according to the research problem and purpose, and the situation is examined in depth with the most suitable participants who will contribute to the research (McMillan, 2004, Patton, 2002). For this reason, while creating purposeful sampling , it was thought that it would be appropriate to determine the prospective teachers who took General Mathematics I-II, General Physics I-II -III courses and General Physics Laboratory I-II courses during their undergraduate education as the study group.

The study group of the research consists of 3rd grade students studying in the Science Teaching program of a state university in the 2021-2022 fall semester and taking the Science Teaching Laboratory Applications I course. The research was conducted with 32 students who wanted to participate in the study among 76 students who took this course. A student was removed from the study group because he/she did not submit the second scale. The study group of the research consisted of 31 pre-service science teachers.

Data Collection Tools

In the study, two scales consisting of open-ended questions created by the researcher were used as data collection tools. The 1st scale consists of five questions: two questions one which includes a drawing about image formation in a convex lens, one question about the definition of the concept of infinity, and two questions about the image formed at infinity in optics.The second scale consists of three questions: one question containing a drawing about image formation in a convex lens, one question about the definition of the concept of infinity, and one question about the image formed at infinity after the convex lens experiment. The open-ended questions used are basically questions with the same scope. A question about the image of an object in focal point of a convex lens was asked exactly before and after the experiment. The content and structure validity of the questions were provided by the opinion of an expert with a master's degree in physics education. Scales were applied to upper grade pre-service science teachers before the application.The questions were corrected in line with the feedback received on language, spelling, expression and understandability.

The open-ended questions in Scale 1 are as follows:

1. Briefly explain the location and characteristics of the image on the screen of the candle placed at a point further from the center distance of the convex lens.
2. If a burning candle is at the focal point of the same lens, draw the location and write down the features of the image of the candle. Make the necessary explanations.
3. What is infinity? Explain.

4. Is an optical image at infinity real or virtual? What do real image and virtual image mean? Explain.
5. Is an image at infinity visible in optics? Explain your answer.
6. The open-ended questions in Scale 2 are as follows:
7. If a burning candle is at the focal point of the same lens, draw the location and write down the features of the image of the candle. Make the necessary explanations.
8. Write your thoughts about the concept of infinity.
9. How do you explain the situation where the image is at infinity in optics?

Collection of Data and Analysis

The data obtained from the scales were analyzed using the descriptive analysis technique. In descriptive analysis, data are interpreted according to determined themes. Descriptive analysis aims to relate and describe themes and interpret the resulting cause-effect relationships (Yıldırım and Şimşek, 2006).

In the drawing parts of the questions involving drawings in the 1st and 2nd scales, the drawing part was evaluated as correct if the refraction of the rays, their directions and the image on the screen were drawn and displayed correctly. Explanations of the pre-service science teachers regarding these questions; They were categorized as codes consisting of the themes of image feature, image position and image status, and the codes were tabulated by giving frequencies and percentages. For the question about the definition of the concept of infinity, codes were created by reading the answers of the pre-service science teachers over and over again, and the resulting codes were categorized as sub-themes and themes. Two questions on the 1st scale and one on the 2nd scale are related to the situation of having an image at infinity, which is expressed in the image formation situations in optics. Since these questions were explanatory questions, the answers of the pre-service science teachers were grouped and coded by descriptive analysis (Tables 4, 5 and 6). Some examples selected from the the pre-service science teachers' answers are given as they are, without correcting any errors in expression, spelling, spelling and punctuation.

Compliance with Ethical Standard

In the research, all rules specified within the scope of the "Higher Education Institutions Scientific Research and Publication Ethics Directive" were followed. None of the actions mentioned under the heading "Actions Contrary to Scientific Research and Publication Ethics" in Article 4 of the directive have been carried out.

Name of the board that made the ethical evaluation: Marmara University Institute of Educational Sciences Research and Publication Ethics Board

Approval date and number of approvals of the evaluation decision: 22.06.2022/05-30

Issue number of the ethical evaluation document: 341245

Findings

The findings obtained in the research are included in this section.

Findings on First Sub-Problem

The findings obtained from Scale 1 and Scale 2 regarding the first sub-problem of the research are given in tables below.

The answers given by the pre-service science teachers to the first question of Scale 1, which asks about the location and image characteristics of the image of the object at a point other than the center point of the convex lens, are given as frequency and percentage in the table below with a simple descriptive analysis.

Table 1

Pre-Service Science Teachers' Answers About the Location and Characteristics of the Image of an Object Outside the Center Point of a Convex Lens

	Answers	**f**	**%**
Image Feature	Virtual	14	45
	Real	15	48
	Not Specifying Features	2	7
Image Location	Between Focus and Center	7	23
	Behind the Lens	6	19
	At Focal Point	1	3
	Image Location Not Specified	17	55
Image Status	Inverted and Small	28	90
	Inverted	3	10

According to Table 1, when we look at the answers of the pre-service teachers about the feature, location and state of the image of an object located further from the central distance of the convex lens, 45% of the pre-service teachers who participated in the research on the image feature answered as virtual, 48% as real, while 7% of the pre-service teachers did not specify any image features. 55% of the pre-service teachers did not specify the image location. 23% of the pre-service teachers stated that the image was between the focal point and the center, while 19% stated that it was only behind the lens. One pre-service teacher stated that the image would be at the focal point. Most of the pre-service teachers stated that the image would be upright down and smaller than the object. The image of an object at outside the center distance is inverted, smaller than the object, and virtual, between the center and focus in the convex lens. When we look at the accumulation of the pre-service teachers' answers, almost half of them stated the image feature incorrectly. While 90% of the the pre-service teachers stated the status of the image correctly, 55% did not specify the location of the image.

Some examples of student answers to this question are presented below:

S_1: Since the object is outside the convex lens, the image is between the focus and the center, inverted, real and its size is smaller than the size of the object.

S_2: The image is upright down, virtual and smaller in size.

S_3: The image is formed on the back. The image is virtual, inverted and small.

The answers given by the prospective teachers to the 2nd question of Scale 1, which asks about the location of the image of the object at the exact focal point of the convex lens and its image characteristics, are given as frequency and percentage in the table below.

Table 2

Pre-Service Science Teachers' Answers About the Location and Characteristics of the Image of an Object at the Focal Point of a Convex Lens

	Answers	**f**	**%**
Drawing	Correct drawing	2	7
	Wrong drawing	18	58
	No drawing	11	35
Image Feature	Virtual	6	19
	Real	6	19
	Not specifying features	19	61
Image Location	At infinity	21	68
	At focal point	1	3
	At the center	1	3
	Not specifying location	8	26

	Infinitely big	4	13
	Upright and big	2	6
	Upright and same size	4	13
	Inverted and big	6	19
Display Status	Inverted and small	2	6
	Same height	2	6
	Dot shaped	4	13
	Not stating	5	16
	No image formed	2	6

According to Table 2, when we look at the drawings and explanations of the pre-service teachers about the feature, location and situation of the image of an object at the point of the convex lens, it is seen that 58% of the drawings are wrong, 35% of the pre-service teachers did not draw, while only two pre-service teachers were able to draw correctly. In their explanations about the image feature, 19% of the pre-service teachers answered as virtual, 19% as real, while 61% did not specify the image feature. 26% of the pre-service teachers did not specify the image location. While 68% of the pre-service teachers stated that the image was at infinity, one pre-service teacher stated that it was at the focal point, and another pre-service teacher stated that it was at the center. Pre-service teachers used different expressions about the status of the image. According to the findings in Table 2, 19% of the pre-service teachers stated that the image was upright down and large, while 16% did not state this. While 13% of the pre-service science teachers stated that the image would be infinite in size, 13% stated that it would be flat and the same size, and 13% stated that it would appear as a dot, 6% of the pre-service science teachers said that the image would not be formed. In a convex lens, the image of the object at the focal point appears larger than the object and inverted at infinity. While the pre-service teachers expressed the position of the image correctly, they gave different answers about the status of the image.

Some answers given by the pre-service teachers to this question are presented below as examples:

S_1: The image is virtual. It's upright down. Its height is greater than the height of the object.

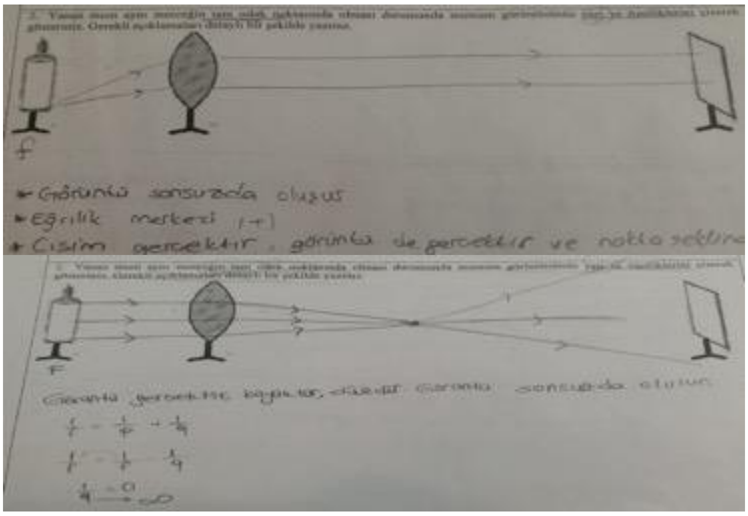
S_2: The image is inverted but larger than the object. The image location is infinite. The exact location cannot be found.

S_3: It occurs at infinity. The rays travel parallel and for this reason they do not intersect. Since they do not intersect, no image is formed.

The pre-service teachers' drawings regarding with the image of the object at the exact focal point of the convex lens contain a large number of errors. The example drawings below include two different pre-service science teachers drawings showing the refraction of the rays coming from the object at the focal point in the convex lens.

Picture 5

Examples of Pre-service Teachers' Drawings of the Image of an Object at the Focal Point of a Convex Lens



The findings of the pre-service teachers' answers to the 1st question in Scale 2 after the image formation demonstration experiment in a convex lens are given in the tables below.

Table 3

The Pre-Service Teachers' Answers About the Location and Characteristics of the Image of an Object at the Focal Point of a Convex Lens

	Answers	**f**	**%**
Drawing	Correct drawing	2	7
	Wrong drawing	20	64
	No drawing	9	29
Image Feature	Virtual	15	48
	Real	6	19
	Not specifying features	10	32
Image Location	In infinity	10	32
	At the back	5	16
	Away	5	16
	Giving distance (in metrics)	3	10
	Not specifying location	8	26
Image Status	Inverted and big	18	58
	Inverted	4	13

	Inverted and small	1	3
	Upright and small	1	3
	Same height	1	3
	Dot shaped	3	10
	Not stating	3	10

Table 3 contains the findings of the pre-service teachers' drawings and explanations about the feature, location and status of the image of an object at the focal point of the convex lens after the image formation demonstration experiment in the convex lens. The same question was asked in Scale 1 (Table 2). When the answers given by the pre-service science teachers to this question in Scale 2 (Table 3) are compared with Scale 1 (Table 2) in this section; While the rate of the pre-service science teachers who do not draw in Scale 1 is 35%, this rate is 29% in Scale 2. While the rate of the pre-service teachers' who drew incorrectly in Scale 1 was 58%, this rate was 64% in Scale 2. This increase shows that some pre-service teachers, who did not draw in Scale 1, drew in Scale 2 but they drew incorrectly. There is no difference in the proportion of the pre-service teachers who draw correctly. In terms of image feature, while the rate of the pre-service teachers who did not specify any image feature before the convex lens experiment was 61%, this rate decreased to 19% after the experiment. While the proportion of the pre-service teachers who stated that the image was real did not change, the proportion of the pre-service teachers who stated that the image was virtual increased from 19% to 48%. At this point, it can be seen that the rate of the pre-service teachers giving correct answers in terms of image features did not change, and this rate was 19% before and after the experiment. While the rate of the pre-service teachers who stated that the location of the image was at infinity before the experiment application was 68%, after the experiment application, almost half of these pre-service teachers used the expressions behind, far away for the location of the image instead of infinity, or stated that it was at a certain distance as in metrics. No difference was detected in the proportion of the pre-service teachers who did not specify their location. For the image status, after the experiment 58% of the pre-service teachers stated that the images were inverted and big while they could not answer correctly before the experiment.

Some answers given by the pre-service teachers to this question are presented below as examples:

S_1: The image is inverted, it is behind the mirror and an apparent image is formed. As the object approaches the lens, the distance in the part where the image is clear becomes longer. The image becomes bigger.

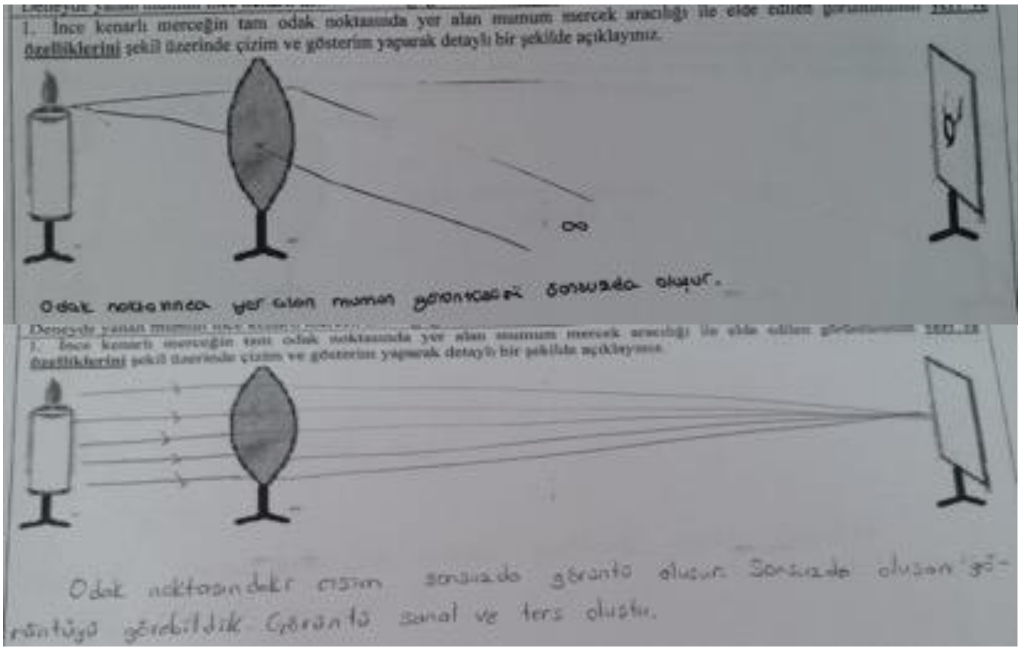
S_2: The image is formed at infinity behind the lens. We see it as a point image. The reflected rays intersect at a single point.

S_3: While the object was in focus, the image appeared very far away, quite larger than the object, and upright down. Before doing the experiment, I thought we could not see the image formed at infinity, but now I have seen what we can see.

The pre-service science teachers generally made incorrect drawings or answered the question without drawing. The sample drawings below include two different pre-service teachers drawings showing the refraction of the rays coming from the object at the focal point in the convex lens.

Picture 6

Examples of Pre-Service Teachers' Drawings of the Image of an Object at the Focal Point of a Convex Lens



The answers given by pre-service science teachers to the 4th and 5th questions in Scale 1, regarding the situation of the image being at infinity, expressed in image formation situations in optics, are grouped and the frequencies and percentages of the codes determined are given in Table 4 and Table 5.

Table 4

Explanations of the Pre-Service Teachers About the Image at Infinity and the Real-Virtual Image in Optics

	Answers	f	%
Image Feature	Virtual	15	48
	Real	6	19
	Uncertain	1	3
	Vary by optical device	9	29
Real and Virtual Image Definitions	True	12	38
	Wrong	13	41
	No answer	1	3
	Explanation with example	4	12
	Description with drawing	1	3

When Table 4 is examined, it is seen that 48% of the pre-service teachers stated that the image formed at infinity is virtual. 19% of the pre-service teachers stated that the feature of the image at infinity was real, 3% stated that it was uncertain, and 29% stated that it would vary depending on the optical device. However, while 38% of the pre-service teachers correctly identified the real and virtual image, 41% identified the real and virtual image incorrectly. It is noteworthy that although 38% of the pre-service teachers correctly stated the definition of real and virtual images, the rate of pre-service teachers who stated that the image of the object in focus at infinity in the convex lens is virtual is 48%, and the rate of pre-service teachers who say that it is real is 19%.

Some answers given by the pre-service teachers to this question are presented below as examples:

> S_1: An image that exists at infinity is virtual. The real image is the image projected on the screen. What appears in optical instruments is a virtual image.
>
> S_2: In optics, an image at infinity is virtual. The image that can be projected onto the screen with the help of rays is real. We cannot observe the virtual image in real life.
>
> S_3: The real image is the image that cannot be seen with the eye and is formed in front of the lens used. The rays themselves intersect. The virtual image is the image that can be seen with the eye and is formed behind the lens. The reflections of the rays intersect. In order for an image to be formed at infinity, the incoming and reflected rays must not intersect. In this case, we cannot say real or virtual. The image is unclear at infinity.

Table 5

The Pre-Service Teachers' Answers on Whether the Image at Infinity Will Be Visible or Not

	Answers	f	%
Image Feature	Visible	8	26
	Invisible	17	55
	No answer	6	19

In Table 5, when the answers of the pre-service teachers about whether the image formed at infinity will be visible or not in the convex lens are examined; it is seen that 55% of the pre-service science teachers answered the question as invisible, 26% answered as visible, and 19% left the question unanswered. The explanations of the pre-service teachers who answered this question as visible include the statement that the image will be pointwise.

Some answers given by the pre-service teachers to this question are presented below as examples:

S_1 : I think the image will not appear. (May appear as dots)

S_2 : The image formed at infinity is not visible. The image will be very small and spotty so it won't be visible.

S_3 : Since the rays will not intersect, no image will be visible to the eye.

The findings of the answers given by the pre-service teachers to the 3rd question in Scale 2 after the image formation demonstration experiment in the convex lens are given in the table below.

Table 6

Explanations of The Pre-Service Teachers About the Image at Infinity in Optics

	Answers	f	%
Image Feature	Visible	13	42
	Invisible	3	10
	Other explanations that are not explicitly stated	15	48

Table 6 , 42% of pre-service science teachers state that the image at infinity can be seen after the experiment. While 10% of the pre-service teachers stated that the image at infinity would not be visible, 48% of them made unclear statements about whether it would be visible or not. Some example expressions are given below.

S_1 : Uncertainty arises in the formation of images at infinity. I thought that the image could not be observed at infinity, but after the experiment, I saw that we could see it.

S_2 : While there will be no image at infinity and there is undetectable, endless perception. In the last experiment, the formation of an image of the object in focus made us wonder whether some perceptions were wrong.

S_3 : Since the image is formed outside the center, an infinite image is formed.

Findings on Second Sub-Problem

The findings obtained from Scale 1 and Scale 2 regarding the second sub-problem of the research are given below in tables.

In order to reveal the pre-service science teachers' understanding of the concept of infinity, the answers given by the the pre-service teacher to the 3rd question of Scale 1, where the conceptual definition is asked, are described and the codes identified and the sub-themes and themes formed by the codes are presented in the table below.

Table 7

The Pre-Service Teachers' Understanding of the Concept of Infinity

Codes	Sub-themes	Themes
Unknown Can't find value Unimaginable Imperceptible multitude Undefined Theoric	Unknown	General
Endless Uncertain ending Endless Without beginning or end Unending Without limit Continually	Having no end	
Infinite number between two numbers Space between numbers That cannot be expressed in numbers	Numerical value	Mathematical
The farthest point of the image The rays do not intersect Rays are parallel When p=q A concept in optics	Distance in optics	Physical
Distance beyond sight The farthest point you can go Unpredictable distance The farthest place	Distance	

When we look at the descriptive analysis of pre-service science teachers' definitions of the concept of infinity in Table 7, it is seen that the definitions are mostly expressed with codes belonging to the sub-themes of "unknown" and "having no end" under the "general" theme. The pre-service science teachers express the concept of infinity under the sub-theme of "unknown" and "having no end" with codes such as unknown, undetectable, undefined, unvalued, endless, unending, endless, limitless, continuous.

Some of the pre-service science teachers made definitions with codes such as infinite number between two numbers, space between two numbers, and cannot be expressed with numbers, which constitute the "mathematical" theme and "numerical value" sub-theme.

On the other hand , some of the pre-service science teachers they tried to define the concept of infinity with codes such as the farthest point of the image, the rays not intersecting, and the rays being parallel, which constitute the sub-theme of "distance in optics", which is discussed under the "physical" theme.

In addition, some of the pre-service science teachers also used codes such as the farthest point that can be reached, unpredictable distance, and the farthest place, which belong to the "distance" sub-theme expressed by the "physical" theme in their definitions.

Some answers given by the pre-service teachers to this question are presented below as examples:

S_1: It is undefined. Endless repetition or the unknown.

S_2: It is the continuous continuation of a situation or event.

S_3: Infinity is the space between one of the objects, numbers, concepts and the one immediately behind it. There are infinite numbers between 1 and 2. But there are also infinite numbers between 1,5 and 1,7. Infinity may be greater than some infinities. Things that have no beginning or end. (It should not be thought of as just numbers.)

The findings of the answers given by the prospective teachers to the second question in Scale 2 after the image formation demonstration experiment in the convex lens are given in the tables below.

Table 8

The Pre-Service Teachers' Understandings About the Concept of Infinity After the Experiment

Codes	Sub-themes	Themes
Incomprehensible	Unknown	General

Known but unobtainable Including everything, every possibility Unexplained things Alternative concept for unexplained things in the Universe Depends on personal perception Imperceptible multiplicity Uncertain, unknown Nonexistent		
Endless Continuity of the event or situation Always ahead Ongoing Something that cannot be limited	Having no end	
Infinite number between two numbers Distance that is difficult to measure Relative, distant compared to short distance Not numerically measurable That cannot be expressed in numbers Without time or dimension Time	Numerical value	Mathematical
Virtual image Virtual image in dot shape In optics, the image is furthest away Out of center distance in optics Reflection of rays moving away from each other The last point where the image can form Failure to detect center distance (R) Endless image When p=q	Distance in optics	Physical
Much further than expected Undetectable distance Very far point The farthest point you can go Unpredictable distance The farthest place	Distance	

Table 8, it can be seen that there is more diversity of expressions in the codes, sub-themes and themes of pre-service teachers' definitions of the concept of infinity after the image formation experiment in the convex lens, compared to the definitions before the experiment, and that there are parallel expressions in the sub-themes and themes. It seems that the themes regarding pre-service teachers' definitions and understanding of the concept of infinity are general, mathematical and physical.

Some answers given by the pre-service teachers to this question are given below as examples:

S_1: To me, infinity means containing everything and containing every possibility. Having endless amounts of it is what makes the infinite infinite.

S_2: It means endless, always ahead. For example, while there is a certain limit within which real images that do not form at infinity will appear, the image formed at infinity will continue to form no matter how far the obstacle in front of it moves.

S_3: Infinity, in general definition, is the multiplicity that even our perception cannot grasp. For optics, infinite rays continue forever, do not intersect, and continue unless an object blocks them.

Discussion and Results

While the codes belong to the pre-service science teachers' definitions below the mathematical and physical themes are lesser before the image formation experiment in the convex lens, it was determined more in their definitions after the experiment. The reason for this difference can be stated that the experience of image formation at infinity was effective in the image formation experiment in the convex lens. While pre-service science teachers mostly try to define infinity with concepts such as unknown, undetectable, unmeasurable, undetectable, unexplained, uncomprehended, they define infinity with general expressions. This understanding of the pre-service science teachers leads them to the misunderstanding that the image of the object at the focal point formed at infinity in the optical lens cannot be seen. This result can be supported by the findings of Kocakülah and Şardağ (2013). However, the infinity in this case can be explained by the fact that the image can be projected onto the screen as a real image at any point placed at certain distances from the lens. Because in the experiment, while the clear image of the object located off-center or at any point between the focal point and the center can be projected onto the screen at a certain location, the image of the object at focal point is reflected on the screen at every point away from the lens. At this point, because the pre-service teachers generally interpreted infinity as unknown, unperceivable, undetectable, very distant, very big, they described the image formed at infinity in the experiment as virtual and invisible before the experiment. While the number of the pre-service science teachers who stated that the image at infinity is real and visible increased after the experiment, the number of the pre-service science teachers who could not clearly express this situation also increased compared to before application. Considering the existence of interpretations infinity such as where the rays move away from each other in geometric optics, not intersecting, and parallel reflection in the definitions in the explanations of the pre-service teachers, it can be explained why the image formed at infinity in the convex lens is expressed as virtual and invisible by the pre-service teachers. Therefore it is possible to mention a misunderstanding that reflected or refracted rays should not intersect in order to form an image in optics. However, according to the particle model of the light, the image is formed when light rays are reflected from any obstacle and reach the eye. Each ray falling on the screen will cause the image to appear on the screen. These results can be supported the results of many studies that found that pre-service science teachers have low conceptual understanding levels on light and optics (Demirci & Ahcı, 2016; Büyükkasap et al. 2013; Yağbasan & Gülçiçek 2003; Goldberg & McDermott, 1987).

While pre-service science teachers' definitions of the concept of infinity created the sub-themes of “unknown”, “having no end”, “numerical value”, “optical distance”, “distance”, these sub-themes were categorized under general, “mathematical” and “physical themes”. The Pre-service science teachers' understanding of infinity corresponds more to the sub-themes of the “unknown” and “having no end”, both before and after the case study application. When the codes of these sub-themes are examined, it is seen that the codes are mostly in the form of interpretations of the concept related to daily life experiences. Therefore, these two sub-themes can be mostly categorized under the “general” understanding theme. In this regard, this result supports the study conducted by Çelik and Akşan (2013). Çelik and Akşan (2013) state that the pre-service science teachers' understanding of the concept of infinity is mainly shaped by their daily life experiences, that they are not successful in explaining the concept correctly, and that they mainly base their explanations of the concept of infinity on the concepts of endless, very large, unlimited and unknown numbers. Some of the pre-service science teachers use expressions corresponding to codes such as space between numbers, cannot be expressed with numbers, space between two numbers, and cannot be measured numerically in their definitions of the concept of infinity. These codes were discussed in the numerical value sub-theme and it was evaluated that a significant portion of the pre-service science teachers had a mathematical understanding of the concept of infinity. As a matter of fact, Singer and Voica (2010) state that infinity is inevitably associated with numbers and draw attention to studies on number perception on this subject. Singer and Voica (2008) discuss the perception of infinity of students of different ages in three categories: processual, topological and spiritual. Beyond these Smith, Solomon, and Carey (2005) found a high level of consistency between the thoughts of children aged 8-

12 about the infinite divisibility of number and weight, and in this study, some of the pre-service science teachers also expressed infinity with some physical quantities in the material world in their definitions. According to the codes gathered under the sub-theme of optical distance, it is seen that the pre-service science teachers have a physical understanding expressed with codes such as "a virtual image in the form of a point, the farthest point of the image in optics, the reflection of rays moving away from each other outside the center distance in optics, the last point where the image can form". Similarly, the sub-theme of "distance", expressed with codes such as "much further than expected, the farthest point to go, unpredictable distance" used by pre-service teachers in their definitions of infinity, was also expressed as a "physical theme". This result can be interpreted as prospective teachers' experiences in the physical universe regarding the concept of infinity are effective. At this point, Date et al. (2017)'s finding that the connections between mathematical and empirical applications of infinity can help the pre-service science teachers to compare finite models of the world with infinite models can be said to support this result.

Recommendations

In this study, it was tried to reveal the understanding of the pre-service science teachers in some cases in optics, one of the physics subjects, about infinity, which is a concept that is difficult to understand and comprehend. The demonstration experiment used in this study is an experiment that is included in the program, theoretically and practically, under the relevant headings of the physics courses of the pre-service science teachers during their secondary and undergraduate education, and is an experiment that they work on or should work on. Considering the results obtained in the study, it is clear that the pre-service science teachers have deficiencies in the topics related to image formation and image properties in the convex lens. The only reason explaining this situation is that the applications, laboratory and experimental activities in physics courses are not sufficient and effective. Therefore, increasing the effectiveness of applications carried out with correct conceptual perception and understanding in physics courses in secondary and higher education programs can be expressed as an important suggestion of this study. In order to develop the conceptual perception and understanding of infinity in the most accurate way in the optical subjects covered in this study, a sample experiment application is also presented to the attention of teachers, students and researchers. The concept of infinity is a concept that is frequently used in philosophy, theology, mathematics, psychology and in our daily lives. Therefore, the definition and meaning of this concept in other fields affects the definition and meaning in science or physics classes. As seen as a result of this research, the fact that the pre-service science teachers made sense of or defined the concept of infinity with general, mathematical and physical knowledge indicates that the definition of the concept in other fields is also important. The pre-service science teachers mostly used the expressions discussed in the general knowledge theme in their definitions. Since infinity, in its most general and simple sense, connotes having no end, it can be stated that the formation of images in lenses and mirrors causes the situations in which infinity is expressed to be incomprehensible. Therefore, these situations in geometric optics should be handled in accordance with the correct definition of the concept in the relevant documents in the textbook and other sources.

Compliance with Ethical Standard

In the research, all rules specified within the scope of the "Higher Education Institutions Scientific Research and Publication Ethics Directive" were followed. None of the actions mentioned under the heading "Actions Contrary to Scientific Research and Publication Ethics" in Article 4 of the directive have been carried out.

Name of the board that made the ethical evaluation: Marmara University Institute of Educational Sciences Research and Publication Ethics Board

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