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Conceptualization, literature review, methodology, implementation, data analysis, translation, and writing.

Abstract

This study aims to investigate mentor teachers' task implementation and instructional dispositions from the perspective of preservice mathematics teachers. Twelve preservice mathematics teachers examined four mentor teachers' task selection and implementation in terms of cognitive demand and instructional dispositions that affect their cognitive demand. The data were gathered from detailed observation notes and focus group interviews of preservice teachers. The results indicated that only one teacher selected most of the tasks at a high level and maintained them without declining their cognitive demand. His instructional dispositions showed us that he displayed an approach that allowed students to explore the tasks, reason, and discuss ideas. Launching fewer problems per lesson, he enabled students to examine the tasks in depth. He discussed student-invented strategies and sometimes focused on multiple solutions and misconceptions. Instead of algorithmic solutions, he emphasized underlying concepts, multiple representations, and daily life situations. On the other hand, other teachers mostly implemented tasks at low-level, however, they had different instructional dispositions towards student explorations, classroom discussions, and utilizing multiple representations. Based on these results, this study highlights the importance of mentor teachers for the professional development of preservice teachers.

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Research Article

How do Mentor Teachers Implement Mathematical Tasks?: The Perspective of Preservice Mathematics Teachers *

Osman BAĞDAT 1 💿

Abstract

This study aims to investigate mentor teachers' task implementation and instructional dispositions from the perspective of preservice mathematics teachers. Twelve preservice mathematics teachers examined four mentor teachers' task selection and implementation in terms of cognitive demand and instructional dispositions that affect their cognitive demand. The data were gathered from detailed observation notes and focus group interviews of preservice teachers. The results indicated that only one teacher selected most of the tasks at a high level and maintained them without declining their cognitive demand. His instructional dispositions showed us that he displayed an approach that allowed students to explore the tasks, reason, and discuss ideas. Launching fewer problems per lesson, he enabled students to examine the tasks in depth. He discussed student-invented strategies and sometimes focused on multiple solutions and misconceptions. Instead of algorithmic solutions, he emphasized underlying concepts, multiple representations, and different instructional dispositions towards student explorations, classroom discussions, and utilizing multiple representations. Based on these results, this study highlights the importance of mentor teachers for the professional development of preservice teachers.

Keywords: Cognitive demand, instructional dispositions, mathematical task, mentor teachers, preservice teachers

1. INTRODUCTION

Preservice teachers (PSTs) as learners make a great effort to attain competencies in the teaching profession by taking many theoretical and practical courses at university. Eventually, they experience the teaching last years of their teaching practice. One of the most crucial courses at the undergraduate level is teaching practice, which provides the behaviors required by the teaching profession and includes the theoretical and practical dimensions of educational sciences (Schatz-Oppenheimer, 2017). Therefore, it should be managed comprehensively within the university-school collaboration, ensuring the most beneficial professional development for PSTs. According to the Journal of Announcements, which is the official publication of Turkish Ministry of Education, the goal of the teaching practice course is "to prepare PSTs for the teaching profession, to develop the competence to use the knowledge, skills, attitudes, and habits related to the general culture, special field education, and teaching profession in a real school setting" (Ministry of National Education [MNE], 1998, p.1360). As a part of this course, "PSTs gain a better understanding of school structure, administration, and daily life in schools, examine educational settings, engage in extracurricular activities, observe experienced teachers, work with students practicum in the individually or in small groups, and gain

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 short-term teaching experience". During this period, mentor teachers have an important role in PSTs' identity, skill, and competence development (Loughran, 2006; van Es & Sherin, 2008).

Mentors are expert teachers who provide assistance to PSTs within the scope of teaching practice. They play a significant role in the training of PSTs both inside and outside of the classroom. Butler and Cuenca (2012) emphasized the three important roles of mentors. First, as emotional supporters, helping them gain their teacher identity. PSTs "often enter student teaching unsure of their abilities. Novice teachers also have uncertainties about what it means to be a teacher, and from a supportive perspective, the mentor's purpose is to help the novice teacher move past these fears (p. 300). Second, as socializing agents, influencing PSTs' educational views. The third and the most significant, is as instructional coaches, they "observe and evaluate instructional practice and provide constructive feedback aimed at improving the methods and techniques of PSTs" (p. 299). For instance, they can support PSTs with daily and annual planning, effective classroom practice, assessment, and classroom management. Previous studies have shown that experienced mentor teachers can greatly contribute to PSTs' professional growth (Hudson, 2012). PSTs may gain valuable experience from the classroom setting and instructional decisions of mentors. They should therefore pay close attention to mentors' practices during the lessons.

One of the factors affecting the practices in the classroom is undoubtedly the quality of the mathematical tasks selected and implemented by mentor teachers. Studies indicated a deep relationship between implemented tasks and students' understanding of mathematics. In this context, mentor teachers' task preference, the nature of their classrooms while implementing these tasks, and embedded classroom norms directly affect their teaching quality (Hiebert & Wearne, 1993; Stein & Lane, 1996), and consequently, the professional development of PSTs under their mentorship. Based on this importance, this study aimed to reveal the instructional dispositions of mentor teachers by examining their task selection and factors affecting task implementations from PSTs' perspectives.

1.1. Theoritical Framework

1.1.1. Mathematical tasks framework

A mathematical task is defined as a classroom activity or problem that focuses students' attention on an important mathematical idea (Stein, Grover, & Henningsen, 1996). In terms of the level of reasoning required by students, Stein et al. (1996) classified mathematical tasks into four categories, two of which are high-level (doing mathematics (DM) and procedures with connections (PwC) and two of which are low-level (procedures without connections (PwoC) and memorization (Mm) (Stein, Smith, Henningsen, & Silver, 2000). This classification, named cognitive demand, is expressed as students' mental effort to accomplish a task (Boston, 2013). The cognitive demand of a selected mathematical task is related to the degree to which that task prompts the student to think. Mm tasks involve learning or reproducing facts, rules, formulas, or definitions which don't require relating to the underlying mathematical ideas of concepts. PwoC tasks often require recalling algorithms in order to complete a task with little cognitive effort. These tasks aim to find the correct answer rather than emphasize the meaning of mathematical ideas. PwC tasks require an effort to comprehend the underlying meanings of mathematical concepts in order to complete tasks and connections may be established between multiple representations, providing an opportunity for deep mathematical understanding. These tasks usually have general instructions that suggest a strategy for the solutions. DM tasks require complex and non-algorithmic thinking and mentally imbalanced situations for deep understanding. These tasks provide students to generalize a mathematical concept, solution process, and relationships (Stein et al., 2000). Table 1 presents examples of task classification.

Task level	Sample examples
Memorization	What are the decimal and percent values of 1/2 and 1/4? (Students rehearse the known facts)
Procedures without connections	Convert the fraction $\frac{3}{8}$ to decimals and percentages. (Students use procedural operations)
Procedures with connections	Using a 10 x 10 table, determine the decimal and percentage values of $\frac{3}{5}$. (Student use table representation to focus on the meaning of the percentage idea)
Doing mathematics	Color 6 squares in a 4 x 10 rectangular table. Using the rectangle, find the percentage, decimal, and fractional value of the colored area. (Students may come up with multiple solution strategies)

Table 1. Sample examples for cognitive demand levels (Stein et al., 2000, p. 13).

The mathematical task framework (MTF) in Figure 1 (Stein et al., 1996, p. 459) represents the phases from task selection to task setup, implementation, and student learning. According to this framework, a teacher firstly selects a mathematical task from a curriculum material, then sets up the task in line with his goal, pedagogical and content knowledge or beliefs, and then implements it in the classroom, and finally, student learning occurs.



Figure 1. Mathematical tasks framework (Stein et al., 2000)

As seen in Figure 1, the cognitive demand of a mathematical task may change from selecting the task from the curriculum source to its implementation due to some factors depending on teacher knowledge, belief or attitude, student behaviors, or characteristics of the task. A level of a cognitively demanding task can be declined or maintained, or a low-level task can be raised to a high-level during implementation. There are several studies investigating teachers' implementation of mathematical tasks in the classroom (e.g. Ader, 2020; Henningsen & Stein, 1997; Stein et al, 1996; Stein & Lane, 1996), their impact on the learning opportunities of students (e.g., Hiebert & Wearne, 1993; Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013; Ni, Zhou, Li, & Li, 2014; Özmantar & Aslan, 2017; Stein & Kaufman, 2010) examining the relationship between teacher-student behaviors, beliefs, content knowledge, pedagogical content knowledge, and implementation of tasks (e.g., Charalambous, 2010; Garrison, 2011; Wilhelm, 2014). Research has shown that the cognitive demand of mathematical tasks in the curriculum resources is mostly high (Ubuz, Erbaş, Çetinkaya, & Özgeldi, 2010), but the majority of teachers can't maintain the cognitive demand of the tasks at a high-level during implementation (Henningsen & Stein, 1997; Stein et al., 2000) because of various factors explained in the next section.

1.1.2. Instructional dispositions of teachers in task implementation phase

Many factors originating from the teacher, students, or classroom setting affect the cognitive demand of a mathematical task in the implementation phase. An important part of these factors are directly related to teachers' classroom practices. The cognitive demand of tasks is often declined by a teacher who routinizes tasks without focusing on the underlying meaning and does not give students enough time to think and reason (Henningsen & Stein, 1997). A teacher who implements mathematical tasks at a high level constructs a setting where students can think and reason, gives enough time for exploring tasks, takes into account the previous learning of students, pays attention to students' explanations and justifications in a classroom discussion, provides students opportunities to establish conceptual relationships and to make connections among multiple representations (Stein,

846

Engle, Smith, & Hughes, 2008). To constitute such a classroom setting, a teachers should first select a cognitively demanding task that allows for multiple representations and solutions, and make detailed planning to implement and maintain it at a high level (Henningsen & Stein, 1997). These factors affecting the cognitive demand of mathematical tasks were framed as a result of the research carried out within the scope of the QUASAR project (Quantitative understanding: Amplifying Student Achievement and Reasoning) (Silver & Stein, 1997; Stein, et al., 1996). Teachers' instructional dispositions can be inferred from whether or not they have these factors that affect the cognitive demand of mathematical tasks. In the current study, these choices such as providing exploration time to solve the task, building task on prior knowledge, focusing on student solutions, setting a whole class discussion and making connections are considered as teachers' instructional dispositions. Subsequent studies (e.g., Ader, 2020; Boston & Smith, 2009) mostly focused to enhance teachers' task implementation to maintain cognitively demanding tasks and to change instructional dispositions through professional development programs. This study aims to investigate the tasks implemented by mentor teachers from the perspective of PSTs. For this purpose, firstly, cognitive demand levels of selected and implemented tasks were examined, and then the instructional dispositions of PSTs that affect the cognitive demand of tasks were analyzed.

2. METHOD

2.1. Research Design

This study utilized a qualitative case study method. Case studies, according to Yin (2003), should be centered on "how" and "why" questions. Merriam (2013), on the other hand, emphasizes the importance of thoroughly describing the phenomenon explored in case studies. This study aimed to seek mentor teachers' task implementation and the reasons for the factors affecting the tasks' CD in detail from thep perspective of PSTs.

2.2. The Context and Participants

This research was carried out as a part of a Teaching Practice course in an elementary mathematics education program at a Turkish university. Within the Teaching Practice course, PSTs in their last year observe mentor teachers for 4 hours per week for 12 weeks. During this period, they accomplish many objectives, such as interviewing with the principal, collecting curriculum information, their mentor's annual plan, and curriculum documents, examining the mentor teacher's lesson plan, observing methods, approaches, classroom management, and student interactions. The current study comprised 12 middle school PSTs and four mentor mathematics teachers, all of whom were given pseudonyms in Table 2. Teachers Akin and Ceren work in a rural part of the city, where they teach in a socioeconomically deprived school with a lower student population than other teachers. Duru and Ali, on the other hand, work in two different schools that are located in the urban part of the city, have a higher socioeconomic status, and have a larger student population. Parents are more involved in their children's lessons, and the majority of students in these schools attend private lessons in addition to their courses.

Table 2.	The	participants
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Teacher	Experience (year)	PSTs paired with teachers
Akin	14	PST1, PST2, PST3
Ceren	14	PST4, PST5, PST6
Ali	23	PST7, PST8, PST9
Duru	7	PST10, PST11, PST12

2.3. Data Collection

In the current study, PSTs were trained by the researcher of this study on the CD levels, task implementation, and factors that affect the implementation of these tasks. In weekly meetings held with the PSTs, they characterized the CD levels of sample tasks and evaluated certain cases where

situations caused the CD of the tasks to decline or maintain during the setup and implementation phases. During this time, their justifications were discussed in order to obtain a consensus on task characterization and case analysis. Following the training, PSTs used the observation form to observe their mentors' lessons, indicating how the teacher planned, from which source s/he selected the tasks, and whether he made any setups in the observations. They were expected to respond to such questions as best they could, incorporating as many dialogues as possible: How did the teacher launch the task during the implementation phase? How many tasks did he select on average? Were any student(s) selected to share their ideas? Was there a whole-class discussion? Did different solution strategies emerge, and how were these solutions shared? Was there a task exploration time? Was there any use of material, technology, or other means? They were, then asked to evaluate the CD of tasks during the selection, setup, and implementation phases, as well as the factors affecting the CD. They were required to give a detailed explanation of their reasoning. During this period, PSTs observed two different lessons, each of which had a different grade level and subject. Furthermore, focus group interviews with PSTs who observed the same mentors were done as part of the study's scope to reveal the general instructional dispositions of the mentors. These focus group interviews were also designed to provide triangulation of data to support trustworthiness and verification, which are critical for the validity and reliability of qualitative research.

2.4. Data Analysis

In the current study, the phases from selecting a mathematical task to the completion of implementation are determined as a unit of analysis and if a mathematical task has sub-items that are related to one another, they were all determined as a single task. However, if the sub-items were independent of one another, each task was evaluated as a separate task. Within this period, PSTs analyzed a total of 138 tasks. They characterized the CD of tasks selected and implemented by the mentors through the Task Analysis Guide (Stein et al., 2000) and explained the factors affecting the CD of the tasks with their reasons. On the other hand, as a researcher, I coded the CD of tasks described in detail in the observation forms and met with PSTs to reach a consensus on contradictions. By subjecting the summary of reasons of the PSTs in the observation forms to content analysis, I coded for the factors affecting the CD of the tasks and combined them under certain themes. A sample student observation and coding are shown in Appendix 1.

3. FINDINGS

3.1. Cognitive Demand of Selected and Implemented Tasks

In the current study, each of the mentor teachers was observed by three PSTs for a total of six hours. As shown in Table 3, Akin implemented an average of 2.5 Ceren 3.6, Ali 7.3, and Duru 9.5 tasks per lesson during these six-hour observations. He selected higher-level tasks (86.6%) than other teachers and almost maintained their cognitive demand. Furthermore, he implemented a DM task that wasn't observed in other teachers' classrooms. Although Ceren implemented more tasks (27.2%) at a high level than the other two teachers, she mainly selected and implemented PwoC level tasks. Duru selected a total of 24 high-level tasks (4 PwC, 5 DM, and 15 PwC) but was able to implement only 4 (7%) of them at the PwC level. In other words, while Duru was successful in selecting high-level tasks, she struggled to maintain them at a high level. Ali, on the other hand, almost completely selected low-level tasks (95.5%) and implemented them at a low level. Table 3 shows the cognitive demand levels of selected and implemented tasks.

848

		Task Selection	Task Implementation	Akin	Ceren	Duru	Ali
	Maintained	DM	DM	1 (6.6%)	-	-	-
	the cognitive demand	PwC	PwC	11 (73.3%)	6 (27.2%)	4 (7%)	2 (4.5%)
Implemented at a high level	Increased the cognitive demand	PwoC	PwC	1 (6.6%)	-	-	-
	Total			13 (86%)	6 (27.2%)	4 (7%)	2 (4.5%)
	Declined the	DM	PwoC		-	5 (8.8%)	-
	cognitive demand	PwC	PwoC	1 (6.6%)	3 (13.6%)	15 (26.3%)	-
Implemented at a low level	Maintained	PwoC	PwoC	1 (6.6%)	12 (54.5%)	32 (56.1%)	40 (91%)
	the cognitive demand	Mm	Mm	-	1 (4.5%)	1 (1.7%)	2 (4.5%)
	Total			2 (13.3%)	16 (72.7%)	53 (93%)	42 (95.5%)
Implemented	task per a lesso	on		2.5	3.6	9.5	7.3

Table 3. Teachers' task selection and implementation

3.1.1. Instructional dispositions of teachers

Teachers make many important decisions before and during the lesson such as giving time to solve the task, focusing on student solutions, and making connections. These instructional dispositions affect the level of implemented tasks. Table 4 shows the factors affecting the cognitive demand levels of the tasks implemented by the mentor teachers. These factors are supported by qualitative data and presented under sub-headings for each teacher.

				Teacher (Nu	nber of implen	nented tasks (n))
Fa	Factors affecting the cognitive demand of the tasks			Akin (n= 15)	Ceren (n= 22)	Duru (n= 57)	Ali (n= 44)
<u>ಟ್</u>	Providing	Providing	Giving feedback	5 (33.3%)	-	-	-
Exploring tasks	exploration	adequate time	No feedback	8 (53.3%)	5 (22.7%)	5 (8.8%)	2 (4.5%)
xplo tas	time	No exploration time		2 (13.3%)	17 (77.2%)	52 (91.2%)	42 (95.5%)
Ĥ	Building on p	rior knowledge		10 (66.7%)	6 (27.2%)	-	1 (2.2%)
	Selecting	Focusing on one strategy		9 (60%)	3 (13.6%)	-	-
gua	student- invented	Focusing on multiple strategies		3 (20%)	2 (9.1%)	-	2 (4.5%)
invented strategies Selecting the	Focusing on misconceptions		3 (20%)	-	-	-	
Sel sol	$\frac{1}{2}$ Selecting the expected correct solution strategy		-	2 (9.1%)	14 (24.5%)	16 (36.3%)	
Selecting no solution			3 (20%)	17 (77.2%)	43 (75.4%)	26 (59%)	
	Setting a whole class discussion		10 (66.7%)	13 (59.1%)	1 (1.7%)	13 (29.5%)	
Summarizing	Connection	Emphasizing u behind concepts	nderlying meaning	13 (86.7%)	12 (54.5%)	5 (8.8%)	18 (40.9%)
		Connecting with	n representations	10 (66.7%)	12 (54.5%)	8 (14%)	19 (43.1%)
		Connecting with	n daily life situations	9 (60%)	6 (27.2%)	4 (7%)	-
Sı	Focusing or connecting	n a procedural	solution without	2 (13.3%)	9 (40.9%)	42 (73.6%)	24 (54.5%)

Table 4. Factors affecting the cognitive demand of the tasks implemented by teachers

Akin teacher: "Student-oriented lessons"

PST1: Akin has a master's degree in creative drama. He has chosen drama training, being influenced by Brian Way's question, who is the pioneer of drama, of "why children run out when the bell rings".... He claimed that he used as well as other methods and strategies in his lessons. Most of the time, he uses a questioning technique that allows students to participate in the lesson and solve problems by reasoning.

The above excerpt reveals PST1's views on Akin's approach. Compared to other teachers, Akin fulfills the requirements of a student-oriented approach, gives importance to student strategies, and

discusses ideas. He encourages them to explore, selects high-level tasks, and implements these tasks at a high level (see Appendix 1). As shown in Table 4, In most tasks (86.7%), Akin allowed enough time for exploration, set 66.7% of the tasks on the students' prior knowledge, monitored students in 33.3% of the tasks, and provided feedback on their solutions. The following dialogue presents PSTs views:

PST1: *He waits so long (while monitoring solutions) that we feel bored, but eventually (students) come up with their own solutions.*

PST3: *He monitors students one by one while they are solving tasks. When they have challenges in solving the tasks, he changes the problem's context, provides clues or gives examples from their prior knowledge.*

PST2: If the task is too abstract, he immediately sets it up by changing numbers, establishing an analogy, or simplifying it, and indeed, we noticed that he get better feedback from the students.

As the dialogue shows, Akin examines the solutions while monitoring the task, provides formative feedback, and makes modifications to the problem's context as needed.

He frequently examined student solutions by launching a few activities (an average of 2.5 per lesson), and the majority of the time (66.7%) they were investigated jointly in a whole-class discussion. In 80% of the problems, he selected students' solutions, while in 20% of the tasks, he conducted the lesson without selecting a student solution. In 60% of the tasks, he selected a single student solution, and in 20% of the tasks, he selected multiple solution strategies. He also highlighted misconceptions in 20% of the tasks, even though this was never observed in other teachers' classrooms. Akin's approach to multiple solution strategies is outlined in the following excerpt:

PST3: *His enacted tasks usually have multiple steps and require detailed thinking. He usually opens the task on the smartboard, provides some time for students to explore it, and implements only one or two tasks in 40 minutes. He provides feedback to the students while monitoring their strategies, and they examine different strategies together. He builds lessons on their prior knowledge without giving information directly. I don't think he declined the cognitive demand of any tasks.*

The following excerpt explains Akin's view of classroom discussions:

PST3: *He purposefully let students explain their correct or incorrect solutions. There is often a discussion-based setting beyond show and tell. For example, he asks, "What do you think about your friend's solution? Is it correct or incorrect, and why?*

Except for the 2 tasks (13%), he made connections in all of the other tasks. He emphasized the underlying meaning of concepts in 86.7%, utilized multiple representations in 66.7%, and established connections with daily life in 60% of tasks. According to PST2 "*The students realize the generalization at the end of the discussion. He never explained a procedure, as our teachers did, like "equalize denominators when adding fractions"*. As stated, Akin aims for students to reach generalizations instead of giving routine procedures directly and frequently builds a setting to make connections.

Ceren teacher: "Discussion-based lessons"

PST5: She doesn't wait long for the task to be explored. **PST4:** However, she usually uses the questioning method. She asks how to do it; for example, when a student shares his idea, she usually asks if anyone else thinks about it. **PST4:** She doesn't launch tasks requiring too much time to solve.

PST6: She obviously employs invention method. She frequently gives several examples, enabling students to notice the contrasts between them and, as a result, they build their conceptions.

The above excerpt reveals PST4, PST5, and PST6's views about Ceren's lessons. Compared to Akin, Ceren mostly prefers directly discussing ideas without monitoring student-invented strategies, however, as shown in Table 3, she mostly selects low-level tasks.

Ceren provided exploration time in 22.7 % of the tasks, as seen in Table 4, but she just waited for student solutions without monitoring or providing feedback in these tasks. She built 27.2% of tasks on their prior knowledge, focused on a student-invented single solution for 13.6% and multiple solutions for 9.1% of tasks. While she selected expected correct student solutions to be shared on the board in 9.1%, this wasn't the case in 77.2%. For example, in a task where different strategies emerged, the teacher modeled the algebraic operation"(2x + 1). (2x + 3)", then asked, "How do we do the multiplication?". One of the students applied the distributive property as "(2x + 1). (2x + 3) =(2x + 1).2x + (2x + 1).3", while the other one applied as "(2x + 1).(2x + 3) = 2x.2x + 2x.3 +1.2x + 1.3". As a result, both students explained their ideas, and they discussed how these two ideas had the same result.

Ceren built a discussion-based setting in 59.1% of the tasks, emphasized the underlying meaning behind concepts in 54.5%, made connections with representations in 54.5%, and with daily life in 27.2%. The following excerpt shows PSTs views about Ceren's lessons:

PST5: She connected algebraic identities with the area of the square.

PST6: *She proved the algebraic identities by using models.*

PST5: The feature I like the most is her examples from daily life. For example, she brought a hula hoop to the classroom today to explain the intersection set.

The PSTs stated that Ceren initiates discussions on establishing conceptual relationships and gives examples from daily life situations. However, as seen in Table 4, Ceren focused on algorithmic solutions in 40.9% of the tasks. Below is an excerpt about this situation:

PST4: *I like her lecturing. She utilizes concrete materials to make connections without giving direct formula. However, she may prefer more higher-level tasks or set a problem-based atmosphere.*

PST5: Solving a lot of questions or focusing on a few? I think she should either select high-level tasks or set up these tasks to improve their cognitive demand level.

PST4: Instead of asking too many similar tasks, emphasizing meaning may be better.

PST5: *There are a lot of questions based on reminding the algorithm or procedure.*

As stated, while Ceren generally selects high-level tasks for launching a new subject, she prefers low-level algorithmic tasks for reinforcing a subject. However, PSTs suggest that Ceren should give more emphasis on high-level tasks in her lessons.

Duru teacher: "Exam-based lessons"

PST8: She immediately launches the subject and solves many questions. **PST9:** She gives 100-150 questions per week as homework. **PST8:** She wants all question types to be experienced, like memorizing solution paths.

The above dialogue shows the views of PSTs about Duru's lessons. She usually aims to teach solution paths without allowing students to engage in high cognitive effort and gives high-level tasks as homework. While solving these tasks in the classroom setting, she mainly emphasizes routine procedures without focusing on the underlying meaning of these tasks.

As shown in Table 4, while Duru didn't provide any exploration time for the students in 91.2% of the tasks, she provides time in very few tasks (8.8%) without monitoring and giving any feedback. There wasn't any student-invented strategy and she get students to the board to write the correct solutions of tasks that aim to reinforce the algorithm in 24.5% of the tasks. Furthermore, in 75.4% of the tasks, she directly explained solutions without selecting any student to share his idea. The following excerpt shows PST9's views about Duru's classes:

International e-Journal of Educational Studies (IEJES)

PST9: She sometimes provides exploring time approximately 5 minutes, but mostly for routine low-level tasks. For example, she firstly demonstrated the solution method of a task, and students applied it to others. They then came up to the board to write solutions, but she explained these solutions one more time. Students who have erroneous solutions also come to the board as well, but she corrects them and explains them the correct solution.

Duru set up whole-class discussions for only 1.7% of tasks. She emphasized underlying concepts in 8.8% of the tasks, employed multiple representations in 14% of the tasks, and made connections with real-life situations in 7% of tasks. She also focused on algorithmic solutions in 73.6% of the tasks. Consider the following quote:

PST8: *There is no discussion in the classroom.* **PST9:** *The majority of students haven't mentally been involved in classroom activities.*

As the quotation indicates, just a few students in Duru's class were participating in classroom activities, and they hardly participated in whole-class discussions. The following quotation reflects the PSTs' views on the extent of Duru's tendency to make connections.

PST8: She usually instructs low-level tasks.

PST9: Tasks at the end of the units mostly had high cognitive demand, but she declined their level since she didn't make connections. Even if tasks require making connections, she declined their levels by attempting to solve many problems quickly and preferring to have students memorize solution types rather than conceptual learning. I hardly noticed her making connections with daily life or using multiple representations. I don't think she purposefully utilized models. She introduced modeling as an alternative solution once the solution was complete, rather than encouraging students to use the model to explore the task. For example, she instructed the rules of addition operations in algebraic expressions and then demonstrated using models as an alternative approach without reasoning.

PST8: When dealing with integer operations, students first performed routine procedures taught to them by their teachers, then drew number counters. For instance, the teacher calculated the result of +12:3 and then drew a suitable model for it. They should have examined the operation with models first, then generalized it, in my opinion.

As the excerpts show, Duru's didactic approach limits her from establishing an atmosphere to build relationships. While the task requires making connections with models, Duru prefers to teach algorithmic procedures rather than focusing on the underlying meaning of the task and making connections.

Ali teacher: "Model it and solve"

PST10: The teacher selects cognitively low demanding tasks, but instead of memorizing the rule, he explains it on the models. He recommends students to "model and solve".

Ali prefers to spend more time on solutions on the smartboard, instead of allowing students to explore the tasks, As shown in Table 4, Ali did not allow time for students to explore tasks in 95.5 percent of tasks. In the tasks he spent time on (4.5%), he waited until the tasks were solved instead of monitoring student solutions and giving feedback. He also built 2.2% of the tasks on students' prior knowledge. In only 4.5% of the tasks, he allowed student-invented strategies, and in 36.3% of the tasks, he selected the expected correct solution strategies to be shared with the class. In 59% of the tasks, he selected no solution and solved the problems on the board himself. Consider the quote of PST10 below:

PST10: First of all, he solves the task of ensuring that students comprehend the subject, without allowing exploration time to the students. In his demonstration, he focuses on the concept, uses models, and then moves on to solving too many low-level routine tasks.

Ali, as mentioned by PST10, usually prefers to begin by presenting a task's solution method and then follow up with activities to reinforce his solution methods. As shown in Table 4, he provided a discussion-based setting in 29.5% of the tasks, emphasized the concept underlying the tasks in 40.9%, and used multiple representations in 43% of the tasks. In 54.5% of the tasks, on the other hand, he focused on algorithmic solutions.

4. DISCUSSION and CONCLUSION

In this research, one of our goals was to uncover mentor teachers' task selection and implementation routines in terms of cognitive demand. According to the findings, although teachers' task selection and implementation routines varied, three of the teachers (Ceren, Duru & Ali) either decreased the cognitive demand level of the high-level tasks during the implementation or already selected and implemented low-level tasks. Research has shown that the cognitive demand of mathematical tasks included in the curriculum materials are considerably high (Ubuz et al., 2010), but teachers either select low-level tasks or decline their level during the implementation phase (Henningsen & Stein, 1997; Stein et al., 2000). In this sense, their task selection and implementation behaviors are similar to those observed in studies (Güzel, Bozkurt, & Özmantar, 2021). On the other hand, unlike the other teachers, one of the teachers (Akin) mostly selected high-level tasks and maintained the cognitive demand of these tasks during the implementation phase. This approach is among the intended behaviors in teachers in many studies, recent research has focused on improving teachers' task selection and implementation quality through professional development (Ader, 2020; Arbaugh & Brown, 2006; Boston, 2013; Boston & Smith, 2009, 2011).

Stein et al. (1996) claim that "teachers' and students' habits and dispositions refer to relatively enduring features of their pedagogical and learning behaviors that tend to influence how they approach classroom events" (p. 461). From this point of view, it may be noted that even though Ceren, Duru, and Ali have various task preferences and implementation routines in the classroom, their instructional disposition can be identified as a "teacher-driven" approach. In teacher-driven classrooms, there are settings built on whole class discussion and making connections as well but these discussions tend to be more concerned with the solutions selected by the teacher than those emerged by the students. This is precisely why Akin's approach has been described as "student-oriented".

4.1. Teacher-Driven Approach

In a teacher-driven setting, teachers usually adopt an approach in which their ideas and preferences are emphasized rather than a course based on student ideas. As shown in Table 4, various factors were observed affecting cognitive demand tasks implemented by these teachers in the study. For example, teachers mostly avoided giving sufficient time to explore the task, didn't base their tasks on students' prior knowledge, dismissed student-invented strategies, and accordingly didn't focus on misconceptions. They usually prompted students to write the desired correct solution when they were called to the smartboard, so they concentrated on instruction instead of discussing student-invented strategies. Teachers' lack particular content knowledge or the ability to identify critical topical understandings (Wallin & Amador, 2018) or "teachers' orientations-their beliefs, values, and preferences-influence their actions (Stockero, Leatham, Ochieng, Van Zoest, & Peterson, 2019). Therefore, in recent years, the number of professional development research such as improving teachers' noticing skills has been increasing (e.g., Guner & Akyuz, 2019).

Although Ceren, like the other two teachers, had a teacher-driven orientation, she favored a "discussion-based method" as pre-service teachers mentioned, and she strove to emphasize underlying concepts, multiple representations, and daily life situations throughout these discussions. However,

selecting low-level tasks and not allowing students enough time to explore led teachers to implement these tasks at a low level. In fact, a discussion-based approach in a reform-oriented classroom is deemed important in studies (Stein, et al., 2008), but it was stated in interviews with pre-service teachers that Ceren's discussions are far from revealing students' ideas. Studies have emphasized the importance of quality of tasks for creating a rich classroom discussion atmosphere (Stein et al., 2008), higher-order questioning (Ni et al., 2014), and shaping students' mathematical thinking (National Council of Teachers of Mathematics [NCTM], 1991).

Duru, on the other hand, did not enable students to discuss their ideas or make connections, and she generally routinized tasks by removing their problematic parts, despite selecting cognitively demanding tasks. As a result, she declined the cognitive demands of tasks, as in earlier research (e.g., Hong & Choi, 2019). Although teaching procedures in mathematics is necessary, research has shown that focusing solely on procedures and ignoring mathematical meaning is ineffective in ensuring students' learning and mathematical progression (Stein & Lane, 1996; Stigler & Hiebert, 1999). Unlike Duru, Ali emphasized multiple representations and the meaning of concepts underlying tasks, but he used representations to make his instruction more understandable rather than as a tool for students to explore the task in a sufficient time.

4.2. Student-Oriented Approach

In this study, a student-oriented classroom is expressed as a classroom climate where the teacher mostly tries to reveal student ideas, set whole-discussions and tries to make connections between these ideas. Akin, compared to other teachers, launched fewer tasks per lesson and provided students to explore them in-depth. He constructed the majority of the tasks on students' prior knowledge, provided adequate time to explore tasks, and frequently gave formative feedback to students. He led to discuss student-invented strategies in the classroom and occasionally focused on multiple solutions and misconceptions. He focused students' attention on the underlying meaning of concepts, making connections with multiple representations and daily life situations, rather than algorithmic solutions. Accordingly, he mostly selected high-level tasks and maintained the cognitive demand of these tasks during the implementation phase. Akin's approach is in line with the approaches of teachers who implement the reform-based curriculum (Ader, 2020; Ni et al., 2014; Stein et al., 1996). In the current study, no research was conducted to measure students' learning, but it is stated that providing support and using their existing or prior knowledge can give students confidence (Hong & Choi, 2019) and ensure success (Stein & Lane, 1996), and there are quite a few evidence that CDTs provide all students with important learning opportunities (e.g., Stein & Lane, 1996; Zohar & Dori, 2003).

4.3. Summary and Implications

The results of the current study indicate that the mentor teachers observed by PSTs had a range of instructional dispositions in terms of factors such as task selection, giving task exploration time, selecting on student-invented strategies, building the task on students' prior knowledge, setting wholeclass discussion and making connections. The level of cognitive demand of tasks in the implementation phase is directly impacted by these factors. While some of the PSTs gained experience in classrooms with teachers who primarily favor low-level tasks, pay little attention to student ideas, and concentrate on reinforcing algorithmic procedures; a number of PSTs gained experience in classrooms with mentors who select high level tasks and maintain their level by allowing students time to explore tasks, listening and discussing their ideas, and making connections. It should be no surprise that the second group of PSTs would notice a number of crucial abilities that qualified teachers' possess according to the literature on mathematics education. This finding highlights that each mentor teacher had different instructional orientations, which would generate lasting traces in the formation of PSTs' professional identities. Therefore, selecting a mentor teacher is crucial for PSTs who would have just experienced teaching before beginning the profession. It is of great importance for PSTs' training to have mentors who use reform-based activities, take into account

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strategies, recognize cognitively demanding tasks and can implement tasks without declining their cognitive demand. As a matter of fact, Akin's mentees acknowledged that this supervision provided them with a great experience. At this point, it is recommended that educational faculties may consider criteria for selecting mentor teachers, such as getting feedback from previous pre-service teachers, postgraduate-doctoral education status, participation in professional development programs, and experience.

The data for this study was gathered through observations and interviews conducted by preservice teachers. Because detailed observations in a scientific study require professionalism, observations performed by novice teachers may be considered a limitation. To overcome this limitation, the researcher employed a tool that allows pre-service teachers to make comprehensive observations and conducted in-depth focus group interviews with them. In addition, based on preservice teachers' grasp of the cognitive demand framework, this study attempted to explain the nature of mentor teachers' classroom atmosphere. As a consequence, through observations and perspectives of PSTs.

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Appendix 1. The participants sample student obser	vation and coding
body into parts. This procedure helps painters to check whether the drawn parts are compatible. For example, they sketch the head as $\frac{1}{8}$ of the total length, arm length as $\frac{3}{8}$ of the total length, and the leg length as $\frac{4}{8}$ of the total length. If the length of a human body sketched in accordance with the above measurements is 40 centimeters, can you find how many centimeters the lengths of the head, arms, and legs are sketched? If these measurements were accurate for all people, how many centimeters would a person with a height of 176 centimeters have in head, arm, and leg length? Source: MNE (2019, pp. 107) The summary of task implementation After the teacher reminded prior knowledge about fractions, he asked them to sketch a human body by counting the squares in the notebook. T: Look at how long a square in your notebook is with a ruler. S1: 0.5 cm. T: So, 2 squares equal 1 cm. If the man's height is 24 cm, how many squares do we need? S2: I need 48 squares. The teacher provided approximately 10 minutes for the students' sketch, but most could not sketch it. So, the teacher started to sketch it on the board himself by questioning students. Because the students had given no response, the teacher compared the situation to a piece of baklava (a kind of sweet), divided 24 into 8 equal portions,	This task aims to find the fractional part of a whole. Before the solution, the teacher reminded prior knowledge on fraction and unit fraction concepts. He explained that to find the fractional part of a whole, it is necessary to find the unit fraction and multiply it by the number of parts. To be better understood, he let students make connections by exploring daily life situations . It is a PWC type of task because the students realized the underlying concepts of proportional reasoning by comparing quantities. While monitoring students' solutions, he provided feedback on their mistakes . Since he provide a classroom setting to enable students to think and reason , there was no change in the CD level during the implementation of tasks. Researcher's comments The task aims to enable students to use proportional reasoning skills connecting with fractional quantities in a real-life context. In the second part of the task, they have to rethink the proportional quantities they discovered for real human dimensions. Because the second part of the task is complicated and cannot be solved immediately, it causes anxiety in students, seeks to regulate cognitive processes and demands high-level cognitive functions. Hence, the task is at the DM level . The teacher allowed for a while for the task to be solved, then completed it on the board with help of the students. Even though he did not provide enough time for the task. Therefore, there has been no change in the cognitive demand level of the task. The factors affecting cognitive demand, emphasized by the PST, as can be seen in the bold signs, were determined as establishing a relationship with daily life, conceptual connections, building on preliminary knowledge, reasoning, and providing students with formative feedback.
how many apples are given to each friend of Ayse. The summary of task implementation The teacher provided students time to explore the question, however, he didn't monitor their solutions. He then initiated a whole-class discussion, emphasized on equal partitioning and focused on two different solutions. In the first solution, after distributing 25 apples to 5 people, the remaining two apples were divided into $\frac{2}{5}$ for each friend. In the second solution, each apple was divided into 5 slices, so they discussed on 27 slices of $\frac{1}{5}$, or $\frac{27}{5}$. Then, by connecting these solutions, he made them realize that $5\frac{2}{5}$ is equal to $\frac{27}{5}$. Goal: Does addition and subtraction in algebraic	This task includes a daily life context, aiming to make sense of a conversion of a compound fraction to a mixed fraction and to convert a mixed fraction to a compound fraction; hence it is a PWC task. The task was launched by the teacher in two different ways . He provided adequate time to explore and implement it without declining the cognitive demand of the task by constantly questioning . Researcher's comments The task aims to establish a conceptual relationship and connect with daily life context; hence it is a PWC task. Although the teacher didn't keep the monitoring time longer, he focused on different strategies, enabled students to establish conceptual relationships, and make judgments in a whole-class discussion. Therefore, the task was maintained at the PWC level in the implementation phase. The factors affecting cognitive demand, emphasized by the PST, as shown in the bold signs, were determined as making conceptual connections, associating with daily life, building on preliminary knowledge, providing exploration time, emphasizing various strategies, and orchestrating whole-class discussions. The CD levels and reasons determined by the PST
expressions. Observer: PST11; Class: 7-E, Teacher: Duru	This task involves the student making operations and modeling in algebraic expressions. It is a PWC task as it requires connecting algebraic expressions with model representation .

				Osmun DAGDA
Task: Add The summa The teacher in the first giving the s expressions addition op	ary of task in asked a stud and second tudents any t 2x-3 and -x eration (2x-3	models on the state interval to explore. T + 5 and the teach) + (- $x + 5$) = x	ebraic expressions martboard without 'he student said the	the correct solution using the routine algorithms she taught, it caused a decline in the cognitive demand of the tasks during implementation. The factors affecting cognitive demand emphasized by the PST, as shown in the bold signs, were determined as not providing enough time to explore
				focusing on the correct answer, and routinizing the task.
Observer: Task: $\frac{3}{4}$ of you have ea The summa The teacher whole. He to the smart it by first ca kg, and then kg. He then	PST8; Class: the oranges in the 2 kg of o ary of task in provided chu provided a st tboard to con alculating the n calculating subtracted th	5-D; Teacher: A in the basket is 6 ranges, how man mplementation the that they shot tudent some time mplete the work. The weight of one p the total weight, the eaten portion to	5 kilograms. Since y kilos are left? ald go from part to e and directed him The student solved biece, which was 2 which was $4x2 = 8$ o get the solution.	whole relationship, it is a PWoC level task. The teacher did not give students any time to explore the task throughout the implementation but instead aimed to reinforce a routine procedure without reasoning, therefore it was maintained at the PWoC level. The factors affecting cognitive demand, emphasized by the PST, as shown in the bold signs, were determined as routinizing, providing clues, focusing on the correct answer, and not making connections.
		pts of variable, c	onstant terms, and	The CD levels and reasons determined by the PST
		s: 7-E; Teacher: in the table. Constant terms	Duru Coefficients	Recalling the rules or formulas of variables, constant terms and coefficients in algebraic expressions is a memorization task that does not involve any procedure and reasoning. The teacher's statement that the unknown is a variable led to the misconception that each letter is a variable. During the task's
a+3	a	3		implementation, there was no change in the level of cognitive
2x+3y			2, 3	demand.
5mn				Researcher's comments The task requires students to recall definitions, facts, rules, or
The teacher	asked about		hat should be used ng trouble finding	formulas in their minds without the need for thinking, reasoning or using any procedure. Therefore, it is a memorization task The factors affecting cognitive demand, amphasized by the