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Testing the Significance of Regression Coefficients in Liu Type Estimators

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Highlights

- This paper focuses on significance testing of regression coefficients of biased estimators.
- A test is obtained to test the significance of the regression coefficients of the Liu-type estimator.
- Performances of the model coefficients' significance for Ridge, Liu, and Liu-type estimators.

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Abstract

In the linear regression model, the multicollinearity problem arises when there is a linear relationship between independent variables. This situation causes the variance of the estimations of the model parameters obtained by the Least Squares Estimator method to increase and move away from the true value, resulting in unstable and incorrect results. Biased Estimator methods are developed to eliminate the adverse effects caused by multicollinearity. In this study, a test statistic is obtained to test the significance of the model coefficients for the Liu-Type Estimator using the test statistic method suggested in the study of Halawa and El-Bassiouni (2000). With a simulation study, the significance of the model coefficients of the Ridge, Liu, and Liu type biased estimators in different situations is tested; the type I errors and power values of the estimators are calculated; the results are compared. In addition, a real data application is performed to better understand the test procedure.

1. INTRODUCTION

In the linear regression model, the Least Squares (LS) method is frequently used for estimating the regression parameters. Estimations obtained by LS method give reliable results in case of certain assumptions are met. One of these assumptions is that there is no relationship between independent variables. In the linear regression model, a multicollinearity problem arises when a complete or nearly complete linear relationship exists between one or more independent variables. This increases the variance and standard error of the LS estimator, causing the incorrect results of model estimations and different test statistics. One of the most common methods to solve the multicollinearity problem is biased estimators.

Hoerl and Kennard (1970) proposed the Ridge estimator in their study on biased estimation methods and analyzed it according to the Mean Square Error (MSE) values [1]. Liski (1982) included the preference criteria between LS estimators and biased estimators according to their MSE values [2]. Liu (1993) proposed the Liu-Kejian estimator with a single bias parameter as an alternative to the Ridge estimator [3]. Akdeniz and Kaçıranlar (1995) defined this estimator as the Liu estimator [4]. Halawa and El-Bassiouni (2000) used the t-test statistic to test the significance of the regression model coefficient based on the ridge estimator. The simulation study compared the test statistic results obtained for LS and Ridge estimators [5]. Kibria (2003) compared some Ridge regression and LS estimators through a simulation study [6]. It has been observed that biased estimation methods with a single bias parameter do not give good results when multicollinearity is severe. For this reason, studies on biased estimation methods with two parameters have

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been conducted in the literature. Liu (2003) proposed the Liu-type estimator as a two-parameter biased estimator using k and d parameters. He also showed that in the case of severe multicollinearity, it gives better results than the Ridge regression estimator according to the MSE criterion [7]. Ebegil, Gökpınar, and Ekni (2006) compared the Ridge and Liu estimators with simulation techniques by using the test statistic obtained in the work of Liski (1982, 1983) [2, 8, 9]. Özkale and Kaçıranlar (2007) proposed a new estimator by studying estimators with two bias parameters [10]. Subsequently, Sakallıoğlu and Kaçıranlar (2008) proposed a different two-bias parameter estimator, which they defined as a k-d class estimator [11]. Ebegil and Gökpınar (2012) used a test statistic established under a necessary and sufficient condition to compare LS and Liu-type estimators [12].

Gökpınar and Ebegil (2016) improved the work of Halawa and El-Bassiouni (2000) for the Ridge estimator and examined the k estimators that give the best results for the Ridge estimator. They compared the test statistics according to experimental type I error rates and power values of the test by performing simulation studies under different conditions. As a result, they tried to determine the best k values for the Ridge estimator [13]. Wilcox (2019) proposed a test statistic to test the significance of the parameters of the Ridge regression estimator in the case of varying variance [14]. In the study by Kibria and Melo (2021), a test statistic was obtained using the test statistic proposed in Halawa and El-Bassiouni (2000) for the Liu estimator. They simulated this test statistic under different situations and compared the performance of type I errors and the power of the test [5,15].

Although there are many studies in the literature on estimating the biasing parameters, there are few studies on testing the significance of the model coefficients obtained with these estimation methods.

In this study, a test statistic for testing the significance of the regression coefficients of the Liu-type estimator was obtained using the test statistic proposed for the Ridge estimator in Halawa and El-Bassiouni (2000). With the simulation study, the significance tests of the regression coefficients of the Ridge, Liu, and Liu-type estimators under different conditions were performed, and the experimental type 1 error rates of the estimators were compared with the performance of the power of the test. A real data application was made on Hald's Portland Cement data [16].

2. MATERIAL METHOD

2.1. Regression Model

When p is the independent variable, and n is the number of observations, the general form of the multiple regression model can be shown as in Equation (1)

$$Y = X\beta + \varepsilon. \tag{1}$$

Here Y is the dependent variable vector centered around the $(nx1)$ dimensional mean, where $q = p + 1$, X is the (nxq) dimensional observation matrix of non-stochastic independent variables, averaged and scaled to obtain $(X'X)$ correlation form; β, is $(qx1)$ dimensional vector of unknown parameters; ε is $(nx1)$ dimensional zero mean $\sigma^2 I$ random error vector with variance.

Since the $C = X'X$ matrix is a qxq dimensional positive definite matrix, there is an orthonormal matrix P that diagonalizes the $C = X'X$ matrix in the form $P'CP = \Lambda$. Here Λ is a (qxq) dimensional diagonal matrix whose elements are the positive eigenvalues $(\lambda_1, \dots, \lambda_q)$ of the $(X'X)$ matrix [2,9].

Where $Z = XP$ and $\alpha = P'\beta$, the canonical form of the model in Equation (1) is obtained as in Equation (2)

$$Y = XPP'\beta + \varepsilon = Z\alpha + \varepsilon. \tag{2}$$

In regression analysis, the LS estimator is commonly used to estimate the parameters of the model given in Equation (1). The LS estimator for the model in Equation (1) is given in Equation (3)

$$\hat{\beta}_{LS} = (X'X)^{-1}X'Y. \tag{3}$$

The hypothesis established to test the significance of the regression coefficients is as in Equation (4)

$$\begin{aligned} &H_0: \beta_i = 0 \\ &H_1: \beta_i \neq 0. \end{aligned} \tag{4}$$

The test statistic based on the LS estimator is given in Equation (5)

$$t_{i(LS)} = \frac{\hat{\beta}_{i(LS)}}{S(\hat{\beta}_{i(LS)})}. \tag{5}$$

$\hat{\beta}_{i(LS)}$ is the i-th element of $\widehat{\beta}_{LS}$, and $S^2(\hat{\beta}_{i(LS)})$ is the i-th diagonal element of the variance estimate of $\hat{\beta}_{LS}$. The related equation is given in Equation (6)

$$Var(\hat{\beta}_{LS}) = \sigma^2(X'X)^{-1}. \tag{6}$$

Here, when σ^2 is unknown, the estimation of σ^2 based on the LS estimator isused and given in Equation (7)

$$\hat{\sigma}^2 = \frac{(Y - X\,\hat{\beta}_{LS})'(Y - X\hat{\beta}_{LS})}{n-q-1}. \tag{7}$$

The test statistic given in Equation (5) has a t-distribution with $(n - q - 1)$ degrees of freedom under H_0 hypothesis. This test is referred to as the LS test.

Some assumptions are needed to obtain reliable values with the LS estimation method. One of them is that no linear relationship should be between the independent variables. If there is a linear relationship between the independent variables in the model, the problem of multicollinearity arises. In this case, the inverse of the $X'X$ matrix cannot be obtained; therefore, parameter estimation cannot be performed. In cases where the inverse is found, the estimations based on the regression model may be inconsistent and unstable since the variance will be too large.

If multicollinearity is encountered in the regression model, among the biased estimators developed to eliminate this situation, Ridge and Liu estimators having a single bias parameter are used. When the severity of multicollinearity increases, a biased estimation method having two bias parameters, such as the Liu-type estimator, should be used.

2.2. Test Statistics for Regression Coefficients based on Ridge and Liu Estimation Methods

The Ridge regression estimator is given in Equation (8)

$$\hat{\beta}_R = (X'X + kI)^{-1}X'Y\,, \;\; k \geq 0. \tag{8}$$

According to Hoerl and Kennard (1970), the k value that minimizes the MSE of the Ridge estimator is obtained from Equation (9) [1]

$$\hat{k}_{HK} = \frac{\hat{\sigma}^2}{\hat{\beta}_{max}^2}. \tag{9}$$

$\hat{\beta}_{max}^2$ is expressed as the largest element of the $\hat{\beta}_{LS}$ estimator.

The *k* value developed by Hoerl, Kennard, and Baldwin (1975) with a different method is given in Equation (10) [17]

$$\hat{k}_{HKB} = \frac{q\hat{\sigma}^2}{\hat{\beta}_{LS}{}'\hat{\beta}_{LS}}. \tag{10}$$

To test the null hypothesis in Equation (4), the test statistic based on the Ridge estimator, which is a non-exact t-type test, is given in Equation (11) [5]

$$t_{i(R)} = \frac{\hat{\beta}_{i(R)}}{S(\hat{\beta}_{i(R)})}. \tag{11}$$

$\hat{\beta}_{i(R)}$ is the i-th element of $\hat{\beta}_R$ and $S^2(\hat{\beta}_{i(R)})$ is the i-th diagonal element of variance estimation of $\hat{\beta}_R$, as given in Equation (12)

$$\mathrm{Var}(\hat{\beta}_R) = \hat{\sigma}^2(X'X + kI)^{-1}(X'X)(X'X + kI)^{-1}. \tag{12}$$

When σ^2 is unknown, the estimation of σ^2 can be used based on the Ridge estimator given in Equation (13)

$$\hat{\sigma}^2{}_R = \frac{(Y - X\hat{\beta}_R)'(Y - X\hat{\beta}_R)}{(n - q - 1)}. \tag{13}$$

The test statistic given in Equation (11) has a t-distribution with $(n - q - 1)$ degrees of freedom under H_0 hypothesis. The test statistic in Equation (11) based on $\hat{k}_{HK}$ is named HK, and the test statistic in Equation (11) based on $\hat{k}_{HKB}$ is named the HKB test.

Liu (1993) proposed the Liu estimator as an alternative to the Ridge estimator. The Liu estimator is given in Equation (14) [3]

$$\hat{\beta}_L = (X'X + I)^{-1}(X'Y + d\hat{\beta}_{LS}), \quad 0 \le d \le 1. \tag{14}$$

Here d is defined as the bias parameter of the Liu estimator. Which of the Ridge and Liu estimators is more effective in estimating the regression coefficients depends on the unknown parameters k and d. It is more practical in application to use their estimations instead of the unknown parameters. In this respect, many formulas have been developed for estimating the biasing parameter d, as in the estimation of the k bias parameter of the Ridge estimator. Liu (1993) proposed the estimator in Equation (15) for the bias parameter d, which minimizes the MSE value [3]

$$\hat{d}_L = 1 - \hat{\sigma}^2 \left[\frac{\sum_{i=1}^{q} \frac{1}{\lambda_i(\lambda_i+1)}}{\sum_{i=1}^{q} \frac{\hat{\alpha}_i^2}{(\lambda_i+1)^2}} \right]. \quad (15)$$

Here λ_i represents the positive eigenvalues of the $(X'X)$ matrix,α_i $(i = 1, .., q)$ represents the i-th element of α, which expresses the $(qx1)$ dimensional regression coefficient in the canonical form in Equation (2).

To test the null hypothesis in Equation (4), the test statistic based on the Liu estimator, which is a non-exact t-type test, is given in Equation (16) [15]

$$t_{i(L)} = \frac{\hat{\beta}_{i(L)}}{S(\hat{\beta}_{i(L)})}. \quad (16)$$

$\hat{\beta}_{i(L)}$ is the i-th element of $\hat{\beta}_L$;$S^2(\hat{\beta}_{i(L)})$ is the i-th diagonal element of the variance estimate of $\hat{\beta}_L$, as given in Equation (17)

$$\mathrm{Var}(\hat{\beta}_L) = \hat{\sigma}^2 \, (X'X + I)^{-1}(X'X + dI)(X'X)^{-1}(X'X + dI)(X'X + I)^{-1}. \quad (17)$$

Here, when σ^2 is unknown, the estimation of σ^2 can be used based on the Liu estimator given in Equation (18)

$$\hat{\sigma}^2{}_L = \frac{(Y - X\hat{\beta}_L)'(Y - X\hat{\beta}_L)}{(n - q - 1)}. \quad (18)$$

The test statistic given in Equation (18) has a t-distribution with $(n - q - 1)$ degrees of freedom under H_0 hypothesis. This test statistic is called an L test statistic.

2.3. Test Statistics for Regression Coefficients Based on Liu-Type Estimation Method

When multicollinearity is severe, the Ridge and Liu estimators may be inadequate to address this problem. The bias parameter k of the Ridge estimator is chosen as small in the implementation method. However, to eliminate severe multicollinearity, k should be chosen large. In this case, while the MSE value decreases, the amount of bias increases. Therefore, a second parameter is needed to reduce the bias. Accordingly, the Liu-type estimator with two bias parameters developed by Liu (2003) is given in Equation (19) [7]

$$\hat{\beta}_{LT} = (X'X + kI)^{-1}(X'Y - d\hat{\beta}^*), \quad k > 0, \qquad -\infty < d < \infty. \quad (19)$$

Liu (2003) showed the cases where $\hat{\beta}^*$ is both an LS and a Ridge estimator in his study [7]. In this study, $\hat{\beta}_R$ is used instead of $\hat{\beta}^*$.

Liu (2003) has proposed the parameter k of the estimator in Equation (19) as in Equation (20)

$$\hat{k}_{LT} = \frac{\lambda_{\max} - 100 * \lambda_{min}}{99}. \quad (20)$$

Here $\lambda_{\max}$ denotes the largest eigenvalue of the matrix $X'X$, λ_{min} denotes the smallest eigenvalue of the matrix $X'X$. Accordingly, the calculated $\hat{d}$ values are as in Equation (21) and Equation (22) [7]

$$\hat{d}_{LT1} = \left[\frac{\sum_{i=1}^{q} \frac{(\lambda_i(\hat{\sigma}_R^2 - \hat{k}_{LT}\hat{\alpha}_R^2))}{(\lambda_i + \hat{k}_{LT})^3}}{\sum_{i=1}^{q} \frac{(\lambda_i(\lambda_i\hat{\alpha}_R^2 + \hat{\sigma}_R^2))}{(\lambda_i + \hat{k}_{LT})^4}} \right] \tag{21}$$

$$\hat{d}_{LT2} = \left[\frac{\sum_{i=1}^{q} \frac{(\hat{\sigma}_R^2 - \hat{k}_{LT}\hat{\alpha}_R^2)}{(\lambda_i + \hat{k}_{LT})^2}}{\sum_{i=1}^{q} \frac{(\lambda_i\hat{\alpha}_R^2 + \hat{\sigma}_R^2)}{(\lambda_i + \hat{k}_{LT})^2}} \right]. \tag{22}$$

Here, σ^2 represents the error variance in the multiple regression model, λ_i represents the positive eigenvalues of the $(X'X)$ matrix, and α_i $(i = 1,..,q)$ represents the i-th element of α, which expresses the $(qx1)$ dimensional regression coefficient in the canonical form in Equation (2). Instead of σ^2 and α^2 parameters, the estimators $\hat{\sigma}_R^2$ and $\hat{\alpha}_R^2$ obtained from Ridge regression are used.

Considering the test statistic approach proposed by Halawa and Bassiouni (2000) for the Ridge estimator in their study, the test statistic based on the Liu-type estimator, which is a non-exact t-type test, to test the null hypothesis in Equation (4) is given in Equation (23)

$$t_{i(LT)} = \frac{\hat{\beta}_{i(LT)}}{S(\hat{\beta}_{i(LT)})}. \tag{23}$$

$\hat{\beta}_{i(LT)}$ is the i-th element of $\hat{\beta}_{LT}$, and $S^2(\hat{\beta}_{i(LT)})$ is the i-th diagonal element of the variance estimate of $\hat{\beta}_{LT}$ as given in Equation (24). According to this,

$$\mathrm{Var}(\hat{\beta}_{LT}) = \hat{\sigma}^2 (X'X + kI)^{-1}(I - d(X'X + kI)^{-1})(X'X)(I - d(X'X + kI)^{-1})(X'X + kI)^{-1}. \tag{24}$$

Here when σ^2 is unknown, the estimation of σ^2 can be used based on the Liu-type estimator in Equation (25)

$$\hat{\sigma}^2{}_{LT} = \frac{(Y - X\hat{\beta}_{LT})'(Y - X\hat{\beta}_{LT})}{(n - q - 1)}. \tag{25}$$

The test statistic given in Equation (23) has a t-distribution with $(n - q - 1)$ degrees of freedom under H_0 hypothesis. The test statistic based on $\hat{d}_{LT1}$ in Equation (23) is called the LT1 test statistic, and the test statistic based on $\hat{d}_{LT2}$ is called the LT2 test statistic.

3. SIMULATION STUDY

In this section, the parameter estimation values of LS, Ridge, Liu, and Liu-type estimators will be analyzed through simulation studies. Considering the studies of Mcdonald and Galerneau (1975) and Kibria (2003), the independent variables were formed as in Equation (26) with $i = 1,2,\ldots,n$ and $j = 1,2,\ldots,q$ [18,6]

$$x_{ij} = (1-\rho^2)^{1/2} z_{ij} + \rho z_{iq+1} \,. \tag{26}$$

Here, z_{ij} is the standard normal random variable, and ρ^2 is the correlation between any two independent variables. In this study, simulation studies were performed for ρ=0.85, 0.95, 0.99, n=20, 30, 50, 100, and q=4, 6, 10. Each vector in Equation (27) is centered and scaled

$$y_i = \beta_0 + \beta_1 x_{i1} + \ldots + \beta_q x_{iq} + \varepsilon_i \,. \tag{27}$$

The dependent variable y_i is formed as in Equation (27), where $i = 1,2,\ldots,n$; ε_i is an independent normal distribution with mean 0 and variance σ^2 and is centered around its mean. The standard deviation of error values are taken as σ=0.5,1,2. In the simulation study, 1000 repetitions were performed for each condition.

In the simulation study, firstly, the experimental type I errors of the tests are calculated, and these values are given in Tables 1 and 9.

Table 1. *Experimental Type I Error Rates of Tests with ρ=0.85 and σ=0.5*

Test	q=4				q =6				q=10			
	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100
LS	0.0492	0.0500	0.0504	0.0502	0.0491	0.0499	0.0501	0.0494	0.0500	0.0504	0.0499	0.0502
HK	0.0528	0.0555	0.0573	0.0590	0.0502	0.0540	0.0542	0.0552	0.0484	0.0522	0.0523	0.0539
HKB	0.0608	0.0763	0.0834	0.0861	0.0479	0.0619	0.0667	0.0710	0.0327	0.0485	0.0540	0.0572
L	0.0830	0.1152	0.1289	0.1385	0.0614	0.1042	0.1346	0.1486	0.0473	0.0738	0.1109	0.1467
LT1	0.2125	0.2314	0.2488	0.2650	0.0876	0.1082	0.0986	0.0979	0.0269	0.0724	0.0867	0.0839
LT2	0.0221	0.0214	0.0206	0.0177	0.0376	0.0416	0.0425	0.0422	0.0075	0.0426	0.0457	0.0475

Table 2. *Experimental Type I Error Rates of Tests with ρ=0.95 and σ=0.5*

Test	q=4				q=6				q=10			
	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100
LS	0.0493	0.0502	0.0491	0.0499	0.0495	0.0497	0.0511	0.0508	0.0494	0.0498	0.0514	0.0510
HK	0.0525	0.0563	0.0549	0.0562	0.0514	0.0528	0.0557	0.0553	0.0482	0.0518	0.0550	0.0539
HKB	0.0590	0.0771	0.0818	0.0863	0.0457	0.0549	0.0612	0.0633	0.0322	0.0452	0.0515	0.0537
L	0.0692	0.0895	0.0948	0.1036	0.0578	0.0862	0.1042	0.1164	0.0479	0.0650	0.0886	0.1152
LT1	0.0963	0.1137	0.1219	0.1168	0.0716	0.0997	0.1185	0.1262	0.0934	0.0768	0.1007	0.1241
LT2	0.0400	0.0467	0.0467	0.0469	0.0281	0.0435	0.0482	0.0493	0.0012	0.0331	0.0447	0.0499

Table 3. *Experimental Type I Error Rates of Tests with ρ=0.99 and σ=0.5*

Test	q=4				q=6				q=10			
	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100
LS	0.0500	0.0496	0.0514	0.0493	0.0507	0.0510	0.0493	0.0507	0.0508	0.0498	0.0506	0.0491
HK	0.0523	0.0549	0.0573	0.0553	0.0523	0.0532	0.0529	0.0546	0.0494	0.0523	0.0528	0.0523
HKB	0.0541	0.0620	0.0665	0.0650	0.0436	0.0494	0.0511	0.0534	0.0328	0.0432	0.0472	0.0497
L	0.0549	0.0593	0.0646	0.0623	0.0529	0.0611	0.0628	0.0676	0.0503	0.0542	0.0601	0.0661
LT1	0.2871	0.3049	0.2998	0.3167	0.2998	0.2879	0.2871	0.2940	0.3379	0.3737	0.3621	0.3569
LT2	0.0304	0.0465	0.0528	0.0542	0.0144	0.0421	0.0505	0.0577	0.0000	0.0297	0.0478	0.0615

Table 4. *Experimental Type I Error Rates of Tests with ρ=0.85 and σ=1*

Test	q=4				q=6				q=10			
	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100
LS	0.0497	0.0488	0.0505	0.0506	0.0502	0.0504	0.0499	0.0503	0.0497	0.0492	0.0508	0.0501
HK	0.0521	0.0521	0.0536	0.0555	0.0505	0.0530	0.0527	0.0543	0.0478	0.0505	0.0531	0.0527
HKB	0.0513	0.0564	0.0594	0.0627	0.0438	0.0515	0.0538	0.0574	0.0321	0.0435	0.0492	0.0518
L	0.0557	0.0662	0.0716	0.0760	0.0484	0.0629	0.0695	0.0760	0.0467	0.0489	0.0594	0.0712
LT1	0.1917	0.2484	0.2585	0.2665	0.0847	0.1071	0.1061	0.1058	0.0227	0.0692	0.0907	0.0942
LT2	0.0224	0.0201	0.0185	0.0201	0.0378	0.0411	0.0416	0.0424	0.0075	0.0412	0.0466	0.0475

Table 5. *Experimental Type I Error Rates of Tests with ρ=0.95 and σ=1*

Test	q=4				q=6				q=10			
	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100
LS	0.0490	0.0501	0.0505	0.0495	0.0508	0.0497	0.0506	0.0492	0.0493	0.0488	0.0491	0.0507
HK	0.0512	0.0535	0.0547	0.0537	0.0516	0.0535	0.0545	0.0540	0.0474	0.0503	0.0521	0.0528
HKB	0.0495	0.0556	0.0589	0.0594	0.0435	0.0499	0.0526	0.0534	0.0319	0.0416	0.0473	0.0502
L	0.0521	0.0594	0.0655	0.0650	0.0497	0.0558	0.0627	0.0677	0.0480	0.0478	0.0549	0.0677
LT1	0.1103	0.1122	0.1211	0.1181	0.0592	0.0892	0.1033	0.1179	0.0973	0.0553	0.0665	0.0869
LT2	0.0410	0.0461	0.0473	0.0468	0.0266	0.0424	0.0470	0.0478	0.0007	0.0295	0.0404	0.0469

Table 6. *Experimental Type I Error Rates of Tests with ρ=0.99 and σ=1*

Test	q=4				q=6				q=10			
	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100
LS	0.0493	0.0497	0.0506	0.0498	0.0490	0.0489	0.0497	0.0495	0.0494	0.0511	0.0507	0.0512
HK	0.0515	0.0538	0.0554	0.0550	0.0501	0.0529	0.0530	0.0541	0.0480	0.0525	0.0531	0.0537
HKB	0.0488	0.0529	0.0557	0.0561	0.0418	0.0476	0.0496	0.0511	0.0315	0.0437	0.0478	0.0495
L	0.0498	0.0517	0.0542	0.0543	0.0484	0.0507	0.0527	0.0545	0.0488	0.0498	0.0524	0.0565
LT1	0.2304	0.2150	0.2106	0.2168	0.2534	0.2268	0.2153	0.2012	0.3142	0.3606	0.3356	0.3155
LT2	0.0235	0.0401	0.0461	0.0491	0.0085	0.0324	0.0419	0.0477	0.0000	0.0192	0.0354	0.0448

Table 7. *Experimental Type I Error Rates of Tests with ρ=0.85 and σ=2*

Test	q=4				q=6				q=10			
	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100
LS	0.0508	0.0483	0.0505	0.0490	0.0497	0.0509	0.0500	0.0505	0.0496	0.0510	0.0498	0.0506
HK	0.0514	0.0513	0.0539	0.0531	0.0503	0.0533	0.0537	0.0539	0.0481	0.0525	0.0517	0.0532
HKB	0.0483	0.0511	0.0548	0.0551	0.0428	0.0495	0.0519	0.0531	0.0317	0.0435	0.0466	0.0500
L	0.0503	0.0543	0.0595	0.0602	0.0454	0.0521	0.0551	0.0568	0.0457	0.0452	0.0472	0.0523
LT1	0.1863	0.2398	0.2586	0.2725	0.0867	0.1071	0.1081	0.1096	0.0230	0.0732	0.0905	0.0964
LT2	0.0250	0.0191	0.0190	0.0189	0.0389	0.0428	0.0427	0.0425	0.0077	0.0418	0.0454	0.0481

Table 8. *Experimental Type I Error Rates of Tests with ρ=0.95 and σ=2*

Test	q=4				q=6				q=10			
	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100
LS	0.0502	0.0508	0.0500	0.0498	0.0496	0.0485	0.0502	0.0492	0.0506	0.0489	0.0505	0.0508
HK	0.0515	0.0536	0.0541	0.0543	0.0510	0.0521	0.0540	0.0530	0.0491	0.0506	0.0526	0.0532
HKB	0.0482	0.0526	0.0534	0.0549	0.0428	0.0479	0.0506	0.0510	0.0326	0.0426	0.0460	0.0490
L	0.0497	0.0535	0.0547	0.0563	0.0471	0.0485	0.0533	0.0548	0.0487	0.0453	0.0495	0.0526
LT1	0.0860	0.1164	0.1185	0.1211	0.0571	0.0863	0.1066	0.1139	0.0860	0.0483	0.0653	0.0798
LT2	0.0403	0.0465	0.0469	0.0469	0.0278	0.0417	0.0465	0.0471	0.0009	0.0296	0.0398	0.0459

Table 9. *Experimental Type I Error Rates of Tests with ρ=0.99 and σ=2*

Test	q=4				q=6				q=10			
	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100
LS	0.0505	0.0512	0.0497	0.0516	0.0507	0.0501	0.0498	0.0508	0.0500	0.0499	0.0496	0.0495
HK	0.0523	0.0545	0.0538	0.0560	0.0508	0.0536	0.0536	0.0551	0.0493	0.0514	0.0523	0.0522
HKB	0.0486	0.0523	0.0525	0.0556	0.0426	0.0486	0.0499	0.0521	0.0325	0.0422	0.0467	0.0483
L	0.0503	0.0514	0.0505	0.0527	0.0494	0.0495	0.0500	0.0519	0.0493	0.0478	0.0484	0.0498
LT1	0.1839	0.1894	0.1900	0.2054	0.2316	0.1953	0.1894	0.1832	0.3080	0.3368	0.3196	0.3088
LT2	0.0232	0.0388	0.0439	0.0489	0.0075	0.0316	0.0400	0.0454	0.0000	0.0156	0.0309	0.0414

According to the simulation results, for different ρ values when σ is 0.5, the experimental type I error values of the LS, HK, and HKB tests are close to 0.05 according to Table 1, Table 2, and Table 3. It can be said that the experimental type I error values of the LT2 test generally approach 0.05 as the sample size increases. When Table 7 is examined, it is observed that while the experimental type I error rate of the LT2 test is considerably lower than 0.05 when q=4, it is affected by the increase in ρ and approaches 0.05. The LT1 test statistic shows the worst-performing test compared to the other tests. As can be seen from the tables, the experimental type I error rate of the test is considerably larger than 0.05.

When Tables 4, 5, and 6 are examined for different ρ values where σ is 1, it is observed that the experimental type I error values of the LS, HK, and HKB tests give values close to 0.05. In addition, it is observed that the experimental type I error values of the L test gave values close to 0.05, especially as the ρ value increased. The LT1 test performed worse than the other tests with values greater than 0.05. The LT2 test shows that the experimental type I error value approaches 0.05 as the number of variables and ρ value increase.

When Tables 7, 8, and 9 are examined for different ρ values when σ is 2, it is observed that the experimental type I error values of the LS, HK, HKB, and L tests are close to 0.05. The LT2 test is negatively affected by the increase in σ and the number of variables in small sample sizes. In these cases, the experimental type I error values for the tests are significantly lower than 0.05. The LT1 test, as in other cases, gives values considerably greater than 0.05.

When the simulation results are evaluated in general, it is observed that the experimental type I error values of the LS, HK, and HKB tests gave the results close to 0.05. It is also observed that the L test gave values closer to 0.05, significantly as the value of ρ increased. The experimental type I error values of the LT1 test are above 0.05. It is observed that the LT2 test gave better results than the LT1 test. Regardless of the standard deviation, in cases where the ρ value is low, the experimental type I error values approach 0.05 as the number of variables increases. In cases where the ρ value is high, the experimental type I error value is not affected by the number of variables and gives values close to 0.05.

The power values of the tests related to the simulation results are given in Tables 10 and 18. Cases where the experimental type I error values of the tests were significantly far from the nominal α value were not taken into account in the calculation of the power values of the tests. Experimental type I error values in the range of 0.025 and 0.075 were considered in the calculation [19]. Values outside this range are indicated by (*), and the power values of the tests were not calculated.

Table 10. *Powers of tests when ρ=0.85 ve σ=0.5*

Test	q=4				q=6				q=10			
	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100
LS	0.0895	0.1028	0.1055	0.1025	0.0696	0.0844	0.0838	0.0856	0.0559	0.0651	0.0678	0.0705
HK	0.1186	0.1462	0.1488	0.1506	0.0803	0.1030	0.1052	0.1077	0.0567	0.0724	0.0771	0.0804
HKB	0.1821	*	*	*	0.0877	0.1383	0.1459	0.1552	0.0410	0.0746	0.0869	0.0950
L	0.1585	*	*	*	0.1004	*	*	*	0.0548	0.0999	*	*
LT1	*	*	*	*	0.1464	*	*	*	*	*	*	*
LT2	0.1035	0.1159	0.1168	0.1197	0.0699	0.1061	0.1085	0.1120	*	0.0703	0.0861	0.0959

Table 11. *Powers of tests when ρ=0.95 ve σ=0.5*

Test	q=4				q=6				q=10			
	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100
LS	0.0923	0.0935	0.1004	0.1041	0.0707	0.0780	0.0798	0.0844	0.0558	0.0640	0.0673	0.0683
HK	0.1204	0.1332	0.1505	0.1550	0.0822	0.0970	0.1015	0.1098	0.0551	0.0724	0.0772	0.0800
HKB	0.1679	*	*	*	0.0972	0.1367	0.1516	0.1650	0.0398	0.0812	0.0954	0.1069
L	*	*	*	*	0.1097	*	*	*	0.0524	0.1192	*	*
LT1	*	*	*	*	*	*	*	*	0.0331	0.1221	*	*
LT2	*	*	*	*	0.0604	0.0718	0.0692	0.0713	*	0.0649	0.0715	0.0732

Table 12. *Powers of tests when ρ=0.99 ve σ=0.5*

Test	q=4				q=6				q=10			
	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100
LS	0.0905	0.1007	0.1048	0.1083	0.0716	0.0841	0.0844	0.0875	0.0561	0.0655	0.0697	0.0725
HK	0.1136	0.1334	0.1438	0.1499	0.0808	0.1008	0.1015	0.1050	0.0564	0.0714	0.0767	0.0796
HKB	0.1673	0.2231	0.2487	0.2631	0.0813	0.1166	0.1234	0.1302	0.0394	0.0674	0.0780	0.0846
L	0.1137	0.1437	0.1558	0.1662	0.0810	0.1135	0.1231	0.1335	0.0561	0.0761	0.0919	0.1071
LT1	*	*	*	*	*	*	*	*	*	*	*	*
LT2	0.2114	0.2617	0.2758	0.2869	*	0.1938	0.2127	0.2303	*	0.1159	0.1596	0.1825

Table 13. *Powers of tests when ρ=0.85 ve σ=1*

Test	q=4				q=6				q=10			
	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100
LS	0.0923	0.0935	0.1003	0.1040	0.0687	0.0794	0.0858	0.0845	0.0561	0.0639	0.0665	0.0694
HK	0.1128	0.1193	0.1291	0.1380	0.0751	0.0936	0.1018	0.1010	0.0551	0.0697	0.0733	0.0771
HKB	0.1360	0.1654	0.1822	0.1943	0.0760	0.1117	0.1246	0.1285	0.0393	0.0685	0.0793	0.0868
L	0.1405	0.1800	0.2006	*	0.0787	0.1332	0.1539	*	0.0528	0.0782	0.1046	0.1293
LT1	*	*	*	*	*	*	*	*	*	0.1123	*	*
LT2	*	*	*	*	0.0581	0.0672	0.0736	0.0720	*	0.0609	0.0670	0.0709

Table 14. *Powers of tests when ρ=0.95 ve σ=1*

Test	q=4				q=6				q=10			
	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100
LS	0.0949	0.1011	0.1041	0.1044	0.0691	0.0824	0.0834	0.0848	0.0547	0.0661	0.0693	0.0697
HK	0.1184	0.1300	0.1408	0.1406	0.0772	0.0969	0.0990	0.1009	0.0541	0.0725	0.0750	0.0776
HKB	0.1488	0.1944	0.2182	0.2228	0.0759	0.1116	0.1179	0.1233	0.0375	0.0681	0.0777	0.0826
L	0.1372	0.1758	0.1955	0.2044	0.0796	0.1300	0.1471	0.1624	0.0529	0.0776	0.1024	0.1209
LT1	*	*	*	*	0.1273	*	*	*	*	0.0977	0.1311	*
LT2	0.1019	0.1077	0.1140	0.1149	0.0585	0.0935	0.0983	0.1010	*	0.0591	0.0744	0.0822

Table 15. *Powers of tests when ρ=0.99 ve σ=1*

Test	q=4				q=6				q=10			
	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100
LS	0.0895	0.1008	0.1077	0.1052	0.0755	0.0777	0.0855	0.0870	0.0562	0.0649	0.0681	0.0698
HK	0.1117	0.1310	0.1384	0.1390	0.0830	0.0909	0.0999	0.1017	0.0565	0.0718	0.0751	0.0771
HKB	0.1457	0.1946	0.2106	0.2183	0.0789	0.0975	0.1111	0.1162	0.0382	0.0655	0.0736	0.0786
L	0.1057	0.1339	0.1479	0.1502	0.0820	0.0957	0.1125	0.1213	0.0555	0.0699	0.0804	0.0911
LT1	*	*	*	*	*	*	*	*	*	*	*	*
LT2	*	0.2187	0.2333	0.2387	*	0.1430	0.1679	0.1804	*	*	0.1079	0.1309

Table 16. *Powers of tests when ρ=0.85 ve σ=2*

Test	q=4				q=6				q=10			
	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100
LS	0.0900	0.0945	0.1012	0.1035	0.0707	0.0795	0.0832	0.0863	0.0551	0.0643	0.0664	0.0699
HK	0.1042	0.1140	0.1269	0.1287	0.0777	0.0918	0.0960	0.0997	0.0546	0.0689	0.0728	0.0770
HKB	0.1154	0.1390	0.1571	0.1621	0.0743	0.1000	0.1075	0.1137	0.0387	0.0646	0.0736	0.0802
L	0.1145	0.1476	0.1651	0.1743	0.0732	0.1058	0.1191	0.1321	0.0517	0.0659	0.0800	0.0918
LT1	*	*	*	*	*	*	*	*	*	0.1057	*	*
LT2	0.0390	*	*	*	0.0611	0.0654	0.0720	0.0745	*	0.0591	0.0665	0.0703

Table 17. *Powers of tests when ρ=0.95 ve σ=2*

Test	q=4				q=6				q=10			
	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100
LS	0.0958	0.0986	0.1039	0.1075	0.0725	0.0810	0.0825	0.0853	0.0557	0.0640	0.0686	0.0697
HK	0.1136	0.1234	0.1331	0.1361	0.0804	0.0926	0.0957	0.0999	0.0544	0.0689	0.0748	0.0751
HKB	0.1325	0.1625	0.1796	0.1870	0.0766	0.0974	0.1057	0.1121	0.0378	0.0635	0.0742	0.0772
L	0.1232	0.1519	0.1694	0.1815	0.0759	0.1072	0.1205	0.1349	0.0535	0.0662	0.0828	0.0954
LT1	*	*	*	*	0.1086	*	*	*	*	0.0817	0.1099	*
LT2	0.0972	0.1061	0.1136	0.1152	0.0585	0.0869	0.0929	0.0980	*	0.0521	0.0699	0.0760

Table 18. *Powers of tests when ρ=0.99 ve σ=2*

Test	q=4				q=6				q=10			
	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100	n=15	n=30	n=50	n=100
LS	0.0919	0.1030	0.1039	0.1065	0.0725	0.0835	0.0869	0.0877	0.0566	0.0666	0.0702	0.0708
HK	0.1107	0.1321	0.1329	0.1383	0.0807	0.0947	0.1003	0.1012	0.0569	0.0717	0.0766	0.0771
HKB	0.1340	0.1792	0.1882	0.2016	0.0763	0.0985	0.1078	0.1103	0.0401	0.0654	0.0744	0.0768
L	0.1071	0.1322	0.1382	0.1451	0.0760	0.0992	0.1103	0.1168	0.0556	0.0693	0.0781	0.0855
LT1	*	*	*	*	*	*	*	*	*	*	*	*
LT2	*	0.2004	0.2123	0.2224	*	0.1270	0.1486	0.1604	*	*	0.0916	0.1079

According to the simulation results, when Tables 10, 11, and 12 are analyzed for different values of ρ when σ is 0.5, in most cases, the LS test shows lower power values than the other tests. According to the retrieved results, while the number of variables is generally the same for each test, the power values of the tests increase as the sample size increases. When examined in terms of the number of variables, it is seen that the power values of the tests decrease as the number of variables increases. For example, when Table 12 is examined, while the power value of the LT2 test is 0.2758 for q=4, n=50, it decreased to 0.2127 at q=6, n=50 and to 0.1596 at q=10, n=50. In cases where the ρ value is small, and the number of variables is low, the power value of the HK test is high among the tests, while the power value of the HKB test increased within itself as the number of variables increased. When the ρ value increases to 0.95, the HK test gives better results when the number of variables is low, while the power value of the HKB test increases as the number of variables increases. When the ρ value is 0.99, the power value of the LT2 test is higher than the other tests.

When Tables 13, 14, and 15 are examined for different values of ρ when σ is 1, in general, while the number of variables is the same, the power values of the tests increased as the sample size increased. When analyzed in terms of the number of variables, the power values of the tests decrease as the number of variables increases. In cases where the ρ value is 0.85, the power value of the L test is higher than the other tests. When the ρ value increases to 0.95, the power value of the L test is generally higher than the other tests as the number of variables increases. When ρ is 0.99, the power values of the LT2 test are higher than the other tests.

When Tables 16, 17, and 18 are analyzed for different values of ρ when σ is 2, it gives similar results to cases where σ is 0.5 and 1. When the simulation results are evaluated in general for the power values of the tests, it is seen that the power values of the tests decrease as the number of variables increases. When the number of variables is the same, the power values of the tests increase as the sample size increases in all cases. For ρ value of 0.85 and large values of σ, the power value of the L test is higher than the other tests. When ρ increases to 0.95, the power value of the HKB test is higher in cases where the number of variables is low, while the power value of the L test is higher when the number of variables increases. When ρ value increases to 0.99, the power value of the LT2 test is higher than the other tests.

In summary, it can be generalized that the power value of the HKB test is high in cases where ρ and σ values are low, the power value of the L test is high at moderate ρ values, and the power value of the LT2 test is higher at high ρ values.

4. NUMERICAL EXAMPLE

In this section, the widely used Hald's Portland Cement data is studied. This dataset has been used in the application of many studies examining multicollinearity in the literature. The dataset consists of 4 independent variables and 13 observations in this example. The independent variables are tricalcium aluminate (X_1), tetracalcium silicate (X_2), tetracalcium alumino ferrite (X_3), and dicalcium silicate (X_4). The dependent variable Y represents the amount of heat released for 1 gram of cement in calories. The dataset is given in Table 19. The purpose of applying the numerical example is to calculate the parameter estimation values of LS, Ridge, Liu, and Liu-type estimators from these data and compare the results obtained [16].

Table 19. *Portland Cement Data*

X_1	X_2	X_3	X_4	Y
7	26	6	60	78.5
1	29	15	52	74.3
11	56	8	20	104.3
11	31	8	47	87.6
7	52	6	33	95.9
11	55	9	22	109.2
3	71	17	6	102.7
1	31	22	44	72.5
2	54	18	22	93.1
21	47	4	26	115.9
1	40	23	34	83.8
11	66	9	12	113.3
10	68	8	12	109.4

In this study, the data were used in their standardized form. While Y values are standardized around the mean, X values are standardized according to the unit length scaling method. The $X'X$ correlation matrix showing the linear relationship between two variables is obtained as follows

$$X'X=\begin{bmatrix} 1.000 & 0.2286 & -0.8241 & -0.2454 \\ 0.2286 & 1.000 & -0.1392 & -0.9730 \\ -0.8241 & -0.1392 & 1.000 & 0.0295 \\ -0.2454 & -0.9730 & 0.0295 & 1.000 \end{bmatrix}. \tag{26}$$

Accordingly, while there is an inverse relationship with a magnitude of 0.973 between the variables X_2 and X_4, an inverse relationship with a magnitude of 0.824 can be seen between X_1 and X_3 variables. The relationship between the variables is high, according to the $X'X$ matrix; however, this is not a sufficient criterion for determining multicollinearity. For this reason, the eigenvalues of the $X'X$ matrix are studied. The eigenvalues of $X'X$ matrix are calculated as λ_1=2.2357, λ_2=1.5761, λ_3=0.1866, and λ_4=0.0016. As λ_1 and λ_4 are the maximum and minimum eigenvalues of the $X'X$ matrix, respectively, the number of conditions is calculated as approximately 1397,3125 according to the formula $CN=\frac{\lambda_1}{\lambda_4}$. It is a fact that this result is greater than 1000, indicating a high degree of multicollinearity problem. For this data, the results obtained from the LS, Ridge, Liu, and Liu-type estimators of the linear regression model in case of severe multicollinearity are given in Tables 20, 21, 22, 23, and 24, respectively.

Table 20. *Test results of regression coefficients based on the LS estimator*

i	$\widehat{\boldsymbol{\beta}}_{i(LS)}(S(\widehat{\boldsymbol{\beta}}_{i(LS)}))$	$t_{i(LS)}$	$p_{i(LS)}$
1	31.6060(15.1785)	2.0823	0.0709
2	27.4972(39.0215)	0.7047	0.5010
3	2.2600(16.7480)	0.1349	0.8960
4	-8.3563(41.1192)	-0.2032	0.8440

Table 21. *Test results of regression coefficients based on Ridge estimator*

	HK		
i	$\widehat{\boldsymbol{\beta}}_{i(R)}(S(\widehat{\boldsymbol{\beta}}_{i(R)}))$	$t_{i(R)}$	$p_{i(R)}$
1	27.3977(5.0299)	5.4470	0.0006
2	17.3914(8.6149)	2.0188	0.0782
3	-2.2881(5.1351)	-0.4456	0.6677
4	-18.9737(8.9584)	-2.1180	0.0670

Table 22. *Test results of regression coefficients based on Ridge estimator*

	HKB		
i	$\hat{\beta}_{i(R)}(S(\hat{\beta}_{i(R)}))$	$t_{i(R)}$	$p_{i(R)}$
1	26.4798(4.1524)	6.3770	0.0002
2	16.1666(4.8107)	3.3606	0.0099
3	-3.1510(4.0843)	-0.7715	0.4626
4	-20.2144(4.8578)	-4.1613	0.0032

Table 23. *Test results of regression coefficients based on Liu estimator*

i	$\hat{\beta}_{i(L)}(S(\hat{\beta}_{i(L)}))$	$t_{i(L)}$	$p_{i(L)}$
1	29.6813(14.0676)	2.1099	0.0679
2	25.9809(36.1097)	0.7195	0.4923
3	1.2534(15.5168)	0.0808	0.9376
4	-8.8875(38.0498)	-0.2336	0.8212

Table 24. *Test results of regression coefficients based on Liu-type estimator*

	LT1			LT2		
i	$\hat{\beta}_{i(LT)}(S(\hat{\beta}_{i(LT)}))$	$t_{i(LT)}$	$p_{i(LT)}$	$\hat{\beta}_{i(LT)}(S(\hat{\beta}_{i(LT)}))$	$t_{i(LT)}$	$p_{i(LT)}$
1	29.6645(5.5994)	5.2978	0.0679	25.8599(3.8563)	6.7059	0.0002
2	18.1758(9.2225)	1.9708	0.4923	15.7756(3.4924)	4.5172	0.0020
3	-0.4829(5.6808)	-0.0850	0.9376	-3.6703(3.7466)	-0.9796	0.3560
4	-18.4107(9.5697)	-1.9239	0.8212	-20.5655(3.3999)	-6.0488	0.0003

5. CONCLUSION

In this study, a test statistic was obtained to test the significance of the model coefficients for the Liu-type estimator. It is aimed to reveal which of the tests obtained in the case of multicollinearity is better in statistical inference. For this purpose, the significance tests of the model coefficients of the Ridge, Liu, and Liu-type biased estimators in different situations were performed with a simulation study, and the type I errors and power values of the tests were calculated. According to the results of the simulation study, it has been observed that the HKB test is stronger than the other tests at moderate multicollinearity and small values of σ. In addition, it has been identified that the L test is stronger than the other tests at moderate multicollinearity and high values of σ.In cases of severe multicollinearity, the LT2 test appears to be stronger than the other tests.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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