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Research Article

Time Series Analysis Methodology for Damage Detection in Civil Structures

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ABSTRACT

Structural health monitoring (SHM) methodologies employing data-driven techniques are becoming increasingly popular for detection of structural damage at the earliest stage possible. With measured vibration signals from the structure, time series modeling methods provide quantitative means for extracting such features that can be utilized for damage diagnosis. In this study, one-step prediction error of an autoregressive (AR) model over a data set is used as damage indicator. In particular, the difference between the prediction of the AR model that is fit to the measured acceleration signal obtained from the intact structure and actual measured signals collected for different damage states of the structure are interrogated for diagnosis purposes. More specifically, the standard deviation of the residual error is employed to locate the damaged region. Singular-value decomposition (SVD) is employed to find the optimal order for an AR model created using the impulse responses of the system. Numerical simulations are carried out using the impulse responses acquired from a four-story frame structure contaminated with additive noise including single and multiple damaged elements. The results of the simulations demonstrate that the method can be effectively employed to detect and locate damage. The performance of the proposed procedure are further demonstrated using the impact data acquired from a reinforced concrete frame for real applications.

Introduction

The symptoms of deterioration or damage manifest themselves with the changes in the dynamic state of the structural system and are reflected as discrepancies in its anticipated vibration response. This is the fundamental principle upon which any vibration-based method is based. Vibration-based damage detection is a field of study within the general framework of structural health monitoring (SHM) that primarily focuses on extracting damage-sensitive features from the vibration measurements and developing approaches that can be employed for damage detection and localization purposes [1-4].

Within this context, time series analysis is involved with analyzing a sequence of response signals collected over a time period to extract the pattern in the data and develop an appropriate model to describe it accurately. With their potential to process high volumes of sensor data in an efficient and easily automated manner and to produce robust and reliable results without the requirement of physics based models, time series analysis methods lend themselves quite suitable for SHM applications [5-10] especially when they are cast in conjunction with a statistical pattern recognition framework such as novelty or outlier detection.

Autoregressive models (AR) are among the parametric methods used for time-series representations of stationary signals that are regularly sampled and acquired from a dynamic system assuming that the structure is subjected to random excitation. Methods based on the time series representations have received increased attention for SHM purposes and the corresponding model coefficients as well as the residual signals have been explored as damage detection measures [8-14.]

Selection of the model order and the sensitivity of the model to measurement noise are the among the critical issues that must be dealt with for robust and reliable application of the model as well as the success of the damage detection methodology. Akaike information criterion (AIC), Bayesian information criterion (BIC) and final prediction error (FPE) are the most widely used conventional approaches to select model order. However, these methods usually cannot guarantee the correct model order [15] and furthermore, for a transient signal, the applicability of these criteria is questionable.

In this study, singular value decomposition (SVD) of the response matrix obtained with a unit delay time is utilized for finding the optimal order of the model. With the effective singular values and the associated vectors, the SVD-based low-rank approximation is obtained to reconstruct the signal from the reduced number of components. The SVD allows for the correct estimation of the model order as well as to help reduce the measurement noise. AR model is then fit to this 'filtered' version of the data acquired at the baseline state of the structure. The proposed method uses this AR model to predict the response measurements acquired at different damaged states of the system. The difference between the predicted response and the measured responses are utilized for detecting and locating damage.

The remainder of the paper is organized as follows: Next section reviews the basic principles of the AR time series modeling and the singular value decomposition. The proposed approach implemented for detection and localization based on the damage index is discussed next. The results of the investigation carried out with numerically simulated data on a four-story frame structure and the experimental implementation on a reinforced concrete test frame is presented in the following section. The final discussion on the proposed methodology and the concluding remarks are summarized in the last section of the manuscript.

Theoretical Background

Autoregressive (AR) Models

An AR model is in the form of stochastic difference equation regresses the output variable based on its own previous values and are used in the analysis of stationary time series processes. A univariate AR model of order p is represented as AR(p) and can be written as

$$x_t = \varphi_1 x_{t-1} + \varphi_2 x_{t-2} + \cdots + \varphi_p x_{t-p} + e_t \qquad (1)$$

where $x_t, \cdots, x_{t-p}$ are the current and previous values of the time series, $\varphi_1, \cdots, \varphi_p$ are the unknown AR coefficients and e_t is the error term with zero mean and constant variance. The values of φ_i are estimated by fitting the AR model to the time history data using Burg's method. The method estimates the coefficients recursively up to the selected order p by minimizing forward and backward predictions as p-linear least squares problem. It is computationally simple and the estimated coefficients are guaranteed to be stable [16].

Singular Value Decomposition (SVD) and Model Order Determination

The decomposition schemes with the fundamental idea of decomposing a complicated signal in to simpler yet similar components provides insight into the data and the underlying system by revealing which components are the most important for describing the original data. This allows for major data compression as well as facilitating removal of measurement noise and extraction of features.

Singular value decomposition has been used effectively in a variety of applications, including, signal denoising, data compression and fault diagnosis. The first step to decompose a one-dimensional signal is transforming it into a trajectory matrix form through the so called 'embedding process' [17, 18]. One of the most widely used form of matrix transformation is the Hankel matrix due to its zero phase shift characteristic [19].

For a discrete signal $x = [x(1), x(2), \cdots, x(n)]$ a Hankel matrix can be formed as

$$A = \begin{bmatrix} x(1) & x(2) & \dots & x(m) \\ x(2) & x(3) & \dots & x(m+1) \\ \vdots & \vdots & \vdots & \vdots \\ x(k) & x(k+1) & \dots & x(n) \end{bmatrix} \qquad (2)$$

The SVD of matrix $A \in R^{k\times m}$ leads to the following factorization:

$$A = U\Sigma V^T \qquad (3)$$

where the two orthogonal matrices$U = [u_1, u_2, \dots, u_m] \in R^{k\times k}$ and $V = [v_1, v_2, \dots, v_m] \in R^{m\times m}$ are the left and the right singular vectors, and each of these column vectors are the eigenvectors of the covariance matrix, AA^T and A^TA, respectively. Σ is a diagonal matrix of size $k \times m$ in which the entries of the leading diagonal are the signular values of A. The diagonal entries, $[s_1, s_2, \dots, s_p]$ where p=min(k, m), are the non-negative square roots of the eigenvalues of the covariance matrix, A^TA.

It is possible to rewrite eqn.3 using summation as :

$$A = \sum_{i=1}^{p} s_i\, u_i v_i^T \qquad (4)$$

As expected, the number of non-zero singular values coincides with the rank of $[A]$. Since zero singular values can be interpreted as 'small' values, smaller than a certain tolerance, due to the noise effects in the measurements, the number of significant singular values can be used to determine the model order.

One can obtain low-rank approximation of matrix A, $\tilde{A}$, by truncating the contributions of the small singular values and the associated singular vectors in order to remove the noise and unrelated components from the measured signal. With the truncated matrix $\tilde{A}$ in Hankel form, the associated time-series signal $\tilde{X}$ can be reconstructed using reverse Hankel construction approach which essentially takes the first row of the matrix and pads it with the values in the last column starting from the second row. More specifically if:

$$\tilde{A} = \begin{bmatrix} \tilde{x}(1) & \tilde{x}(2) & \dots & \tilde{x}(m) \\ \tilde{x}(2) & \tilde{x}(3) & \cdots & \tilde{x}(m+1) \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{x}(k) & \tilde{x}(k+1) & \dots & \tilde{x}(n) \end{bmatrix} \qquad (5)$$

then the reconstructed time series is:

$$\tilde{X}_t = [\tilde{x}(1)\ \tilde{x}(2)\ \dots \tilde{x}(m)\ \tilde{x}(m+1) \dots \tilde{x}(n)] \qquad (6)$$

An important issue that requires clarification regarding the Hankel matrix representation is number of rows and columns to be included in the matrix. Based on [20], the optimal number of matrix columns should be selected based on the maximum energy of the singular values since the energy of the singular values are inherently related to the information richness of the trajectory matrix. It is proven in [36] that the Hankel matrix is square or close to square, the corresponding energy in the singular values is maximized. This means that if the signal length n is even, selecting the number of columns, $m = n/2$ and the number of rows, $k = n/2 + 1$ will maximize the energy of the singular values. If n is odd, the energy can be maximized by selecting $m = (n+1)/2$ and $k = m$.

Damage Localization Methodology

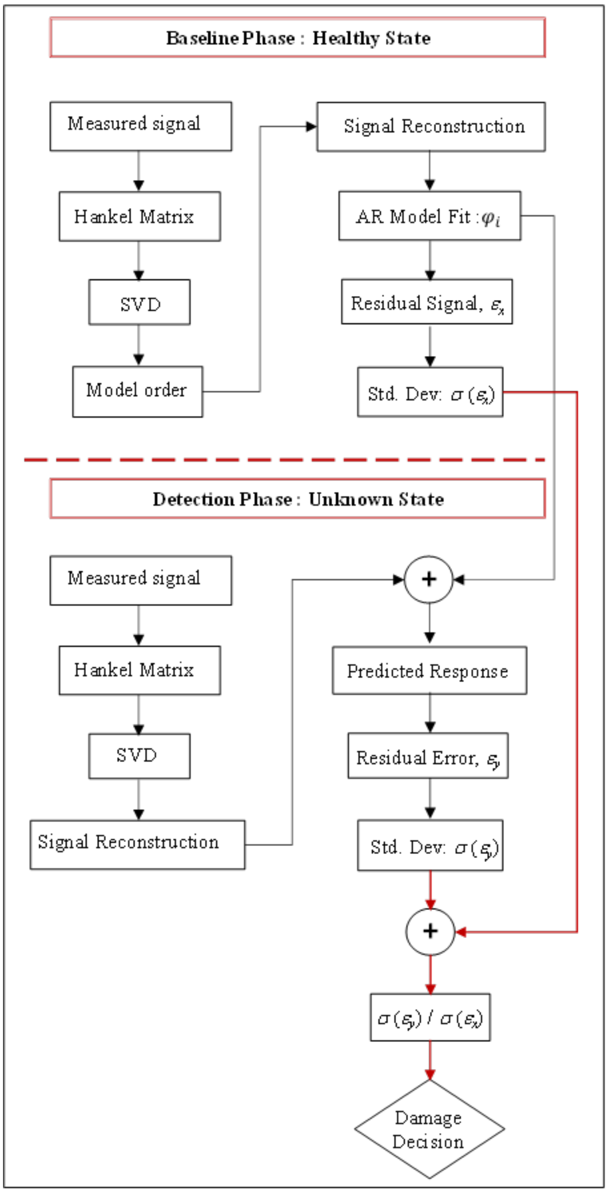

Figure 1. Extraction of damage sensitive features

Fig 1. illustrates the steps of the methodology employed to extract the damage-sensitive features. The initial processing of the sensor data from the structural system at any state is identical. The data is transformed into the Hankel form and using SVD the optimal order for the AR model is determined. Once the system order is decided based on the response signal obtained at the baseline state, it is assumed as set for the system at any state. Using the respective singular values and vectors corresponding to the system order, the signal is reconstructed following equations (3)-(5). The residual error defined as the difference between the predicted signal using the AR-model established for the baseline state and the reconstructed signal of the unknown state. The ratio of the standard deviation of the residual errors, the unknown state to that of the baseline state, $\sigma(\varepsilon_y)/\sigma(\varepsilon_x)$, is exploited as the damage-sensitive feature. Clearly, in order to arrive at a decision regarding the existence of damage, some threshold value for this ratio should be specified. This can be achieved by acquiring sensor data under different operating conditions and carrying out a statistical analysis.

Among various statistical tools for detecting anomalies in the data, an outlier detection algorithm which essentially classifies a value that is more than three scaled median absolute deviations (MAD) from the median as an anomaly is implemented in this study. Proven to be a resilient statistical tool for outliers, MAD, for a vector x of length N, is defined as:

$$MAD = median(|x_i - median(x)|) \quad i = 1{:}N \quad (7)$$

It should be noted that damage detection step can be implemented at the sensor level; that is each sensor information can be processed individually. However, localization of damage requires fusing the information collected from all sensor positions. In this regard, the same index can be used to locate damage with the assumption that the residual errors are maximized at the sensors closest to the damaged region.

Numerical Simulations

This section presents the numerical simulations conducted on a frame structure to investigate the performance of the proposed algorithm and the damage index. The 4-story moment resisting frame with 24 degrees-of-freedom (DOF) is depicted in Figure 2. The parameters of the model are arbitrarily chosen so that the fundamental period of corresponding to the first translational mode is 1 sec. Mass of the system is assumed to be lumped along the translational DOF and 2% proportional damping is assigned for all the modes. The damage scenarios including single and multiple damaged elements in the form of plastic hinge formations are also depicted in the same figure. The analytically computed natural frequencies corresponding to the translational modes for all the simulation cases are listed in Table 1. It is assumed that a total of four acceleration sensors, one in each floor measuring in the lateral direction, are deployed through the structure to measure the output signals. At each different health state of the structure, a total of 50 simulations are performed and impact data is generated. To simulate the variations during normal operating conditions, sensor noise is contemplated using a random number generator with a level ranging 5-10% of the root-mean-square (RMS) of the response measured at the

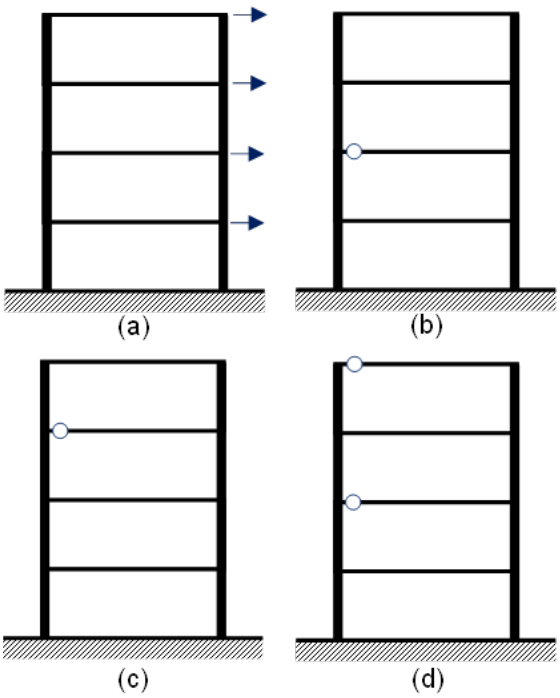

Figure 2. (a) Healthy State, (b) Damage Case 1, (c) Damage Case 2, (d) Damage Case 3

associated sensor such that signal-to-noise ratio is defined as

$$SNR = \sigma_{noise}^2 / \sigma_{signal}^2 \qquad (8)$$

where σ_{noise}^2 and σ_{signal}^2 are the variance of the noise and response signals.

Following the data acquisition stage, extraction of damage sensitive features has to take place. To achieve that, the first task is to estimate the order and the parameters of the AR-model using the data acquired at the baseline state. Following the proposed procedure shown in the flowchart of Fig.1, each of the simulated signals is transformed into Hankel matrix form and SVD is carried out. Examination of the singular values reveals that the suitable model order for the system can be selected as eight. A sample singular value plot is provided in Fig. 3.

Table 1. Natural frequencies corresponding to the translational modes

Mode	Healthy	DC 1	DC 2	DC3
1	1.00	0.88	0.93	0.86
2	3.42	3.42	3.14	3.12
3	6.44	6.08	6.43	5.74
4	8.96	8.96	8.90	8.86

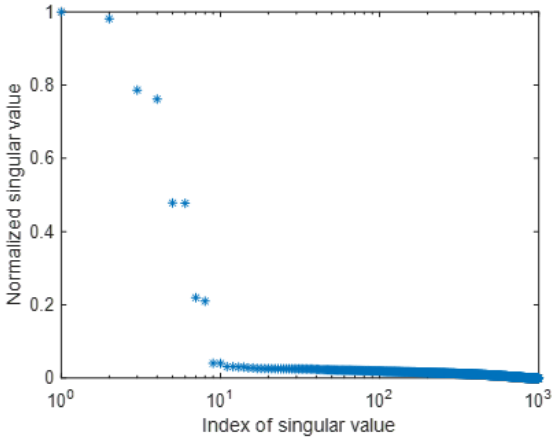

Figure 3. SV plot for a sample signal

With the model order determined, the signal is reconstructed using these eight singular values and vectors. AR model is then fit to this reconstructed signal and eight AR coefficients are extracted. Using these parameters, the predicted signal is computed and the difference between the predicted and the original signal is stored as the residual error. The standard deviation of this residual error constitutes the first part of the damage-sensitive feature. The same process is repeated for all the sensor data and all the simulations and the statistics of the residual error and the fit model are recorded for later use in the damage detection and localization stages.

Next stage starts with data processing at an unknown health state of the structural system. Using the order of the system determined for the baseline state, first SVD process is utilized and the signals are reconstructed with the determined number of singular values and vectors. based. Using the AR-model created at the baseline state, response signal is predicted. The discrepancy between the predicted response and the actual response gives the residual error for the unknown state.

At this stage detection algorithm can be invoked for the SHM activity. Using the three scaled MADs from the median as the threshold for the classification, all the simulation cases are successfully classified as ‘novelty’. For these cases proven to be ‘not-belonging’ to the baseline state, damage localization must be carried out. The ratio of the standard deviation of the residual error obtained for the unknown state to that of the baseline state, $\sigma(\varepsilon_y)/\sigma(\varepsilon_x)$, is defined as the damage index and the normalized value of this index for all the simulation cases are presented in the form of a box-plot for damage cases 1-3 in Fig. 4 (a)-(c), respectively. On each box in these figures, the central mark indicates the median value and the bottom and top edges of the box indicate the 25th and 75th percentiles, respectively. The outliers are plotted individually with circle markers. It follows that for the single member damages at second and third floors, damage cases 1 and 2, damage index successfully isolates the damaged region. For damage case 3 involving multiple damaged elements on second and fourth floors, damage index in descending order arrives at correct ranking list, however, cannot single out the damaged floors.

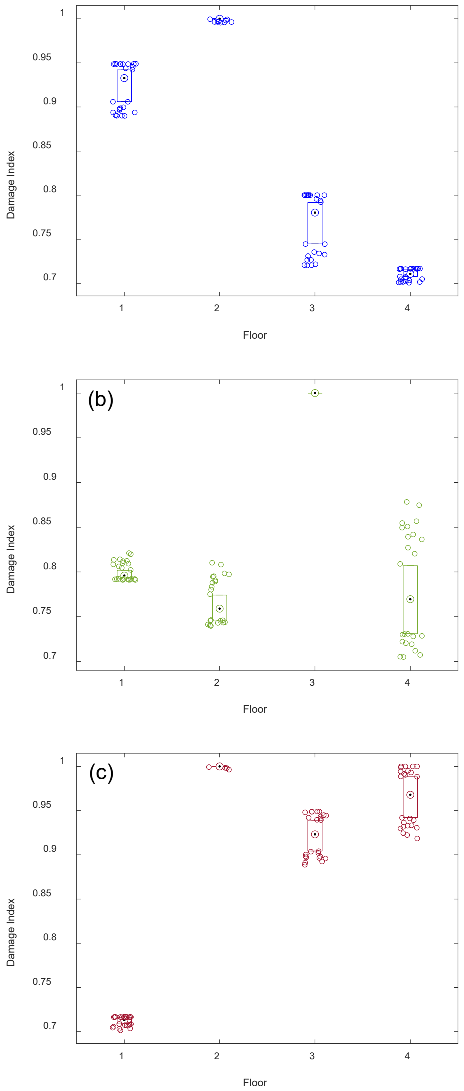

Figure 4. Box plot of damage index: (a) Damage case 1, (b) Damage case 2, (c) Damage Case 3.

Experimental Verification

A one-story one bay reinforced concrete frame with a height of 1.5m. and a span length of 2m. shown in Fig. 5 is subjected to lateral loading at the top level of the column in a quasi-static cyclic manner. The loads were applied in a displacement-controlled manner in sets of three cycles and damage is inflicted on the system. The completion of the load test at the predetermined drift levels is followed by vibration tests using impact hammer and accelerometer data is acquired at seven different locations with a sampling frequency of 500 Hz (dt=0.002s). The impact locations are chosen to coincide with these accelerometer locations. Further details of the experimental procedure can be found in [20]. In this study, the associated vibration data corresponding to the baseline state before any lateral loads

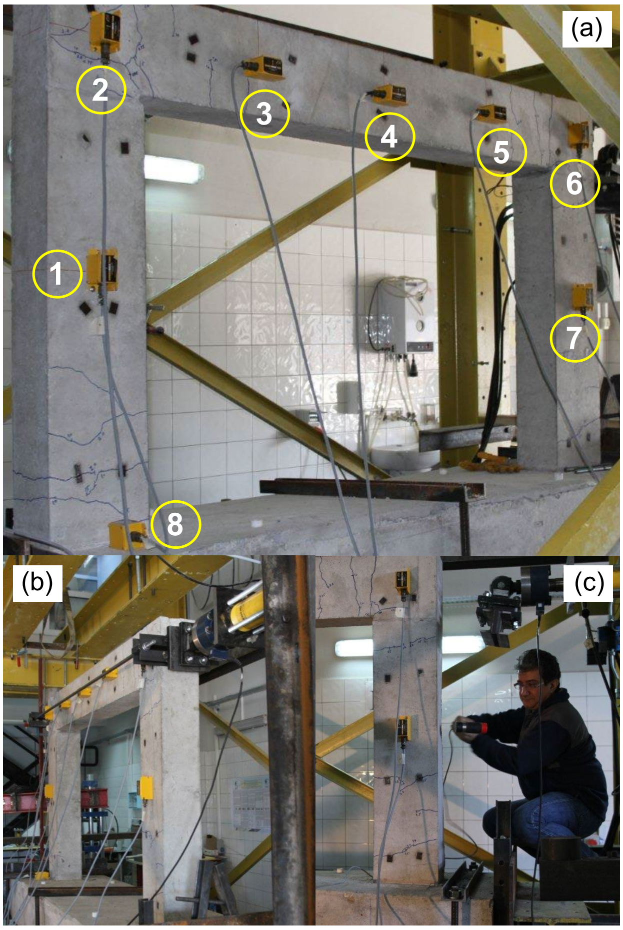

Figure 5. (a) Test-specimen and accelerometer locations (b) load test set-up (c) impact testing

are applied and the one after the structure is pushed to a drift level of 2%, are processed to investigate the applicability of the proposed approach for localizing the damaged region.

Following the flowchart presented in Fig. 1, the first stage of data processing involves fitting an AR model to the sensor data recorded at the baseline state of the structure.

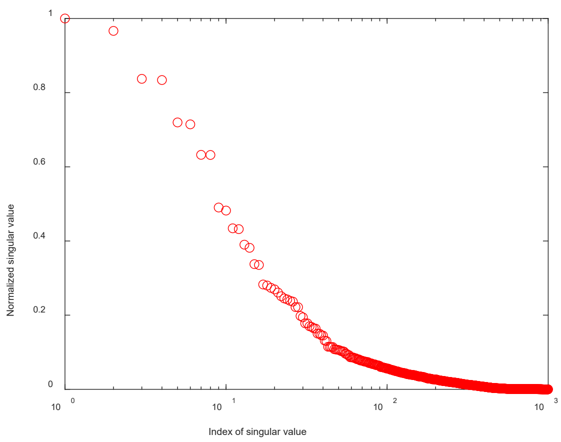

Figure 6. Singular value plot for a sample acceleration record

Interrogation of the singular values, shown in Fig. 6, the system order is selected as 16. With the associated singular values and vectors, data is reconstructed and AR model fit is obtained. The difference between the predicted response based on this model and measured response defines the residual signal. The ratio of standard deviations of the residuals, damaged state with respect to the baseline state, as damage index computed at all seven sensor locations is presented in Fig. 7.

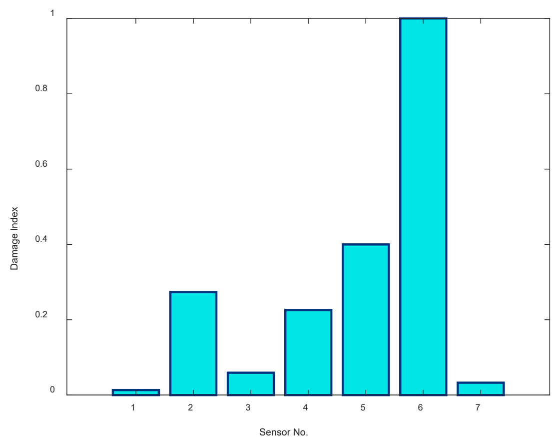

Figure 7. Damage localization index

Region around accelerometer 6, which is the beam column connection is identified as the potential damage location. This is in fact consistent with the observed damage at this drift level. The visual inspections however, also revealed damage infliction in the proximity of sensor 2. Examining the ranking of the members based on the damage index, sensor 2, although is among the top three potential damage locations, it does not distinctly stand out from two of the remaining six locations.

Concluding Remarks

A methodology to detect and locate damage that using the measured vibration data, that can operate on noisy transient signals is presented. The method has been shown to be successfully applied on simulated acceleration data from a frame type structure subjected to impact type unmeasured excitation. The experimental work on the reinforced concrete frame demonstrated that although a truly damaged region is successfully located, multiple damage locations represents a challenge with the current form of the methodology.

Selection of appropriate model order and the sensitivity of the AR model parameters, the two critical issues effecting the reliability of the model, are overcome through the SVD approach. The singular values defining the system order although form a clearly separable cluster with the simulated data; they are not as well separated for the measured data from the test specimen.

As for the damage index, the standard deviations of the residual error between the measured and predicted signals, more specifically the ratio between the unknown state and that of the baseline state as damage sensitive feature has proven its potential to localize damage for the simulated damage cases. With several advantages it offers, such as; detection of damage being carried out in an unsupervised learning mode and in a decentralized manner where data processing takes place individually at each senor, requiring only the output signals and its robustness in the presence of noise, the proposed methodology appears to be a promising signal-based approach for damage diagnosis. The improvement in the efficiency of the methodology when dealing with multiple damage locations from a real structure is required which is reserved for future work.

Ethics committee approval and conflict of interest statement

There is no need to obtain permission from the ethics committee for the article prepared.

There is no conflict of interest with any person / institution in the article prepared

Authors' Contributions

-Study conception and design: BG, OG

-Acquisition of data: OG, BG

-Analysis and interpretation of data: BG, OG

-Drafting of manuscript: BG

-Critical revision: OG

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