Journal of Transportation and Logistics
JTL 2024, 9 (1), 1–19
DOI: 10.26650/JTL.2024.1365336

RESEARCH ARTICLE

Enhancing Network Performance in Mitigating the Impact of Traffic Accidents with Full User Information

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ABSTRACT
This paper presents a novel approach aimed at bolstering network resilience against traffic accidents, particularly when users possess complete information about accident locations. Two new measures are introduced to evaluate the performance of the network and the importance of a link within the network. In addition, an objective function is designed to quantify optimal trip scheduling following an accident that guides investment decisions in network infrastructure. Additionally, we propose a model for addressing the Accident Improvement Problem and put forth a heuristic algorithm to solve this model. To illustrate the feasibility and effectiveness of the proposed methodologies, we present and analyze two illustrative examples, one at a small scale and the other at a medium scale. The findings underscore how the occurrence of accidents can markedly alter the importance of a link within the network during time. Unlike the prevailing trend in existing studies, which often overlook the repercussions of accidents on traffic flow along other links, our research highlights the importance of considering the impact of newcomers on the routes of existing travelers within the network. These findings demonstrate that such considerations can significantly influence the overall performance of the network in the event of an accident.

Keywords: Network performance measure, Link importance, Accident improvement problem, Accident prevention.

1. Introduction

Traffic accidents leave considerable undesirable effects in terms of property losses, injuries, and deaths, as well as family disruptions and societal economic disutility. In addition to personal costs, there are significant social damages that include those related to Emergency medical services, hospitals and legal (Karatas and Yakıcı, 2021). An accident can make the performance of the transportation network unsustainable and results in noticeable costs upon other users of the networks in the form of traffic delays and congestion.

Traffic accidents exact a heavy toll, resulting in significant property damage, injuries, fatalities, and profound societal and economic consequences. These consequences encompass personal hardships, including physical injuries and emotional trauma, as well as broader societal burdens, such as increased demands on healthcare systems and legal processes. Moreover, traffic accidents disrupt the normal functioning of transportation networks, leading to noticeable costs for other users, such as traffic delays and related inconveniences (Ji et al., 2019; Zhang and Xiong, 2017).

The occurrence of traffic accidents not only reduces road capacity but also unpredictably extends travel times for network users, resulting in a diminished level of service and increased traffic congestion. Traffic congestions as well affect the performance of the network and its reliability and sustainability, significantly (Sharma and Chauhan, 2022). To circumvent the uncertainties associated with accident-related delays and the anxiety of potentially missing scheduled arrivals, travelers are inclined to seek alternative routes as soon as they become aware of an accident, irrespective of the severity of the delay (Qiao et al., 2014). Consequently, users are willing to incur certain costs to avert uncertain delays, which they perceive concerning their origin-destination (O-D) travel time. In cases where suitable alternative routes are unavailable, travelers are forced to endure prolonged delays caused by the accident, resulting in traffic congestion and highly inefficient urban transportation network performance, a detriment to both network managers and users (Mahmoudi et al., 2019). On the other hand, the absence of viable substitute routes not only

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Submitted: 28.09.2023 • **Revision Requested:** 31.12.2023 • **Last Revision Received:** 15.01.2024 • **Accepted:** 15.01.2024 • **Published Online:** 04.03.2024

affects those directly impacted by the accident but also causes unwarranted delays for users on the designated alternative routes. Depending on the duration of the accident and its residual effects on traffic flow, this may create a second wave of substitute route seekers, and so on. In this process, a portion of the O-D trips becomes unsuitable, adding to the overall disruption (Poorzahedy and Bushehri, 2005). Therefore, it is imperative to conduct further research to comprehensively explore and analyze the ramifications of traffic accidents on the performance of urban transportation networks.

Research on the evaluation of street network performance under uncertain future conditions can be broadly categorized into two main groups (Chen et al., 2011; Lam, 1999). These groups address distinct aspects of network performance and disruptions. The first group of studies focuses on recurrent congestion, which arises from random fluctuations in demand and supply (capacity) within the network. In particular, the fluctuations in supply may, in part, result from user behavior. In recurrent congestion scenarios, users typically experience less time pressure because they have the opportunity to employ preventive strategies, such as selecting alternative routes or adjusting their departure times to mitigate anticipated delays (Huang et al., 2018). Conversely, the second group of studies aims to assess network performance in the presence of non-recurrent congestions. Non-recurrent congestions manifest as disruptions caused by events with both long-term and short-term effects. Long-term effects can stem from events like earthquakes, floods, or network maintenance operations, while short-term effects often result from incidents such as traffic accidents (Yu et al., 2016). In scenarios characterized by non-recurrent congestion, travelers face greater time constraints and a higher degree of uncertainty, as these disruptions are typically unexpected. Unlike recurrent congestion, where travelers have the opportunity to plan ahead, non-recurrent congestion events necessitate real-time decision-making and adaptation to the immediate circumstances.

The modeling of non-recurrent congestion in transportation networks initially emerged as an endeavor to quantify the adverse consequences stemming from capacity reductions on specific network links (Chen et al., 2002; Dhahir and Hassan, 2019). Within the realm of urban transportation networks, various types of network disruption problems, each targeting different objectives or outcomes, have been explored (Adelé et al., 2019). Some studies aim to pinpoint the links most in need of improvement or protection as a resource-allocation measure (Murray-Tuite and Mahmassani, 2004; Poorzahedy and Bushehri, 2005; Sohn, 2006). While significant work has been undertaken to evaluate and enhance network performance in non-recurrent congestion scenarios characterized by long-term effects, the literature is relatively sparse when it comes to non-recurrent congestion with short-term impacts, such as traffic accidents. There is a dearth of research addressing both the analysis of network performance and the development of measures to improve it in such situations. Sansó and Soumis (1991) introduced a method for assessing network performance under uncertain conditions, presenting a 3T model specifically designed to analyze traffic accidents in transportation networks. This model encompasses three distinct phases: before the accident, immediately after the accident, and after users have received information about the accident occurrence. Poorzahedy and Bushehri (2003) formulated two problems related to accident management in urban street networks, namely Accident Prevention (AP) and Accident Mitigation (MP). AP focuses on enhancing critical links to maximize a network performance metric, while MP seeks to optimize network connectedness by maximizing a different performance measure. Both of these problems are grounded in the concept of link importance. Huang et al. (2018) conducted an analysis of the impact of non-recurrent congestion on drivers' speed choices and aggressive behaviors using a substantial dataset of taxi floating car data.

A widely adopted approach in the literature for enhancing the resilience of transportation networks against the impacts of accidents involves the identification of critical nodes or links within the network and allocating resources to them strategically. In this regard, Zhou and Wang (2017) integrated a ranking method with an innovative mesoscopic model aimed at assessing the performance of urban transportation networks and identifying critical links within these networks, considering the vulnerability and potential of each link. The proposed model was based on the cell transmission model and was applied to real-scale networks, demonstrating its practicality and effectiveness in improving urban transportation systems. Drawing on graph theory, transportation network design, and urban planning principles, Psaltoglou and Calle (2018) proposed a methodology for identifying critical points in urban transportation networks, with the goal of optimizing the allocation of city resources. They defined the criticality of a node as the connectivity and activity density of that node over time. To evaluate the connectivity of a node, they took into account various factors, including vehicle capacity, distance, and frequency. For assessing activity density, they relied on indicators such as population density, land use, and urban form. Ghavami (2019) developed a multi-criteria integrated model for identifying the most crucial between cities roads within the network to inform decision-making in disaster scenarios. The proposed method was an integrated Geographic Information System (GIS) and Multi-Criteria Decision Making (MCDM) approach to sort data, identify evaluation criteria and rank links using the Analytic Hierarchy Process (AHP) technique. Vodák et al. (2019) introduced a deterministic algorithm designed to identify the most critical links within the network. The proposed algorithm was centered on the search for the shortest cycles and introduced a measure based on standard deviation to assess the vulnerability of a link. The main advantage of their algorithm was being rapider than algorithms that necessitate a comprehensive scan of the entire network, such as the brute-force algorithm. Tian et al. (2019) developed an approach to identify key links within urban transportation networks. Their method primarily relied on the temporal-spatial distribution of traffic congestion and encompassed three key phases: identifying congestion states, computing the time distribution of congestion states, and determining critical links within the network. They utilized time-interval coverage as a metric to classify a link as either critical or non-critical. Lin and Lin (2022) focused on the evaluating the vulnerability of transportation systems and proposed a novel framework to identify the most vulnerable components in transportation networks. They developed a nonlinear programming considering equilibrium constraints and users' route choices. They also suggested a short-term planning and a long-term planning vulnerability measure to identify vulnerable components. Vodák et al. (2022) investigated the problem of identifying combinations of road links that can cause severe network disruption when blocked simultaneously. To this end, they used three modified metaheuristic algorithms, including Simulated Annealing, Guided Local Search, and Variable Neighborhood Search, and compared their performance. Applying these algorithms to four real world networks, they showed that Simulated Annealing outperformed other algorithms. Iliopoulou and Makridis (2023) a multi-objective optimization was introduced for identifying critical disruption scenarios in public transport networks. They employed a transit assignment model to gauge passenger reactions to these disruptions. The method generates combinations of transit network link failures that maximize unsatisfied demand and additional travel time, encompassing passengers with no viable travel alternatives and those whose optimal paths are changed. Their results indicated that optimization-based attacks can reveal scenarios with significant passenger disconnections and elevated detour costs, outperforming single-objective and centrality-based approaches in identifying critical disruptions.

In this paper, we introduce a novel method aimed at enhancing the performance of urban networks during instances of non-recurrent congestion, with a particular emphasis on traffic accidents. Our approach centers on the identification of crucial network links, and subsequently, the optimization of these links using the limited available resources to achieve the maximum improvement in the performance of the network. In comparison to existing research, this study offers several notable contributions:

- In contrast to most existing studies, which often assume that the alteration of travelers' routes due to accidents on their intended paths has a negligible effect upon the level of service offered to other travelers, so that they keep on using their usual paths and that their trips remain suitable (Ghavami, 2019; Poorzahedy and Bushehri, 2003, 2005), this paper takes a more realistic approach. We acknowledge that in the real world, the redirection of travelers can have significant repercussions on network performance and user experiences. As such, we address this assumption to better align our model with real-world scenarios.
- some prior research has focused on evaluating trip suitability based on the moment when the network state changes due to an accident (i.e., transitioning to state). These studies have utilized the metric of "remaining time-hour" to gauge network performance after the accident (Ghavami, 2019; Poorzahedy and Bushehri, 2005). In this study, we introduce an improved measure, the mean total "suitable trip-hour" in the network after accident occurrence, to more accurately assess network performance, considering the suitability of trips over time.
- Previous approaches solely measure link importance by considering the presence or absence of that link (Ghavami, 2019; Poorzahedy and Bushehri, 2005). This study not only assess a link's importance in facilitating suitable paths for network users but also gauge its significance in providing alternative routes to users who have another accident-struck link on their intended paths as well.
- Our proposed approach in this paper places a strong emphasis on network reliability in the face of incidents hampering flows. We introduce a relevant and effective importance index for network links, with the primary objective of solving a network design problem aimed at enhancing the network's resilience to such incidents. While the problem we address is of significant importance to cities worldwide, especially those grappling with congestion, to the best of the authors' knowledge only a limited number of studies have explored this issue to date (El-Maissi et al., 2020; Iliopoulou and Makridis, 2023; Ma et al., 2022). This underscores the novelty and importance of our research in addressing a critical challenge faced by urban transportation networks.

According these discussions, Table 1 presents a brief comparison between the current study and previous studies.

- **Realistic Approach to Traffic Redirection:** Acknowledges the significant impact of traffic redirection on network performance and user experience, challenging the assumption that traffic alterations due to accidents are negligible.
- **Advanced Network Performance Measurement:** Introduces the "suitable trip-hour" metric, providing a more accurate assessment of network performance post-accident over time.
- **Comprehensive Link Importance Assessment:** Evaluates link importance not only by their presence but also by their role in providing alternative routes during accidents.
- **Network Reliability and Resilience:** Focuses on enhancing network resilience to incidents, introducing a novel importance index for network links to solve network design problems related to congestion and accidents.
- **Accident Improvement Model:** A new model is proposed for addressing the Accident Improvement Problem.
- **Heuristic Algorithm:** The paper proposes a heuristic algorithm as a practical solution to the model, highlighting its efficacy through illustrative examples.

Table 1. A brief comparison between the current study and previous studies

Main features	Previous Research	Current study
Network Performance Measures	Limited to standard metrics like travel time and delay	Introduction of new measures considering full user information
Approach to Traffic Accidents	Focus on long-term effects or general congestion	Specific focus on accident impact and immediate response
Traffic Redirection Impact	Assumes negligible impact of traffic redirection due to accidents	Realistically assesses significant impacts of traffic redirection on network performance and user experience
Link Importance Assessment	Measures link importance based on presence or absence	Evaluates link importance in facilitating suitable paths and providing alternative routes during accidents
Modelling Techniques	Traditional models without real-time data integration	Novel model integrating real-time accident information
Solution Methods	Conventional algorithms	Advanced heuristic algorithm for practical application
Impact Analysis	Overlooks the dynamic nature of accidents	Detailed analysis of the temporal impact of accidents on links

The remainder of this paper follows this structure: Section 2 introduces the relevant definitions, underlying assumptions, and the notation employed. In Section 3, we present our proposed model. Section 4 outlines the solution procedure suggested to solve the proposed model. To demonstrate the practicality and effectiveness of our algorithm, we solve and analyze several numerical examples of varying scales, encompassing both small and medium sizes, in Section 5. Lastly, in Section 6, we conclude the study and outline potential avenues for future research.

2. Notations

Suppose that accidents take place in the network links (intersections can be represented by links). The state of the network at any given time can be represented by c=(...,c_{ij}...), where $c_{ij} = 1/0$ shows that in the meanwhile *no/an accident* has occurred in link (i,j) . Therefore, c°=(1,...,1,...,1) is the state of the network with all links in no accident condition. This state is called the prevalent state. The following notations are used in the modelling the problem in this study:

V	the set of nodes
A	the set of links
$N(V, A)$	the urban transportation network
O	the set of origins
D	the set of destinations
n	the number of nodes where $n = \|V\|$
P	the set of O-D pairs
(k, s)	a pair of O-D nodes
d^{ks}	demand of (k,s)
c°	the current state of the network
c	the state of the network after an accident
t^{ks}	shortest travel time of in prevalent state
$t^{ks}(c)$	the travel time from origin k to destination s in state c ($t^{ks}(c^{\circ}) = t^{ks}$)
$t^{js}(c)$	the average travel time from node j to destination s when the network is in state c .
p^{ks}	the set of paths in the network from origin k to destination s
x_p^{ks}	the user equilibrium flow in path p during prevalent state where $p \epsilon p^{ks}$ and $(k, s) \epsilon P$
x_{ij}	the user equilibrium flow in link (i,j)
t_{ij}	the average travel time on link (i,j)
m_{ij}	the number of passengers on link (i,j)
m_p^{ks}	the number of passengers travelling between origin k and destination s on path p
m_{ij}^{ks}	the number of passengers on link (i,j) originating from k and heading to s
m_{ij}^{s}	the number of passengers on link (i,j) going to s , where $m_{ij}^{s} = \Sigma_{k \epsilon o} m_{ij}^{ks}$
P_{ij}	the probability of no accidents occurring in link (i,j) during a specific time period, e.g. a peak period or rush hour
C_{-ij}°	the state of the network in which only link (i,j) has been affected by an accident

3. The proposed model

In this section, we present a model designed to enhance network performance when accidents occur. The core concept of our model revolves around the identification of *important* links in the network, and the allocation of resources to these links within the resource constraints. Investing in the *important* links is expected to yield the most substantial improvements in network performance.

3.1. *Network's Performance Measure*

Assume that the probability of an accident occurring in a particular link during the analysis period is known. This probability can be estimated based on historical data from previous periods. Additionally, we presume that all users within the network receive accident information promptly following its occurrence, typically through channels such as radio broadcasts or variable message signs. It is a common trend that travelers who initially planned to traverse the link affected by the accident on their routes to their respective destinations often opt to modify their routes, irrespective of the accident's severity. This study considers the following assumptions:

Assumption 1. The probability of an accident occurring over a link is uniformly distributed along the length of that particular link. If this condition is not met, the links are redefined by segmenting them into smaller links to meet this condition.

Assumption 2. Accident events are assumed to be independent of one another, both within the network as a whole and within individual links of the network.

Assumption 3. Travelers along a given path are considered to be uniformly distributed along the entire path. Based on this assumption, travelers are uniformly distributed along every individual link within a path. Furthermore, it can be demonstrated with ease that the number of trip-makers on a link within a path is directly proportional to the travel time associated with that specific link on the path. Additionally, the number of trip-makers traveling along a particular path between an O-D pair is proportional to the travel time associated with that specific path (Detailed proofs can be found in the Appendix).

To facilitate further discussions, we classify trips into two distinct groups: *suitable* trip and *unsuitable* trips. For a predefined standard threshold $\theta^{ks}(e.g., \theta^{ks} = 1.2)$, trip from origin k to destination s in state c is *suitable* if $t^{ks}(c) < \theta^{ks}.t^{ks}$; otherwise, it is categorized as *unsuitable*. t^{ks} is shortest travel time from k to s in *prevalent* state. A *suitable* trip with a duration of $t^{ks}(c)$ has t^{ks} *suitable trip-hours.*

Regarding the accident occurring within a link, trip-makers within the network can be classified into three distinct groups. The first group consists of travelers currently situated within the accident-affected link. If the accident lies ahead of these travelers, they have no choice but to proceed through it, rendering their trips unsuitable. Conversely, if the accident is located behind them as they travel, they continue their journeys unimpeded, ultimately reaching their destinations suitably. The second group includes travelers who have the accident-affected link on their routes from origin to destination and receive immediate accident notifications. Upon receiving this information, they make route decisions based on their perceptions of link travel times before the accident, despite the possibility of these times changing due to route adjustments made by other travelers in response to the accident. The third group consists of travelers whose routes do not include the accident-affected link. These travelers maintain their original pre-accident routes. However, their travel times can fluctuate, either increasing or decreasing, as a result of route modifications made by passengers who initially had the accident-affected link on their pre-accident routes. Consequently, the journeys of travelers in this group may become *unsuitable* due to these alterations. Previous studies have typically overlooked the impact of newcomers on the routes of existing travelers within the network.

To effectively gauge user satisfaction and subsequently improve network performance during traffic accidents, the development of a robust objective or evaluation criterion is essential. The commonly used objective function in previous studies, namely the *total suitable* trips in the network after an accident, lacks the required precision. This criterion fails to distinguish between short and long trips impacted by accidents. Therefore, in this study, we introduce a new metric as our objective function, termed *Total Suitable Trip-Hours*. This measure provides a more nuanced and accurate assessment of the network's performance by considering not only the suitability of trips but also their duration, ultimately offering a more comprehensive evaluation of user satisfaction.

Let $E(c)$ and $E(c^\circ)$ represent the *mean total suitable trip-hours* in the network in states c and c° , respectively, where c denotes the current state of the network and c° is the state of the network without any accidents across the entire network. The Performance Index (PI) of the network in state c can be defined as follows:

$$PI(c) = E(c)/E(c^\circ) \tag{1}$$

Based on the assumptions, the *mean total suitable trip-hours* of the travelers on link (*i,j*) heading to destination *s* in state *c* of the network, $E^{s}_{ij}(c)$, can be determined as follows:

$$E^{s}_{ij}(c) = \sum_{k \epsilon O} \left[m^{ks}_{ij} t^{ks} - m^{ks}_{ij} \left(t^{ks}/2 \right) \bar{Z}_{ij}(c) \right] Z^{jks}(c) \tag{2}$$

where $\bar{Z}_{ij}$ (c) is a binary variable, taking the value of 1 if an accident occurs in link (*i,j*) in state *c*, and 0 otherwise. Z^{jks} (c) is a binary variable equal to 1 if a trip from *k* to *s* passing through node j remains suitable in state c, and 0 otherwise. Eqs. (1) and (2) result the following formulation to obtain *PI(c)* :

$$PI(c) = \sum_{j \epsilon V} \sum_{s \epsilon D} \sum_{k \epsilon O} \left(\Phi^{jks} - \psi^{jks}(c) \right) . Z^{jks}(c) \tag{3}$$

where:

$$\Phi^{jks}(c) = \sum_{\rho \epsilon \rho^{ks}} \sum_{(i,j) \epsilon B^{j}} X^{ks}_{\rho} . t_{ij} . \delta^{ks}_{ij}, \rho . t^{ks} / \sum_{(k,s) \epsilon P} d^{ks} . \left(t^{ks} \right)^{2} \tag{4}$$

$$\psi^{jks}(c) = \sum_{\rho \epsilon \rho^{ks}} \sum_{(i,j) \epsilon B(j)} X^{ks}_{\rho} . t_{ij} . \delta^{ks}_{ij}, \rho . \left(t^{ks}/2 \right) . \bar{Z}_{ij}(c) / \sum_{(k,s) \epsilon P} d^{ks} . \left(t^{ks} \right)^{2} \tag{5}$$

additionally, B(*j*) is the set of links with their head node *j* . $\delta^{ks}_{ij,p}$ is a binary variable, equal to 1 if link (i,j) belongs to path ρ from origin *k* to destination s, and 0 otherwise. Please refer to the Appendix for a comprehensive explanation and detailed proofs of eqs. (2) and (3).

With these considerations in place, we define the *Expected Performance Measure (EPM)* of a network under the effects of traffic accidents as follows:

$$EPM = E[PI] = \sum_{c \epsilon C} p(c) . PI(c) \tag{6}$$

where *p(c)* is the probability of state *c* of the network, and *C* is the set of the states that may happen to the network.

3.2. Link Importance Measure

In order to effectively enhance the performance of an urban transportation network in its ability to withstand the impacts of traffic accidents while operating under full user information, it is crucial to possess a guiding measure for identifying the weak links of the network. Improving these vulnerable links can lead to a substantial upgrade in the overall network performance. The importance of link (*m,n*) in the network, I_{mn} , can be defined as the rate of change of *EPM* with respect to the change in the probability of non-failure o the prevalent situation on this link as follows:

$$I_{mn} = \partial(EPM) / \partial(P_{mn}) \tag{7}$$

Based on this definition, the following approximation can be employed to quantify the importance of a link within the network:

$$I_{mn} \approx (p(c^{\circ})/p_{mn}) \left\{ \left[1 - PI(c^{\circ}_{-mn}) \right] + \Sigma_{(u,v) \epsilon A} \left[\left(1 - P_{uv} \right) / Puv \right] \left[PI \left(c^{\circ}_{-uv} \right) - PI \left(c^{\circ}_{-uv,-mn} \right) \right] \right\} \tag{8}$$

where $(p(c^{\circ})/p_{mn}) \left[1 - PI(c^{\circ}_{-mn}) \right]$ measures the reduction in the network performance when only link (m,n) experiences failure due to traffic accidents. This term, indeed, represents the importance of link (m,n) in providing suitable paths for network users. However, $(p(c^{\circ})/p_{mn})\Sigma_{(u,v)\epsilon A}\left[\left(1-P_{uv}\right)/Puv\right]\left[PI\left(c^{\circ}_{-uv}\right)-PI\left(c^{\circ}_{-uv,-mn}\right)\right]$ signifies the pair-wise importance of link (m,n) in combination with the other links in the network. Hence, this term shows the importance of link in providing alternative paths to the destinations of the users deprived of access to another accident-struck link. Essentially, it indicates whether link *(m,n)* belongs to one or more effective alternative paths. Detailed proofs for eq. (8) can be found in the Appendix.

Substituting eq. (3) in will result:

$$\begin{aligned}\left(\mathrm{p(c^{\circ})}/P_{mn}\right)\left[1-PI(c^{\circ}_{-mn})\right] &= \left(\mathrm{p(c^{\circ}}/P_{mn})\right)\left[1-\sum_{j\epsilon V}\sum_{s\epsilon D}\sum_{k\epsilon O}\Phi^{jks}.Z^{jks}(c^{\circ}_{mn})\right] \\ &+\left(\mathrm{p(c^{\circ}}/P_{mn})\right)\left[\sum_{j\epsilon V}\sum_{s\epsilon D}\sum_{k\epsilon O}\psi^{jks}(c^{\circ}_{-mn}).Z^{jks}(c^{\circ}_{mn})\right]\end{aligned} \tag{9}$$

where in eq. (9), the first term measures the importance of link *(m,n)* in providing suitable paths for network users, while the second term reflects the importance of link *(m,n)* in facilitating *suitable* trips for passengers using this link. In addition, based on eqs. (6) and (7), the changes in the *EPM* over a specific alternative improvement action of w for any link can be obtained as follows:

$$d(EPM)=\sum_{(m,n)}\frac{\partial(EPM)}{\partial(P_{mn})}d(P_{m,n})d(p_{mn})=P_{mn}\alpha^{w}_{m,n,\forall w}\sum_{(m,n)}\frac{\partial(EPM)}{\partial(P_{mn})}P_{m,n}\alpha^{w}_{mn}=\sum_{(m,n)}I_{mn}P_{mn}\alpha^{w}_{mn} \tag{10}$$

where an alternative improvement action refers to a candidate action to reduce the likelihood of a traffic accident occurring on a specific link, consequently enhance its probability of survival.α^{W}_{mn} is a ratio that quantifies the extent to which the survival probability of link *(m,n)* is augmented through the implementation of the alternative action w.

3.3. *The Proposed Model for Accident Improvement Problem*

In this section, building upon the measures introduced in the previous sections, we formulate a mathematical programming, named *Accident Improvement Problem (AIP)*, to improve the network's performance in mitigating the adverse effects of traffic accidents as follows:

$$\begin{aligned}&AIP: \\ &Max\sum_{(m,n)\epsilon A}\sum_{w=1}^{W_{mn}}\left[I_{mn}P_{mn}\alpha^{w}_{mn}\right]Z^{w}_{mn} \\ &s.t. \\ &\quad\Sigma^{W_{mn}}_{W=1}Z^{w}_{mn}\leq 1 && \forall(m,n)\epsilon A \\ &\quad\Sigma_{(m,n)}\Sigma^{w_{mn}}_{w=1}c^{wl}_{m}nZ^{w}_{mn}\leq B^{1} && \forall l=1,\ldots,L \\ &\quad Z^{w}_{mn}\epsilon\{0,1\} && \forall(m,n)\epsilon Aw\epsilon\{1,\ldots,W_{mn}\}\end{aligned} \tag{11}$$

where Z^{w}_{mn} is a binary variable, which is 1 if action *w* is selected for link *(m,n)*, 0 otherwise. W_{mn} is the total number of alternative improvement actions to reduce the likelihood of traffic accidents on link *(m,n). L* denotes the number of distinct resource types available for the implementation of improvement actions.B^{l} represents the available quantity of recourse type *l*, and c^{wl}_{mn} is the resource requirement of resource type *l* necessary for the execution of alternative action w on link *(m,n)* . In the context of the AIP, decision-makers must determine which alternative actions should be applied to each link in order to achieve the maximum improvement in the network's performance against the impact of traffic accidents. Hence, the objective function in model (11) maximizes the cumulative enhancement in the survival probabilities of the candidate links within the network, with weights assigned based on their importance in the network. The first set of constraints in this model ensures that, for each candidate link, at most one action can be chosen. The second set of constraints represents the resource constraints, ensuring that the utilization of resources does not surpass the available quantities.

4. A solution procedure

Most of the problems related to transportation planning and network design at the network level are highly complex, making it impossible to solve any proposed mathematical model for real-world networks (Mahmoudi et al., 2022; Mahmoudi, 2019). Various algorithms have been employed within the transportation science literature to address the inherent complexity of the proposed mathematical programming model (Agrawal et al., 2022; Vodák et al., 2022). In this section, a solution algorithm is developed to solve the suggested *AIP* model. This algorithm requires the flow levels on each link of the network subsequent to the occurrence of the accident(s). These post-accident flow volumes are then utilized to calculate the travel times experienced by passengers in the aftermath of the accident state of the network, *c*. Then, the obtained travel times must be compared with the corresponding values of the travel times in the *prevalent* state, c°, to determine whether the travel times for these passengers have become *unsuitable* or not.

During this process, it becomes imperative to determine new routes for passengers who initially intended to travel via the accident-affected link(s) but can no longer do so due to the accidents. Moreover, the solution procedure must estimate the increase or decrease in flow volumes on the links of the new and previous paths, stemming from the redirection of these passengers to the new paths. To this end, the algorithm must estimate the O-D demand for the impacted passengers after the occurrence of the accidents. Given that these passengers opt for alternative paths after the accidents, their trip origins no longer align with their original trip origins. In such cases, the new trip origins can be established as the first nodes at which passengers arrive immediately following the accidents. As a result, the proposed solution algorithm necessitates the following pieces of information:

$T_{mn,uv}^{js}$ the demand for destination s, reaching node j as the first node immediately following the accident incidents in link *(m,n)* and *(u,v)* ϵA, *(j,s)*$\epsilon V \times D$

$X_{ij}^{+mn,+uv}$ additional flow on link *(i,j)*, due to failures of links *(m,n)* and *(u,v)*

$X_{ij}^{mn,uv}$ the portion of the trip volume on link *(i,j)* that was initially intended to pass through links *(m,n)* or *(u,v)* before the accident occurrence on these links, but the accidents prevented this from happening

Taking into account that $x_{\rho}^{ks}\left(\frac{t_{ij}}{t^{ks}}\right)$ and $\frac{1}{2}x_{\rho}^{ks}$ are the demand and expected demand, for destination s reaching node *j* in link *(i,j)*, in state $c_{-mn,-uv}^{\circ}\left(\bar{Z}_{ij}(.)=0\right)$, and is hit by an accident , respectively, $T_{mn,uv}^{js}$ is estimated as follows:

$$T_{mn,uv}^{js} = \sum_{i\epsilon B(j)}\sum_{k\epsilon O}\sum_{\rho\epsilon\rho^{ks}}\frac{x_{\rho}^{ks}t_{ij}}{t^{ks}}\left[1-\frac{\bar{Z}_{ij}\left(c_{-mn-uv}^{\circ}\right)}{2}\right]\delta_{ij,\rho}^{ks}.\delta_{mn,uv,\rho}^{ks} \tag{12}$$

where $\delta_{mn,uv,p}^{ks}$ is equal to 1 if links *(m,n)* or *(u,v)* belong to path ρ from origin *k* to destination *s*, and 0 otherwise. $x_{ij}^{+mn,+uv}$ will be estimated as the algorithm outputs. Finally, $x_{ij}^{mn,uv}$ is estimated using the following equation:

$$X_{ij}^{mn,uv} = \sum_{(k,s)\epsilon P}\sum_{\rho\epsilon\rho^{ks}} x_{\rho}^{ks}\delta_{ij,\rho}^{ks}\delta_{mn,uv,\rho}^{ks} \tag{13}$$

The proposed algorithm is a heuristic procedure to compute the importance of the links and select the appropriate actions to improve their performances, all while adhering to resource constraints. The algorithm can be broken down into the following steps:

Step 0. **Preparation.** Define *N(V,A)* by specifying *V* and *A*. Input the survival probabilities, P_{uv} , link cost functions, $t_{uv}\forall(u,v)\epsilon A$, and the O-D demands, d_{ks} , and travel time suitability standards, $\theta^{ks}, \forall(k,s)\epsilon P$.

Step 1. **Equilibrium flow computation before accident occurrence.** Solve the user equilibrium flow problem for the provided demand d_{ks}, and compute the path flows $\left(x_{\rho}^{ks}, \forall\rho\epsilon\rho^{ks}, \forall(k,s)\epsilon P\right)$, and the corresponding equilibrium link flows and travel times $\left(x_{uv}, t_{uv}, \forall(u,v)\epsilon A\right)$

Step 2. **Specification of the trip condition after the accident.** For each link $(m,n)\epsilon A$, follow these steps to determine the network's condition in a state where two links have failed due to accident occurrences, with one of these links being link *(m,n)*:

(a) Obtain Φ^{jks} for each $k\epsilon O, \psi^{jks}(c_{-mn}^{\circ})$ for each $k\epsilon O, T_{mn,uv}^{js}$ for each $(u,v)\epsilon A$ and $x_{ij}^{mn,uv}$ for each *(ij)* and $(u,v)\epsilon$, A for all $(j,s)\epsilon V\times D$, by using eqs. (4)-(5), (12) and (13), respectively.

(b) For each link $(u,v)\epsilon A$, using the equilibrium link travel times t_{uv} in *Step 1*, for all $(j,s)\epsilon V\times D$, assign $T_{mn,uv}^{js}$ to the network $(V, A_{-mn,-uv})$, excluding links *(m,n)* and (u,v) , by using employing all-or-nothing procedure. Compute the additional flow resulting from the failures of links *(m,n)* and $(u, v), x_{ij}^{+mn,+uv}$, for all links $(i, j)\epsilon A_{-mn,-uv}$. Determine the link flows and travel times after the accidents in links *(m,n)* and *(u,v)* , for each link $(i, j)\epsilon A_{-mn,-uv}$:

$$x_{ij}^{new} = x_{ij} - x_{ij}^{mn,uv} + x_{ij}^{mn,+uv} \tag{14}$$

$$t_{ij}^{new} = t_{ij} - \left(x_{ij}^{new}\right) \tag{15}$$

(c) For each $(j, s)\epsilon V \times D$ compute the shortest (j,s) path travel time before and after the accident occurrences, $t^{js}(c^{\circ})$ and $t^{js}(c^{\circ}_{-mn,-uv})$, using t_{ij} (from *Step 1*) and t_{tj}^{new} (from Eq. (15)), respectively.

(d) If $\left(t^{js}(c^{\circ}_{-mn,-uv}) - t^{js}(c^{\circ})\right) \leq \left(\theta^{ks} - 1\right)t^{ks}$, where the right-hand-side is the maximum allowable delay tolerance for (k,s) pair, set $Z^{jks}(c^{\circ}_{-mn,-uv})$ (i.e., declare the trips of the passengers from *k* to *s* , who visit node *j* as the first node on their path right after the simultaneous accident occurrences in links (m,n) and (u,v) , are *suitable*), otherwise, set $Z^{jks}(c^{\circ}_{-mn,-uv})$ (that is, these trips end *unsuitable*).

Step 3.* Compute the importance of link(m,n)* . Compute the importance of link *(m,n)* in mitigating the impacts of accidents in the network, I_{mn} , by using Eq. (8) and the information obtained in *Step 2.*

***Step 4.* Compute link importance for all links.** Repeat *Steps* 2 and 3 for all links in the network to calculate their respective importance values.

***Step 5.* Solve AIP model.** Solve AIP model using $I_m n, \forall(m.n)\epsilon A$ and an appropriate integer programming solution method.

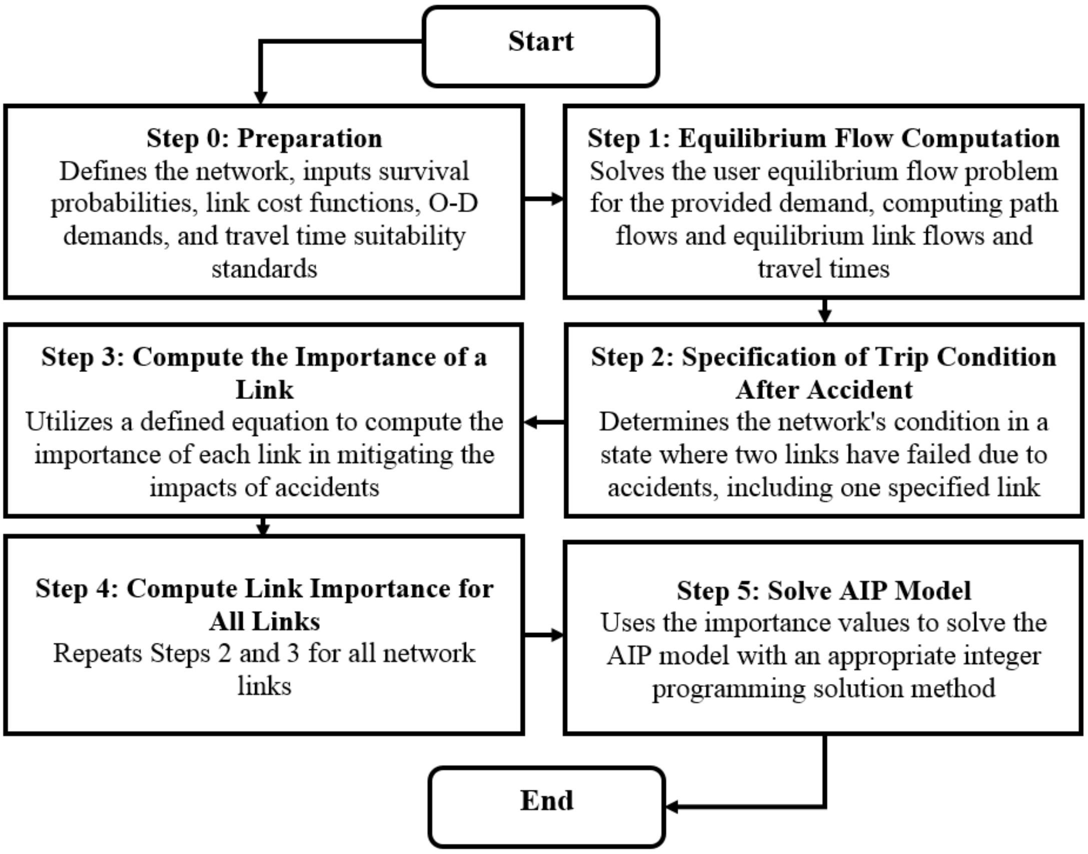

Figure 1. The flowchart of the proposed algorithm

5. Illustrative examples

In this section, we demonstrate the practicality of the proposed approach and examine its outcomes through the solution of two numerical examples in small and medium sizes. The provided algorithm has been implemented in Gauss software and executed on a PC with a 2.26 gigahertz Core i7 CPU and Windows Seven, utilizing 4 GB of RAM.

5.1. *A small Scale Example: Improving Network Performance by Preventive Actions*

Figure 2 shows an example network with 10 nodes and 18 links. Table 2 presents the parameters of the link travel time functions in the form of $t_{ij} = a_{ij} + b_{ij}x_{ij}^4$. The link survival probabilities (representing the likelihood of no accidents occurring in a specific time period) are assumed to be 0.98 for all links within the network. The demand from the two origins 1 and 2, heading to the two destinations 6 and 7 is identical and amounts to 3500 trips per hour. In this example, we consider an average vehicle occupancy of 1 person, and $\theta^{ks} = 1.1, \forall(k,s)\epsilon P$, meaning that a passenger's travel time is deemed unsuitable if it increases by more than 10% due to an accident in the network.

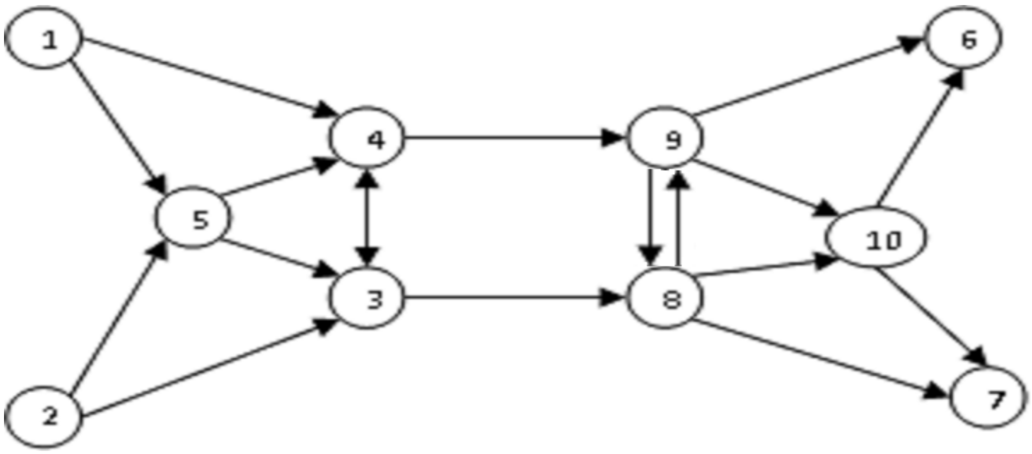

Figure 2. Example network 1

Table 2. The network specification of Example1.

Link (i,j)	$a_{ij}(\times 10^{-2}hr)$	$b_{ij}(\times 10^{-4}hr/(1000veh/hr)^4)$	Link (i,j)	$a_{ij}(\times 10^{-2}hr)$	$b_{ij}(\times 10^{-4}hr/(1000veh/hr)^4)$
(1,4)	5	0.030	(5,4)	3	0.030
(1,5)	3	0.030	(8,7)	5	0.030
(2,3)	5	0.030	(8,9)	3	0.030
(2,5)	3	0.030	(8,10)	3	0.030
(3,4)	3	0.030	(9,6)	5	0.030
(3,8)	3	0.030	(9,8)	3	0.030
(4,3)	3	0.030	(9,10)	3	0.030
(4,9)	3	0.010	(10,6)	3	0.030
(5,3)	3	0.030	(10,7)	3	0.030

Three alternative actions are available to mitigate accident levels in a link. The first action involves the deployment of patrol police, where the presence of 1 unit of patrol police on a street increases its "survival" probability (the likelihood of no accidents occurring) by 1% of its prevalent state. The second alternative includes investing in the enhancement of street infrastructure, such as signs, signals, markings, geometric design, and maintenance, in addition to choosing the first alternative. In this case, when 1 unit of patrol police is present, 1 unit of investment boosts the survival probability by 1.5% of the prevalent state. Lastly, the third alternative is the utilization of an "immediate response system," which is implemented in conjunction with the previous two alternatives. This system rapidly detects accidents and promptly clears the accident scene, effectively making the accident almost unnoticeable or bearable to other passengers. This results in a near 100% survival probability. In addition, it is assumed that $B^1 = 4$ units of patrol police, $B^2 = 3$ units of money to invest for the street condition enhancement, and $B^3 = 2$ units of the immediate response system are available resources for implementing these actions.

The results of the proposed algorithm for this problem are reported in Table 3. In this table, I_{mn}^a shows the importance of link (*m,n*) in providing *suitable* trips for passengers within this link, I_{mn}^b indicates the importance of link (*m,n*) in providing *suitable* trips to travelers in the network, and I_{mn}^{c} represents the importance of link (*m,n*) in providing alternative routes to the passengers whose original routes have been disrupted by traffic accidents. The heuristic Effective Gradient procedure proposed by Ahmed (1983) is applied to solve the AIP model. This procedure determines a set of links for investment and the best action for each selected link, in order to maximize the enhancement in the survival probabilities of vulnerable links within the network.

Table 3. Results of the proposed algorithm for Example1.

Link (m,n)	$t_{mn}(hr)$	$x_{mn}(1000veh\ /\ hr)$	I_{mn}^{a}	I_{mn}^{b}	I_{mn}^{c}	I_{mn}	Z_{mn}^{*}
(1,4)	0.0529	5.5930	0.0522	0.0000	0.0179	0.0701	-
(1,5)	0.0300	1.4070	0.0080	0.0000	0.0025	0.0105	-
(2,3)	0.0517	4.9105	0.0452	0.0000	0.0133	0.0585	-
(2,5)	0.0301	2.0895	0.0119	0.0000	0.0039	0.0158	-
(3,4)	0.0300	0.0000	0.0000	0.0000	-0.0003	-0.0003	-
(3,8)	0.0348	6.3175	0.0361	0.1225	0.0558	0.2143	3
(4,3)	0.0300	0.0000	0.0000	0.0000	-0.0030	-0.0030	-
(4,9)	0.0335	7.6825	0.0439	0.2747	0.0963	0.4149	3
(5,3)	0.0300	1.4070	0.0080	0.0000	0.0039	0.0120	-
(5,4)	0.0301	2.0895	0.0119	0.0000	0.0062	0.0181	-
(8,7)	0.0517	4.9070	0.0451	0.0000	0.0235	0.0687	-
(8,9)	0.0300	0.0000	0.0000	0.0000	0.0000	0.0000	-
(8,10)	0.0300	1.4105	0.0081	0.0437	0.0183	0.0700	-
(9,6)	0.0529	5.5895	0.0522	0.0000	0.0296	0.0818	-
(9,8)	0.0300	0.0000	0.0000	0.0000	0.0000	0.0000	-
(9,10)	0.0301	2.0930	0.0120	0.0648	0.0281	0.1049	1
(10,6)	0.0300	1.4105	0.0081	0.0598	0.0237	0.0915	-
(10,7)	0.0301	2.0930	0.0120	0.0887	0.0358	0.1365	2

Note. Z_{mn}^{*} represents choice of alternative action *w* to upgrade link (m,n) survival probability.

Here are some key findings from the analysis:

a. As it is clear from Table 3, in Example Network 1, links (3,8) and (4,9) in Figure 2 have the highest importance. This is because passengers traveling from origins 1 and 2 to destinations 6 and 7 must pass through either of these two links. These links play a critical role in providing *suitable* and alternative routes to connect O-D pairs in the network. Link (4,9) (with $b_4 9$) has a higher importance index than link (3,8) (with $b_3 8$) due to its higher capacity.

b. Links (1,4), (1,5), (2,3), and (2,5), which connect origins 1 and 2 to the network, have low importance values. This is partly because travelers who have not yet started their trips after the accident will only do so if their trips become *suitable*. For these links, I_{mn}^{b}, which is the importance of link (*m,n*) for providing suitable routes to the trip-makers in the network, is zero. This is another reason for their low importance levels.

c. Links leading to destinations 6 and 7, such as links (8,7), (10,7), (10,6), and (9,6), have higher importance than links starting at origins 1 and 2. This is because a larger number of passengers in the network rely on these links at the time of the accident to end their trips suitably, compared to the links starting at origins. Recall that only passengers who are certain of completing their trips suitably will begin their trips.

d. Links (3,4), (4,3), (8,9), and (9,8), have zero equilibrium flow in the prevalent state, as shown in Table 3. Hence, the respective values of I_{mn}^{a} (for within the link passengers) and I_{mn}^{b} (for the network passengers) are zero. Moreover, the corresponding values of I_{mn}^{c} for links (3,4) and (4,3) are negative. This indicates that these two links not only fail to contribute to forming alternative routes for passengers when accidents occur in other links in the network but, by redirecting new flow to other congested links, they turn some previously suitable routes into unsuitable ones.

e. The congestion resulting from the redirection of passengers due to traffic accidents can either increase or decrease the importance of certain links in the network. This is because new route choices can lead to increased congestion on some links while reducing it on others, thereby making previously suitable paths unsuitable or vice versa. Table 4 shows the importance values of the links in example 1 when the congestion effects of redirected flows over the links, whether positive or negative, are not considered. To compute the importance of all links in this case, x_{ij}^{new} in eq.(30) is replaced by x_{ij}. Links (3,8) and (4,9) are two most important links in the network that together carry all demand in the network. Comparing the respective link importance values for these two links in Table 3 and 4 shows that the importance values with due considerations of the congestion effects of redirected flows over the links in Table 3 are higher than the respective values without these effects, shown in Table 4. This is because when either of these links is involved in an accident, its flow diverts to other link, increasing congestion and the likelihood of more trips becoming *unsuitable* in the network. This phenomenon does not happen when the algorithm does not consider this diversion in Table 4. As another example, consider links (3,4) and (4,3), which have lower importance values with the diversion of flow in Table 4 (negative values) than without this diversion in Table 4. The reason is that when traffic diversion occurs, some passengers choose these links in alternative paths, causing congestion on other paths and rendering their trips *unsuitable*.

f. Table 3 reveals that model recommends action 3 for the most important links, (3,8) and (4,9), and actions 2 and 1 for the next most important links, (10,7) and (9,10), respectively.

Table 4. The importance of links in example 1 when the congestion effect of the redirected flows in the network is not considered.

Link (m,n)	I^a_{mn}	I^b_{mn}	I^c_{mn}	I_{mn}	Link (m,n)	I^a_{mn}	I^b_{mn}	I^c_{mn}	I_{mn}
(1,4)	0.052	0.000	0.016	0.068	(5,4)	0.012	0.000	0.006	0.018
(1,5)	0.008	0.000	0.003	0.011	(8,7)	0.045	0.000	0.021	0.067
(2,3)	0.045	0.000	0.014	0.059	(8,9)	0.000	0.000	0.000	0.000
(2,5)	0.012	0.000	0.004	0.016	(8,10)	0.008	0.044	0.02	0.071
(3,4)	0.000	0.000	0.000	0.000	(9,6)	0.052	0.000	0.025	0.077
(3,8)	0.036	0.106	0.049	0.192	(9,8)	0.000	0.000	0.000	0.000
(4,3)	0.000	0.000	0.000	0.000	(9,10)	0.012	0.065	0.027	0.104
(4,9)	0.044	0.128	0.059	0.231	(10,6)	0.008	0.06	0.026	0.094
(5,3)	0.008	0.000	0.005	0.013	(10,7)	0.012	0.089	0.036	0.136

5.2. *A Medium Scale Example: Sioux Falls Network*

The purpose of solving this example problem is to demonstrate the effectiveness of the proposed solution algorithm in solving the AIP problem in larger networks. Figure 3 shows the Sioux Falls network, with 24 nodes and 76 links, which is a widely recognized test network in transportation studies. The network specifications, the O-D demands, and the link survival probabilities are taken from Poorzahedy and Bushehri (2003). For this example it is assumed that the average car occupancy in the network is 1, $\theta^{ks} = 1.1, \forall(k,s)\epsilon P, B^1 = 30, B^2 = 15$ and $B^3 = 5$.

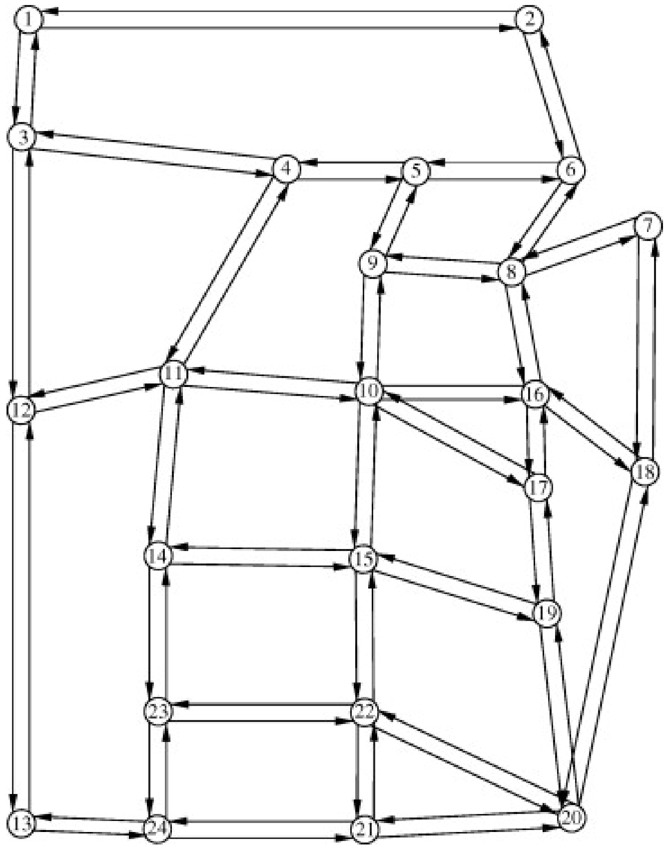

Figure 3. The urban street network of Sioux Falls

When the proposed solution algorithm is applied to this problem, taking into account the available resources, it yields the link importance values as presented in Table 5. This table also shows the selected actions for the chosen links. These actions aim to maximize link survival, ensuring that passenger trips remain suitable to the greatest extent possible by strengthening the links to prevent accidents or mitigate the impacts of accidents that have occurred, thereby maintaining the network's performance close to the *prevalent* state.

Table 5. Results of the proposed solution algorithm for AIP problem in Sioux Falls.

Link (m,n)	I_{mn}	z^*_{mn}	Link (m,n)	I_{mn}	z^*_{mn}
(1,2)	0.0056	0	(2,1)	0.0097	0
(1,3)	0.0034	0	(3,1)	0.0192	0
(2,6)	0.0086	0	(6,2)	0.0111	0
(3,4)	0.0127	0	(4,3)	0.0309	2
(3,12)	0.0220	1	(12,3)	0.0091	0
(4,5)	0.0255	1	(5,4)	0.0481	3
(4,11)	0.0039	0	(11,4)	0.0066	0
(5,6)	0.0156	0	(6,5)	0.0072	0
(5,9)	0.0219	1	(9,5)	0.0362	2
(6,8)	0.0182	0	(8,6)	0.0282	2
(7,8)	0.0123	0	(8,7)	0.0172	0
(7,18)	0.0112	0	(18,7)	0.0085	0
(8,9)	0.0307	2	(9,8)	0.0220	1
(8,16)	0.0328	2	(16,8)	0.0303	2
(9,10)	0.0116	0	(10,9)	0.0023	0
(10,11)	0.0073	0	(11,10)	0.0034	0
(10,15)	0.0164	0	(15,10)	0.0182	0
(10,16)	0.0319	2	(16,10)	0.0196	0
(10,17)	0.0089	0	(17,10)	0.0219	1
Link (m,n)	I_{mn}	z^*_{mn}	Link (m,n)	I_{mn}	z^*_{mn}
(11,12)	0.034	2	(12,11)	0.0231	1
(11,14)	0.0363	2	(14,11)	0.0169	0
(12,13)	0.0121	0	(13,12)	0.0069	0
(13,24)	0.0197	1	(24,13)	0.0282	1
(14,15)	0.0045	0	(15,14)	0.0234	3
(14,23)	0.0244	1	(23,14)	0.0252	1
(15,19)	0.0176	0	(19,15)	0.0094	0
(15,22)	0.0106	0	(22,15)	0.0069	0
(16,17)	0.0073	0	(17,16)	0.0036	0
(16,18)	0.0058	0	(18,16)	0.0131	0
(17,19)	0.0201	1	(19,17)	0.0165	0
(18,20)	0.0216	1	(20,18)	0.0113	0
(19,20)	0.0244	1	(20,19)	0.0118	0
(20,21)	0.0176	0	(21,20)	0.0182	0
(20,22)	0.0345	2	(22,20)	0.0267	1
(21,22)	0.0265	3	(22,21)	0.0246	3
(21,24)	0.0169	0	(24,21)	0.0269	3
(22,23)	0.0163	0	(24,22)	0.0196	1
(23,24)	0.0107	0	(24,23)	0.0094	0

6. Conclusions and future research directions

Advancements in information technology have paved the way toward the availability of online information and even navigation aids to circumvent congestion and flow obstacles. Many modern vehicles, both private and public, now come equipped with a range of audio-visual technologies as standard features. These include safety essentials like seat belts and airbags, as well as advanced systems like automatic braking, global positioning system, and connectivity to online apps via vehicle infotainment systems and mobile phones. These technological leaps support a scenario where network users can receive immediate updates about accidents.

This paper first introduces measures to quantify the network's performance and the importance of a specific link. Then, a mathematical programming and a heuristic algorithm are proposed to improve network performance to withstand traffic accident impacts under users' full information. It introduces an objective function that measures *suitable trip-hours* after accident occurrences in the network. This measure is a more suitable and efficient objective for the network design problem than the total vehicle hours in the conventional network design problem or the total suitable trips in the previous studies. Moreover, the "suitability" of trips is a well-known concept in network reliability, and trip-hour is related to consumer surplus, which is a measure of consumer welfare (Poorzahedy and Bushehri, 2005).

In the *AIP*, the managers of an urban transportation network mainly try to prevent accident occurrence in the important links of the network under resource constraints. The new link importance measure introduced in this study is applicable to determining the importance of the link in providing *suitable* trips to the passengers within the link, the passengers in the network as a whole, and the passengers who are confronted by accidents in other links of the network. To show the mechanism of the suggested solution algorithm in solving the developed mathematical programming for *AIP*, as well as the properties of the measures used by the algorithm, *AIP* is solved for a small-scale example network and a medium-sized network, and the results are analyzed. Results showed how the importance of a link can be increased or decreased after an accident occurs in the network. Also, results showed how the impacts of newcomers on the routes of existing travelers within the network can affect the performance of the network when an accident occurs.

Future research in this area could explore several promising directions. To begin with, the approach presented in this study could be applied to real-world case studies. While the current study mainly assumes deterministic parameters and variables, actual urban transportation networks typically involve vast datasets with some elements in probabilistic, stochastic, or missing form. Therefore, a valuable avenue for future research would involve devising suitable models that account for these probabilistic or stochastic variables/parameters and handle missing data effectively. Furthermore, in this study, a heuristic algorithm was developed to address the proposed model problem. Future research efforts could focus on developing alternative heuristic and metaheuristic algorithms that are not only different but also more efficient in solving the model. Comparative studies between these various solution approaches could provide valuable insights into their relative merits and performance characteristics.

Peer Review: Externally peer-reviewed.

Author Contributions: Conception/Design of Study- S.N.S.B., R.M.; Data Acquisition- S.N.S.B., R.M.; Data Analysis/ Interpretation- S.N.S.B., R.M.; Drafting Manuscript- S.N.S.B., H.P., R.M.; Critical Revision of Manuscript- S.N.S.B., H.P., R.M.; Final Approval and Accountability- S.N.S.B., H.P., R.M.

Conflict of Interest: Authors declared no conflict of interest.

Financial Disclosure: Authors declared no financial support.

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REFERENCES

Adelé, S., Tréfond-Alexandre, S., Dionisio, C., Hoyau, P.-A., 2019. Exploring the behavior of suburban train users in the event of disruptions. Transportation Research Part F: Traffic Psychology and Behaviour 65, 344-362.

Agrawal, A.K., Yadav, S., Gupta, A.A., Pandey, S., 2022. A genetic algorithm model for optimizing vehicle routing problems with perishable products under time-window and quality requirements. Decision Analytics Journal 5, 100139.

Ahmed, N.U., 1983. An analytical decision model for resource allocation in highway maintenance management. Transportation Research Part A: General 17, 133-138.

Chen, A., Yang, H., Lo, H.K., Tang, W.H., 2002. Capacity reliability of a road network: an assessment methodology and numerical results. Transportation Research Part B: Methodological 36, 225-252.

Chen, A., Zhou, Z., Chootinan, P., Ryu, S., Yang, C., Wong, S., 2011. Transport network design problem under uncertainty: a review and new developments. Transport Reviews 31, 743-768.

Dhahir, B., Hassan, Y., 2019. Probabilistic, safety-explicit design of horizontal curves on two-lane rural highways based on reliability analysis of naturalistic driving data. Accident Analysis & Prevention 123, 200-210.

El-Maissi, A.M., Argyroudis, S.A., Nazri, F.M., 2020. Seismic vulnerability assessment methodologies for roadway assets and networks: A state-of-the-art review. Sustainability 13, 61.

Ghavami, S.M., 2019. Multi-criteria spatial decision support system for identifying strategic roads in disaster situations. International Journal of Critical Infrastructure Protection 24, 23-36.

Huang, Y., Sun, D.J., Zhang, L.-H., 2018. Effects of congestion on drivers' speed choice: assessing the mediating role of state aggressiveness based on taxi floating car data. Accident Analysis & Prevention 117, 318-327.

Iliopoulou, C., Makridis, M.A., 2023. Critical multi-link disruption identification for public transport networks: A multi-objective optimization framework. Physica A: Statistical Mechanics and its Applications 626, 129100.

Ji, X., Ban, X., Zhang, J., Ran, B., 2019. Moment-based travel time reliability assessment with Lasserre's relaxation. Transportmetrica B: Transport Dynamics 7, 401-422.

Karatas, M., Yakıcı, E., 2021. A multi-objective location analytics model for temporary emergency service center location decisions in disasters. Decision Analytics Journal 1, 100004.

Lam, W.H., 1999. Network reliability and transport modelling. Journal of Advanced Transportation 33, 121-123.

Lin, J., Lin, W., 2022. Transportation system vulnerability assessment considering environmental impact. Journal of advanced transportation 2022.

Ma, C., Alam, S., Cai, Q., Delahaye, D., 2022. Critical links detection in spatial-temporal airway networks using complex network theories. IEEE Access 10, 27925-27944.

Mahmoudi, R., Shetab-Boushehri, S.-N., Emrouznejad, A., 2022. Sustainability in the evaluation of bus rapid transportation projects considering both managers and passengers perspectives: A triple-level efficiency evaluation approach. International Journal of Sustainable Transportation 16, 1059-1077.

Mahmoudi, R., Shetab, N., Hejazi, R., Emrouznejad, A., 2019. Determining the relative importance of sustainability evaluation criteria of urban transportation network. sustainable cities and society.

Mahmoudi, R.S., Nader; Hejazi, Reza; Emrouznejad, Ali; Rajabi, Parisa;, 2019. A hybrid egalitarian bargaining game-DEA and sustainable network design approach for evaluating, selecting and scheduling urban road construction projects. Transportation Research Part E: Logistics and Transportation Review 130, 161-183.

Murray-Tuite, P.M., Mahmassani, H.S., 2004. Methodology for determining vulnerable links in a transportation network. Transportation Research Record 1882, 88-96.

Poorzahedy, H., Bushehri, S.N.S., 2003. Project Selection in Traffic Accident Prevention and Mitigation. EJTIR 3, 241-261.

Poorzahedy, H., Bushehri, S.N.S., 2005. Network performance improvement under stochastic events with long-term effects. Transportation 32, 65-85.

Psaltoglou, A., Calle, E., 2018. Enhanced connectivity index – A new measure for identifying critical points in urban public transportation networks. International Journal of Critical Infrastructure Protection 21, 22-32.

Qiao, W., Lu, Y., Xiong, C., Haghani, A., 2014. A game theory approach for the measurement of transport network vulnerability from the system prospective. Transportmetrica B: transport dynamics 2, 188-202.

Sansó, B., Soumis, F., 1991. Communication and transportation network reliability using routing models. IEEE Transactions on Reliability 40, 29-38.

Sharma, M.K., Chauhan, B.K., 2022. Analytical assessment of implementation aspect of regional rapid transit system. Decision Analytics Journal 4, 100093.

Sheffi, Y., 1985. Urban transportation networks.

Sohn, J., 2006. Evaluating the significance of highway network links under the flood damage: An accessibility approach. Transportation research part A: policy and practice 40, 491-506.

Tian, Z., She, W., Li, S., Wang, Y.-W., Liu, W., Zai, G.-J., Jia, L.-M., Qin, Y., Dong, H.-H., 2019. Key links identification for urban road traffic network based on temporal-spatial distribution of traffic congestion. Modern Physics Letters B 33, 1950307.

Vodák, R., Bíl, M., Svoboda, T., Křivánková, Z., Kubeček, J., Rebok, T., Hliněný, P., 2019. A deterministic approach for rapid identification of the critical links in networks. PloS one 14.

Vodák, R., Křivánková, Z., Bíl, M., 2022. Identification of road links with the gravest network impacts when blocked concurrently. European Journal of Transport and Infrastructure Research 22, 63-92.

Yu, R., Wang, X., Yang, K., Abdel-Aty, M., 2016. Crash risk analysis for Shanghai urban expressways: a Bayesian semi-parametric modeling approach. Accident Analysis & Prevention 95, 495-502.

Zhang, L., Xiong, C., 2017. A novel agent-based modelling framework for travel time reliability analysis. Transportmetrica B: Transport Dynamics 5, 78-95.

Zhou, Y., Wang, J., 2017. Critical link analysis for urban transportation systems. IEEE Transactions on Intelligent Transportation Systems 19, 402-415.

How cite this article

Shetab Boushehri, S.N., Poorzahedy, & H., Mahmoudi, R. (2024). Enhancing network performance in mitigating the impact of traffic accidents with full user information. *Journal of Transportation and Logistics, 9*(1), 1-19. https://doi.org/10.26650/JTL.2024.1365336

Appendix

Lemma A.1. Based on assumption 3, travelers are uniformly distributed over each link of a path ρ.

Proof. It can be claimed that $x_{ij} = \Sigma_{(k,s)\epsilon\rho}\Sigma_{\rho\epsilon\rho^{ks}}x_p^{ks}.\delta_{ij,\rho}^{ks}$, where $\delta_{ij,\rho}^{ks} = 1$ if $(i,j)\epsilon\rho\epsilon\rho^{ks}$, and 0 otherwise. Since $x^{ks}\rho$ is uniformly distributed over link (*i,j*) for all $\rho\epsilon\rho^{ks}$ and all $(k,s)\epsilon P$, such that $\sigma_{ij,\rho}^{k,s} = 1$. So, x_{ij} is, also, uniformly distributed, by flow additivity over link (*i,j*).

Lemma A.2. The number of trip-makers on a link (*i,j*) of path ρ is proportional to the travel time of that link of the path: $m_{ij} = x_{ij}.t_{ij}$. Furthermore, the number of trip-makers on a path ρ from k to s is proportional to the travel time of the path: $m_\rho^{ks} = x_\rho^{ks}.t_\rho^{ks}$.

Proof. Assuming constant flow speed v_{ij} in link (*i,j*) with length (l_{ij}) and flow x_{ij} , based on Lemma A.1, link (*i,j*) has a uniform passenger (trip) density d_{ij} over its length. Using the speed-density-flow relationship, , $x_{ij} = d_{ij}.v_{ij}, x_{ij}$ can be calculated by $x_{ij} = (m_{ij}/l_{ij}).\left(l_{ij}/t_{ij}\right)$, where $m_{ij} = \Sigma(k,s)\epsilon P^{m_{ij}^{ks}}$ is the number of passengers (trips) on link (i,j), which are uniformly distributed over l_{ij}. This results $m_{ij} = x_{ij}.t_{ij}$, which proves the first statement. Now, let m_{ij}^{ks},ρ be the number of passengers on link (*i,j*) on path ρ from k to s. Since the flow is additive, therefore $m_{ij}^{ks},\rho = \left(x_\rho^{ks}.t_{ij}\right).\delta_{ij,\rho}^{ks}$, where $\delta_{ij,\rho}^{ks} = 1$ if $(i,j)\epsilon\rho\epsilon\rho^{ks}$, and 0 otherwise. Hence, the number of passengers on path ρ from k to s may be computed as $m_\rho^{ks} = \Sigma_{(i,j)\epsilon\rho}x_\rho^{ks}.t_{ij}.\delta_{ij,\rho}^{ks} = x_\rho^{ks}\Sigma_{(i,j)\epsilon\rho}t_{ij}.\delta_{ij,\rho}^{ks} = x_\rho^{ks}.t_\rho^{ks}$, which proves the second statement.

Lemma A.3. can be determined as follows:

$$E_{ij}^s(c)\sum_{k\epsilon O}\lfloor m_{ij}^{ks}t^{ks} - m_{ij}^{ks}(t^{ks}/2)\bar{Z}_{ij}(jks)(c)\rfloor Z^{jks}(c) \tag{A.1}$$

Proof. Consider link (*i,j*) in Figure A1, where an accident has occurred at point e on this link.

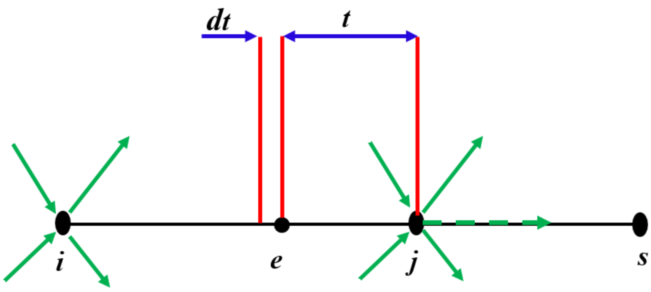

Figure 4. Accident occurrence in a link of network on the way to destination *s*

For a constant average speed in link (*i,j*) , according to Assumption 3, $\Sigma_{k\epsilon O}\left(m_{ij}^{ks}/t_{ij}\right)$dt number of travelers are traveling to destination s in a equivalent length of the link, forming $\Sigma_{k\epsilon O}\left(m_{ij}^{ks}/t_{ij}\right)t^{ks}.dt$ trip-hours (passenger-hours) of the network. By Assumption 1, for $\bar{Z}_{ij}$(c)=1 , there is a probability of t/t_{ij} and $1 - t/t_{ij}$ for the accident to take place in *e* to *j* and *i* to *e* portions of link *(i,j)*, respectively, where $0 \leq t \leq t_{ij}$. Thus, the probability of non-occurrence of an accident on *e* to *j* portion of link (*i,j*) is $1 - (t/t_{ij}).\bar{Z}_{ij}(c)$, and the mean suitable trip-hours of the passengers in link (*i,j*) , in state c of the network when an accident happens at point *e*, is equal to:

$$\Sigma_{k\epsilon O}(m_{ij}^{ks}/t_{ij})t^{ks}.dt.\left[1 - (t/t_{ij}).\bar{Z}_{ij}(c)\right].Z^{jks}(c) \tag{A.2}$$

$E_{ij}^s(c)$ can be obtained by integrating over t to consider all possible locations:

$$E_{ij}^{s}(c)=\int_{0}^{t_{ij}}\Sigma_{k\epsilon O}\left(m_{ij}^{ks}/t_{ij}\right)t^{ks}\left[1-\left(t/t_{ij}\right).\bar{Z}_{ij}(c)\right].Z^{jks}(c)dt \tag{A.3}$$

Lemma A4. To obtain PI the following equation can be used:

$$PI(c)=E(c)/E(c^{\circ})=\sum_{j\epsilon V}\sum_{s\epsilon D}\sum_{k\epsilon O}\left(\Phi^{jks}-\psi^{jks}(c)\right).Z^{jks}(c) \tag{A.4}$$

where:

$$\Phi^{jks}=\sum_{\rho\epsilon\rho^{ks}}\sum_{(i,j)\epsilon B(j)}x_{\rho}^{ks}.t_{ij}.\delta_{ij,\rho}^{ks}.t^{ks}/\sum_{(k,s)\epsilon P}d^{ks}.\left(t^{ks}\right)^{2} \tag{A.5}$$

$$\psi^{jks}(c)\sum_{\rho\epsilon\rho^{ks}}\sum_{(i,j)\epsilon B(j)}x_{\rho}^{ks}.t_{ij}.\delta_{ij,\rho}^{ks}.\left(t^{ks}/2\right)\bar{Z}_{ij}(c)/\sum_{(k,s)\epsilon P}d^{ks}.\left(t^{ks}\right)^{2} \tag{A.6}$$

Proof. Based on Lemma A2, $m_{rho}^{ks}=x_{\rho}^{ks}t_{\rho}^{ks}$ is the number of passengers on path ρ from k to s, at any point in time, and this quantity for link (i,j) due to path ρ from k to s is $m_{ij,rho}^{ks}=x_{\rho}^{ks}.t_{ij}.\delta_{ij,\rho}^{ks}.$. Then, the number of passengers on link (i,j) who are destined to s can be calculated as $m_{i}^{s}j=\Sigma_{k\epsilon O}\Sigma_{\rho\epsilon\rho}x_{\rho}^{ks}.t_{ij}.\delta_{ij,\rho}^{ks}$. According to Lemma A3 and substituting $m_{ij}^{ks}\Sigma_{\rho\epsilon\rho^{ks}}x_{\rho}^{ks}.t_{ij}.\delta_{ij,\rho}^{ks}$, the mean suitable trip-hours of the users in link (i,j) in state c, can be computed as follows,

$$E_{ij}^{s}(c)=\Sigma_{k\epsilon O}\Sigma_{\rho\epsilon\rho^{ks}}\left\{\left(x_{\rho}^{ks}.t_{ij}.\delta_{ij,\rho}^{ks}.t^{ks}\right)Z^{jks}(c)\left(x_{\rho}^{ks}.t_{ij}.\delta_{ij,\rho}^{ks}\right)\left(t^{ks}/2\right)\bar{Z}_{ij}(c)Z^{jks}(c)\right\} \tag{A.7}$$

Then, the mean suitable trip-hours to destination s that meet j as the first node on their paths after state c occurrence, E^{js}(c), can be calculated as:

$$E^{js}(c)=\Sigma_{(i,j)\epsilon B(j)}E_{ij}^{s}(c) \tag{A.8}$$

For the whole network, therefore, we have:

$$E(c)=\sum_{j\epsilon V}\sum_{s\epsilon D}E^{js}(c) \tag{A.9}$$

When the state of the network is c° and x is the user equilibrium flow, t^{ks} is the minimum travel time, $\forall(k,s)\epsilon P$. By Kuhn-Tucker optimality conditions for user equilibrium (Sheffi, 1985), all used paths for a given O-D pair have equal travel times, and this time is less than the time for an unused path. According to Lemma A2, $m_{\rho}^{ks}=x_{\rho}^{ks}.t_{\rho}^{ks}$ is the number of passengers on the path ρ from k to s . Each passenger spends t^{ks} units of time to reach s from k. This trip time is a *suitable* trip time. Hence, based on the above statements, the suitable trip-hours for the passenger traveling in path ρ from k to s in state c°, $E_{\rho}^{ks}(c^{\circ})$, is:

$$E_{\rho}^{ks}(c^{\circ})=x_{\rho}^{ks}.t_{\rho}^{ks}.t^{ks}=x_{\rho}^{ks}\left(t^{ks}\right)^{2} \tag{A.10}$$

The value of this measure for a specific O-D pair, , can be determined as:

$$E^{ks}(c^{\circ})=\sum_{\rho\epsilon\rho^{ks}}x_{\rho}^{ks}\left(t^{ks}\right)^{2}=\left(t^{ks}\right)^{2}\sum_{\rho\epsilon\rho^{ks}}x_{\rho}^{ks}=d^{ks}.\left(t^{ks}\right)^{2} \tag{A.11}$$

and for the whole network:

$$E(c^{\circ}) = \sum_{(k,s)\epsilon P} D^{ks}.\left(t^{ks}\right)^{2} \tag{A.12}$$

Eqs. (A.9) and (A.12) results:

$$PI(c) = E(c)/E(c^{\circ}) = \Sigma_{j\epsilon V}\Sigma_{S\epsilon D}E^{js}(c)/\Sigma_{(k,s)\epsilon P}d^{ks}.\left(t^{ks}\right)^{2} \tag{A.13}$$

Substituting eq. (A.7) in eq. (A.8), and substituting the results in eq. (13), would complete the proof. □

Lemma A5. The following expression is an approximate measure of link importance measure:

$$I_{mn} \approx \left(p(c^{\circ})/P_{mn}\right)\left\{\left[1 - PI\left(c^{\circ}_{mn}\right)\right] + \Sigma_{(u,v)\epsilon A}\left[\left(1 - P_{uv}\right)/P_{uv}\right]\left[PI\left(c^{\circ}_{uv}\right) - PI\left(c^{\circ}_{uv,-mn}\right)\right]\right\} \tag{A.14}$$

Proof. First, it should be noted that:

$$EPM = E[PI] = E[PI(m,n)survives].p_{mn} + E[PI(m,n)fails].(1 - P_{mn}) \tag{A.15}$$

Taking derivative of both sides of eq. (A.15) with respect to p_{mn} would result:

$$\partial(EPM)/\partial(p_{mn}) = E[PI(m,n)survives] - E[PI(m,n)fails] \tag{A.16}$$

If the probability of a link failure during a period of time, such as rush hour, is low, based on Assumption 2, the probability of concurrent traffic accidents in two or more links becomes very low. Thus, as an approximate computation of I_{mn} for a link *(m,n)* , the state of the network in which more than one link is subject to failure simultaneously is ignored.

$p(c^{\circ}), E[PI(m,n)survives]$ and $E[PI(m,n)fails]$ can be obtained as follows:

$$p(c^{\circ}) = \Pi_{(i,j)\epsilon A}P_{ij} \tag{A.17}$$

$$\begin{aligned}E[PI(m,n)survives] &= \frac{p(c^{\circ})}{p_{mn}}.PI(c^{\circ}) \\ &+ \sum_{\substack{(u,v)\epsilon A \\ (u,v)\neq(m,n)}} \frac{p(c^{\circ})(1-p_{uv})}{P_{mn}P_{uv}}.PI(c^{\circ})_{-uv}) + \sum_{\substack{(u,v),(g,r)\epsilon A \\ (u,v)\neq(m,n) \\ (g,r)\neq(m,n) \\ (u,v)\neq(g,r)}} \frac{p(c^{\circ})(1-P_{uv})(1-P_{gr})}{P_{mn}P_{uv}P_{gr}}.PI(c^{\circ}_{-uv,-gr}) + \dots\end{aligned} \tag{A.18}$$

$$\begin{aligned}E[PI(m,n)fails] &= \frac{P(c^{\circ})}{P_{mn}}.PI(c^{\circ}_{mn}) \\ &+ \sum_{\substack{(u,v)\epsilon A \\ (u,v)\neq(m,n)}} \frac{P(c^{\circ})(1-P_{uv})}{P_{mn}P_{uv}}.PI(c^{\circ}_{-mn,-uv}) + \sum_{\substack{(u,v),(g,r)\epsilon A \\ (u,v)\neq(m,n) \\ (g,r)\neq(m,n) \\ (u,v)\neq(g,r)}} \frac{P(c^{\circ})(1-P_{uv})(1-P_{gr})}{P_{mn}P_{uv}P_{gr}}.PI(c^{\circ}_{-mn,-uv,-gr}) + \dots\end{aligned} \tag{A.19}$$

Substituting eqs. (A.18) and (A.19) in eq. (A.16), noting that , results:

$$E[PI] = \frac{p(c^{\circ})}{p_{mn}}.\left[1 - PI(c^{\circ}_{mn})\right] + \sum_{\substack{(u,v)\epsilon A \\ (u,v) \neq (m,n)}} \frac{p(c^{\circ})(1-p_{uv})}{p_{mn}p_{uv}}.\left[PI(c^{\circ}_{uv}) - PI(c^{\circ}_{-mn,-uv})\right]$$

$$+ \sum_{\substack{(u,v),(g,r)\epsilon A \\ (u,v) \neq (m,n) \\ (g,r) \neq (m,n) \\ (u,v) \neq (g,r)}} \frac{p(c^{\circ})(1-p_{uv})(1-p_{gr})}{p_{mn}p_{uv}p_{gr}}.\left[PI(c^{\circ}_{-uv,-gr} - PI(c^{\circ}_{-mn,-uv,-gr})\right] + \dots \tag{A.20}$$

The probability of an accident occurrence in a link is low, therefore the probability of the concurrent occurrences of accidents in two links(

$$\sum_{\substack{(u,v),(g,r)\epsilon A \\ (u,v) \neq (g,r)}} \frac{p(c^{\circ})(1-p_{uv})(1-p_{gr})}{p_{uv}p_{gr}} \times [PI(c^{\circ}_{-uv,-gr} - PI(c^{\circ}_{-mn,uv,-gr})]$$

)becomes negligible. It should be noted that $[PI(c^{\circ}_{-uv,-gr} - PI(c^{\circ}_{-mn,uv,-gr})]$ is a very small value, especially in large scale real networks. Therefore, by a similar reasoning, it can be claimed that the third term and the higher-order terms, in eq. (A.20) are negligible. □