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Examination of Seventh Grade Students' Van Hiele Geometric Thinking Levels and Their Mistakes on 'Quadrilaterals'

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Abstract. In this study, it was aimed to examine seventh grade students' Van Hiele geometric thinking levels and their mistakes on 'Quadrilaterals'. The research was conducted with ten seventh grade students and the case study model, one of the qualitative research methods, was used. The Van Hiele geometric thinking levels test developed by Usiskin and the diagnostic test developed by Başısık were rearranged and administered to the students. According to the research findings, seventh grade students have misconceptions about diagonal, height concepts and special quadrilaterals. One of the most important of these is that the square positioned on one corner cannot be recognized by students at the analysis level, but is recognized by students at the visualization level. Students at the analysis and visualization levels have different misconceptions regarding the trapezoid. While there is a misconception that all side lengths must be different from each other as a condition for being a trapezoid, there is also a misconception that having four sides is sufficient. Another one is related to the concept of diagonal. There is a misconception that shapes with equal side lengths also have equal diagonal lengths. While no errors or misconceptions were encountered in students at the informal level, misconceptions were encountered in students at the analysis level, and more errors were encountered in students who were visualization and could not be assigned to any level.

Keywords. Van Hiele geometry levels, quadrilaterals, misconceptions and errors.

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Geometry is a branch of science that makes it easier for us to understand the world we live in and even the universe from different perspectives. Geometry helps individuals gain high-level thinking skills by enabling them to comprehend the relationships between shapes and objects (Türnüklü and Berkün, 2013, p.337-356). Students' ability to recognize the geometric shapes and objects they see around them, to distinguish the similarities and differences between shapes and objects, improves their logical thinking and improves their problem-solving skills in all areas of life. For this reason, teaching the geometry objectives included in the curriculum is of great importance.

It is accepted by almost all educators that the first three levels of Van Hiele geometric thinking levels of the achievements in the primary education curriculum should be acquired by seventh grade students (Teppo, 1991, pp. 210-221). Considering that Van Hiele's levels of geometric thinking progress hierarchically, teaching geometry requires great importance and care. Students should be guided by providing necessary guidance so that they can move up to the next level. Despite all the studies carried out, it is known that students have misconceptions while learning the subjects, which prevents them from understanding the subject (Ersoy, 2019; Özkan and Bal, 2018). A student with a misconception cannot reach the desired level in Van Hiele geometric thinking levels (Ersoy; 2019).

Students come to class with a certain amount of knowledge, and what is happening around them helps them make sense of the new topics they learn. However, this interpretation is not always complete. They may experience mental confusion during this interpretation. The formations that cause confusion in their minds are misconceptions (Baki, 2006). Ubuz (1999) defines misconception as the student's perception of a scientifically defined concept as different from its scientific definition, while error is defined as inaccuracies in the answers.

Van Hiele geometric thinking model is one of the models that describe individuals' perception and interpretation of shapes and objects geometrically. Thanks to this model, students' perception of geometric shapes and objects emerges (Duatepe Paksu , 2016). While developing this model, based on the difficulties experienced by students in geometry, the reasons were revealed and solution suggestions were offered. According to this model, learning geometry occurs at five levels. These levels are hierarchical within themselves; one cannot move on to the next without completing one level. In order to complete the levels, the necessary geometric experiences must be experienced. According to the Van Hiele model, since teaching without considering the students' levels makes it difficult for students to learn geometry, it is recommended that teachers take their students' levels into consideration when planning their lessons (Usiskin, 1982).

Van Hiele Geometric thinking model is called visualization level, analysis level, informal inference level, formal inference level and systematic thinking levels. At the Visualization Level, students can distinguish the desired shape among mixed shapes. When making this distinction, they benefit from their experiences, not definitions. (Van De Walle, 2013; cited in Ersoy, 2019). Considering quadrilaterals specifically, students at this level can choose square, rectangle, trapezoid, parallelogram and rhombus among the mixed shapes. At the level of analysis, students now become interested in the properties of shapes. They know the properties of shapes in detail. For example; “A square has four angles and four sides.” They can make definitions such as: At the informal inference level, associations between shapes begin. Definitions have become meaningful for students in this period, but logical inferences cannot be made in this period. The student in this period can also define a square as a rectangle. At the inference level, students can go beyond the relationship between shapes and create proofs and create theorems. At the level of systematic thinking, students can also perceive non-Euclidean geometry. They can easily work and prove different axiomatic systems (Usiskin, 1982).

The primary school mathematics curriculum objectives are examined, it is seen that square and rectangle are emphasized and their general properties are aimed to be taught. Considering the achievements, a student who completes primary school is expected to be at the level of analysis, which is the second level of Van Hiele's geometric thinking levels. When the secondary school achievements of the primary school mathematics curriculum are examined, it is seen that in the fifth grade, the general properties of quadrilaterals are given in detail and associations are made by emphasizing that the square is a special case of the rectangle. This association comes to the fore even more in seventh grade achievements (MoNE, 2018).

Quadrilaterals and polygons, which are basic concepts in geometry teaching, and Van Hiele Geometric thinking levels are examined, it is found that the subject of quadrilaterals and polygons is frequently discussed and misconceptions on this subject are revealed (Ersoy, 2019; Mutluoğlu and Erdoğan, 2020; Öksüz and Başışık, 2010; Özkan and Bal, 2018). In the study conducted by Öksüz and Başısık in 2010, it was concluded that the general properties of quadrilaterals could not be expressed correctly by fifth grade students. In addition, they observed that there were a limited number of students who succeeded in associating quadrilaterals. It was concluded that students tend to recognize geometric shapes in the first form they learned, but when the shapes are rotated, some students cannot recognize them. When Van Hiele was associated with geometric thinking levels, it was stated that most students remained at the visualization level. In their study, Mutluoğlu and

Erdoğan (2020) again discussed the subject of quadrilaterals and examined the geometric reasoning results of sixth grade students on this subject. According to the results of the research, it was observed that students with lower level reasoning power could not go beyond the models in which they were first taught geometric shapes, while students at higher levels benefited from concepts in geometric reasoning, but were still under the influence of the model they first learned. It was stated that students had difficulty in recognizing the images formed when the shapes were rotated. A study was conducted by Ersoy with seventh grade students in 2019 on the relationship between success in quadrilaterals and Van Hiele geometric thinking levels. The Van Hiele geometric thinking levels of the students in the study were lower than they should have been. It was concluded that there is a significant relationship between success in quadrilaterals and Van Hiele geometric thinking levels.

Quadrilaterals are a basic concept in teaching geometry and it can be said that it is one of the most important concepts for teaching geometry. According to the literature, it is seen that a quantitative study has been conducted examining the relationship between 7th grade students' achievements in quadrilaterals and Van Hiele geometric thinking levels with the relational screening model (Ersoy, 2019). According to this study, the students who participated in Van Hiele Although geometric thinking levels were lower than they should be, a moderate relationship was found between the scores obtained from both tests. In this study, which aims to examine this relationship qualitatively, the Van Hiele geometric thinking levels of seventh grade students and the mistakes made about quadrilaterals were examined.

Method

In this section, the type of research, selection of participants, data collection tools, data collection, process and data analysis are presented.

Research Model

Which aimed to examine the Van Hiele geometric thinking levels of 7th grade students and the mistakes made about quadrilaterals, the case study method, one of the qualitative research designs, was used. Case study is defined as a method in which the existing situation is revealed by examining one or more events in depth (Büyüköztürk, Kılıç Çakmak, Akgün, Karadeniz, Demirel; 2019, p.268). In this study, it was aimed to correlate students' misconceptions about quadrilaterals with their Van Hiele geometric thinking levels.

Study Group

The study group of the research consists of 10 seventh grade students studying in the Çal district of Denizli province. Clinical interviews are to be held with the students determined according to the results of the Van Hiele geometric thinking test and the diagnostic test on quadrilaterals. While forming the study group, students' first semester mathematics course grades were taken as basis. It is aimed to create a study group that includes students at all success levels. The information about the students' mathematics achievement and in-class participation is given in the table below.

In the table created according to the end-of-term report card grades, students whose grade point average is in the range of 0-50 are low achieving student, students whose grade point average is in the range of 51-65 are student at near-intermediate level, students who are in the range of 66-80 are mid-level successful student , students whose grade point average is in the range of 81-90 are successful student, students whose grade point average is in the range of 91-95 are high achieving student and students whose grade point average is in the range of 96-100 are high successful student.

Table 1.

Students' Success in Mathematics Course

Student	Math Grade	Mathematics Achievement
Ö1	100	High successful student (BİLSEM student)
Ö2	95	High achieving student
Ö3	95	High achieving student
Ö4	85	Successful student
Ö5	85	Successful student
Ö6	70	Mid-level successful student
Ö7	60	Student at near-intermediate level
Ö8	60	Student at near-intermediate level
Ö9	40	Low achieving student
Ö10	40	Low achieving student

Data Collection Tools

Van Hiele geometric thinking test was used to collect the data of the research and a diagnostic test was used to reveal misconceptions about quadrilaterals. Van Hiele Geometry test was developed by Usiskin in 1982 and adapted to Turkish by Duatepe in 2000 after validity and reliability tests were performed. In this test, there are five questions corresponding to each level, for a total of 25 questions. Since the research was limited to seventh grade students, the first fifteen of twenty-five questions were used. According to the Van Hiele Geometry test, a student who answers three out of every five questions correctly can move to the next level. Hierarchical order is important. For example, even if a student cannot reach the number of three correct numbers at the second level, but reaches the third level, he should be considered to be at the first level. He cannot move to the next level.

In order to detect misconceptions about quadrilaterals, the questions equivalent to the relevant achievements of the diagnostic test on the subject of quadrilaterals, developed by Başısık in 2010 by checking its validity and reliability, were arranged by the researcher and a 17- question diagnostic test was created by taking the opinions of field experts. This test consists of open-ended questions. For this reason, the verbal expressions that students give to the questions become very important.

Data Analysis

The Van Hiele test, students who answer at least three of the questions in the range of 1-5 correctly are at the visualization level, students who answer at least three of the questions in the range of 6-10 are at the analysis level, students who answer at least three of the questions in the range of 11-15 correctly are at the informal inference level. accepted at the level. Content analysis was used to analyze the diagnostic test. Open-ended questions were examined one by one and the obtained data were presented in tables. As a result of the association of Van Hiele and diagnostic tests, clinical interviews were conducted with students selected from the first three levels. The most important feature of the clinical interview is that it enables the researcher to communicate directly with the people to whom the research is applied (Ginsburg, cited in Karataş and Güven, 2003). In this way, a better understanding of misconceptions is achieved. The clinical interview was recorded, paying attention to the confidentiality of the individuals, and the results obtained were conveyed directly without changing the sentences using the descriptive analysis method.

Results

This section includes the findings of the research. First, the findings of the Van Hiele geometric thinking levels test are included. Afterwards, the students' answers to the diagnostic test were discussed in terms of their Van Hiele levels, and finally, the errors and misconceptions in the diagnostic test were mentioned.

Findings on Seventh Grade Students' Van Hiele Geometric Thinking Levels

In order to determine the students' Van Hiele thinking levels, the first 15 questions of the Van Hiele test were applied to the students and the results are shown in Table 2. is also presented.

Table 2.

Van Hiele Geometric Thinking Levels Test

Student	in the range of 1-5	in the range of 6-10	in the range of 11-15	Current Level
Ö1	5	5	5	Informal Inference
Ö2	3	4	2	Analysis
Ö3	4	4	2	Analysis
Ö4	5	3	1	Analysis
Ö5	3	5	0	Analysis
Ö6	2	1	0	Could Not Assign to Level
Ö7	2	1	1	Could Not Assign to Level
Ö8	2	1	1	Could Not Assign to Level
Ö9	3	1	2	Visualization
Ö10	3	1	0	Visualization

In the student names in Table 2, the letters G, A, İ were used to represent the levels they were in during the subsequent naming. The first letter is the first letter of the geometric level it is in. No letters were used for students at the zeroth level who could not be assigned to any level. Codes were given to the students using the letter G for the visualization level, A for the analysis level, and I for the informal inference level. When Table 2 is examined, students coded Ö2, Ö3, Ö4 and Ö5 were included in the analysis level, which is the level expected from seventh grade students. Ö1 participated at the informal level by answering all questions correctly. It can be said that 4 students could not reach any level and were at the pre-visualization level. Two students participated at the visualization level.

Diagnostic Test Visualization Level Findings

Among the questions in the diagnostic test, the relevant questions addressing the visualization level are analyzed one by one and presented in this section.

Table 3.

Distribution of Student Answers to the Second Question of the Diagnostic Test

Student / Question	İÖ1	AÖ2	AÖ3	AÖ4	AÖ5	Ö6	Ö7	Ö8	GÖ9	Ö10
Rectangle Recognizes	+	+	+	+	+	+	+	+	+	+
Square Recognizes	+	+	+	+	+	+	+	–	–	+
Parallelogram Recognizes	+	+	+	+	+	+	+	+	–	–
Recognizes Rhombus	+	+	+	–	–	+	–	+	–	–

When Table 3 is examined, it is seen that AÖ4 and AÖ5, who are at the analysis level, cannot recognize rhombus. In addition, Ö6, who is at the zeroth level, recognized all the shapes. Again, Ö7, who is at the zeroth level, could not recognize the rhombus but could recognize other shapes. Ö8 at the zeroth level could not recognize the square. It was observed that GÖ9 could only recognize rectangles, even though they were at the visualization level.

Table 4.

Distribution of Student Answers to the Third Question of the Diagnostic Test

Student / Question	İÖ1	AÖ2	AÖ3	AÖ4	AÖ5	Ö6	Ö7	Ö8	GÖ9	Ö10
Can Draw A Square On Dotted Paper	+	+	+	+	+	+	+	+	+	+
Can Draw a Rhombus on Dotted Paper	+	+	+	+	−	+	+	+	−	−

When we examined Table 4, it was seen that all students were able to draw a square on dotted paper. AÖ5 at the analysis level and GÖ9 at the visualization level could not draw the rhombus. In addition, while Ö10, who was at the zeroth level, could not draw, the other zeroth level students, Ö6, Ö7, and Ö8, were able to draw.

Table 5.

Distribution of Student Answers to the Fourth Question of the Diagnostic Test

Student / Question	İÖ1	AÖ2	AÖ3	AÖ4	AÖ5	Ö6	Ö7	Ö8	GÖ9	Ö10
Recognizes the Square	+	+	+	+	+	+	+	+	+	+
Recognizes Parallelogram	+	+	−	+	+	+	+	−	−	−
Recognizes the Square Positioned on its Corner	+	−	−	−	−	+	−	+	+	−
Recognizes Rhombus	+	+	−	−	−	−	−	+	−	−

According to Table 5, all students recognized the square given in the classical form in the fourth question, but while İÖ1, Ö6, Ö8 and Ö9 recognized the square positioned on its corner, the students at the analysis level could not. In this question, while the rhombus was recognized by İÖ1, AÖ2 and Ö8, it was not recognized by the other students.

Table 6.

Distribution of Student Answers to Questions Five, Six and Seven of the Diagnostic Test

Student / Question	İÖ1	AÖ2	AÖ3	AÖ4	AÖ5	Ö6	Ö7	Ö8	GÖ9	Ö10
Recognizes the Trapezoid in Question 5	+	+	+	—	—	—	+	—	—	—
6. Recognizes the Trapezoid Question	+	+	—	—	—	—	—	—	—	—
Recognizes the Trapezoid in Question 7 (Daily life example)	+	+	—	—	—	—	—	—	—	—

According to Table 6, while İÖ1, who is at the informal level, and AÖ2, who is at the analysis level, recognized the trapezoid in all three questions, the other students could not give sufficient answers on this subject.

Table 7.

Distribution of Student Answers to the Eleventh Question of the Diagnostic Test

Student / Question	İÖ1	AÖ2	AÖ3	AÖ4	AÖ5	Ö6	Ö7	Ö8	GÖ9	Ö10
Can Distinguish Parallelogram and Trapezoid.	+	+	+	+	+	—	—	—	—	—

According to Table 7., students at the informal level and analysis level were able to distinguish parallelograms and trapezoids, but other students could not distinguish them.

Table 8.

Distribution of Student Answers to the Thirteenth Question of the Diagnostic Test

Question \ Student	**İÖ1**	**AÖ2**	**AÖ3**	**AÖ4**	**AÖ5**	**Ö6**	**Ö7**	**Ö8**	**GÖ9**	**Ö10**
Recognizes Non-Rectangular Shapes	+	+	+	−	−	−	+	−	−	−
Recognizes a Trapezoid Positioned On One of its Corners	+	+	+	−	−	−	−	+	−	−
Recognizes The Trapezoid Given in Classical Form	+	+	+	−	−	−	−	−	−	−
Recognizes the Square	+	+	+	+	−	−	−	−	−	−
Recognizes a Square Positioned On One of its Corners	+	−	−	−	−	−	−	−	−	−
Recognizes Rhombus	+	+	+	−	−	−	−	−	−	−

According to Table 8., trapezoid and rhombus were recognized only by İÖ1, AÖ2 and AÖ3, but they were not recognized by the other students. The square positioned on one corner could only be recognized by İÖ1, who was at the informal level. Detection of non-rectangular shapes could only be made by İÖ1, AÖ2, AÖ3 and Ö7.

Table 9.

Distribution of Student Answers to the Fourteenth Question of the Diagnostic Test

Student / Question	İÖ1	AÖ2	AÖ3	AÖ4	AÖ5	Ö6	Ö7	Ö8	GÖ9	Ö10
Recognizes Rectangle	+	+	+	+	+	+	+	+	+	+
Recognizes A Parallelogram Positioned On One of its Corners	+	+	+	+	+	–	–	–	–	+
Recognizes a Rectangle positioned on a Corner	+	+	–	–	–	–	–	–	–	–
Recognizes Rhombus	+	–	–	–	–	+	+	+	+	–

According to Table 9, while only İÖ1, who was at the informal level, and AÖ2, who was at the analysis level, recognized the rectangle positioned on one corner, all students were able to recognize the rectangle given in classical form. While the students at the informal level and analysis level could recognize the rectangle positioned on one corner, the student at the visualization level could not. Among the students at the zeroth level, only Ö10 was able to recognize it. The rhombus in this question could not be recognized by the students at the analysis level and by Ö10.

Table 10.

Distribution of Student Answers to the Fifteenth Question of the Diagnostic Test

Student / Question	İÖ1	AÖ2	AÖ3	AÖ4	AÖ5	Ö6	Ö7	Ö8	GÖ9	Ö10
Can draw the Diagonals of Quadrilaterals.	+	+	+	+	–	+	–	–	–	–

According to Table 10, it was seen that the concept of diagonal in a quadrilateral was known by İÖ1, AÖ2, AÖ3, AÖ4 and Ö6, but was not known by the other students.

Findings of Diagnostic Test Analysis Level

Among the questions in the diagnostic test, the relevant questions addressing the analysis level are analyzed and presented in this section.

Table 11.

Distribution of Student Answers to the First Question of the Diagnostic Test

Question \ Student	İÖ1	AÖ2	AÖ3	AÖ4	AÖ5	Ö6	Ö7	Ö8	GÖ9	Ö10
Defines the Square	+	+	+	+	+	+	+	+	+	+
Defines the Rectangle	+	+	+	+	+	+	+	–	+	+
Defines Quadrilateral	+	+	+	–	–	–	–	–	–	–
Defines Rhombus	+	+	+	+	+	+	+	–	+	+
Defines Parallelogram	+	+	+	+	+	–	+	–	–	–
Defines the Trapezoid	+	+	+	–	–	–	–	–	–	–
It completes the Diagonal	+	+	+	–	–	–	–	–	–	–
Defines the Concept of Parallelism	+	+	+	–	–	–	–	–	–	–
Defines the Concept of Height	+	+	+	–	–	–	–	–	–	–

The first question requires being able to define the given concepts. When Table 11 is examined, all students were able to define Square. While Ö8 could not define rectangle and rhombus, all other students could. While the concepts of trapezoid, diagonal, parallelism, quadrilateral and height could be defined by İÖ1, AÖ2, AÖ3, they could not be defined by other students. While the definition of parallelogram could be defined by the students at the informal level and analysis level and by Ö7 at the zeroth level, it could not be defined by the other students.

Table 12.

Distribution of Student Answers to the Fifth and Sixth Questions of the Diagnostic Test

Student / Question	İÖ1	AÖ2	AÖ3	AÖ4	AÖ5	Ö6	Ö7	Ö8	GÖ9	Ö10
It may Explain why the Shape is not Crooked	+	+	+	–	–	–	–	–	–	–

İÖ1, AÖ2, AÖ3 and AÖ4 were able to explain together which of the shapes in the questions were not crooked and why. They mentioned that at least two edges should be parallel.

Table 13.

Distribution of Student Answers to the Eighth Question of the Diagnostic Test

Student / Question	İÖ1	AÖ2	AÖ3	AÖ4	AÖ5	Ö6	Ö7	Ö8	GÖ9	Ö10
Diagonal Lengths of Square are Equal	+	+	+	+	+	–	+	–	+	+
Diagonal Lengths of a Rectangle are Equal	+	+	+	–	–	+	+	–	–	+
Diagonal Lengths of a Rhombus Are Not Equal	+	+	+	–	–	–	–	–	–	–

All students except Ö6 and Ö7 concluded that the diagonal lengths of the square are equal. AÖ4, AÖ5, Ö8 and GÖ9 could not comprehend that the diagonal lengths of a rectangle are equal. The students who can deduce that the diagonal lengths of the rhombus are not equal are İÖ1, AÖ2 and AÖ3. Other students thought that the diagonal lengths in a rhombus were equal.

Table 14.

Distribution of Student Answers to the Ninth Question of the Diagnostic Test

Student / Question	İÖ1	AÖ2	AÖ3	AÖ4	AÖ5	Ö6	Ö7	Ö8	GÖ9	Ö10
Diagonal Lengths Determine Unequal Shapes	+	+	+	−	−	+	+	−	+	+

AÖ4, AÖ5 and AÖ8 could not correctly answer the question of identifying shapes with unequal diagonal lengths on the visualization. In this question, it can be assumed that other students know that the diagonal lengths of squares and rectangles are equal, and that they are aware that the diagonal lengths of parallelograms and trapezoids are not equal.

Table 15.

Distribution of Student Answers to the Tenth Question of the Diagnostic Test

Student / Question	İÖ1	AÖ2	AÖ3	AÖ4	AÖ5	Ö6	Ö7	Ö8	GÖ9	Ö10
Draws Height in Rectangle	+	+	+	−	−	+	+	−	+	−
Draws Elevation in Rhombus	+	+	+	−	−	+	−	−	+	−
Draws Elevation in a Parallelogram	+	+	+	−	−	+	−	−	+	−
Draws the Elevation of a Rhombus Positioned on one of its Vertices	+	−	−	−	−	−	−	−	−	−

The height of the rhombus positioned on one of its corners could only be shown by drawing İÖ1, while the other students could not. It was observed that AÖ4, AÖ5, Ö7, Ö8 and Ö10 did not fully know the concept of height.

Table 16.

Distribution of Student Answers to the Sixteenth Question of the Diagnostic Test

Student / Question	İÖ1	AÖ2	AÖ3	AÖ4	AÖ5	Ö6	Ö7	Ö8	GÖ9	Ö10
Determines Shapes with Equal Diagonal Lengths	+	+	+	+	–	+	+	–	–	–
Explains why	+	+	+	+	–	–	–	–	–	–

İÖ1, AÖ2, AÖ3, AÖ4, Ö6 and Ö7 were able to show shapes with equal diagonal lengths, but Ö6 and Ö7 could not explain the reason.

Diagnostic Test Informal Findings for the Inference Level

Informal questions included in the diagnostic test relevant questions addressing the inference level are analyzed and presented in this section.

Table 17.

Distribution of Student Answers to the Sixth Question of the Diagnostic Test

Student / Question	İÖ1	AÖ2	AÖ3	AÖ4	AÖ5	Ö6	Ö7	Ö8	GÖ9	Ö10
One can say that a Rhombus is also a Trapezoid.	+	–	–	–	–	–	–	–	–	–

Only İÖ1 could say that a rhombus is also a trapezoid.

Table 18.

Distribution of Student Answers to the Twelfth Question of the Diagnostic Test

Question \ Student	İÖ1	AÖ2	AÖ3	AÖ4	AÖ5	Ö6	Ö7	Ö8	GÖ9	Ö10
Difference between Rhomb Square	+	—	+	—	—	—	—	—	—	—
Distinction between Rectan Parallelogram	+	+	+	—	—	—	—	—	—	—
Diagonal Properties of a Trapezoid	+	+	+	+	+	—	—	—	—	—
Diagonal Properties in Parallelogram	+	+	+	—	—	—	—	—	—	—

Only İÖ1 and AÖ3 could distinguish between a rhombus and a square. İÖ1, AÖ2 and AÖ3 were able to deduce that the rectangle is also a parallelogram. İÖ1, AÖ2, AÖ3, AÖ4 and AÖ5 were able to say that the diagonal lengths of the trapezoid are not always equal. Only İÖ1, AÖ2 and AÖ3 could say that the diagonal lengths of the parallelogram are not equal.

Table 19.

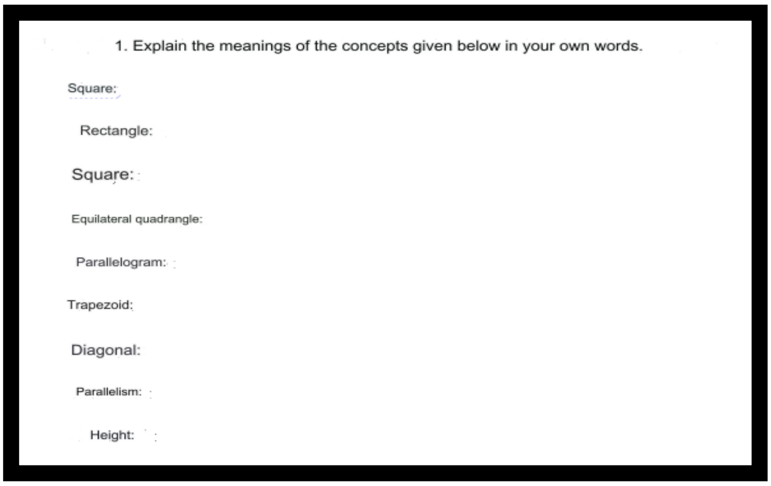
Distribution of Student Answers to the Seventeenth Question of the Diagnostic Test

Question \ Student	İÖ1	AÖ2	AÖ3	AÖ4	AÖ5	Ö6	Ö7	Ö8	GÖ9	Ö10
Knows that Rectangles and Squares are also Parallelograms.	+	+	+	—	+	—	—	—	—	—

İÖ1, AÖ2, AÖ3 and AÖ5 deduced that rectangles and squares are also parallelograms.

Findings from Clinical Interviews

The questions in the diagnostic test were examined, the questions with errors and misconceptions were evaluated, and clinical interviews were conducted with students determined at different levels.



1. Explain the meanings of the concepts given below in your own words.

Square:

Rectangle:

Square:

Equilateral quadrangle:

Parallelogram:

Trapezoid:

Diagonal:

Parallelism:

Height:

Figure 1. Question One of the Diagnostic Test.

When Figure 1 is examined, it is seen that the concepts of quadrilateral, trapezoid, diagonal and parallelism in the first question, which requires writing the definitions of the concepts, could not be answered by AÖ4 and AÖ5 at the analysis level. In the clinical interview, the first question was first asked again to the student coded AÖ4 and the following answers were received.

Researcher: For a quadrilateral, "All sides are equal." You used the expression.

AÖ4: Yes, I think so.

Researcher: Can you show the quadrilateral by drawing a figure?

AÖ4: Draws squares and rhombuses.

AÖ4 has the misconception that only shapes with four equal sides are quadrilaterals. This misconception also emerged in the thirteenth question. There again, he only expressed square and rhombus as quadrilaterals.

Researcher: For trapezoid, "None of its sides are equal." You said, can you show it by drawing?

AÖ4: Draw quadrilaterals with different side lengths.

Because of this misconception about the trapezoid, AÖ4 could not adequately answer all the trapezoid questions in the diagnostic test.

A clinical interview was also conducted with AÖ5, who left blank the concepts of diagonal, parallelism and height in the first question of the diagnostic test. As a result of the interview, it was answered that these concepts were not known to the student and therefore they were left blank. It was observed that AÖ5, who could not express height verbally, could draw the heights of the quadrilaterals in the tenth question.

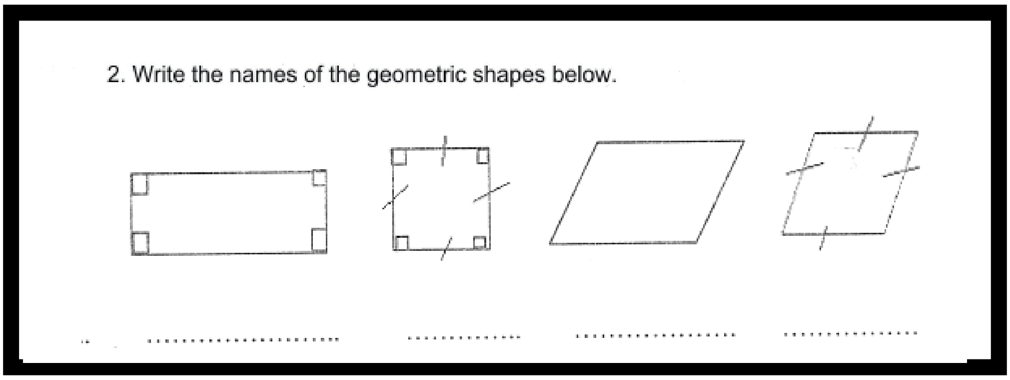


Figure 2. Second Question of the Diagnostic Test.

In the second question, it was tried to determine whether the shapes given in prototype form could be recognized by the students. AÖ4 used the expression parallelism for the figure in the fourth place, but when asked the question again in the clinical interview, he said rhombus.

Researcher: Why did you say parallelism for the fourth figure in the question?

AÖ4: When I saw the word "parallelism" in the first question, I thought it was the name of that shape.

AÖ4 made a mistake by expressing the rhombus as parallelism. AÖ5 also made the same mistake and called the figure in the fourth row a parallelism.

Researcher: Why did you say parallelism for the fourth figure in the question?

AÖ5: I couldn't find anything to say. It looked like a parallelogram, but it wasn't a parallelogram because all its sides were equal. When I saw the word parallelism in the first question, I thought it was that.

When the question was asked a second time in the clinical interview, both students corrected their mistakes and said rhombus.

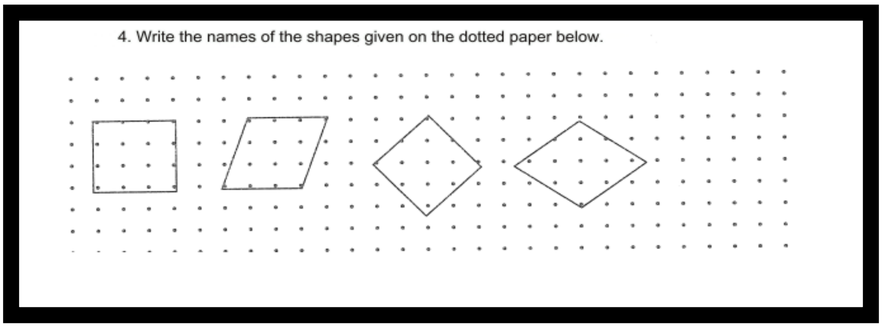


Figure 3. Fourth Question of the Diagnostic Test.

In the fourth question, it was aimed to determine whether they could recognize the square given in classical form and the square positioned on one of its corners, and whether they could distinguish between a parallelogram and a rhombus. Students at the analysis level AÖ2, AÖ3, AÖ4, AÖ5 could not recognize the square positioned on one corner. They made a misconception by calling it a rhombus. During the clinical interview, all four students admitted that the angles were ninety degrees, but they stated that they thought that way because their posture resembled a rhombus. There is a misconception that all squares have the form given in the first figure. It is seen that the students at the analysis level associate the shapes with their prototypes. There is a tendency to name shapes according to their stance. In addition, student Ö9, who was at the visualization level, and Ö6 and Ö8, who could not be assigned to any level, were able to recognize this shape.

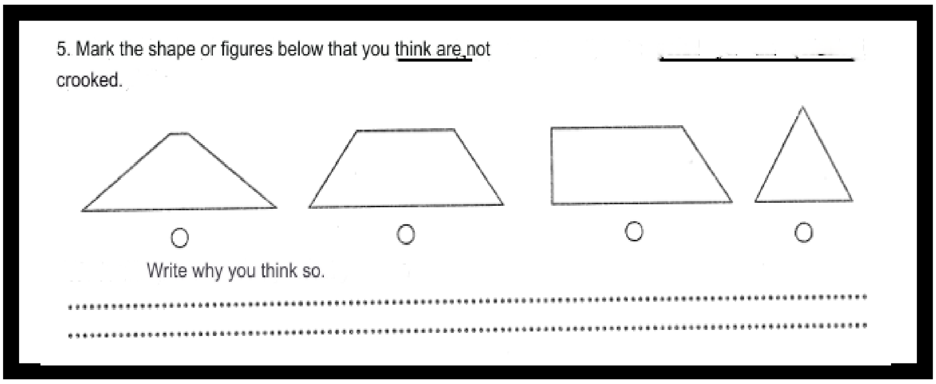


Figure 4. Fifth Question of the Diagnostic Test.

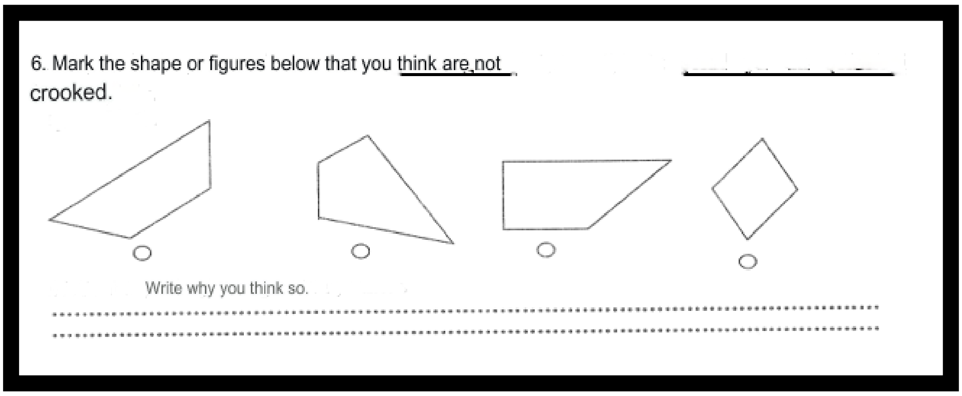


Figure 5. Question Six of the Diagnostic Test.

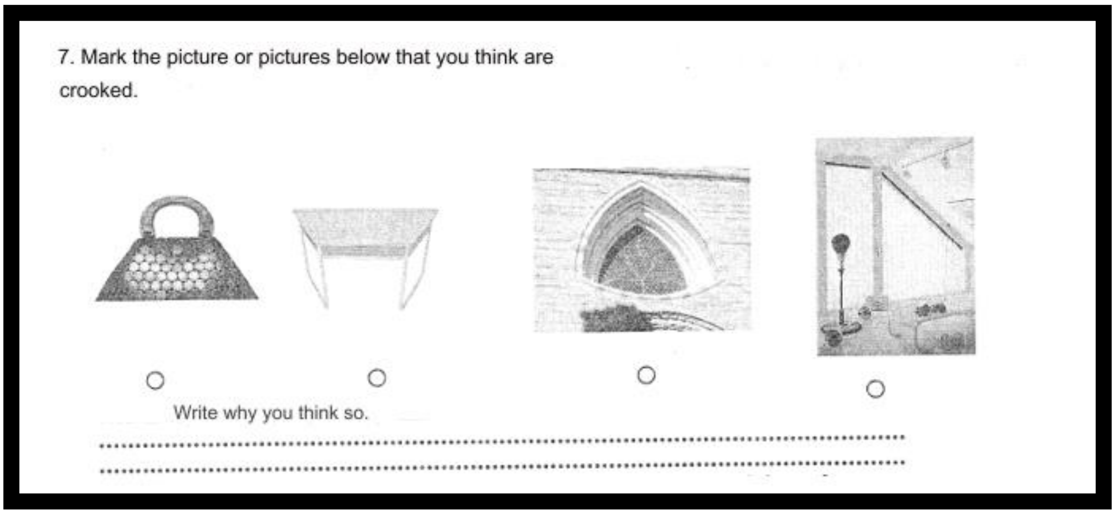


Figure 6. Seventh Question of the Diagnostic Test.

In the fifth and sixth questions, it was aimed to measure whether the trapezoid given in different shapes could be distinguished, and in the seventh question, it was aimed to measure whether the trapezoid could be recognized in the examples given from daily life. As a result of the clinical interview, it was determined that AÖ4 and AÖ5, who were at the analysis level, had misconceptions about the trapezoid. AÖ4 defined a trapezoid as 'a shape with no equal sides'. In AÖ4, there is a misconception that a trapezoid is a quadrilateral with all side lengths different from each other. He tends to describe any rhombus as a trapezoid. A similar misconception also exists in student AÖ5. He also deemed it sufficient to have four sides as a condition for being a trapezoid. He makes the mistake of describing any rhombus as a trapezoid.

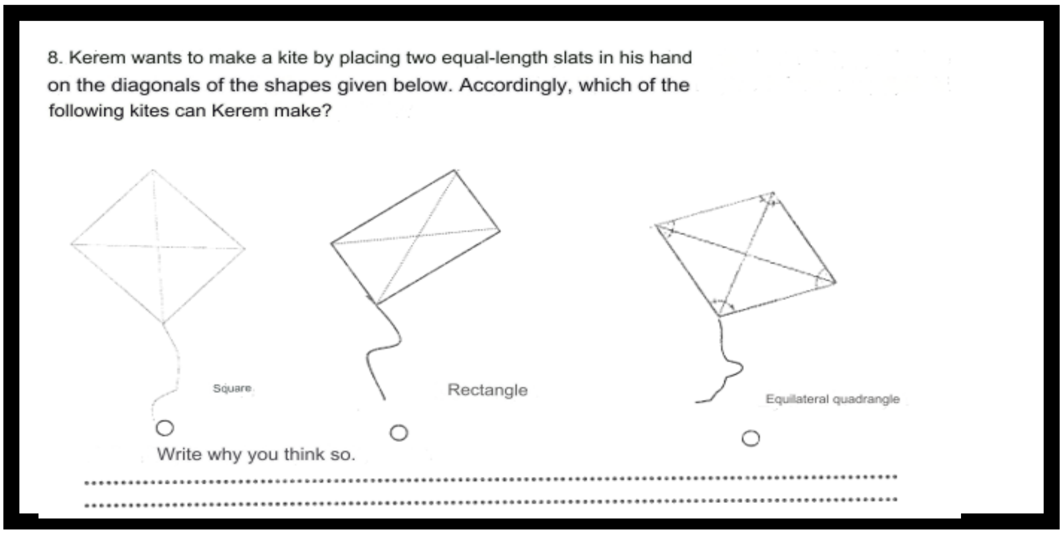


Figure 7. Eighth Question of the Diagnostic Test.

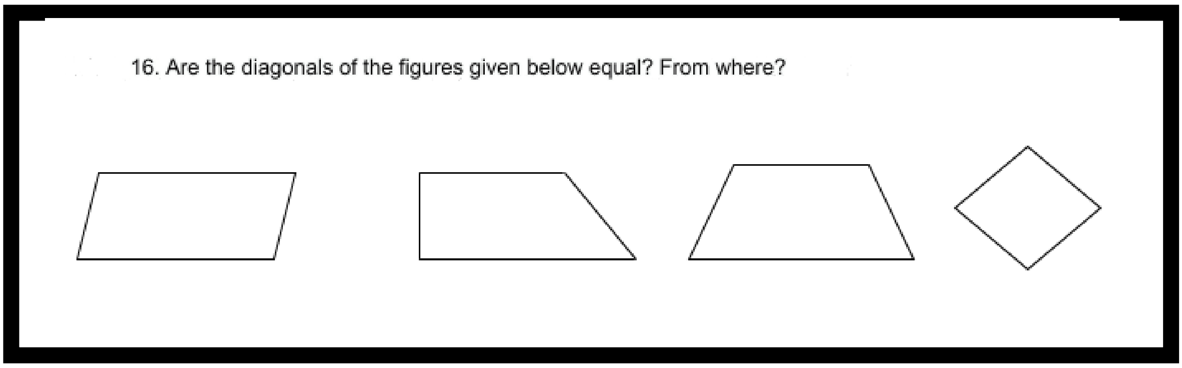


Figure 8. Sixteenth Question of the Diagnostic Test.

The eighth and sixteenth questions are about being able to express shapes with and without equal diagonal lengths. As a result of clinical interviews conducted on this question, different misconceptions emerged. AÖ5 “If the diagonals bisect each other, the diagonal lengths are equal.” He stated. AÖ4 said, "Diagonal lengths of shapes with equal side lengths are equal." has the misconception.

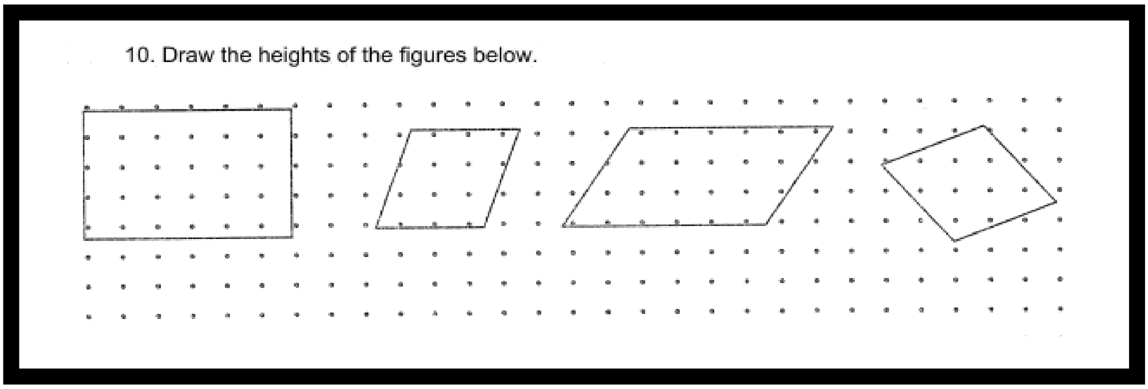


Figure 9. Tenth Question of the Diagnostic Test.

Rectangles and parallelograms had difficulty drawing the height of rhombuses and made incorrect drawings. They said that the reason for this was that they had not encountered height drawings in rhombuses before. They appear to be more familiar with drawing elevations in other quadrilaterals. AÖ3, who was at the analysis level, was able to make the correct drawing when the question was asked again in the clinical interview.

Researcher: You drew it correctly this time, why couldn't you draw it before?

AÖ3: It seemed different to me. Other shapes were easy, but that one was difficult.

Researcher: Why do you think you had difficulty?

AÖ3: I had drawn other shapes before, but the fourth shape looked different. But when you asked again, I was able to draw it perpendicularly from corner to edge and it worked.

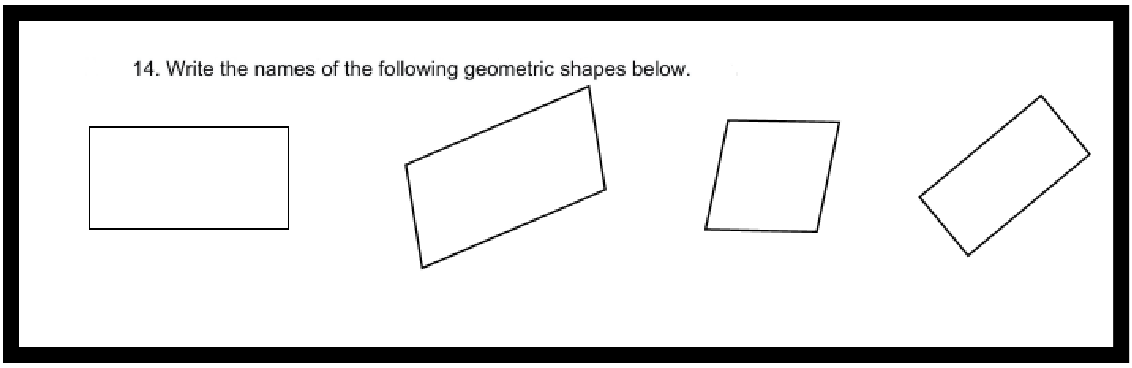


Figure 10. Fourteenth Question of the Diagnostic Test.

AÖ4 and AÖ5, who express the rectangle positioned on one of its corners as a parallelogram, have misconceptions in this regard. They stated that it looked like a rectangle, but because it looked oblique, they thought it was a parallelogram.

Discussion and Conclusion

In the study, which aimed to examine the Van Hiele geometric thinking levels of 7th grade students and their mistakes about quadrilaterals, as a result of the Van Hiele test, one out of ten students in the study group was found to be at the informal level. No errors or misconceptions were found in the diagnostic test applied to the student who was at the informal level. Four students were at the level of analysis. While the level expected from the seventh grade is to complete the analysis level, it was observed that the students at the analysis level had misconceptions about quadrilaterals. The incidence of misconceptions has increased in students who were assigned to the visualization level and who could not be assigned to any level. According to Van Hiele, they are not expected to use a scientific definition at the visualization level. At this level, they are not expected to know the properties of shapes, they are only expected to recognize shapes intuitively based on their experiences. Accordingly, depending on their visualization experience, mistakes and misconceptions about quadrilaterals can be considered natural. The fact that these ten students, who are at the same age and at the same grade level, are assigned to different levels, or even students who cannot be assigned to the level, reveals how important the experiences of the students in their daily lives outside of formal education are, regardless of the curriculum. Of course, it should not be forgotten that these students have different mathematical achievements. The student coded Ö6, who was averagely successful according to his mathematics grade, could not be assigned to any level. Accordingly, it can be said that mathematics achievement and Van Hiele geometric thinking levels do not show parallelism in the study group of this research.

Which are given as classical forms of quadrilaterals positioned on an edge, are generally recognized by students, misconceptions and errors occur regarding shapes with new appearances created by rotating them at certain angles.

If we consider the quadrilaterals one by one, it has been observed that the square given in classical form is recognized by the students, while the square positioned on one of its corners is referred to as a rhombus by the students at the analysis level. In this sense, it is similar to the study conducted by Başısık in 2010. Başısık encountered a similar result in his research to determine fifth graders' misconceptions about quadrilaterals.

While the rhombus given in its classical form, that is, positioned on its edge, was recognized by the students, it was observed that they confused the rhombus positioned on its edge with the parallelogram. Kemankaşlı and Gür encountered a similar situation in their study in 2005, and Başısık in their study in 2010.

Four different side lengths". In addition, in the question where the naming of different quadrilaterals is measured, the quadrilateral that is not a special quadrilateral and does not have parallel sides within itself was also named as a trapezoid by the students at the analysis level. It is not thought that the two sides of a trapezoid must be parallel and this causes misconceptions about the trapezoid.

At the analysis level, there are students who have the misconception that the diagonal lengths of shapes with equal side lengths are also equal. In this case, they have the misconception that the diagonal lengths of the square and rhombus are equal, but the diagonal lengths of the rectangle, parallelogram and rhombus are not equal. Some students who are at a visualization level made mistakes by confusing the concepts of corner and diagonal.

It is thought that the reason why they make mistakes when drawing diagonals in a rhombus is that the height of the rhombus is not included much in classroom activities.

Recommendations

- Van Hiele geometry levels test can be applied to students before teaching the subject of quadrilaterals. Lessons can be planned in line with the results.
- Before explaining the topic, a readiness diagnostic test should be applied to identify any misconceptions about the topic and eliminate them.
- After the subject is explained, a diagnostic test should be applied and different teaching methods should be applied in order to eliminate errors and misconceptions caused by incomplete learning, if any.
- The hierarchy of rectangles should be emphasized and plenty of activities should be done on this subject.
- While geometry teaching, the focus should not only be on prototype shapes, but also shapes with different stance positions should be frequently included in examples.

- A new study can be designed and research can be conducted to eliminate the identified misconceptions.
- The research can also be conducted to determine the relationship between misconceptions in other geometry subjects and Van Hiele levels.

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Conflict of Interest

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Ethical Standards

We have carried out the research within the framework of the Helsinki Declaration.

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