

## Problem Solving Skill in Primary Mathematics Curricula Documents of the Republican Period

*Cumhuriyet Dönemi İlkokul Matematik Dersi Öğretim Programlarında Problem Çözme Becerisi*

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### *Abstract*

The aim of this study is to examine problem solving skill attended to in primary mathematics curricula put into practice during Turkish Republican period. The study employs a document analysis method and the official curricula documents constitute the data. The documents are examined via content analysis technique. There emerge four main themes of mathematical problem solving through the analysis: (1) the notion of mathematical problem, (2) prescribed problem solving steps, (3) instructional arrangements for teaching-learning process and (4) role definitions regarding problem solving. These themes are comparatively analysed along nine different primary mathematics curricula that have been put into effect since 1926. The findings shed some light on the conceptualisation of mathematical problem and problem solving as well as approaches adopted to enhance students' problem solving skills and the expected duties of teachers. The findings are discussed in a historical context and suggestions are made for future program development efforts.

*Keywords:* heuristics, historical analysis, mathematics curriculum, mathematical problem solving

### *Özet*

Bu çalışma kapsamında, Cumhuriyet tarihi boyunca uygulamaya konulan ilkökul matematik dersi öğretim programlarının, problem çözme becerisine dönük düzenlemeler bağlamında karşılaştırmalı olarak incelenmesi amaçlanmıştır. Araştırmada doküman incelemesi yöntemi kullanılmıştır. Çalışmanın verilerini ise Cumhuriyet döneminde uygulamaya konulan öğretim programlarının yazılı metinleri oluşturmuştur. Dokümanlar içerik analizine tabi tutularak incelenmiştir. Analizler sonucunda problem çözme konusunda dört ana tema belirlenmiştir: (1) matematiksel problem kavramı, (2) önerilen problem çözme adımları, (3) öğrenme-öğretme sürecine ilişkin eğitsel düzenlemeler ve (4) problem çözmeye ilişkin rol tanımlamaları. Cumhuriyet tarihi boyunca uygulamaya konulan dokuz öğretim programının bu temalara dayalı olarak karşılaştırmalı analizleri gerçekleştirilmiştir. Elde edilen sonuçlar matematiksel problemin ve problem çözme kavramının ele alınış şekli, öğrencilerin problem çözme becerilerinin gelişimi için benimsenen yaklaşımlar ile öğretmenlere bu süreçte tayin edilen roller konusunda aydınlatıcı bilgiler sunmaktadır. Bulgular tarihsel bağlamda karşılaştırmalı olarak irdelenmekte ve gelecekte yapılacak program geliştirme çalışmalarına ilişkin bir takım önerilerde bulunmaktadır.

*Anahtar kelimeler:* matematik öğretim programı, matematiksel problem çözme, problem çözme adımları, tarihsel inceleme

## **Introduction**

A problem might be defined as a situation given quantitatively or in different patterns without any apparent way or method to reach a solution (Holth, 2008). As this definition suggests memorized rules or known methods would not be sufficient to produce a solution (Hiebert et al., 1997); rather problem solving requires students to synthesize prior knowledge and use reasoning processes to produce a solution (Krulik & Rudnick, 1987). In other words, previously learned knowledge, skills, and understandings are synthesized into a new or different situation during the problem solving process.

Problem solving is a complicated activity and involves higher order thinking skills; it, therefore, has a significant place in mathematics education. When arranged properly, problem solving activities cultivate positive attitudes in students for mathematics in general and problem solving in particular (Krulik & Rudnic, 2000). Encountering with real life problems in the teaching learning process helps students to understand the significance of mathematics (Kennedy et al., 2008). Problems with real life contexts have also potential to form a positive atmosphere in the classrooms and also help students develop problem solving skills required in real world (Krulik & Rudnic, 1987). Furthermore, it serves to achieve a holistic understanding by establishing internal connections between distinct and seemingly disconnected topics and ideas in mathematics; and it motivates students towards learning mathematics as it is more exciting, challenging and interesting than exercises (Wilson et al., 1993). As briefly explained above, problem solving serves to many purposes in mathematics education and therefore has long been one of the main research areas in the field.

Studies in the area of problem solving focus their attention on cognitive, affective and social components that influence the development of students' problem solving proficiency (Santos-Trigo, 2007). Problem solving as a practice domain is studied through several important issues such as design and implementation of curriculum documents, course syllabi, teaching materials and activities. Students' performance in routine and non routine problems, their problem solving approaches and the development of mathematical thinking facilitated by various kinds of problems have also been among the research topics in the field (Santos-Trigo, 2014). A significant proportion of research attention has focused on the understanding of mathematicians' problem solving approaches. With such studies, researchers aim to understand the thinking skills of experts, which were then used to create certain instructional approaches of problem solving process by dividing it into stages or steps known as heuristics. Teaching problem solving skill in mathematics education is often related to the heuristics that Polya (1945) proposed: understanding the problem, developing a solution plan, implementing the plan and evaluating the solution. Research conducted on the heuristics has generally offered certain generic means and methods (usually shaped around Polya's problem solving steps) that can be used to tackle (especially non-routine) problems (see, for examples, Maccini & Hughes, 2000; Burton, 1984; Bransford & Stein, 1984).

Mathematical problem solving has gained an impact beyond research undertakings and led to influential reform movements emerged in different countries. In the USA, for instance, the Back to Basic wave that emerged as a reaction to the New Math movement of the 1960s evolved into a process centring on problem solving with the contribution of NCTM. In this regard, NCTM (1980, pp. 2-4) advised that "Problem solving must be the focus of school mathematics in the 1980s". The NCTM's emphasis more clearly reflects itself on a curriculum framework structured around a problem solving proposed in 1989 as Curriculum and Evaluation Standards. In the UK, the well-known Cockcroft Inquiry also stated that "mathematics for all" could be achieved via problem solving and investigational work which were hence recommended to shape the curricular materials. In the Netherlands, problem solving has an important place in the theory of Realistic Mathematics which has had a great influence on the curricular work of such countries as Germany, Denmark, Portugal, South Africa and Brazil (de Lange, 1996). The reform movement of the 1950s' in Soviet Russia, known as Khrushchev's education reform and continued by Kolmogorov starting from the 1970s, paid more attention to mathematical deductions using formal and rigorous mathematical language. Three critical properties of school mathematics of this movement have come to the forefront: rigorousness, abstractness, and application. Problem solving and discussions about what kind of mathematics students must learn have been at the core of the movement (Karp & Vogeli, 2010). In their study, Cai & Nie (2007) conducted a historical analysis about the place of problem solving in mathematics education in China. They point out that the role of problem solving in mathematics education in China was strongly affected by the Soviet approach adopted in the 1950s. The authors also state that Chinese problem solving approach was shaped on the basis of experience and practice rather than cognitive elements and that the problem solving emerging with such an approach was considered as a fundamental goal, according to which the curricula were arranged.

In Turkish context, mathematical problem solving has been considered as an important skill that curricula documents at every level aim to develop. However, we have a very limited knowledge base as to how and in what ways mathematical problem solving give directions to the curriculum innovation efforts in Turkey. There is scant research regarding how different curricula documents conceptualise mathematical problem and problem solving; what approaches adopted to enhance learners' problem solving skills and performance; what kinds of instructional arrangements prescribed; and what changes were experienced in these issues historically during the Turkish Republican period. To the best of our knowledge, only did one study examine problem solving skill in the curricula from a historical perspective (Dinç-Artut & Tarım, 2016). This study limits itself to the consideration of the quality of the problems, problem solving studies and homeworks.

With the present study, we aim to fill the aforementioned gap by examining problem solving skill in primary mathematics curricula within a historical context during the Republican period. This research, we believe, is of particular importance as it provides information, through a historical comparison of the documents, about the effects of curricula changes on mathematical problem solving. There is a specific reason

for examining the curricula at the primary school level. Young learners are encountered with formal school mathematics for the first time during this period. This period has a critical effect on the attitudes that students develop with regard to mathematics and on their understanding of what mathematics is all about. The approaches regarding the development of the problem solving skill of this age group in the history of the Republic may provide significant information about our ideas and practices of mathematical cultivation of children at early ages.

With this aim in mind, the paper is structured as follows. Firstly, brief background information on the main curricula changes taking place in the history of the Republic will be presented. Then the research method and the analysis process will be delineated in detail. The paper ends with a discussion of the issues emerging from the analysis along with several suggestions for future program development efforts.

### **Mathematics curricula developed in the history of the Republic**

Mathematics curricula have changed 10 times during the history of the Republic, and hence 10 different curricula have been implemented to date. The years in which the curricula were developed are 1924, 1926, 1936, 1948, 1968, 1983, 1990, 1998, 2005 and 2015 (Ergün et al., 2015). Immediately after declaration of the Republic, the 1924 curriculum was put into effect. It was a kind of transitory curriculum which was soon revised in 1926. The new curriculum was often criticized due to the disconnections encountered in the transitions, particularly between the grades. The curriculum was in use for 10 years and then, in 1936, it was renewed. The 1936 curriculum was also criticized on the grounds that it did not reflect the scientific understanding for curriculum development of the time. 12 years later, in 1948, a new curriculum was issued with a heavy content load and with a particular concern for cognitive development of students. The 1948 curriculum was probably the most harshly criticised yet longest used one. It has been in use for about 20 years until 1968. In that year, education system witnessed the first fruits of scientific curriculum development in the history of the Republic (Gözütok, 2003) with a number of novelties such as preparation and planning in teaching units and topics, emphasis on research and inquiry aspects of learning, responsibilities for students to become self regulated learners, argumentation as a classroom practice, and introduction of the assessment and evaluation concepts as an integral part of the curricula.

It is seen that the curriculum development endeavours for all school subjects progressed jointly until 1968. Afterwards, curriculum of the each school subject was separately developed. The 1968 program was revised in 1983 and implemented starting from the 1985-1986 academic year (Demirel, 1999). The 1983 curriculum targeted at facilitating the measurement of the objectives, which were hence expressed in terms of behaviour statements.

Curricula development efforts during the 1990s have taken primary education as a whole of eight-year schooling and developed the curricula accordingly. The 1990 curriculum viewed primary education as an eight-year continuum, and the distribution of topics and units was arranged based on this continuum. In 1997, compulsory education was framed as an eight-year continuous education, and thus a new curriculum encompassing the eight year education as a whole was developed. The 1990 and 1998

curricula had great similarities with the 1983 curriculum in terms of the contents and approaches. The curricula developed in the 1990s have been the target of the criticisms. The critiques found these programs wanting with regard to the reflection of the latest scientific developments on both content and instructional approaches (Karakaya, 2004). Taking these criticisms into consideration, new curriculum development studies gained impetus at the beginning of the 2000s. A new curriculum was developed in 2004, piloted for one year and then put into effect nationwide in 2005. This curriculum was terminated with the implementation of the last curriculum prepared in 2015 in accordance with the fragmented basic compulsory education organized as 4+4+4. The 2015 curriculum features as the first and single curriculum in which primary education was framed as a 4-year period (historically it has been a 5-year period) since the declaration of the Republic.

### **Method**

This is a qualitative study and employs document analysis method. Document analysis refers to a systematic research process which includes evaluating and interpreting printed and electronic materials. Written data in the form of books, diaries, programs, and documents about an organization or an event are examined in depth in line with the research problems; and then meanings are revealed, understandings are developed and views are explored through document analysis method. Furthermore, analysing documents comparatively helps to construct meanings about change and development (Bowen, 2009).

The data for this study are composed of the 1926, 1936, 1948, 1968, 1983, 1990, 1998, 2005 and 2015 primary mathematics curricula documents. During analysis, particular attention is paid to the general explanations of the written documents. The dataset of the research was examined through content analysis technique. Open, axial and selective coding processes were followed in the content analysis (Neuman, 1991). Open coding is the first kind of coding employed on the raw data; axial coding is the phase in which the codes are arranged, connections are made and main analytical categories are explored. Selective coding is the last phase in which the data supporting the categories are determined and previous codes are examined for selection (Neuman, *ibid.*).

The data about each curriculum were analyzed line by line in accordance with the purpose of the study in the analysis process. The codes about problem solving were constructed depending on the meanings emerged directly or indirectly. The codes serving similar tasks and purposes in the problem solving process were grouped under the categories of problem solving. As a result of this process, the codes regarding problem solving were combined under 14 categories. The categories regarding problem solving were constructed by reviewing the related literature and their features. As a result of analysis, there emerged four main themes: (1) the notion of mathematical problem, (2) prescribed problem solving steps, (3) instructional arrangements for teaching-learning process of problem solving and (4) role definitions regarding problem solving. The findings about the themes, categories, and codes concerning problem solving skill were presented comparatively by evaluating primary mathematics curricula put into use since 1926. In what follows, we present codes and categories under

each theme with the exemplary citations from the curricula documents. Table 1 below presents codes and categories of the first theme.

**Table 1. Descriptions of Codes and Categories for the Notion of Problem**

Cat.	Codes	Exemplary Citations from the Curricula Documents
Problem Definition	Undefined	No definitions were provided in the curriculum document
	Situations requiring the use of four operations	Problems are the situations which necessitate students to use four operations (1990, p.27; 1998, p. 13) ...These situations must be new for students. In other words, the situations presented as problems must certainly be different from the ones solved in the classroom and present in the textbooks (1998, p.27)
	Novel situations	In order for a mathematical situation become a problem, solution must not be apparent and require students to use their reasoning skills and prior knowledge (2005, p.11)
	Situations without obvious solutions and require reasoning skills	
The Characteristics of the Problem	Daily life problem situations	The teacher must always take into consideration children's daily experiences in the problems...(1990, p.27)
	Motivating students	Problems must be vivid and attractive so that children feel the urge to solve (1968, p.16)
	Alignment with prior learning	...problems must be selected such that students can solve it with their prior learning... (1936, p.159)
	Alignment with student developmental levels	The teacher must consider students' intelligence, ability and development levels while assigning homework to be done outside the classroom (1948, p. 182)
	Inclusion of narration	Problems must be long and detailed in such a way that narrates a situation and helps children acquire some information at the same time...(1948, p.183)
Problem types	Clarity	Problems must be clearly stated and help students obtain some information meanwhile...(1998, p.14)
	Routine problems	Problems which can be solved through four operations and have a single correct answer (2015, p.6)
	Non routine problems	Problems which do not have only one solution and the answers change depending on the person and situation (2015, p.6)

Content analysis of the documents yielded three main categories under the theme “notion of problem”: problem definitions, characteristics of the problem and problem types. The citations from the documents exemplify the emerging codes.

**Table 2. Descriptions of Codes and Categories for Problem Solving Steps or Heuristics**

Cat.	Codes	Exemplary Citations from the Curricula Documents
Understanding the Problem	Expressing the problem in their own words	Before starting to carry out calculations, the teacher must have students re state the problem in their own words and must have a conviction that they understand the problem (1968, p.16)
	Determining what is given and what is required	Students must be able to differentiate between the things that are known and that need to be found in problems (1968, p.17)
	Expressing the problem with figures, drawings and symbols	Importance must be given to the restatement of the problems to be solved with symbols, figures and drawings (1948, p.183)
	Summarizing the problem	Writing the problem in summary (1998, p.14)
	Emphasizing directly the significance of understanding the problem	Students must conceive...the significance of understanding the problem (2005, p.11)
Developing a plan	Dividing the problem into stages	Children must be able to divide a problem into stages and anticipate what is done at every stage and what more will be done in other stages (1983, p.19)
	Anticipating what can be done at each stage	
	Anticipating which operations need to be done at each stage	The teacher...must engage students in finding out what operations they should carry out in every stage to solve the problem... (1968, p.17)
	Anticipating the result of the problem	The teacher must have children to anticipate the result of the problem ...(1948, p.183)
	Guiding students to devise a way for the solution	The teacher must have students to find out what paths/ways they need to pursue for the solution (1968, p.17)
	Emphasizing the significant points to be carried out for the solution	It must be assured that students know the significance of understanding the problem, making plans, checking the result and using different strategies... (2005, p.11)
	Stating and writing the operation(s) to be used in problem solving with the reasons	Stating and writing what operation or operations must be applied with their reasons in problem solving (1998, p.14)
	Emphasizing directly the significance of planning for problem solving	It must be ensured that students must grasp the significance of understanding the problem, making plans, checking them and using different strategies (2005, p.11)

Implementing the plan	Executing the operational steps correctly	The teacher... must have students to find out what operations they need to apply in separate stages in problem solving process and to carry out these operations correctly (1968, p.17)
	Stating and writing the result upon the solution	Executing the solutions process and then stating and writing the result (1998, p.14)
	Selecting and implementing problem solving strategies	Selecting and implementing problem solving strategies (2005, p.12)
Controlling	Checking the accuracy of the result	The teacher must have students to anticipate the result of the problem approximately and to check whether the result is correct or not on their own (1968, p.17)
	Checking the suitability and plausibility of the solutions for the problems	Checking the suitability and plausibility of the solutions for the problems...
	Explaining whether the result was correct or not with reasons	Determining the reason behind whether the solution of the problem is correct or not; and if not, then stating the incorrect aspects (1998, p.14)
	Drawing attention to the significance of controlling the result	...having students to understand...the significance of checking the solution (2015, p.6)
Error analysis	Analysing the incorrect aspects by benefiting from a correct solution	...before having students to solve the problem on the board, the teacher must engage students in solving the problem on their own and help them see the incorrect aspects on the board afterwards (1983, p.20)
	Determining inaccuracies by whole class evaluation	Mistakes must be thoroughly examined; the teacher and students must work on them as necessary and give importance to the thinking process followed to reach the solution (1983, p.21)
Problem solving strategies	No mention of the problem solving strategies	Problem solving strategies are not included either directly or indirectly.
	Indirect emphasis on the problem solving strategies	Writing the problem in summary (1998, p.14) Anticipating the result of the problem, stating and writing it (1998, p.14)...
	Direct emphasis on the problem solving strategies	It must be ensured that students must grasp the significance of understanding the problem, making plans, checking them and using different strategies (2005, p.11)

Table 2 presents codes and categories for the theme of problem solving steps. Content analysis under this theme leads to determination of six categories: understanding the problem, developing a plan, implementing the plan, controlling, error analysis and problem solving strategies. Each code under the categories is exemplified with a brief citation from the documents on the right hand side of the table.



**Table 3. Descriptions of Codes and Categories for Instructional Arrangements**

Cat.	Codes	Exemplary Citations from the Curricula Documents
Problem solving arrangement	Supporting students to find out the solution on their own	The teacher must have children to find out what ways to follow in problem solving and to think on this ... (1948, p.181)
	Helping students when necessary and only as much as needed	The teacher must have students to solve problems on their own as far as possible and interfere with them when needed (1990, p.27)
	Deciding on the solution method with whole class discussion	Discussing how calculation operations will be performed must be paid attention to (1926, p.48)
	Choosing the easiest/shortest way while deciding on the solution	...The teacher must make a comparison between these ways and have students to choose the easiest, truest and shortest way (1968, p.17)
	Getting mental calculations performed to anticipate the result	Mental calculation skill has a significant place in anticipating the result in problem solving (1990, p.27)
	Focusing more on the thinking process than the result	... attention must be paid to the thinking process rather than the result (1948, p.182)
	Supporting the development of original solutions	... the teacher must appreciate the original ways that students find out on their own (1948, p.181; 1968, p.17)
	Evaluating different ways of solution in problem solving	...different solutions must be evaluated (1990, p.27)
	Using think aloud technique	Thinking aloud on the problems in the classroom and stating this both facilitate problem solving for students and enable teachers to detect students' thinking styles and the difficulties they encounter (1948, p.183)
Activity based arrangements	Assigning research and inquiry homework regarding the problems	...teachers can give students homework which require research and inquiry out of the classroom (1968, p.18)
	Assigning homework regarding the problems in non extensive amounts for home	It must be considered that exercises and problems given to students outside the classroom are not a lot (1990, p.27; 1983, p.20; 1968, p.18)
	Assigning homework regarding the problems in non extensive amounts and within student capacity	It must be paid attention to that the problems assigned to second term children are not a lot and do not excess their capacity (1948, p.182)
	Presenting problems in order from easy to hard ones in the teaching learning process	Problems must be given in an arrangement starting from easy ones to hard ones (1968, p.17)

Table 3 presents categories and codes under the theme of instructional arrangements. The analysis suggests two main categories with regard to this theme: problem solving arrangements and activity based arrangements. Under both of these categories, instructional prescriptions with regard to teaching-learning process of problem solving are coded and exemplified with citations from the documents.

**Table 4. Descriptions of Codes and Categories for Role Definitions**

Cat.	Codes	Exemplary Citations from the Curricula Documents
The roles assigned to problem solving	Not mentioned	No explanation is made with regard to problem solving in the curricula documents.
	The main purpose of mathematics	The main purpose of mathematics is to cultivate the habit of solving problems they [students] are faced with in daily life (1983, p.19)
	One of the skills to be developed	It is defined under the heading “problem solving skills” in the 2005 curriculum
Teachers’ role in the problem solving process	Encouraging students to share their ideas about the solution	Attention must be paid to the discussion of how to perform the operation and calculations [for the problems] (1926, p.48)
	Supporting students to become autonomous problem solvers	The teacher must have students to anticipate the result approximately and then to check whether they have done it correctly or incorrectly. Thus students can gain the habit of problem solving on their own (1968, p.17)
	Assisting students to gain problem posing skills	After handling one or more problems on a situation, students must be encouraged to construct problems of the same type (1926, p.48)
	Guiding students through timely and appropriate level of assistance	The teacher must allow students to solve problems on their own way; and unless inevitably necessary, teacher should not interfere with the solution endeavor (1948, p.182).
Students’ role in the problem solving process	Gaining the autonomous problem solving skills	The teacher must have students to find out what ways to be pursued in the solution of the problem... (1948, p.181)
	Developing skills for problem posing	Stating and writing a problem in such a way that require the use of previously learned knowledge (1990, p.28)
	Sharing and developing alternative/multiple solutions	A classroom atmosphere which enables students... to comfortably share their views about problem solving with teachers and peers must be formed (2005, p.12)

Table 4 presents the codes and categories of role definitions with regard to problem solving. Under this theme, there emerged three main categories: the roles assigned to problem solving, teachers' and students' role in the problem solving process.

Within the realm of this study, reliability work has also been performed. First of all, coder reliability was ensured. In this regard, the data set was given to a specialist academician in the field of mathematics education. Agreements and disagreements about the coding were designated through the comparison of the codes suggested by the academician and the researchers. In the case of discrepancies, the coders discussed to reach a consensus. Hence, all the codes and categories have finally emerged with an agreement of all the parties involved in the analysis process. Moreover, how inferences and results were obtained in the analysis process was presented in a detailed way so that an outsider could easily follow the routine. Finally, exemplary citations are also shared in order to provide evidence on the emerging codes and to support the findings.

### Findings

The findings of the study are organised around four themes, each of which will be presented under a separate heading.

#### The notion of Problem in the Primary Mathematics Curricula

The results indicated that there were differences in the details given about the notion of problem in the mathematics curricula. The findings with this regard are provided in Table 5 below. Please note that the ticks and dashes refer, respectively, to existence and non-existence of the corresponding code.

Table 5. Notion of Problem in the Curricula Documents

Cat.	Codes	1926	1936	1948	1968	1983	1990	1998	2005	2015
Problem Definition	Undefined	√	√	√	√	√	-	-	-	√
	Situations requiring the use of four operations	-	-	-	-	-	√	√	-	-
	Novel situations	-	-	-	-	-	√	-	-	-
	Situations without obvious solutions and require reasoning skills	-	-	-	-	-	-	-	√	-
Characteristics of the Problem	Daily life problem situations	√	√	√	√	√	√	√	√	√
	Motivating students	-	√	√	√	√	√	√	√	√

	Alignment with prior learning	-	√	√	√	√	√	√	√	√
	Alignment with student developmental levels	√	√	√	√	√	√	√	√	-
	Inclusion of narration	-	-	√	√	√	√	-	-	-
	Clarity	-	-	√	√	√	√	√	-	-
Problem Types	Routine problems	√	√	√	√	√	√	√	√	√
	Non routine problems	-	-	-	-	-	-	√	√	√

As can be seen from Table 5, the notion of problem was considered in the documents with regard to definition, characteristics, and types. The problem definition was provided within three curricula: 1990, 1998 and 2005. The 1990 curriculum defined problem as new situations involving the use of four operations and placed an emphasis on the novelty of the situations. In the 1998 curriculum, emphasis on the novelty was withdrawn. In the 2005 curriculum, the problem was conceptualised as the situations without obvious solution and associated problem solving with the use of reasoning skills. With this definition, the emphasis on the situations, which require using four operations, was eliminated from the 2005 curriculum.

With regard to the characteristics, the findings suggest that all the curricula documents somehow mention about the problem features. Association of problems with daily life situations, for instance, appears to be a concern for all the curricula. Likewise, alignment with student developmental level can be marked as a feature mentioned by all but 2015 curriculum. Motivation and prior knowledge have also been among the consistently emphasised features since 1936. Taken holistically, it may be suggested that the 1926 curriculum is the one, which covers the fewest characteristics of the problem, and that there are considerable similarities among the documents issued during the period of 1948-1990.

The findings about the types of problems revealed that two different types were mentioned in the primary mathematics curricula. Of these, routine problems have found a place since 1926. After 1998, however, non-routine problems (along with the routine ones) appear to have found its way into the curricula.

### Problem Solving Steps or Heuristics in the Curricula Documents

The second theme emerged from the document analysis is the problem solving steps or heuristics prescribed by the curricula. A comparative analysis of the heuristics was presented in Table 6.

**Table 6. Problem Solving Steps in the Curricula Documents**

		Similarity groupings of the curricula in terms of heuristic approaches								
		1926 - 1935	1936-1989				1990-2004		2005 - 2014	2015
Cat.	Codes	1926	1936	1948	1968	1983	1990	1998	2005	2015
Understanding the Problem	Expressing the problem in their own words	-	√	√	√	√	-	-	-	-
	Determining what is given and what is required	-	√	√	√	√	√	√	-	-
	Expressing the problem with figures, drawings and symbols	-	√	√	√	√	√	√	√	√
	Summarizing the problem	-	-	-	-	-	√	√	-	-
	Emphasising directly the significance of understanding the problem	-	-	-	-	-	-	-	√	√
Developing a plan	Dividing the problem into stages	-	√	√	√	√	-	-	-	-
	Anticipating what can be done at each stage	-	√	√	√	√	-	-	-	-
	Anticipating which operations to be done at each stage	-	√	√	√	√	-	-	-	-
	Anticipating the result of the problem	-	√	√	√	√	√	√	-	-
	Guiding students to devise a way for the solution	-	√	√	√	√	√	√	√	√
	Emphasizing the significant points to be carried out for the solution	-	√	√	√	√	-	-	√	√
	Stating and writing the operations to be used in problem solving with the reasons	-	-	-	-	-	√	√	-	-

	Emphasising directly the significance of planning for problem solving	-	-	-	-	-	-	√	√
Implementing the plan	Executing the operational steps correctly	-	√	√	√	√	-	-	-
	Stating and writing the result upon the solution	-	-	-	-	-	√	√	-
	Selecting and implementing the problem solving strategies	-	-	-	-	-	-	-	√
Controlling	Checking the accuracy of the result	-	√	√	√	√	-	-	-
	Checking the suitability and plausibility of solutions for the problems	-	-	-	-	-	-	-	√
	Explaining whether the result was correct or not with reasons	-	-	-	-	-	√	√	-
	Drawing attention to the significance of controlling the result	-	-	-	-	-	-	-	√
Error analysis	Analysing the incorrect aspects by benefiting from a correct solution	-	√	√	√	√	-	-	-
	Determining inaccuracies by whole class evaluation	-	√	√	√	√	-	-	-
Problem solving strategies	No mention of the problem solving strategies	√	-	-	-	-	-	-	-
	Indirect emphasis on the problem solving strategies	-	√	√	√	√	√	√	-
	Direct emphasis on the problem solving strategies	-	-	-	-	-	-	-	√

As seen in Table 6, with regard to the prescription of heuristics, five different periods have been identified; these are: 1926-1935, 1936-1989, 1990-2004, 2005-2014 and 2015 to present day. In the first period (1926-1935), the 1926 curriculum was in effect and it was found that the document specified neither problem solving steps nor the regulation of the problem solving process. In the second period (1936-1989) there were four main curricula changes taking place: 1936, 1948, 1968 and 1983. All these curricula documents directly prescribed a heuristic

akin to Polya's problem solving steps. The distinctive characteristic of this period is that the documents did not impose a strict order of the steps to be followed while solving a mathematical problem. It is worth noting that despite four major curricula changes in this period, the prescriptions for the heuristics remained almost the same.

In the third period (1990-2004), the curricula underwent two important changes, resulting in the publication of the 1990 and 1998 documents. These two curricula have had almost the same spirit in terms of the adopted heuristic approaches. In these curricula too, attention was paid to problem solving steps similar to those of Polya's. However, the heuristics of problem solving were presented systemically step-by-step and students (or teachers for that matter) were expected to follow these steps in the given order. It is observed, hence, that the problem solving steps were presented in a linear fashion. This approach continued in the 1998 curriculum; and in fact this linearity was marked as the distinctive characteristic of the third period. Further to this, several novel features unique to this period for problem solving steps were noticed. These are; summarising the problem (as part of understanding the problem), stating and writing the operation(s) to be used during the solution with reasons (as part of developing a plan), stating and writing the result upon the solution (as part of implementing the plan). Verbal and written statements with reasons have come to the fore as another marking feature of this period.

In the fourth period (2005-2014), the linear approach was abandoned in the 2005 curriculum, which instead put emphasis on the importance of heuristics to reach at a solution. Another marking feature of this program is that it referred to the problem solving strategies directly. In the curriculum, problem solving strategies were listed for the first time, and much importance was attached to making evaluations in selecting and implementing the strategies. Furthermore, checking and interpreting the suitability and plausibility of the solutions to the problem was highlighted. This feature was observed only in this program.

The final period begins with the 2015 curriculum, which is still in effect to the present day. This curriculum does not mention about error analysis. In this curriculum too, the studies supporting problem solving was emphasized similar to the approach adopted in the second period. Moreover, the selection and implementation of the problem solving strategies were attended to in the curriculum directly. The significance of defining the problem, developing plans and controlling is directly stressed; and metacognitive skills are integrated into the curriculum for the first time in this program.

### Instructional Arrangements for Problem Solving in the Curricula

The third theme detected through the analysis of the documents is that of instructional arrangements for problem solving. The characteristics of instructional arrangements for the problem solving are presented comparatively in Table 7.

**Table 7. Instructional Arrangement for Problem Solving in the Curricula Documents**

Codes	1926	1936	1948	1968	1983	1990	1998	2005	2015
Supporting students to find out the solution on their own	√	√	√	√	√	√	√	√	√
Helping students when necessary and only as much as needed	√	-	√	√	√	√	√	√	-
Deciding on the solution method with whole class discussion	√	-	√	√	√	-	-	√	-
Choosing the easiest way while deciding on the solution	-	-	√	√	√	√	√	√	-
Getting mental calculations performed to anticipate the result	-	-	-	-	-	√	√	-	-
Focusing more on the thinking process than the result	-	-	√	√	√	-	-	√	-
Supporting the development of original solutions	-	√	√	√	√	√	√	√	√
Evaluating different ways of solution in problem solving	-	√	√	√	√	√	√	√	√

Problem solving arrangements



	Using think aloud technique	-	-	√	√	√	√	-	-	-
	Assigning research and inquiry homework regarding problems	-	-	-	√	√	-	-	√	-
Activity based arrangements	Assigning homework regarding the problems in non extensive amounts	-	-	-	√	√	√	√	-	-
	Assigning homework regarding the problems in non extensive amounts and within student capacity	√	√	√	-	-	-	-	-	-
	Presenting problems in order from easy to hard ones in teaching learning process	-	-	√	√	√	√	√	-	-

Three important features appear striking in Table 7. First, since 1926, all the curricula documents expect teachers to support students in reaching a solution on their own way. Second, elicitation of original solutions from the students was promoted (except for 1926). Third, all the curricula but the 1926 expected students to propose different solutions. These three features are relevant to autonomous problem solving skills, which we discuss later.

The use of mental calculations for anticipating the result in the problem solving process was encompassed in the 1990 and 1998 curricula; and the idea to pay more attention to the process rather than the results in the problem solving process was emphasized in the 1948, 1968, 1983 and 2005 curricula. It was observed that emphasis was put on the think-aloud technique in the 1948, 1968, 1983 and 1990 curricula in order to collect information about students' thinking processes, facilitate problem solving, and designate learning difficulties.

Presenting problems from the easy to the hard ones was directly accentuated in the 1948, 1968, 1983, 1990 and 1998 curricula; however, in the 2005 and 2015 curricula, no findings were obtained about stressing this principle directly in the

problem solving process. The documents until 2005 curriculum also advised the use of homework on problem solving.

### Role Definitions Regarding Problem Solving Skill

The final theme emerged in the study were the role definitions. In this regard, three categories with several codes were determined. Findings of the comparative analysis of the documents are presented in Table 8 below.

**Table 8. Role Definitions for Problem Solving in the Curricula Documents**

Cat.	Codes	1926	1936	1948	1968	1983	1990	1998	2005	2015
The role assigned to problem	Not mentioned	-	√	-	-	-	-	-	-	-
	The main purpose of mathematics	√	-	√	√	√	√	√	-	-
	One of the skills to be developed	-	-	-	-	-	-	-	√	√
Teachers' role in the problem solving process	Encouraging students to share their ideas on the solution	√	√	√	√	√	-	-	√	√
	Supporting students to become autonomous problem solvers	√	√	√	√	√	√	√	√	√
	Assisting students to gain problem posing skills	√	-	√	√	√	√	√	√	√
	Guiding students through timely and appropriate level of assistance	√	-	√	√	√	√	√	√	-
Students' role in the problem solving process	Gaining the autonomous problem solving skills	√	√	√	√	√	√	√	√	√
	Developing skills for problem posing	√	-	√	√	√	√	√	√	√
	Sharing and developing alternative/multiple solutions	√	√	√	√	√	-	-	√	√

When Table 8 is examined, it is seen that a role was attributed to problem solving in all of the curricula with the exception of the 1936 curriculum. In the 1926 curriculum, problem solving was defined as one of the main purposes of mathematics. In the curricula documents published during the period of 1936-1998, however, problem

solving was specified as the main purpose of mathematics. Yet this strong emphasis changed later on and it was accepted as a mathematical process in the 2005 and 2015 curricula.

Certain observations were also made about teachers' roles either directly or indirectly in the curricula. The roles attributed to teachers are seen to be similar in the problem solving process in the 1926, 1948, 1968, 1983 and 2005 curricula. But historically speaking, the curricula documents appeared to have much concern with sharing ideas about the solution, autonomy of students while problem solving, and development of problem posing skills. In this developmental process, teachers were expected to support students with timely and appropriate level of assistance.

### **Discussion**

The discussion section is organized around the four themes that the study designated: notion of problem, heuristics, instructional arrangements and role definitions. Each theme will be discussed under a separate heading below.

#### **The Notion of Problem in the Curricula Documents**

Given the way mathematical problems are dealt with in the curricula (see Table 5), it can be noticed that the most striking aspect is the emphasis on choosing problems from among the ones faced in daily life. As Baş (2016) states, the idea behind the use of daily-life problems is to cultivate a thinking style, which may help students solve the problems encountered in real-life situations. Furthermore, the research has now well established that one way of meaningful learning and conceptual understanding is through working on real life problems in mathematics (Bransford et al., 1999; Cooper & Harries, 2005; Kennedy et al., 2008). Hence, we believe, consistent emphasis of all the Republican curricula on the use of daily-life problems and the aim of cultivating students in handling such problems have potential to make significant contribution to the quality of mathematics education in Turkey.

However, despite this emphasis, when we consider the types of problems referred to in the curricula documents, an important weakness can be noticed. That is, while all the curricula focus on routine problems, non-routine problems managed to find its way into curricula documents not until 1998. Routine problems are mainly useful for the mastery of certain mathematical features and structures; and, generally speaking, can be solved by implementing certain rules and procedures modelled by teachers. However, their support for the development of students' thinking independently is limited (Hannula, 2014). We believe that including non-routine problems in the curricula in 1998 is an overdue development because the importance of these problems has been stressed since the 1980s (e.g., Schoenfeld, 1985). In addition, we argue that the Republican period of curricula development efforts could not catch up with the latest scientific developments of the time and; for example, was lacking in an important instrument (i.e. non-routine problems) for cultivation of a thinking style that could help students manage real problems faced in their daily lives.

The curricula documents also referred to affective dimension of problem solving by particularly emphasizing the motivational aspect of problems for students. It is delineated in Polya's (1945) classical work that problem solving is a challenging endeavour with an affective dimension. Polya notes that this process encompasses hope, determination and emotions. Affective dimension of problem solving has become a research topic particularly in the 1980s (see, Cobb et al., 1989; Schoenfeld, 1985). The research findings pointed to students' constantly changing affective states in the problem solving process. The findings also revealed that both expert and novice problem solvers experience positive and negative emotions in especially working on non-routine problems and that those who feel positive emotions or do not give up easily become successful. In this sense, given the positive effects of enjoyment on mathematics learning (DeBellis & Goldin, 1997), it can be argued that the curricula's emphasis on the motivational aspect of problems is rather relevant.

It is also observed that the curricula documents mostly treated the notion of problem as if it were a self-evident term and appeared to eschew its definition. It is interesting to see that until 1990, the problem definition was absent. The 1990 and 1998 curricula both cited the same definition of a mathematical problem, i.e. a situation requiring the use of four operations. However such definition narrows the notion of problem by depth and scope and, in fact, misses out some important features. The relevant literature suggests that mathematical problem should not have a clear or apparent solution (method) but rather it ought to require the agent (i.e. student) to synthesize prior knowledge (Holth, 2008) as well as employ reasoning skills (Krulik & Rudnick, 1987). It is the 2005 curriculum that provided a problem definition that reflects these features. When we think of the impact of curriculum documents on a variety of issues ranging from classroom practices to the development instructional materials (including textbooks), evasive nature of the notion of mathematical problem as existed in the documents reflects an important deficiency. We believe that this notion has to be clearly defined to differentiate it from some of the most relied concepts in mathematic instruction such as exercises, examples and drills.

### **Heuristics in the Curriculum Documents**

The findings on heuristics (see Table 6) indicated that curriculum changes of the Republican period evolved in five different phases: 1926-1935, 1936-1989, 1990-2004, 2005-2014 and 2015 to present day. This demonstrates that change and innovation attempts launched in the specified time periods have not brought about much difference in the prescriptions with regard to heuristic approaches adopted in the problem solving process. For example, although three different curriculum changes were launched in the 1936-1989 period (1948, 1968 and 1983), it can be noticed that the details provided with regards to the problem solving process had similar features in these curricula. It is also interesting to see that the importance of heuristics such as understanding, planning and controlling was realised as early as 1936. This is particularly significant as the importance of heuristics appeared to be realized earlier than Polya's (1945) postulation of the four step problem solving approach which was specified through his reflection on his own experiences as a mathematician.

In the problem solving approach of the 1990-2004 period, structured steps were presented for the problem solving process and the prescription was to follow the steps in a linear fashion. The main difficulty involved in such linearity is, as Dolan & Williamson (1983) argue, related to the perception imposed on students that these steps have to be followed in order. Such a view apparently creates obstacles for the development of a flexible and creative problem solving skills.

It was probably due to this concern that the linear approach to problem solving was abandoned in the 2005 curriculum. Instead, it emphasized the importance of heuristics (e.g. understanding, planning and controlling) for a successful problem solution. Furthermore, learning problem solving strategies, which is an invaluable characteristic for problem solving (Suydam, 1987), was clearly brought to the fore in this curriculum. Different from the other curricula, in the 2015 curriculum, the use of metacognitive skills was adopted in the problem solving process. It was, we feel, a significant development to integrate metacognitive skills; however, this integration can be considered as a belated attempt for the development of problem solving skill because metacognition has been around since mid-1970s and included in the curricula of many countries, such as Singapore, much earlier.

When evaluated holistically, it is seen that teaching of heuristics has been set as an aim with different approaches in different periods. The rationale behind the adopted approaches is highly likely to enhance students' problem solving performance. Included in many textbooks and curriculum resources, heuristics can be usefully employed to handle different tasks ranging from exercises based on basic calculations, non-routine or multi-phase problems to the situations including mathematical modelling (van de Walle et al., 2010). Nevertheless, research in this area has produced substantial evidence that teaching heuristics has a rather limited effect to enhance students' problem solving skills (Begle, 1979; Schoenfeld, 1992; Sriraman & English, 2010). Schoenfeld (1992), for example, considers Polya's problem solving steps as a general structure for problem solving; but argued the necessity of teaching specific problem solving strategies and training students to gain (meta)cognitive awareness in order to make problem solving more effective. In a similar vein, Sriraman and English (2010) contend that "understanding" heuristics means when, where, why and how to use heuristics along with some other useful resources including emotional, social and metacognitive tools. Therefore, we believe that mentioning the importance of heuristics and its instruction will not suffice unless we design an approach serving to problem solving as informed by the research findings briefly illustrated above.

### **Instructional Arrangements for Problem Solving in the Curriculum Documents**

Instructional arrangements prescribed by the curricula documents for problem solving (see Table 7) aim to create a learning environment that fosters students' problem solving skills. To this direction, we can safely conclude, on the basis of findings, that all the curricula of the Republican period, in one way or another, envision making students autonomous problem solvers. Within the realm of this vision, the curricula insist on supporting students to find solutions on their own, providing help when necessary and

only as much as needed (except for the 1936 & 2015 curricula), as well as on developing original and different solutions. All these can be taken as indicators of student-centred approach to the problem solving as opposed to teacher-centred one. Teacher-centred approach is often associated with direct instruction (Stephan, 2014).

The literature suggests certain advantages of direct instruction over student-centred teaching of problem solving. Sweller and Chandler (1991), for instance, argue that students reach more readily to the right procedures and knowledge through direct instruction which also reduces the likelihood of developing misconceptions and committing errors. In a similar vein, Kirschner et al. (2006) state that when students work on the problems on their own, they usually employ trial-and-error or means-ends analysis. This, according to the authors, increases the burden on the limited capacity of working memory, which, in turn, might inhibit students to learn new concepts and procedures. Hardiman et al. (1986), on the other hand, claim that direct instruction reduces the disengagement and frustration that could arise from solving problems without teacher assistance.

Despite such arguments underscoring the benefit of direct instruction, the literature also provides compelling evidence on positive effects of student-centred approach to problem solving. To begin with, Kapur (2014) underlines the importance of problem solving experience with students' own efforts. The result of his study shows that the number of student-generated solutions significantly predicted the learning outcomes. Hence, he argues that even if students fail to solve problems on their own efforts, this failure is productive in nature in that students learn from their own failed problem solving attempts. Further to this, Hiebert and Grouws (2007) convincingly argue that even though generation of solutions on their own may place a heavy cognitive load on students, this difficulty could serve to schema assembly and meaningful encodings, resulting in a better learning from the subsequent instruction. DeCaro and Rittle-Johnson (2012) also claim that generating solutions without direct instruction might assist students to realise inconsistencies in their prior knowledge and hence determine their limits. Siegler (2002), on the other hand, states that students could find opportunities to compare their own solutions with that of a correct one; this in turn helps students attend to the critical features of the concepts involved in the solution process. When viewed from the research perspectives, it could be argued that Republican curricula's emphasis on student-centred problem solving approach is rather relevant and appropriate for their development.

When the curricula's emphasis on student-centred problem solving approach is considered along with their focus on elicitation of original and/or alternative solutions, it can be realized that the documents preferred to support student autonomy over heteronomy. Stephan (2014) argues that the most important contribution of education to individuals' life is helping the development of their autonomy. Kamii (1982) considers autonomy as the ability to think for oneself and make independent decisions without the fear of punishments or the expectation of rewards. Such viewed, autonomous individuals can be described as those who govern and make decisions for themselves rather than relying on the rules of others (Stephan, *ibid.*). When the importance of

rearing independent individuals is considered, the curricula's insistence on the development of autonomous problem-solvers become even more apparent.

The curricula's instructional approach to problem solving is certainly dependent on the teachers who would eventually shape the classroom practice. Burkhardt's (1988) findings on the teacher difficulties in teaching problem solving are particularly relevant to our discussion here. According to Burkhardt, in making instructional arrangements in the elicitation of different or original solutions to problems and/or supporting the autonomy of students for problem solving, difficulties that teachers experience have three distinct yet interrelated dimensions: mathematical, pedagogical and personal.

Thinking mathematically, Burkhardt states that teachers should give formative and constructive feedback to students and be able to evaluate different approaches. This requires teachers to focus more on the thinking process than the results, a quality referred to in the 1948, 1968, 1983 and 2005 curricula documents. Thinking pedagogically, according to Burkhardt, teachers should carefully decide upon the interventions and feedback; and the level of assistance should be kept as low as possible. We observe that the curricula documents historically well realized the importance of this dimension and put an emphasis (except for the 1936 and 2015 curricula) on the least possible help to the students. It should be noted that the currently practiced curriculum (issued in 2015) missed out this emphasis and we think of this as a crucial deficiency. Thinking personally, Burkhardt points out that teachers should have experience, confidence and self-awareness in order to be able to evaluate the student solutions without knowing all the answers. Consideration of personal aspect of teacher difficulties impinges upon nationwide teacher preparation approaches and experiences as well as opportunities with professional development. Hence the problem solving skills that the curricula documents aim to develop require a consideration of multi-dimensional teacher preparation and professional development policies.

When evaluated generally, it can be concluded that instructional arrangements that are considered to be important for the development of problem solving skills appeared to have a place in the curricula documents even at early dates. However, the currently practiced curriculum of 2015 unfortunately lagged behind recent research findings (and in fact, in some respects, even behind the documents developed in 1940s). Hence the current program needs to be revised in terms of at least instructional arrangements for problem solving.

### **Role Definitions for Problem Solving in the Curriculum Documents**

When the findings on role definition are examined (see Table 8), it can be realised that problem solving was historically seen as the main purpose of mathematics teaching until 2005. From then on, in parallel to NCTM (2000), mathematical problem solving was stated as a skill to be developed. It is also determined that the curricula documents define such roles for teachers as encouraging students to share ideas about the solution, supporting student autonomy, assisting students to gain problem posing skills and providing guidance. The students were also assigned with the roles of gaining autonomy, posing problems and designating alternative solutions.

Encouraging students to share their solutions is of particular importance as this will help students to know more about their solutions, compare alternatives and thus enhance problem solving skills (Cathcart et al., 2001; NCTM, 2000). All the curricula (except for the 1936) pointed to the importance of problem posing skills and assigned roles to both teachers and students. The research in this area provides evidence that problem posing increases the creative and critical thinking skills, mathematical knowledge and computational fluency of students (English, 1997; Silver & Cai, 1996). In this respect, we feel important that the problem posing has found a place in the documents as early as in 1926. In addition, the emphasis on providing guidance to students through timely and appropriate level of assistance can be again related to the student autonomy; this role is consistent and compatible with the relevant instructional arrangements discussed earlier.

In the light of findings and discussions given hitherto, it could be concluded that many important arrangements for the development of problem solving skills were inserted into the curricula documents of the Republican period. We traced the arrangements back to the earliest curriculum efforts as in 1926. However, some of the significant arrangements and features were abandoned at certain periods with some reasons not apparent to us. Surprisingly enough, the abandoned regulations were later adopted on in different programs. Therefore, one can observe ups and downs in the adoption of certain instructional arrangements and features. We can conclude that curriculum renewal efforts of the Republican period, unfortunately, did not seem to achieve a state of permanent improvement. We feel, finally, obvious that future program development efforts, while deciding upon instructional arrangements to cultivate students' problem solving skills and enhance performances, need to make a more effective use of the research findings in the field.

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