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Estimation of Population Variance: A Ranked Set Sampling Approach in A Finite Population Setting

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ABSTRACT : Ranked set sampling is a sampling technique that provides substantial cost efficiency in experiments where a quick, inexpensive ranking procedure is available to rank the units prior to formal, expensive and precise measurements. Although tbe theoretical properties and relative efficiencies of this approach with respect to simple random sampling have been extensively studied in the literature for the infinite population setting, the use of ranked set sampling methods has not yet been explored widely for finite populations. The purpose of this study is to use a sheep population data to demonstrate the practical benefits of ranked set sampling procedures relative to the more commonly used simple random sampling estimation of the population variance in a finite population. It is shown that the ranked set sample variance estimators are unbiased only with use of the finite population correction factor. The variance estimators provide substantial improvement over its simple random sample counterpart.

Keywords: Concomitant variable, sampling design, judgment ranking, finite population correction, ranked set sampling

ÖZET: Sıralı küme örneklemesi birimlerin, maliyeti düşük basit bir yöntemle sıralanmasından sonra maliyeti yüksek gerçek ölçümlerin alınmasına dayanan maliyet indirgeyici bir örnekleme yöntemidir. Bu yaklaşımın teorik özellikleri ve basit şans örneklemesine göre nisbi etkinliği sonsuz populasyon durumunda literatürde yoğun olarak çalışılmış olmakla birlikte sonlu populasyon durumunda sıralı küme örneklemesi yöntemlerinin kullanılması henüz yeterince araştırılmamıştır. Bu çalışmanın amacı bir koyun populasyonuna ait verileri kullanarak sıralı küme örneklemesi prosedürlerinin bir sonlu popopulasyon için populasyon varyansını tahminde genellikle kullanılan basit şans örneklemesine göre sağladığı kazançları göstermektir. Sıralı küme örneklemesi varyans tahmincilerinin ancak sonlu populasyon düzeltme faktörü ile kullanıldığında sapmasız olabileceği gösterilmiştir. Bu varyans tahmincileri basit şans örneklemesi varyans tahmincisine karşı büyük başarı sağlamışlardır.

Anahtar kelimeler: örnekleme, sonlu populasyon, sıralı küme örneklemesi

INTRODUCTION

In many situations, researchers face a dilemma about whether they should require time-consuming, expensive, precise measurements or accept quick, inexpensive, less precise observations Less precise observations are preferable if the cost of measurement is very high, while precise measurements are ideal if the sampling cost is not a limiting factor for the experiment. On the other hand, statistical analyses of the imprecise observations require strong measurement error modeling assumptions about the relationship between the precise measurements and the imprecise observations. A balance can be struck between these two extremes that require minimal modeling assumptions (almost none) and provides substantial reduction in sampling cost. This balanced strategy uses both inexpensive observations and expensive measurements, but the inexpensive observations on a large number of individuals (or experimental units) determine which smaller number of expensive measurements should be collected. Ranked set sampling provides a collection of techniques with detailed plans for how to collect and analyze these inexpensive observations and expensive measurements. Before collecting costly, expensive measurements, ranked set sampling suggests using an initial sample to rank the sampling units either visually or by any other means that does not require full measurement of units. This ranking information (inexpensive, less precise observations) is used to create an artificial stratification so that homogeneous units can be grouped within each stratum. Since this stratum information is acquired without full measurement of the units, it does not substantially increase the sampling cost. On the other hand, homogeneous observations in each strata, the variation in the sample and permits the use of a smaller sample size for the experiment.

The standard ranked set sampling procedure involves randomly selecting m sets, each having m units, from the population. The units in each set are then judgment ranked either visually or with the help of inexpensive and imprecise measurements on these units. Then the full and more expensive measurement is obtained for the i-th ranked unit from set i, $i=1, \ldots, m$. These quantified measurement constitute a cycle and the m judgment classes in this cycle, only m of them are fully measured. The other $m(m-1)$ units are used to create artificial strata (judgment classes) on the basis of rankings from the inexpensive, imprecise observations.

McIntyre (1952) was the first author to use a ranked set sampling procedure to reduce the sampling cost for assessing yields of pasture plots without actually moving and weighing the hay for a large number of plots. The statistical theory for estimating the population mean was developed by Takahasi and Wakimoto (1968), under the assumption of perfect judgment ranking. Dell and Clutter (1972) considered the imperfect ranking situation and concluded that estimation of the mean is unbiased, regardless of the errors in ranking. Stokes (1977) discussed the use of a concomitant variable for ranking the experimental units to estimate the population mean and concluded that the amount of increase in the precision of the estimator depends on the correlation between the concomitant and fully measured variables.

Stokes (1980) suggested an estimator for population variance in ranked set sampling and showed that the estimator is asymptotically unbiased even in the presence of errors in ranking. Further improvements were proposed to Stokes estimator when the underlying distribution is normal and in the case of perfect ranking by Sinha et al. (1996) and Yu et al. (1999). MacEachern et al., (2002) developed an unbiased estimator of the variance of a population based on ranked set sample and showed that their estimator performs even better than Stokes for small sample sizes.

Because of the cost-efficient nature of ranked set sampling, there has been wide interest over the last three decades in applying it to a broad range of research in science, in general, and environmental and ecological research, in particular. A representative slice of this literature follows: Patil et al., (1994), Kaur et al., (1995) and Patil et al., (1999) provided historical background and perspective for ranked set sampling. Martin et al., (1980) applied ranked set sampling to estimating shrup phytomass in Appalachian oak forests. Cobby et al. (1985) used ranked set sampling to estimate mass herbage in a paddock. Stokes and Sager (1988) discussed the benefits of ranked set sampling in estimating tree volume in a forest. Mode et al., (1999) and Barnett (1999) discussed conditions under which ranked set sampling is a cost-effective sampling method for estimation of the population mean from ecological and environmental field studies. Al-Saleh and Al-Shrafat (2001) assessed effectiveness of ranked set sampling by an application to average milk yield in sheep.

Since McIntyre's 1952 paper on ranked set sampling, the majority of research has concentrated on sampling from infinite populations. Takahasi and Futatsuya (1988) were the first to give a formula for the variance of the ranked set sample mean in the context of a general covariance structure in the finite population setting. They later (Takahasi and Futatsuya, 1988) showed that the ranked set sample estimator of the population mean is more precise than the simple random sample estimator. Patil et al., (1995) derived explicit expressions for the variance and the relative precision of the ranked set sample mean in a finite population setting. They concluded that for a given size n when sampling from an infinite population the relative precision depends only on the set size m. In contrast, when sampling from a finite population without replacement the relative precision depends on the number of cycles (r) as well as the set size m.

Ranked set sampling is an appealing sampling technique to maintain an affordable management strategy for farm animals, such as sheep, cattle and cows, where certain variables (such as milk, meat and wool yields) are measured at regular time intervals over the life span of the animals in order to monitor their growth and to devise a reasonable strategy to maintain a progressive improvement in the population. In these experiments, precise measurements of the traits are time consuming and labor intensive due to the size and physical behavior of the animals. On the other hand, easy to measure variables that provide cheap and inexpensive observations are often available for use in the context of ranked set sampling methodology. This paper investigates the estimation of the population variance for a finite sheep population using simulation. Section 2 describes the data set and provides background information about the population. Section 3 introduces the methodology used in this study. It is shown that ranked set sample variance estimators are not unbiased without the use of a finite population correction factor. Section 4 outlines the sampling protocol to perform the simulation and Section 5 discusses the simulation results. Finally, Section 6 provides some concluding remarks.

DATA SET

The data set in this study contains the weights of 224 sheep at the Research Farm of Ataturk University, Erzurum. We used the dam's weight at mating or birth weight as a single concomitant variable for weight of the sheep at seventh months.

The frequency distributions of the dam's weight at mating, birth weight and seventh-month weight of the sheep population are all approximately symmetric. The correlation coefficients between the seventh-month weight and birth weight and the seventh-month weight and dam's weight at mating are 0.79 and 0.43 respectively. The magnitudes of these correlation coefficients indicate that these variables can successfully be used to rank the sheep at their seventh months before the actual weighing process. In the next section a simulation study shows that the efficiency of the ranked set sampling procedure for a finite population is an increasing function of the magnitude of the correlation coefficient between the concomitant and fully measured response variable just as in the infinite population setting. Even though our simulation study involves positive correlations, similar results hold for strong negative correlations as well.

METHODS

The intent of this study is to show the potential improvement over simple random sampling from using ranked set sampling in estimation of the population variance. Throughout the simulation study, we treat the N = 224 sheep in the sheep data set as the population. Let X be the random variable representing the seventh-month weight of the sheep and let $\Omega = \{x_1,...x_N\}$ denote the set of values for the random variable X on this finite population. For this population the variance is σ^2 = 15,140 kg^2. Throughout the study, all samples are taken without replacement.

Let X_1, ..., X_n be a simple random sample (without replacement) of size n for the random variable X. The simple random estimates of the population variance, σ^2, are given by the sample variance S^2. This estimator is unbiased for the population variance (σ^2).

A ranked set sample from a finite population is obtained in a fashion similar to a ranked set sample from an infinite population. First, $m^2 \times r$ units are selected at random without replacement. Then these $m^2 \times r$ units are divided into r cycles and the units in each cycle are divided into m sets each having m units. The units in each set are subsequently judgment ranked based on one of the concomitant variables, dam's weight at mating or birth weight. Then the full and more expensive measurement of the seventh-month weight that corresponds to the concomitant variable is obtained for the i-th ranked unit from set i in cycle j, $X_{[i:m]j}; i = 1,...,m; j = 1,...,r$, with possible errors in ranking depending on the accuracy of the judgment process. The square bracket in this notation indicates the possible judgment ranking error due to imprecise observations. In order to have equal sample sizes for both simple and ranked set samples, we let $n = r \times m$. For this setting, even though the sets are disjoint, the judgment order statistics are not independent; in fact, they are negatively correlated (see, for example, equation 3.9 and 4.4 in Patil et al., 1995), which provides further improvement over ranked set sample from an equally diffused infinite population. Unfortunately, this negative correlation between the $X_{[i:m]j}$'s also leads to a more complicated ranked set sample statistical analysis than is the case for an infinite population.

For estimate of the population variance, we use two different nonparametric estimators. The firs estimator, due to Stokes (1980), is

$$\tilde{\sigma}_S^2 = \frac{1}{n-1}\sum_{i=1}^{m}\sum_{j=1}^{r}(X_{[i:m]j} - \bar{X}_{RSS})^2.$$

This estimator, even in an infinite population setting, is not unbiased for σ^2 for a finite cycle size r. On the other hand, it is asymptotically (either $r \to \infty$ or $m \to \infty$) unbiased, and is at least as efficient as the sample variance from a simple random sample in an infinite population setting.

Recently, MacEachern et al., (2002) proposed an estimator that is unbiased for σ^2 and at least as efficient as the Stokes variance estimator for any $r > 1$ and m in the infinite population setting. Their estimator is given by

$$\tilde{\sigma}_M^2 = \frac{1}{2m^2r^2}\sum^{m}\sum^{m}_{i\neq l}\sum_{j=1}^{r}\sum_{k=1}^{r}(X_{[i:m]j} - X_{[l:m]k})^2$$

$$+ \frac{1}{2m^2r(r-1)}\sum_{i=1}^{m}\sum_{j=1}^{r}\sum_{k=1}^{r}(X_{[i:m]j} - X_{[i:m]k})^2$$

There is a clear difference between $\tilde{\sigma}_S^2$ and $\tilde{\sigma}_M^2$ in the way that they treat the observations. The Stokes estimator treats each observation the same regardless of which judgment class it comes from. On the other hand, $\tilde{\sigma}_M^2$ combines between-and within-class variations in a linear fashion by giving a slightly higher weight to within-class terms.

The properties $\tilde{\sigma}_S^2$ and $\tilde{\sigma}_M^2$ have not yet been studied in the literature for the finite population setting. The following theorem shows that neither of these estimators is unbiased for σ^2 in this setting, but bias in $\tilde{\sigma}_M^2$ can be removed by use of a simple correction factor.

Theorem 1 *Let* $X_{[i:m]j}, i = 1,...,m; j = 1,...,r$ be a ranked set sample (without replacement) from a finite population of size N, having mean μ and variance σ^2. Then

$$E(\tilde{\sigma}_S^2) = \sigma^2\left[\frac{mrN - (N-1)}{(mr-1)(N-1)}\right]$$

$$+ \frac{1}{m(mr-1)}\left[\sum_{i=1}^{m}(\mu_{[i:m]} - \mu)^2 + \sum_{i=1}^{m}\sigma_{ii:m}\right]$$

and

$$E(\tilde{\sigma}_M^2) = \frac{N}{N-1}\sigma^2,$$

where

$$\sigma_{ii:m} = \text{cov}(X_{[i:m]1}, X_{[i:m2]})$$ and

$\mu_{[i:m]} = EX_{[i:m]j}$.

The bias of the Stokes variance estimator is substantial and depends on the other population characteristics, such as means and covariances. When either the number of cycles r or set size m goes to infinity, the expected value of $\tilde{\sigma}_S^2$ reduces to $\sigma^2 N/(N-1)$, which still needs a correction factor $(N-1)/N$ for a finite population. The new Stokes estimator, with the correction factor, is

$$\hat{\sigma}_S^2 = \frac{N-1}{N}\tilde{\sigma}_S^2.$$

However, when N is large, the Stokes estimator is unbiased for either large cycle or set size, just as for the infinite population results in Stokes (1980).

The variance estimator, $\tilde{\sigma}_M^2$, overestimates the population variance in the finite population setting. However, the amount of bias is substantially smaller than that for the Stokes estimator. This bias does not depend on the cycle size, set or other population characteristics, such as the mean and covariances. Thus it can be corrected easily. In the finite population setting we propose to use

$$\hat{\sigma}_M^2 = \frac{N-1}{N}\tilde{\sigma}_M^2$$

as an unbiased estimator for the population variance σ^2. Analytic computations of the variances of these estimators are complicated due to the negative correlations. Thus, we assess their relative precision in a simulation study from a finite population.

SIMULATION SPECIFICATIONS

In the simulation, overall sample sizes of $n = 4(2)30$ are used to compare the preeision of ranked set sample with that of simple random sample. All samples are taken without replacement. For given overall sample size n, the proper set sizes of $m = 2(1)10$ are used so that the cyele size r is an integer. Ranking is performed on the basis of concomitant variables birth weight or dam's weight at mating.

Simple and ranked set samples are generated without replaeement, as described in Section 3. In each replication, S^2 is computed for the simple random samples, and $\hat{\sigma}_S^2$ and $\hat{\sigma}_M^2$ are computed for the ranked set samples. The simulation size is taken to be 500,000 for each sample size in the simple random sampling, and for each sample size, set size and concomitant variable combination in the ranked set sampling.

To assess the effectiveness of the variance estimators we estimated the bias, which is the average difference between the value of the estimator at each iteration and the population parameter. Estimated biases of simple random sample and McEachern-Ozturk-Wolfe-Stark estimators were practically zero as expected and they are not reported in this article. The biases of Stokes estimator is discussed in Section 5 for selected values of simulation specification combinations. In addition, we also estimated the precisions of the variance estimators by

$$\hat{V}(S^2) = \frac{1}{499999}\sum_{i=1}^{500000}(S_i^2 - \bar{S}^2)^2,$$

$$\widehat{MSE}(\hat{\sigma}_S^2) = \frac{1}{499999}\sum_{i=1}^{500000}(\hat{\sigma}_{S,i}^2 - \sigma^2)^2$$

and

$$\hat{V}(\hat{\sigma}_M^2) = \frac{1}{499999}\sum_{i=1}^{500000}(\hat{\sigma}_{M,i}^2 - \bar{\hat{\sigma}}_M^2)^2,$$

where

$$\bar{S}^2 = \frac{1}{500{,}000}\sum_{i=1}^{500{,}000} S_i^2,$$

$$\bar{\hat{\sigma}}_M^2 = \frac{1}{500{,}000}\sum_{i=1}^{500{,}000}\hat{\sigma}_{M,i}^2.$$

Since the Stokes estimator is not unbiased for the population variance, its precision is evaluated in terms of its mean squared error. The estimated relative precision is then expressed

$$RP_1 = \frac{\hat{V}(S^2)}{\widehat{MSE}(\hat{\sigma}_S^2)}, \quad RP_2 = \frac{\hat{V}(S^2)}{\hat{V}(\hat{\sigma}_M^2)},$$

$$RP_3 = \frac{\widehat{MSE}(\hat{\sigma}_S^2)}{\hat{V}(\hat{\sigma}_M^2)}$$

SIMULATION RESULTS

There are two main features that need to be discussed. The first feature is that the Stokes estimator has a substantial bias, especially for small cycle sizes. These biases are shown in Figure I. In these panels each line represents the bias of Stokes variance estimator for the stated set sizes. It is clear that the biases shrink with increased cycle size, which confirms the findings in Stokes (1980) and MacEachern et al., (2002). For example in the first panel, the bias of $\hat{\sigma}_S^2$ is 2.926 when $n = 4$, $m = 4$ and $r = 1$, and it reduces to 0.329 when $n = 28$, $m = 2$, $r = 14$. Another observation regarding the Stokes estimator is that the amount of bias is related to the magnitude of the correlation coefficient between the concomitant and fully measured random variables. The weaker correlation reduces the biases since in this case the ranked set sample is closer to the simple random sample for which the sample variance is unbiased (see panels 1, 2, and 3 in Figure 1).

The simulation study also revealed that the variance estimator of MacEachern et al. (2002) is unbiased as long as the number of cycles r is greater than 1. When the cycle size r=1 the MacEachern-Ozturk-Wolfe-Stark estimator underestimates the population variance. The reason is that, in this case, within-class variation is not estimable due to lack of replications. The estimator $\hat{\sigma}_M^2$ estimates only between-elass variation, which is smaller than the true population variance. These results are consistent with the conclusion of Theorem 1.

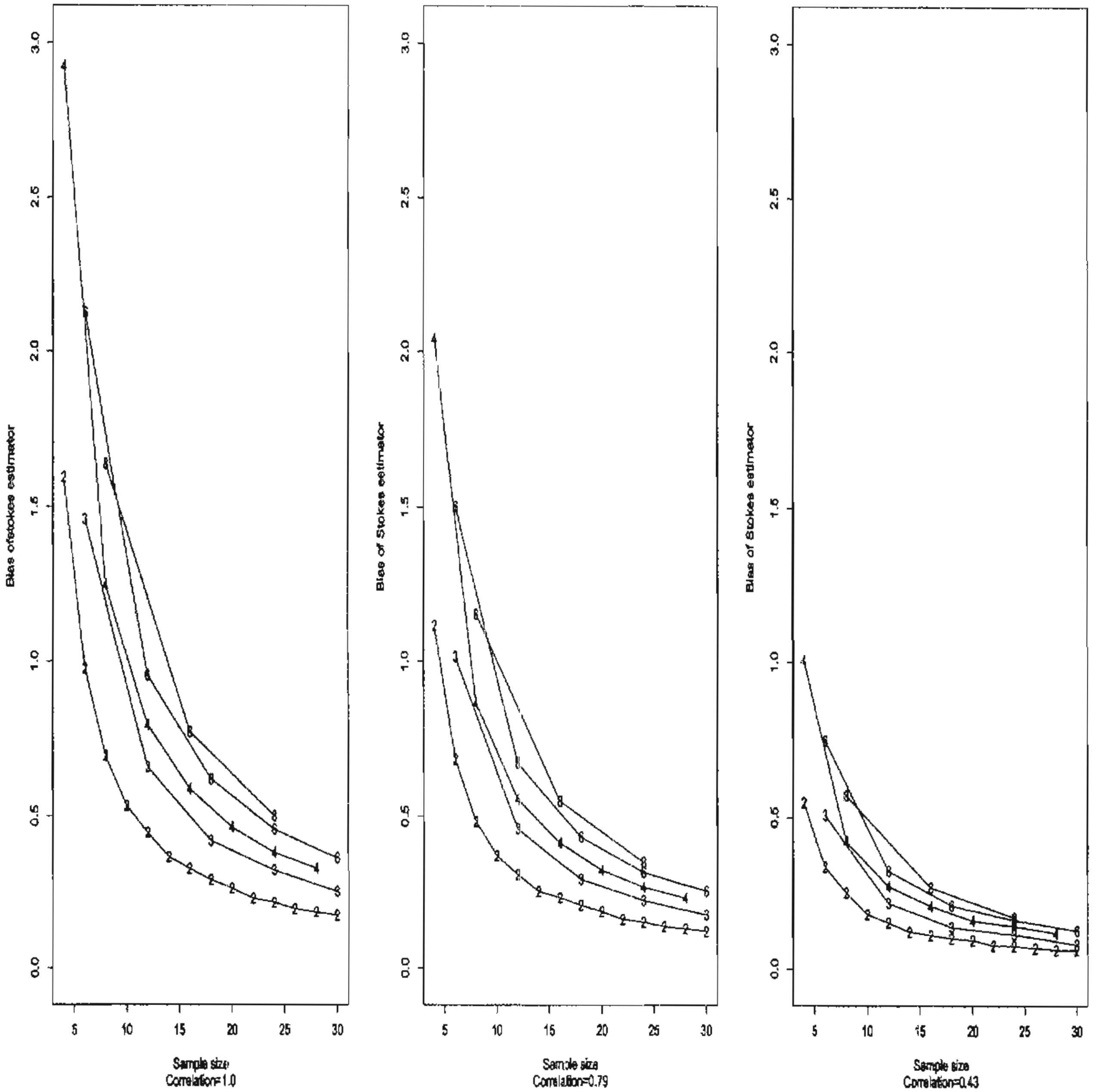

Figure 1. Bias of Stokes variance estimator.

The second feature of the estimators that needs to be discussed is their precision. Figure 2 presents the relative precision of variance estimator. The first row in Figure 2 is the relative preeision of simple random sample variance estimator with respect to Stokes variance estimator (RP1). The second and third rows are the RP2 and RP3, respectively. Each line in all these panels again represents the relative precisions for given values of set sizes. The panels in the first row show that the Stokes estimator performs poorly for small cycle sizes. Its relative precision with respect to the simple random sample variance estimator is less than 1 for small cycle sizes. This is due in small part to bias of Stokes estimator. On the other hand, when the cycle the sizes increase it performs favorably relative to simple random sample variance estimator.

The MacEachern-Ozturk-Wolfe-Stark variance estimator outperforms both the simple random sample and Stokes variance estimators. Its relative precision with respect to that of the simple random sample and Stokes variance estimators. Its relative precision with respect to that of the simple random sample variance estimator increases with set size. The relative precision (PR2) in row 2 in Figure 2 varies between 1.168 (when m=2, n=4 and correlation=1.0) and 1.891 (when m=8, n=24 and correlation=1). Another factor that affects the performance of the $\hat{\sigma}_M^2$ is the correlation between the concomitant and fully measured variables. Weaker correlation reduces the precision as expected (second row, panels 2 and 3 in Figure 2).

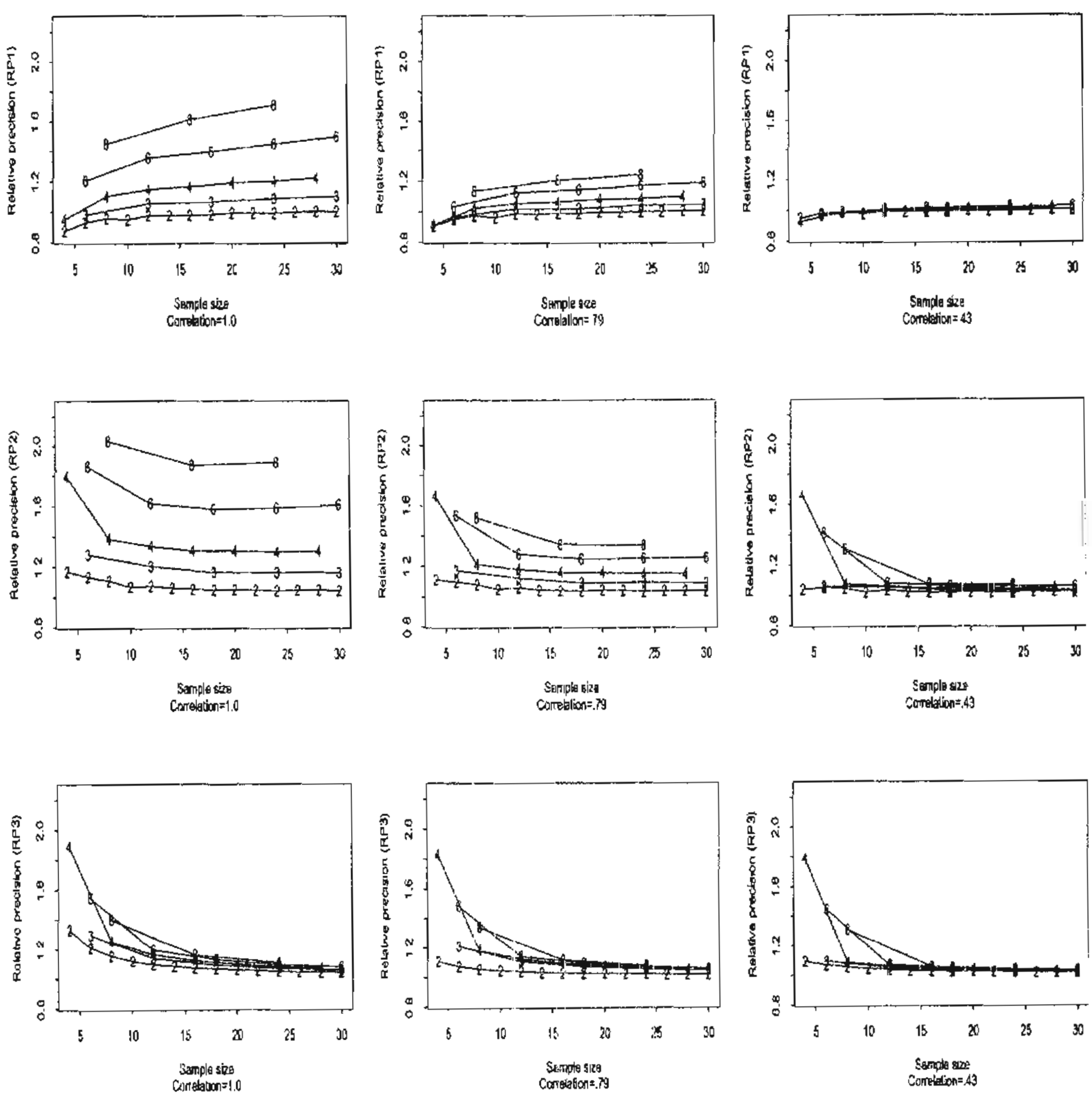

Figure 2. Relative efficiencies of ranked set and simple random sample estimator of the population variance.

The third row of Figure 2 shows that the Stokes estimator performs poorly with respect to the MacEachern-Ozturk-Wolfe-Stark variance estimator, especially for small cycle sizes and moderate-to-large correlations between the concomitant and response variables. On the other hand, the difference between the properties of these two estimators lessens, as the cycle size gets larger since they are asymptotically equivalent. For example, the precision (RP3) of the MacEachern-Ozturk-Wolfe-Stark variance estimator relative to Stokes estimator is as big as 1.332 when n=4, m=2, r=2 and reduces to 1.002 when n=30, m=2, r=15 for the perfect rankings in the first panel in row three of figure 2. Similar observations can be made for panels 2 and 3 for less than perfect correlations between the concomitant and response measurements.

CONCLUSIONS

Theory clearly states that balanced ranked set sampling provides unbiased estimators for the population variance with standard deviations that are at least as small as those of the corresponding simple random sample estimator in an infinite population setting. Furthermore, this is true whether the judgment ranking is perfect or not, which indicates that nothing is lost if we use ranked set sampling where it is applicable. In the worst case, which happens with random judgment, they will be equivalent to the simple random sample estimators. The empirical study of this research demonstrates that the unbiasednees and gain in precision for the population variance estimator remain valid in a finite population setting as well.

The two variance estimators require finite population correction factors; however, these correction factors are both asymptotically equal to one as the population size gets large. It is shown theoretically, and confirmed empirically, that the Stokes variance estimator is substantially biased for small cycle sizes and needs a finite population correction factor. This correction factor for small cycle and set sizes depends on population characteristics, such as the mean and covariances, as well as the cycle, set and population sizes. However, it reduces to $(N-1)/N$ is unbiased for any set size m as long as the cycle size r is greater than 1.

The results of this research show that, in the case of finite population where relatively accurate ranking of sample units can be accomplished easily and inexpensively compared to the cost of full quantification, balanced ranked set sample can be used to reduce the sample sizes to achieve a desired precision for the estimator of the population variance as well as the population mean. We believe that this is especially important in agricultural experiments where the population is usually finite and the measurement process is either time consuming or expensive by nature of the study. We hope that agricultural science researches will adopt this promising technique as a powerful addition to their repertoire of sampling strategies.

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