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A REGRESSION MODEL FOR FINDING OPTIMAL SOLAR PANEL INCLINATION ANGLES

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Abstract

Original scientific paper

Energy efficiency is at the top of the measures created against the ever-increasing energy need for supply. Moreover, the global climate crisis, the protection of the environment, and the improvement of the economy are only possible by optimizing active energy systems. For this purpose, it is a requirement that the solar panels have the appropriate tilt angles and correct positions in order to maximize the efficiency of solar energy systems. In this study, for the province of Van (43°38' N-38°50' E), whose solar energy potential is higher than the average of Turkey, the optimum solar panel angle according to the months of the year was determined for the south-facing solar panels. It has been determined that the annual panel inclination angle is 30.54^0 and in the case of monthly change of panel angles, the annual yield increases up to 14% compared to the horizontal plane, reaching an average annual radiation value of 5090 Wh/m^2-day. 8 different mathematical models have been developed from 4 different variables for estimating the optimum angle value specific to the latitude of the region. The statistical coefficient of certainty (R^2) range of these models varies between 0.9068 and 0.9964. As a result of these findings, the regression model, whose applicability and reliability have been proven, can serve as examples of locations at different coordinates in the same latitude. Furthermore, the results set an example for academics and industrialists in terms of the proposed model's engineering design, eco-energy analysis, and use in optimal design processes.

Keywords: *Optimization, solar energy, PV panels, tilt angle, energy.*

OPTİMAL GÜNEŞ PANELİ EĞİM AÇILARINI BULMAK İÇİN BİR REGRESYON MODELİ

Özet

Orijinal bilimsel makale

Artan enerji ihtiyacı arzına karşı oluşturulan önlemlerin en başında enerji verimliliği gelmektedir. Dahası küresel iklim krizi, çevrenin korunması ve ekonomik kazancın iyileştirilmesi de ancak aktif enerjili sistemlerin optimize edilmesiyle mümkün olmaktadır. Bu amaçla, güneş enerjili sistemlerin verimini en üst düzeye çıkarılmasında güneş panellerinin uygun eğim açılarına ve doğru konumlara sahip olması bir gerekliliktir. Bu çalışmada güneş enerjisi potansiyeli Türkiye ortalamasının üstündeki Van ili (43°38' E-38°50' N) için aylara göre optimum panel açısı güney konumuna bakan yöne göre belirlenmiştir. Yıllık panel eğim açısı 30.540 olduğu tespit edilmiş ve panel açılarının aylık değişimi durumunda yıllık verim yatay düzleme göre %14 'e kadar artarak yıllık ortalama 5090 Wh/m2-day ışınım değerine ulaşmaktadır. Bölgenin sahip olduğu enleme özgü optimum açı değerinin tahmine yönelik 4 farklı değişkenden 8 farklı matematiksel model geliştirilmiştir. Bu modellerin istatiksel olarak belirlilik katsayısı (R2) aralığı 0.9068 ile 0.9964 aralığında değişkenlik göstermektedir. Bu bulgular neticesinde uygulanabilirliğini ve güvenilirliğini kanıtlanan regresyon modeli aynı enlemdeki farklı koordinatlardaki konumlara örnek teşkil edebilir. Ayrıca sonuçlar, önerilen modelin mühendislik tasarımı, eko-enerji analizi ve optimum tasarım süreçlerinde kullanım açısından akademik ve sanayicilere örnek teşkil etmektedir.

Anahtar Kelimeler: *Optimizasyon, güneş enerjisi, PV paneller, eğim açısı, enerji.*

1 Introduction

Throughout the year, due to the Earth's movement within the solar system and its rotation around its axis, the angle at which the sun's rays fall at any point constantly changes. Data used in solar applications are typically calculated based on radiation data hitting a perfectly flat plane. To make the most effective use of solar energy, the tilt angles of the solar panels must be accurately determined. This is especially important in areas with varied weather patterns, as the position of the panels must be adjusted to best capture the available light. Calculations of the tilt angles must be precise in order to ensure maximum energy absorption. In regions where local weather conditions differ greatly, sophisticated software is used to calculate the optimal tilt angles for the solar panels. Accurate information is critical for achieving a higher return on investment in solar energy technology.

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The solar panel tilt angle value refers to the angle at which solar panels could be positioned in a way that maximizes their exposure to sunlight and optimizes their energy generation. The optimum tilt angle is an important factor in maximizing the efficiency of solar panels and optimizing their energy generation. Based on various factors such as solar radiation, latitude, and historical data, various studies have been carried out to develop algorithms and models for calculating the optimum angle of inclination [1–4].

One of these studies, Calabrò (2013), proposed an algorithm for determining the optimum angle of inclination of a solar panel, based on data on solar radiation in the horizontal plane. With this algorithm, by the inclination and orientation of the panel, the theoretical maximum energy generation value is determined. In the study, a linear regression correlation was found between the optimum angle of inclination and latitude. However, he noted that the standard error of mean values increased significantly with latitude, but was unreliable at high latitudes [5]. Soulayman et. al. (2016) presented a set of algorithms for determining the optimum angle of inclination and azimuth angle [(43.45ºS, 66.45ºS) & (43.45ºN, 66.45ºN)] over the mid-latitude regions at the North and South poles. Thus, he developed a mathematical model to estimate solar radiation on an oblique surface and to determine the optimum angle of inclination and orientation for the solar collector at any latitude [6]. Kallioğlu et. al. (2020) examined the empirical calculation of the optimal tilt angle for solar collectors in the northern hemisphere. In this study, primarily for three sample provinces (Antalya, Kayseri, and Trabzon) and then for the Northern Hemisphere, the optimum solar panel angle value was determined according to the months [7]. Sharma et. al., 2021, calculated the optimum tilt angles for different months to maximize solar radiation in the solar collector in the Western Himalayan (L 31° 42′ N) region. In addition, correlation models for the optimal tilt angle specific to the region have been derived [8]. Nassar et. al. (2023) re-analyzed the optimum tilt angle values for 24 different locations in Libya, according to variable albedo values. He states that the correlations created with the data obtained in the study can be used throughout the whole country of Libya [9].

As can be seen from the literature summary, by measuring the radiation coming to the inclined surfaces, the maximum radiation value can be reached by determining the optimum angle.

As a result, monthly, seasonal, and annual optimum panel tilt angles for solar energy applications have been determined or correlative connections have been developed according to regions. This study was conducted with the selection of Van Province (Turkey) as the target area, given that it has a solar energy potential that surpasses the average of Turkey [10]. The purpose of this study was to ensure that the investments in solar energy made in the region were as effective and profitable as possible. By researching the current state of the solar energy sector in the region, the study aimed to identify the most suitable and efficient ways to take advantage of the benefits this renewable energy source can provide. With the right implementation strategy, investments in solar energy can prove to be highly beneficial for all stakeholders involved. The findings of the study can be used to maximize the efficiency of investments in the solar energy sector and to ensure maximum profitability in the long run. In addition, we aimed to create a sustainable model by applying the results calculated with the mathematical equations developed specifically to the region and statistical error tests. In addition, the results are intended to be an exemplary roadmap for the proposed model to be used in engineering design, eco-energy analysis, and optimal design processes.

2 Material and Method

The solar radiation values falling on the PV panels vary according to the tilt angle of the panel compared to full flat (horizontal). In practice, analyses are often carried out with data received from the horizontal plane (β=00) at a given position, while light components falling on an inclined surface are not measured. Getting maximum efficiency from solar panels is possible by positioning them at optimum angles in monthly, annual, and even seasonal intervals. This optimum value varies according to the latitude, the terrain conditions surrounding the panel, the geometry of the solar rays, the movements of the earth on its axis and its orbit around the sun, and the clearness index [11]. In this study, the province of Van (Turkey), located in the temperate zone at coordinates 38°50' N-43°38' E and with an altitude of 1661 m, was analyzed. The average annual temperature of the province is 9.5 (°C), the annual sunshine time is 7.9 hours and the average annual solar radiation value coming to the horizontal plane is 4482 Wh / m^2-day [12]. According to the international climate classification Koppen, the Van region is characterized as "Dsa" Mediterranean-influenced, hot-summer, humid continental climate.

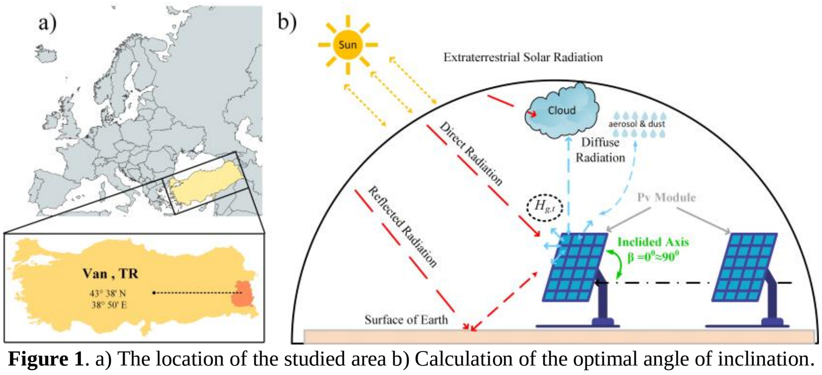

Figure 1. a) The location of the studied area b) Calculation of the optimal angle of inclination.

The monthly, mean daily global irradiance value (H_T) falling on an inclined plane can be calculated using Equation 1. This equation takes into account the latitude, declination, hour angle, sky clearness factor, and tilt angle of the inclined plane, allowing for a precise determination of the irradiance value. The sky clearness factor is especially important, as it accounts for any attenuation of solar radiation due to clouds, haze, dust, or other factors, which can vary significantly depending on the geographic region. With this equation, engineers and scientists can accurately measure the amount of solar radiation energy exposure on any given inclined plane, providing an important tool for understanding and harnessing the power of the sun. This equation consists of the sum of the values of direct solar radiation (H_{Beam}), reflected radiation ($H_{Reflected}$), and scattered radiation ($H_{Scatter}$). In addition, at variable panel tilt angles, it receives different radiant values [13].

$$H_{\mathbf{Total}} = H_{\mathbf{Beam}} + H_{\mathbf{Reflected}} + H_{\mathbf{Scatter}} \tag{1}$$

In order to estimate the solar radiation falling on the panel, various mathematical models were used and in this study, the isentropic open scattering model, which is one of the widely used and easily applied models in the literature, was used [14]. Setting off from equation 1, Equation (2), (3), (4), (5), (6), (7), (8), (9), (10), (11), (12), [15].

$$H_B = (H - H_d)R_b \tag{2}$$

$$R_b = \frac{\cos(\emptyset - \beta)\cos(\delta)\sin(\omega_s') + \omega_s'\left(\frac{\pi}{180}\right)\sin(\emptyset - \beta)\sin}{\cos(\emptyset)\cos(\delta)\sin(\omega_S) + \omega_S\left(\frac{\pi}{180}\right)\sin(\emptyset)\sin(\delta)} \tag{3}$$

$$H_R = \frac{H\,\rho\,(1 - \cos\beta)}{2} \tag{4}$$

$$H_S = H_d\,R_d \tag{5}$$

$$H_d = H\left(1 - 1{,}13\frac{H}{H_0}\right) \tag{6}$$

$$H_o = \left(\frac{24}{\pi}\right)G_{sc}\,k\left[\cos\emptyset\cos\delta\sin\omega_s + \left(\frac{\pi}{180}\right)\sin\emptyset\sin\delta\omega_s\right] \tag{7}$$

$$k = 1 + 0{,}033\cos\left(360\frac{n}{360}\right) \tag{8}$$

$$\delta = 23{,}45^{\circ}\sin\left(360\frac{n + 284}{365}\right) \tag{9}$$

$$\omega_s = \cos^{-1}(-\tan\emptyset.\tan\delta) \tag{10}$$

$$\omega_s' = \min\left[\frac{\omega = \cos^{-1}(-\tan\emptyset\tan\delta)}{\cos^{-1}(-\tan(\emptyset - \beta)\tan\delta}\right] \tag{11}$$

$$R_d = (1 + \frac{\cos\beta)}{2}) \tag{12}$$

The general solar angle calculation equations used in calculating the incoming solar radiation on the inclined surface are shown. The Diffuse radiation calculation used in Equation 2 and Equation 5 is the amount of diffuse radiation in the horizontal plane in Equation 6. In this equation, the amount of radiation emitted from the sky is assumed to be uniform and is an isotropic model [13,16,17]. The albedo value in Equation 4 is taken as 0.2 (soil).

3 Result

3.1 Calculation of the Optimal Panel Tilt Angle

In this study, the measured meteorological data of the Electrical Works Survey Administration (EIE 1991-2020) of the General Directorate of State Meteorological Affairs were used [18]. In the calculations, the optimum point was determined from the total solar radiation values at different angles (0^0-90^0) of the solar panel, utilizing Equation 1-12.

Table 1. Calculation results of Van Province.

Months	H_0	H	H_d	K_T	$\beta_{OPT}(^0)$	H_T
Jan.	5113	1910	1104	0.37	56	2852
Feb.	6459	2840	1429	0.44	48	3727
Mar.	8162	4140	1767	0.51	37	4863
Apr.	9904	5130	2127	0.52	20	5351
May	11116	6630	2162	0.60	6	6652
Jun.	11603	6990	2232	0.60	0	6990
Jul.	11338	7160	2051	0.63	1	7160
Aug.	10346	6050	2052	0.58	16	6213
Sep.	8767	5250	1697	0.60	33	5978
Oct.	6941	3640	1483	0.52	50	5118
Nov.	5419	2310	1197	0.43	56	3487
Dec.	4735	1730	1016	0.37	59	2698

Table 1. shows the values belonging to the province of Van. In this chart, H is the average monthly irradiance amount falling on the horizontal plane (Wh/m^2-day), H_0 is the average non-atmospheric monthly irradiance (Wh/m^2-day), H_d is the monthly average diffuse radiation falling on the horizontal plane (Wh/m^2-day). The optimum panel tilt angle values are $\beta_{OPT}(^0)$ and the amount of radiation falling on the corresponding panel at this angle value is H_T (Wh/m^2-day). These values are shown monthly.

When Chart 1 is analyzed, the optimum angle values and the maximum and minimum points of radiation falling on the panel are 0^0 and 6680 (Wh/m^2-day) in June and 59^0 and 2698 (Wh/m^2-day) in December, respectively.

Upon examination of the region as a whole, it has been determined that the optimal angle values range from 0^0 to 59^0. It has been observed that the highest angle values occur in December and January, whereas the lowest angle values are observed in June and July. Furthermore, it has been noted that the angle values decrease from December to June and increase from June to December. This phenomenon can be attributed to the fact that on June 21st, the sun's rays hit the Northern Hemisphere perpendicularly, and on December 21st, they hit the Northern Hemisphere at the greatest angle. This can be further clarified by the counter graph presented in Figure 2.

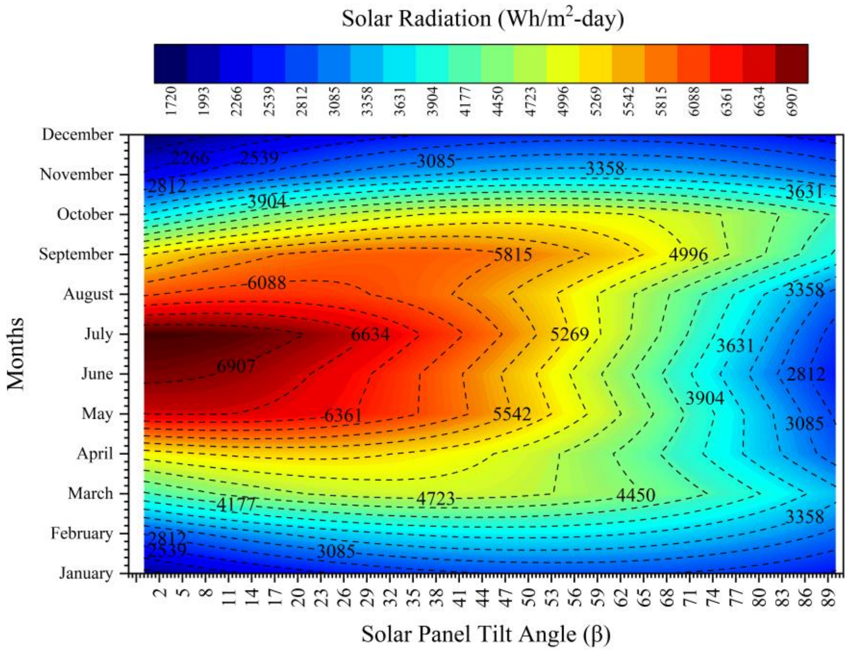

Figure 2. The change of tilt angle-radiation correlation according to months.

3.2 Optimum Angle Prediction Modelling

The developed models specific to the regions where the calculation was performed are shown in Figure 3. In these models, 8 different equations have been developed for finding the optimal tilt angle. In the developed correlational expressions, correlation models were formed with the variables of declination angle (δ), extraterrestrial radiation (Ho), solar radiation coming from the horizontal plane (H), and diffuse radiation (H_d). The models developed specifically for Van (TR) province are given in the form of 13-20. To evaluate the performance of the developed forecasting models, there are many statistical test methods. The most common among these are; determinacy coefficient (R^2), mean deviation error (MBE), square root error (RMSE), and t-test method (t-stat) [7].

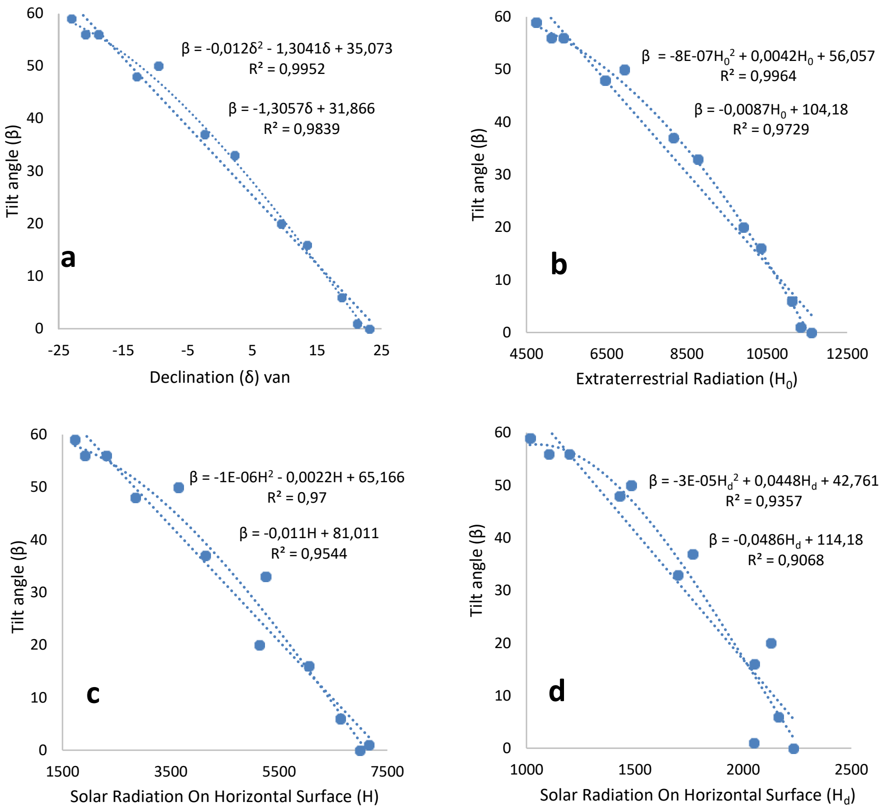

Figure 3. Models specifically designed for Van have been developed. a)δ b)H_0 c)H d)Hd

An in-depth analysis of the correlation between the estimated results of the models developed specifically for the Van region and the calculated values was conducted using various statistical methods. The results of this analysis revealed a strong correlation between the two sets of values, indicating that the models developed were accurate in predicting the outcomes of the region. Furthermore, the models were able to explain any discrepancies between the estimated values and the calculated values, thereby providing further evidence that the models were effective in their predictions. Ultimately, these findings conclusively demonstrate the effectiveness of the models developed for the Van region. A total of 12 models were developed consisting of linear and 2nd-order polynomial correlations. The panel angle of these correlations was the 2nd-degree polynomial, with the best equality in the estimate being 16.

$$\beta = -1.3057(\delta) + 31.866 \quad (13)$$

$$\beta = -0.012(\delta)^2 - 1.3041(\delta) + 35.073 \quad (14)$$

$$\beta = -0.0087(H_0) + 104.18 \quad (15)$$

$$\beta = -0.0000008(H_0)^2 + 0.0042(H_0) + 56.057 \quad (16)$$

$$\beta = -0.011(H) + 81.011 \quad (17)$$

$$\beta = -0.000001(H)^2 - 0.0022(H) + 65.166 \quad (18)$$

$$\beta = -0.0486(Hd) + 114.18 \quad (19)$$

$$\beta = -0.00003(Hd)^2 + 0.0448(Hd) + 42.761 \quad (20)$$

Table 2. Monthly inclination angle values are estimated with the developed equations.

	Equations							
	13	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Jan.	59	57	60	57	60	57	61	56
Feb.	49	50	48	50	50	51	45	46
Mar.	35	38	33	37	35	39	28	28
Apr.	20	22	18	19	25	28	11	2
May	7	6	7	4	8	7	9	-1
Jun.	2	-1	3	-3	4	1	6	-7
Jul.	4	2	6	1	2	-2	15	8
Aug.	14	15	14	14	14	15	14	8
Sep.	29	32	28	31	23	26	32	32
Oct.	44	46	44	47	41	44	42	43
Nov.	57	55	57	55	56	55	56	53
Dec.	62	59	63	58	62	58	65	57

The coefficient of determination with which the statistical analyses were examined, the best (R^2) values, the declination angle (δ) solar radiation from outside the atmosphere (Ho), solar radiation from the horizontal plane (H), and diffuse radiation from the horizontal plane, were respectively, 0.9952, 0.9964, 0.9700 and 0.9356, which are reasonably close to the number "1". In addition to being simple and usable, the location-specific models developed are also powerful in providing statistics. Thus, it is possible to determine the optimum panel angle thanks to meteorological data in cases where it is needed.

Table 3. Statistical analysis of the developed mathematical equations.

Equations	MBE	RMSE	t-statistic	R^2
13	-0.0002	2.7293	0.0003	0.9839
14	-0.0056	1.4872	0.0124	0.9952
15	0.0826	3.5379	0.0774	0.9729
16	1.0220	1.7546	2.3765	0.9964
17	0.1207	4.5928	0.0872	0.9544
18	0.2767	3.7418	0.2459	0.9700
19	-0.0652	6.5647	0.0329	0.9068
20	4.5315	7.3837	2.5781	0.9356

4 Conclusion

The best inclination angle of PV solar modules and solar collectors has been determined to maximize the efficiency of solar energy systems. In this study, Van province, which has a high solar energy investment potential in the eastern Anatolia region of Turkey, was preferred. Thus, reducing energy generation costs will increase the chances of competitiveness with other energy alternatives. However, the forms used to determine the optimal tilt angle in the current literature are either not accurate or require complex calculations. Therefore, this study proposed a new formula to determine the appropriate tilt angle of solar panels for Van province of Turkey and the optimal angle has been determined.

- When the regions were examined in general, the angles of inclination varied according to the months, with the lowest being 0^0 and the highest being around 59^0 degrees.
- The optimum fixed panel angle was measured at 31.83^0 per year, 54.33^0 in Winter, 21.00^0 in Spring, 5.66^0 in Summer, and 46.33^0 in Autumn.
- The yield that can be obtained with the optimum monthly panel angle change varies between 0% and 55.95%. At constant annual value, a yield increase of 13.59% was achieved.
- The highest performance of the developed models was obtained from Equation 16 and its prediction capacity is significantly high.
- All proposed correlations have shown that it can efficiently calculate the optimal tilt angle with high accuracy and therefore can be used safely.
- The data collected from calculations done in a region at latitude (38 $N^0 \pm 3^0$) can be extended to other locations with similar latitudes. Considering the same geographical characteristics, the data can be extrapolated and applied in other areas in the same latitude range. As a result, the region in question can serve as a model for predicting results in similar locations, thus providing valuable insight into the environment of the region.
- Specifically, these regions are comprised of Tabriz in Iranian, Lisboa in Portugal, Napoli in Italy, Athens in Greece, Algiers in Algeria, Los Angeles in the USA, Tashkent in Uzbekistan, Baku in Azerbaijan, Tunis in Tunisia, and Tianjin in China.

Upon analyzing the results, it is evident that by adjusting the panel tilt angles to the value of each month, it is possible to maximize the benefits of solar radiation. This method of optimization takes into account the seasonal fluctuations of the sun's position in the sky and ensures that the solar panels are positioned in the most effective way to absorb as much energy as possible. With this strategy in place, solar energy can be utilized more efficiently and effectively. By maximizing the efficiency of solar energy usage, the cost of production and consumption can be reduced, while also helping to protect the environment. Therefore, it is clear that adjusting panel tilt angles to the value of each month is a beneficial strategy for those looking to make the most of solar energy. In situations where monthly angle adjustment is not an option, it is beneficial to rearrange the system annually or seasonally, at a fixed angle, depending on the purpose of the system. This will allow for the best possible use of the available solar radiation. This adjustment should be done carefully, taking into consideration the local weather patterns and the desired output of the system. By doing this, the solar energy system can be optimized for the most efficient use of energy. For example, if a system is to be installed for a seasonal business that needs to operate with maximum efficiency in the spring or autumn seasons, it is expected that calculating the slope angle accordingly will provide serious economic benefit and efficiency for the particular business in question.

Declaration

The authors declare that the ethics committee approval is not required for this study.

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