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Controller Designs for Nonlinear Systems with Application to 3 DOF Helicopter Model

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Abstract

The paper introduces different control design methods for nonlinear systems with their application to the nonlinear mathematical model of a 3 DOF helicopter. Recent studies on the control design approaches, such as State Dependent Riccati Equation based optimal controller, Model Reference Adaptive Control and Sliding Mode Control, are reviewed and design methodologies are given. Then the mathematical model of a 3 DOF helicopter dynamics is derived and simplified with the software called CIFER. By using the simplified mathematical model of the 3 DOF helicopter, the control algorithms are applied to the model in order to explore the performances of the controllers. The results are given in the simulation environment. The paper is presented in a tutorial manner in order to explain the applications of the proposed nonlinear control methodologies.

1. INTRODUCTION

A 3 DOF helicopter system is designed to research control design methods especially for aerospace studies. In recent years, many researchers use the 3 DOF Helicopter system to test different control algorithms in real-time applications. A supervisory safety controller design for a 3 DOF helicopter was proposed in [1]. A nonlinear dynamic model and linearized model were developed and LQR controller was designed using linearized model for the supervisory controller. LQR based PID controller for a 3 DOF helicopter was discussed in [2]. Based on successive linearization, a model predictive controller was designed in [3]. The reference [4] used Fuzzy Logic and LQR techniques to control the 3 DOF helicopter system. Another method so-called, "explicit model predictive approach" was used for regulation and tracking of 3 DOF helicopter in [5]. The system has nonlinear dynamical model and strongly coupled so that researchers studying nonlinear control techniques tend to use 3 DOF helicopter. A nonlinear adaptive control system was designed and tested in [6]. The reference [7] used robust adaptive LQR control method for 3 DOF helicopter.

In this paper, in order to explore different controller design techniques for nonlinear systems, 3 different control approaches namely "State Dependent Riccati Equation (SDRE)", "Model Reference Adaptive Control (MRAC)" and "Sliding Mode Control (SMC)" methods are designed for a 3 DOF helicopter. State-Dependent Riccati Equation (SDRE) method has become popular since the mid-90's because of providing an effective control algorithm for suboptimal control of nonlinear systems. SDRE first proposed by Pearson [8] and expended in time. In that method, nonlinear dynamics are rearranged and modeled in the form of state vector and the product of a matrix-valued function depends on the state itself so that nonlinear dynamic model is brought to linear-like structure having state dependent coefficient (SDC) matrices [9]. To obtain

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control law, algebraic Riccati equation using the SDC matrices which varying with states and are updated at each time step is solved. Therefore, state dependent algebraic Riccati equation is used for control algorithm. SDRE method has been used for control of different systems. SDRE-based optimal control of cancer dynamics was firs proposed by Itik and Salamci [10]. Çimen documented SDRE methodology and implemented the SDRE method to different systems [9], [11].

Sliding Mode Control (SMC) is well defined for Linear Time Invariant (LTI) Systems in references [12], [13] and [14]. The objective of SMC design is to force the system to a predefined sliding surface which passes through the origin. The sliding surface is defined as the system exhibits stable behavior on it. The sliding surfaces, which are designed for nonlinear systems are either varying with time or nonlinear based on nonlinear dynamics. To design sliding surface optimally, different control methods such as SDRE method can also be used [15].

Model Reference Adaptive Control (MRAC) is one of the main adaptive control approaches to track a reference model with known mathematical model and parameters. The main idea of MRAC methodology is to force the output of the uncertain plant to track the desired output of reference model by updating controller gains. The aim of adjusting the controller gains is to set the tracking error, which is the difference between the output of plant and reference model, zero. MRAC Studies has a wide range of practical applications and MRAC can be combined with other control methodologies to get better performance and to cope with compelling circumstance. For this purpose, SDRE based MRAC architecture is one of the researched areas of it. Stabilizing uncertain nonlinear systems approach has been offered and used by Babaei et al. [16]. The control design of nonlinear reference model is based on SDRE methodology for the MRAC of a parametrically uncertain nonlinear plant in this study.

In this paper, SDRE, MRAC and SMC approaches for tracking control of a 3 DOF helicopter are presented. 3 DOF helicopter system description and nonlinear dynamic model of the system is given in Section 2. Dynamic model rearrangement for using control algorithms and controllers' theories are represented in Section 3. In section 4, simulation results for each control method are conducted. Finally, conclusions are given in Section 5.

2. 3 DOF HELICOPTER MATHEMATICAL MODEL

3 DOF helicopter system is designed to track and regulate the elevation and travel angles of the 3 DOF helicopter that is supplied with a complete mathematical dynamic model, a sampled state-feedback controller and the system parameters. Therefore, the objective of this paper being to develop nonlinear controllers to track elevation and travel angles of the 3 DOF helicopter.

2.1. System Description

Quanser 3 DOF helicopter system is an experimental set-up that basically simulate tandem rotor helicopter model. The system which is shown in the Figure 1 has three degree of freedom about elevation "θ", pitch "ϕ" and travel "ψ" axes and also powered by two individual DC motors attached propellers which are mounted as tandem rotor helicopter.

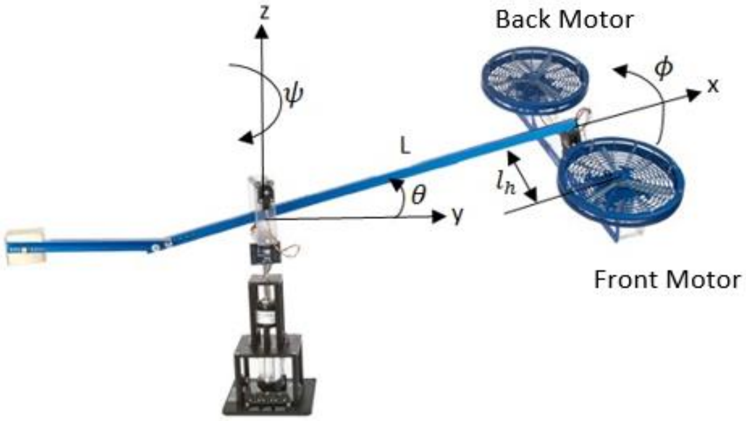

Figure 1. *An illustration of the 3 DOF helicopter [1]*

Motor-propeller systems generate the necessary force according to voltage output of the controller in order to drive model around the axes which is mentioned above. The helicopter frame is suspended from an instrumented joint mounted at the end of a long arm and is free to pitch about its center. The arm is installed on an additional 2 DOF instrumented joint which allows the helicopter body to move in the elevation and travel directions. A counterweight is mounted to the other end of the arm to change the effective mass of the helicopter. Three attitude angles (elevation angle "θ", pitch angle "ϕ" and travel angle "ψ") are measured by using three encoders. More information about 3 DOF helicopter system can be obtained from the Quanser 3 DOF Helicopter User Manual such as motor specifications, encoders specifications, parameters [17].

2.2. Nonlinear Dynamic Model of the 3 DOF Helicopter

In this section, the main nonlinear dynamics of the 3 DOF helicopter using to design controllers is given. The nonlinear dynamic model is derived at the horizontal trim position that center of gravity aim to define at the elevation encoder location where the inertial coordinate axis is placed. That allows Coriolis terms are vanished in order to computational simplicity.

The nonlinear dynamic equations of motion are derived using the Newton's second law of motion (see [1] for details). The equations are then given by [1];

$$J_{yy}\ddot{\theta} = -Mgl_{\theta}\sin(\theta+\theta_0) + (T_L+T_R)L\,cos(\phi)$$
$$J_{xx}\ddot{\phi} = (T_L-T_R)l_h - mgl_{\phi}\,sin(\phi)$$
$$J_{zz}\ddot{\psi} = (T_L+T_R)L\,cos(\theta)\,sin(\phi) + (-T_L+T_R)l_h\,sin(\theta)\,sin(\phi) - Drag$$

where

$$T_L = C_T^L\rho(\Omega_L R)^2\pi R^2$$
$$T_R = C_T^R\rho(\Omega_R R)^2\pi R^2$$
$$Drag = \frac{1}{2}\rho\big(\dot{\psi}L\big)^2(S_0 + S'_0\,sin(\phi))$$

R is the radius of the helicopter blades
Ω is the frequency of the rotating blades (L = left motor and R = right motor)
C_T is the thrust coefficient given by Equation 3.157 of Padfield [22]
S_0 and S'_0 are the effective drag coefficients times the reference area
M is the total mass of the helicopter assembly
m is the mass of the rotor assembly

The nonlinear equations of motion may be simplified by using system identification approaches. Such an identification method is studied in [1] which is called as Comprehensive Identification FrEquency Responses (CIFER) tool developed by NASA. A simplified nonlinear model of the 3 DOF helicopter is then given by [1]

$$\ddot{\theta} = -d_1\dot{\theta} - d_2 \sin(\theta) + d_3\tau_{coll} \cos(-\phi) \quad (1)$$
$$\ddot{\phi} = -b_1(\dot{\phi}) + b_2 \sin(-\phi) - b_3\tau_{cyc} \quad (2)$$
$$\ddot{\psi} = -a_1\dot{\psi} + a_2(\alpha\tau_{coll} + 1) \sin(-\phi) \quad (3)$$
$$\dot{\tau}_{cyc} = -c_1\tau_{cyc} + c_2 \frac{(V_b - V_f)}{2} \quad (4)$$
$$\dot{\tau}_{coll} = -e_1\tau_{coll} + e_2 \frac{(V_b + V_f)}{2} \quad (5)$$

where

θ : Elevation angular position
ϕ : Pitch angular position
ψ : Travel angular position
$\dot{\theta}$: Elevation angular rate
$\dot{\phi}$: Pitch angular rate
$\dot{\psi}$: Travel angular rate
τ_{cyc} : Cyclic thrust
τ_{coll} : Collective thrust

First three equations represent the Euler dynamics of rotation and the other two yields the force generated according to motor voltage. The parameters of the equations are obtained by using the CIFER parameter identification tool.

Parameters are obtained by CIFER tool are given Table 1:

Table 1. *Model parameters [1]*

Parameters	a_1	a_2	b_1	b_2	b_3	c_1	c_2
Value	0.2517	0.2105	0.3290	1.5664	16.2	7.32	1

Parameters	d_1	d_2	d_3	e_1	e_2	α
Value	0.1011	0.504	1.34	6.16	1	4

3. CONTROLLER DESIGNS

This section describes extended linearization, also known as SDC parameterization of the dynamic model of 3 DOF helicopter, augmentation of SDC matrices and SDRE controller design, MRAC controller design and SMC controller design for the tracking of 3 DOF helicopter.

All controller design methods use extended linearization in this paper. Extended linearization is the process of a nonlinear system into a linear-like structure which represented by state-dependent coefficient (SDC) matrices. Before defining State Dependent Coefficient matrices, state space model of the system must be determined with the corresponding states,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} = \begin{bmatrix} \theta \\ \phi \\ \psi \\ \dot{\theta} \\ \dot{\phi} \\ \dot{\psi} \\ \tau_{cyc} \\ \tau_{coll} \end{bmatrix} \tag{6}$$

And the inputs; front motor voltage, back motor voltage

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} V_f \\ V_b \end{bmatrix} \tag{7}$$

Therefore, state-space model is defined as;

$$\dot{x}_1 = x_4 \tag{8}$$
$$\dot{x}_2 = x_5 \tag{9}$$
$$\dot{x}_3 = x_6 \tag{10}$$
$$\dot{x}_4 = -d_1 x_4 - d_2 \sin(x_1) + d_3 x_8 \cos(-x_2) \tag{11}$$
$$\dot{x}_5 = -b_1(x_5) + b_2 \sin(-x_2) - b_3 x_7 \tag{12}$$
$$\dot{x}_6 = -a_1 x_6 + a_2(\alpha x_8 + 1) \sin(-x_2) \tag{13}$$
$$\dot{x}_7 = -c_1 x_7 + \frac{c_2}{2} u_2 - \frac{c_2}{2} u_1 \tag{14}$$
$$\dot{x}_8 = -e_1 x_8 + \frac{e_2}{2} u_1 + \frac{e_2}{2} u_2 \tag{15}$$

Every single term of the model must be state dependent. If model consist of any independent term as trigonometric functions or logarithmic functions that couldn't be separated or defined as the factor of the state variables, some manipulations has to be done before SDC matrices are determined. Inside the mentioned model there exist of trigonometric term as;

$$sin(-x_2) \tag{16}$$
$$sin(x_1) \tag{17}$$

could be defined as;

$$sin(x_1) = \left[\frac{sin(x_1)}{x_1}\right] x_1 \tag{18}$$

which,

$$\frac{sin(x_1)}{x_1} = sinc(x_1) \tag{19}$$

is called cardinal sinus function and goes to "1" while "x_1" converging absolute zero. At the end, these terms are shaped like the factor of states to the SDC parametrization. If "M" and "L" newly described as;

$$M = \frac{\sin(-x_2)}{x_2} \tag{20}$$
$$L = \frac{\sin(x_1)}{x_1} \tag{21}$$

The state-space model of the system becomes;

$$A(x) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -d_2L & 0 & 0 & -d_1 & 0 & 0 & 0 & d_3\cos(-x_2) \\ 0 & b_2M & 0 & 0 & -b_1 & 0 & -b_3 & 0 \\ 0 & a_2(\alpha x_8+1)M & 0 & 0 & 0 & -a_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -c_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -e_1 \end{bmatrix} \tag{22}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\frac{c_2}{2} & \frac{c_2}{2} \\ \frac{e_2}{2} & \frac{e_2}{2} \end{bmatrix} \tag{23}$$

where it can be rearranged as;

$$\dot{x} = A(x)x + Bu \tag{24}$$

SDC parametrized model of the system that has to behave similar to state-space model without losing any information of original dynamics. Because of physical nature of the system, there exist coupled dynamics between pitch and travel and also pitch and elevation axes. The system can be described as under-actuated system because it has only two individual actuators according to three independent dynamics. Therefore, only elevation and travel axes can be driven by reference commands.

In order to perform reference tracking, the SDC parametrized system is augmented as [18];

$$\hat{A}(e) = \begin{bmatrix} A(x) & 0 \\ -C & 0 \end{bmatrix} \quad \hat{B}(e) = \begin{bmatrix} B(x) \\ 0 \end{bmatrix} \tag{25}$$

where for 3 DOF helicopter system:

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{26}$$

Yields error dynamics as;

$$\dot{e} = \hat{A}(e)e + \hat{B}(e)u_e \tag{27}$$

Controllers can be designed by using error dynamics form for tracking of the 3 DOF helicopter. Nonlinear system is stable at every local point which is designed as LTI by the compensation of the full state feedback controller. Unfortunately, it is not possible to measure all of the states for the 3 DOF helicopter. Experimental system has three related encoders in every axis which is only able to measure position of the helicopter. At this point basic first order filters are designed for the purpose of obtaining angular rates by using position data. However, first order filters are not required for simulation based studies and direct derivative of position data can be used to obtain angular rates.

The same situation has been occurred while feeding back the thrust states,

$$\begin{matrix} \tau_{cyc} \\ \tau_{coll} \end{matrix} \tag{28}$$

This is handled by computing the numerical integration of first order thrust dynamics that augmented in state space model.

3.1. SDRE Control Design

SDRE method has become a popular method for suboptimal control of nonlinear systems amongst control researchers in recent years because of a systematic and effective procedure for nonlinear control [19].

Consider a nonlinear system, which can be represented in SDC form as,

$$\dot{x} = A(x)x + B(x)u \tag{29}$$

where $x \in R^n, u \in R^m$ and the SDC matrices are $A(x) \in R^{nxn}$ and $B(x) \in R^{nxm}$. The SDC representation must be a controllable and observable parameterization of the nonlinear system. Therefore, two sufficient tests is to be performed [11].

Controllable of the system can be check by state dependent controllability matrix,

$$M_c(x) = [B(x) \vdots A(x)B(x) \vdots \cdots \vdots A^{n-1}(x)B(x)] \tag{30}$$

has $rank(M_c) = n$ at all x. Similarly, observable of the system can be checked by state dependent observability matrix,

$$M_o = \left[C^T(x) \vdots A^T(x)C^T(x) \vdots \cdots \vdots \left(A^T(x)\right)^{n-1} C^T(x)\right] \tag{31}$$

has $rank(M_0) = n$ at all x.

The objective of the control is to minimize the following cost function,

$$J = \frac{1}{2}\int_0^{\infty} \{x^T Q(x)x + u^T R(x)u\}\, dt \tag{32}$$

where $Q(x)$ and $R(x)$ are state-dependent weighting matrices. However, weighting matrices can be selected constant for simplicity of control design. $Q(x)$ a symmetric positive semi-definite matrix and $R(x)$ is a symmetric positive definite matrix.

To minimize the above cost function, a state feedback control law can be given by

$$u = -K(x)x = -R^{-1}(x)\, B^T(x)\, P(x)x \tag{33}$$

where $K(x)$ is the optimal gain matrix and changes with states. $P(x)$, that is the unique, symmetric and positive-definite solution of the State-Dependent Riccati equation, is obtained from algebraic State-Dependent Riccati equation given by

$$P(x)A(x) + A^T(x)P(x) - P(x)B(x)R^{-1}(x)B^T(x)P(x) + Q(x) = 0 \tag{34}$$

The closed loop dynamics of the nonlinear system is then given by

$$\dot{x} = \left(A(x) - B(x)K(x)\right)x = A_{cl}(x)x \tag{35}$$

where $A_{cl}(x) = \big(A(x) - B(x)K(x)\big)$ satisfies the pointwise Hurwitz condition $Re\ \lambda_i\big(A_{cl}(x)\big) < 0$ [15].

SDC parametrized system is to be augmented and error dynamics model is used (which is represented by Eq.27) to design SDRE controller for tracking of 3 DOF helicopter.

3.2. SDRE based MRAC Design

The experimental setup which is called plant has involved unknown parameters and/or inadequate modelled dynamics. MRAC is a control method based on a reference model whose desired output is taken as a reference trajectory for the plant. In this method, the deviation between the output of plant and reference model is known as tracking error, e. The controller gains are varied by updated online and they are depended on tracking error which is changed considerably by reference command. The main idea of the control design is to adapt the response of the reference model to the plant in order to do that the plant output tracks reference model output asymptotically and e→0 as t→∞ [20]. Repetitively estimation of controller gains is performed using an adaptive law which is derived from the Lyapunov Stability Theorem in this study. Therefore, parameter adaptation law is created by using a positive-definite Lyapunov function.

3.2.1. SDRE based MRAC for uncertain systems with full state feedback

The control design of nonlinear reference model is based on SDRE methodology for the MRAC of parametrically uncertain nonlinear plant in this study. Consider the following nonlinear system as reference:

$$\dot{x}_m(t) = A_m(x_m)x_m(t) + B_m(x_m)u_m(t) \tag{36}$$

where $x_m(t)\ \epsilon\ \mathbb{R}^n$ is the state vector, $A_m(x_m)\epsilon\ \mathbb{R}^{nxn}$ and $B_m(x_m)\epsilon\ \mathbb{R}^{nxq}$ are system dynamics and input SDC matrices respectively for the reference model over and above they are completely known. $u_m(t)\epsilon\ \mathbb{R}^q$ is a bounded external input. The plant is nonlinear system described by:

$$x(t) = A(x)x(t) + B(x)u(t) \tag{37}$$

where $x(t)\ \epsilon\ \mathbb{R}^n$ is the state vector which are all measurable, $A(x)\ \epsilon\ \mathbb{R}^{nxn}$ and $B(x)\ \epsilon\ \mathbb{R}^{nxq}$ are possibly unknown SDC system matrices while $\{A(x),\ B(x)\}$ pair is pointwise controllable and $u(t)\epsilon\ \mathbb{R}^q$ is the control input for the plant. When the SDC matrices are evaluated for a given state vector via the SDRE based method, the nonlinear reference model is regarded as a pointwise LTI which is similar to a frozen system at the state vector. Therefore, the control input is allowed to design with well-known approaches for LTI systems for each state vector [20].

The control input is designed to provide desired states of the reference model, $x_m(t)$, is tracked by the plant states, $x(t)$, as closely as possible and the tracking error, which is defined as $e(t) \triangleq x(t) - x_m(t)$, approaches zero asymptotically. So, the control law can be constituted with SDRE method as follows:

$$u_m(x_m) = -K_m(x_m)x_m \tag{38}$$

where $K_m(x_m)$ is control gain matrix. The control gain matrix is determined for asymptotic stability of pointwise LTI system with LQR strategy. Regulated closed-loop system for the reference model with desired state is in the form

$$\dot{x}_m(t) = [A_m(x_m) - B_m(x_m)K_m(x_m)]x_m(t) = A_{mcl}(x_m)x_m(t) \tag{39}$$

where $A_{mcl}(x_m)$ is pointwise Hurwitz. We consider the following assumptions [20]:

$\{A_m(x),\ B_m(x)\}$ pair is pointwise controllable for reference model.
There exist a $K^*\epsilon\ \mathbb{R}^n$ such that matching conditions of $A_{mcl}(x_m) = A(x) - B(x)K^{*T}$ holds.
There exist a positive definite matrix $G\ \epsilon\ \mathbb{R}^{qxq}$, such that $\hat{B}(x) = B(x)G$ is known.

It should be noted that if the algebraic equation given in the second assumption is satisfied, perfect model following (matching condition) is achieved. By taking into consideration the assumptions, control law for the uncertain nonlinear plant dynamic is proposed as follows:

$$u(t) = -K(x,t)x(t) \tag{40}$$

where $K(x,t)$ is control gain matrix of plant. The adaptation rule for adjusting $K(x,t)$ matrix is created by using the Lyapunov Stability Theorem for the purpose of the error dynamics,$\dot{e}(t)$, which are converged to zero. On the other hand the control parameter error, which is defined $\tilde{K}(x,t) \triangleq K(x,t) - K^*$, is constituted for the error dynamics. The adaptation law is proposed as follows:

$$\dot{K}(x,t) = \Gamma_k x e^T P(x_m)\hat{B}(x),\ \ k(0) = k_0 \tag{41}$$

where Γ_k is positive-valued adaptation rate which is determine the rate of convergence of error to zero and $P(x_m) = P^T(x_m) > 0$ (positive-definite matrix) satisfies the following algebraic Lyapunov equation for stable closed-loop reference model given in (36) for some $Q(x_m) = Q^T(x_m) > 0$ as follows:

$${A_{mcl}}^T(x_m)P(x_m) + P(x_m)A_{mcl}(x_m) = -Q(x_m) \tag{42}$$

It should not be forgotten that the values of $K(x,t)$ are calculated at each iteration in the SDRE based MRAC, so the initial conditions for $K(x,t)$ are updated in every step which is started k_0. Also, constant Q can be selected instead of the state dependent $Q(x_m)$ but chosen matrix must be positive-definite and symmetric in both cases.

Finally the error dynamics, $\dot{e}(t)$, is derived as follows:

$$\dot{e}(t) = \dot{x}(t) - \dot{x}_m(t) = A_{mcl}(x_m)e(t) + B(x)\,\tilde{K}(x,t)\,x(t) \tag{43}$$

3.3. SDRE based SMC Design

Trajectory tracking control is designed for the 3 DOF Helicopter by using Sliding Mode Control technique. The sliding surface is designed to be time varying so that the nonlinear dynamics of the 3 DOF helicopter model is to track elevation and travel axes.

Sliding mode control is applied that the nonlinear model of the 3 DOF helicopter is frozen at each time step so that the nonlinear model can be behaved as Linear Time Invariant System. Sliding surface can be determined by solving State Dependent Ricatti Equation at each time step. The sliding mode controller is designed to move state to sliding surface and to keep states on the moving sliding surface [15].

After sliding motion starts, system dynamic is not affected from parameter changes and disturbances. Sliding mode control can be described within two phases, one is called Reaching phase and the other is called as sliding motion.

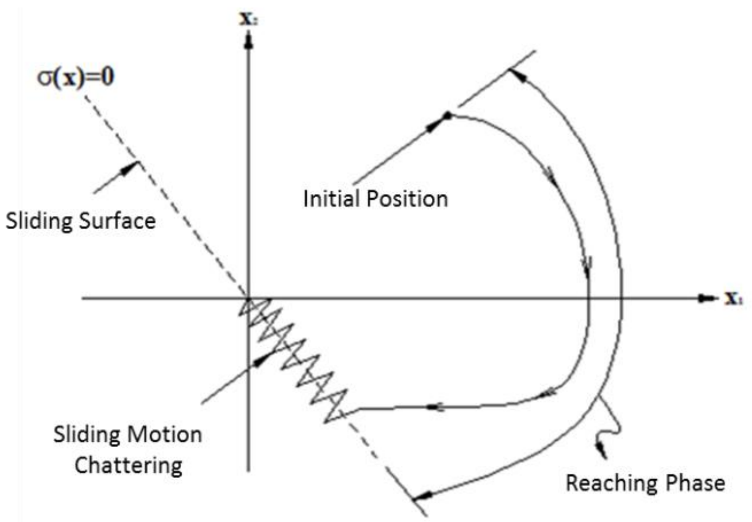

Figure 2. *Phases of sliding mode control*

The system in Eq.44 needs to be transformed into two subsystems to apply sliding mode control. The first subsystem needs to be independent from controller input and second one needs to be dependent from controller input [14], [15].

$$\dot{x} = A(x)x(t) + B(x)u(t), A \in \mathcal{R}^{nxn}, B \in \mathcal{R}^{nxm} x \in \mathcal{R}^{n}, u \in \mathcal{R}^{m} \quad (44)$$

To apply transformation, the system needs to be separated to n-m subsystem, which has controller input and m number subsystem which has no controller input. As well as, the inverse of transformation matrix needs to be calculated.

Transformation matrix can be computable from controllability matrix, Q and weight matrix, W [14].

$$T(x) = (QW)^{-1} \quad (45)$$

The system states can be transform to z coordinate system by using transformation matrix.

$$z(t) = T(x)x(t), x(t) = T(x)^{-1}z(t) \quad (46)$$

$$z(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix}, z_1 \in \mathcal{R}^{n-m}, z_2 \in \mathcal{R}^{m} \quad (47)$$

The system can be transformed into z coordinate system.

$$\dot{z}(t) = T(x)[A(x)T(x)^{-1}z(t) + B(x)u(t)] = T(x)A(x)T(x)^{-1}z(t) + T(x)B(x)u(t) \quad (48)$$

$$A_Z(z) = \begin{bmatrix} A_{11}^z(z) & A_{12}^z(z) \\ A_{21}^z(z) & A_{22}^z(z) \end{bmatrix} \quad (49)$$

$$B_z(z) = T(x)B(x) = \begin{bmatrix} 0 \\ B_2^z(z) \end{bmatrix} \quad (50)$$

The transformed system in matrix form:

$$\dot{z}(t) = \begin{bmatrix} \dot{z_1}(t) \\ \dot{z_2}(t) \end{bmatrix} = \begin{bmatrix} A_{11}^z(z) & A_{12}^z(z) \\ A_{21}^z(z) & A_{22}^z(z) \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B_2^z(z) \end{bmatrix} u(t) \quad (51)$$

The first subsystem without controller input:

$$\dot{z}_1(t) = A_{11}^z(z)z_1(t) + A_{12}^z(z)z_2(t) \quad (52)$$

The second subsystem with controller input:

$$\dot{z}_2(t) = A_{21}^z(z)z_1(t) + A_{22}^z(z)z_2(t) + B_2^z(z)u(t) \quad (53)$$

The sliding surface equation can be described as

$$\sigma(z) = z_2(t) + Cz_1(t) \quad (54)$$

For controllability, $\sigma(z)\dot{\sigma}(z) < 0$ needs to be satisfied. For this purpose, sliding surface scope can be computed by using SDRE, Eq.55 to subsystem 1 [21].

$${A_{11}^z}^T z + zA_{11}^z - zA_{12}^z {A_{12}^z}^T z + Q = 0 \quad (55)$$

The controller input has two parts, one is equivalent control, other is regulator control.

$$u(t) = u_c(t) + u_{eq}(t) \quad (56)$$

where the equivalent control is

$$u_{eq}(t) = -B_2^z(z)^{-1}\left\{\left(A_{21}^z(z)z_1(t) + A_{22}^z(z)z_2(t)\right) + C\left(A_{11}^z(z)z_1(t) + A_{12}^z(z)z_2(t)\right) + \dot{C}z_1(t)\right\} \quad (57)$$

where regulator control is

$$u_c(t) = -B_2^z(z)k\, sgn(\sigma) \quad (58)$$

The problem of the SMC is chattering due to regulator control. The k amplitude determines the reaching phase time and chattering amplitude. To eliminate chattering problem, there are different type of solutions. Instead of signum function, Eq.58 saturation function, Eq.59 and hyperbolic tangent function, Eq.60. A different approach to minimize the chattering problem, the regulator control input can be dependent to errors of states Eq.61.

$$u_c(t) = -B_2^z(z)k\, sat(\sigma) \quad (59)$$

$$u_c(t) = -B_2^z(z)k\, tanh(\sigma) \quad (60)$$

$$u_c(t) = -B_2^z(z)k_a(r(t)^2)(|e(t)| + \varepsilon)\tanh\left(k_s \frac{\sigma(t)}{|e(t)|+\varepsilon}\right) \quad (61)$$

4. SIMULATION RESULTS

In this section, simulation results of tracking of 3 DOF helicopter controlled via SDRE, MRAC and SMC controller are given. All three controller design methods, using in this paper, have same desired state steps for same axes of motion, same (100 Hz) sampling time, same (Runge-Kutta Method, ODE 4) solver and same (125 s) simulation time.

Desired state steps are given,

Elevation angle;
1. Step: 5 degrees at 25th s
2. Step: 10 degrees at 40th s
3. Step: -15 degrees at 65th s
4. Step: -5 degrees at 85th s
5. Step: 10 degrees at 110th s

Travel angle;
1. Step: 15 degrees at 10th s

2. Step: 25 degrees at 40th s
3. Step: -30 degrees at 65th s
4. Step: -10 degrees at 100th s

All the simulations obtained in this studied are performed by using the SIMULINK toolbox of MATLAB. A typical Simulink blocks are represented below in Figures 3-6;

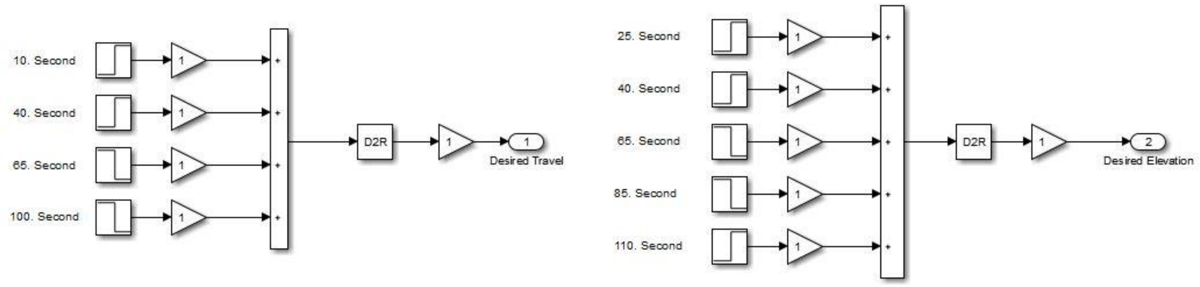

Figure 3. *Desired States Design Scenario*

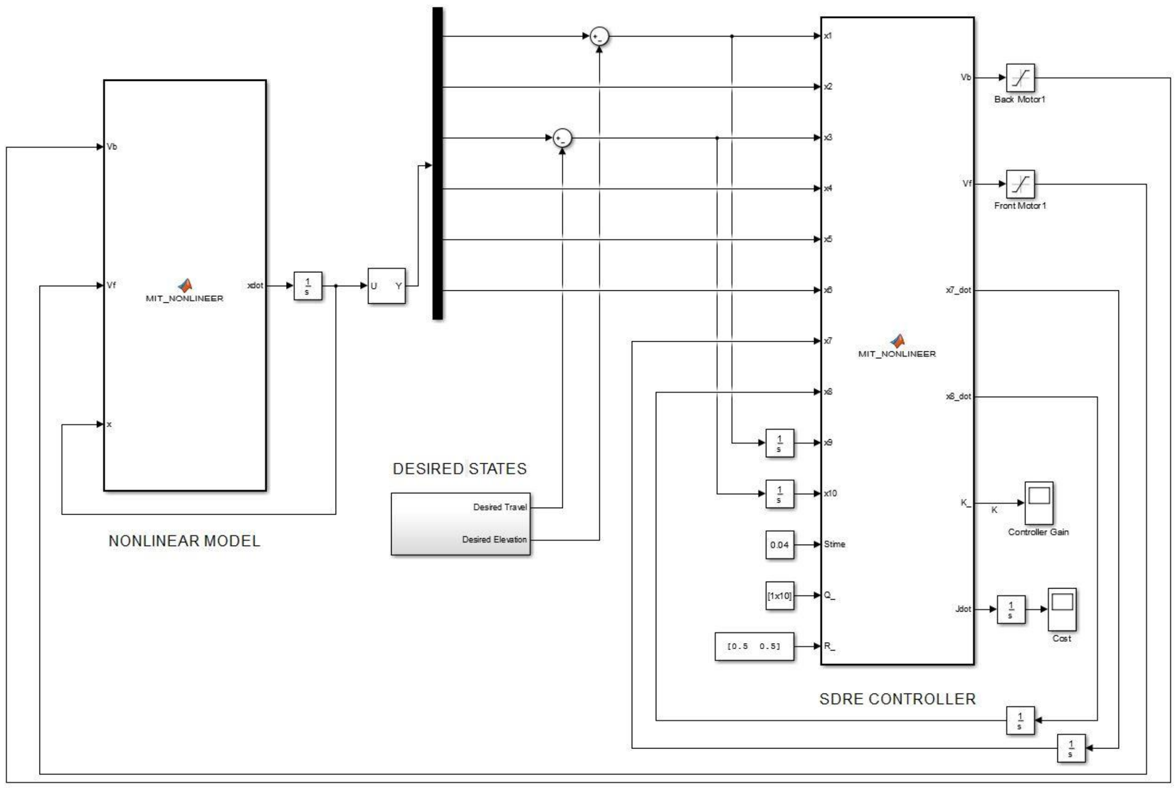

Figure 4. *SDRE Controller Design Scheme with Simulink*

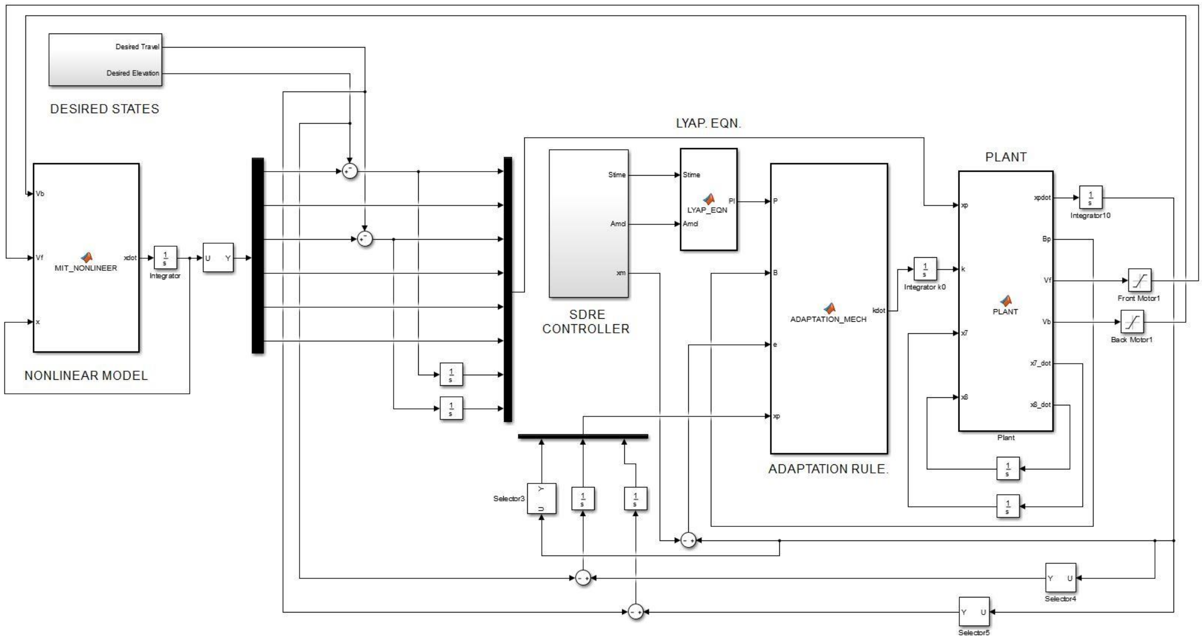

Figure 5. *SDRE Based MRAC Controller Design Scheme with Simulink*

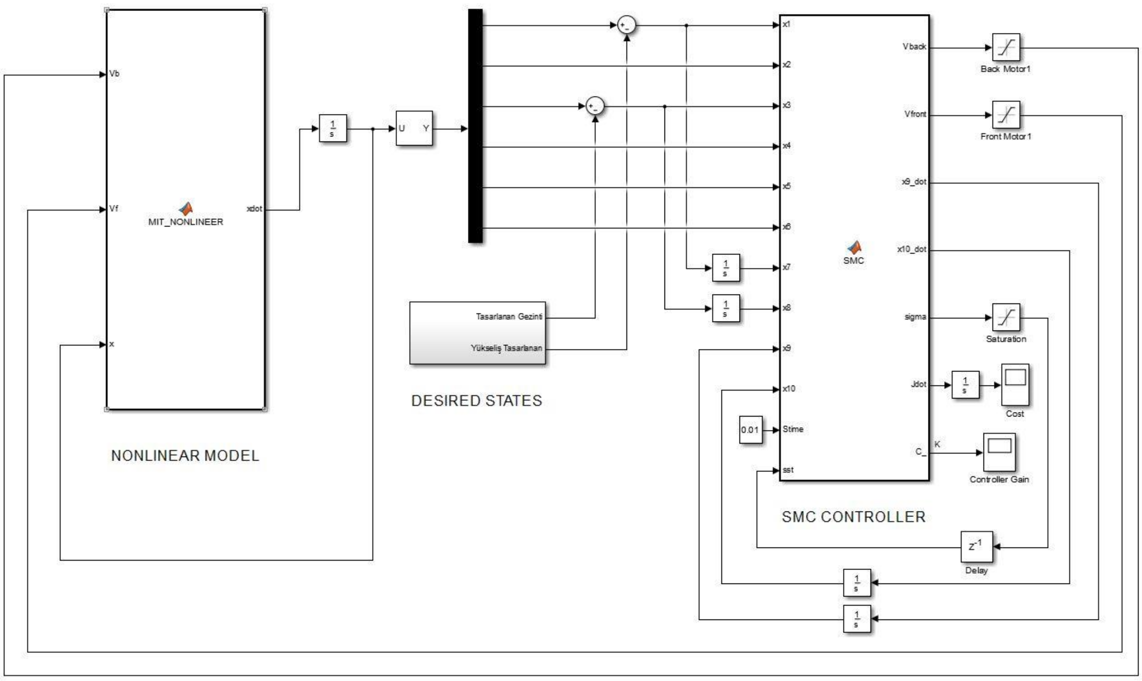

Figure 6. *SDRE Based SMC Controller Design Scheme with Simulink*

4.1. SDRE Results

In this paper, weighting matrices for tracking of 3 DOF helicopter selected as,

$$Q = \begin{bmatrix} 40 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 15 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}, \quad R = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

Figure 7 illustrate the system time responses, namely elevation, travel and pitch angles, to the desired step commands. It is seen that the SDRE control is capable of controlling the helicopter dynamics. The desired elevation and travel commands are followed by the 3 DOF Helicopter as expected. The settling time for each of the step command is small enough to recover the Helicopter for another step commands.

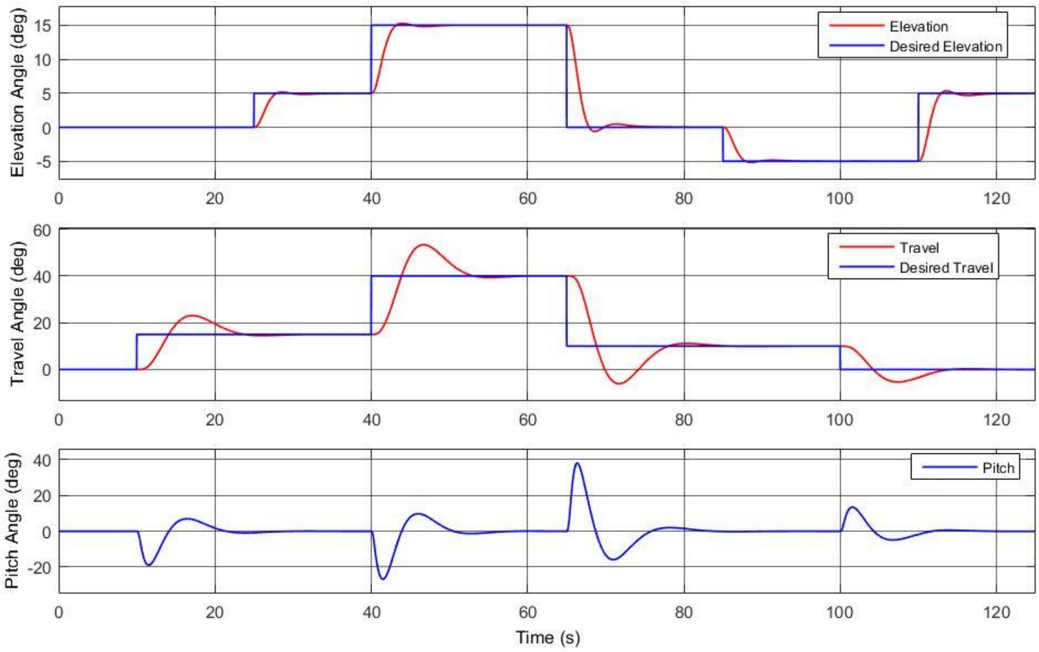

Figure 7. *System Responses of 3 DOF Helicopter for SDRE method*

Figure 8, on the other hand, give the control input signal obtained from the proposed SDRE control design. From the figures, it is seen that the control signal is between ±5 Volts which are compatible with the motors input voltages. It is seen that the voltages vary during the step command changes.

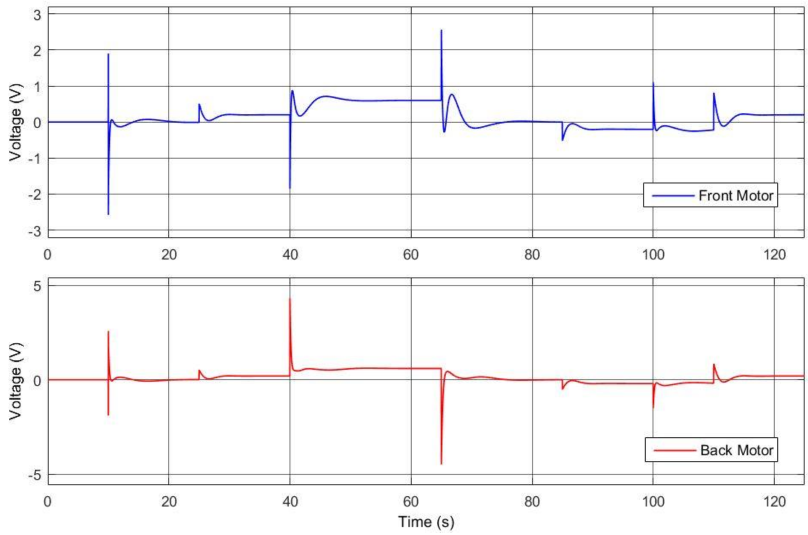

Figure 8. *Input voltages for SDRE method*

4.2. MRAC Results

In this paper, weighting matrix and adaptation rate for tracking of MRAC method are selected as,

$$Q = \begin{bmatrix} 100 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \ \Gamma = 10^{-6}$$

Figure 9 illustrate the system time responses, namely elevation, travel and pitch angles, to the desired step commands for the Model Reference Adaptive Control. It is seen that the MRAC approach is also capable of controlling the helicopter dynamics. The desired elevation and travel commands are followed by the reference model (labeled as “Model”) and the 3 DOF Helicopter (label as “Plant” in the figures). The overshoots are seen in the time responses. However, the desired commands are successfully followed by both the “Model” and the “Plant”.

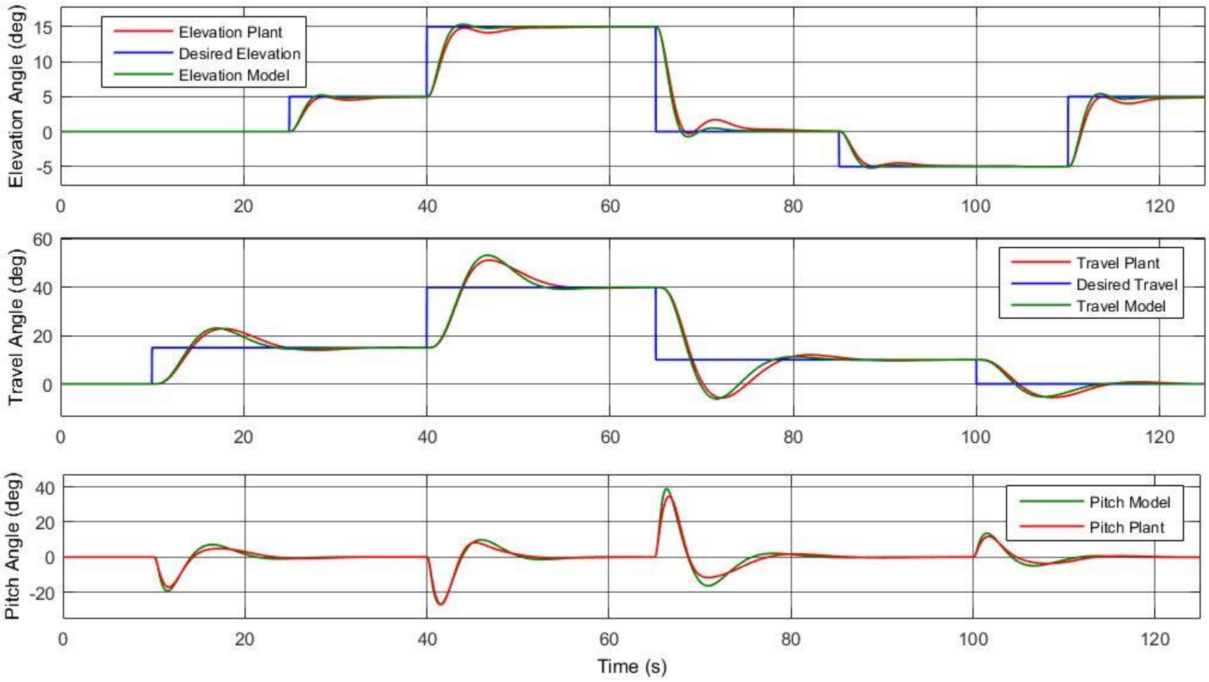

Figure 9. *System Responses of 3 DOF Helicopter for MRAC method*

Figure 10 give the control input signal obtained from the proposed MRAC design. The control signals are again between ±5 Volts which are compatible with the motors input voltages. Similar to the SDRE control cases, the voltages vary during the step command changes.

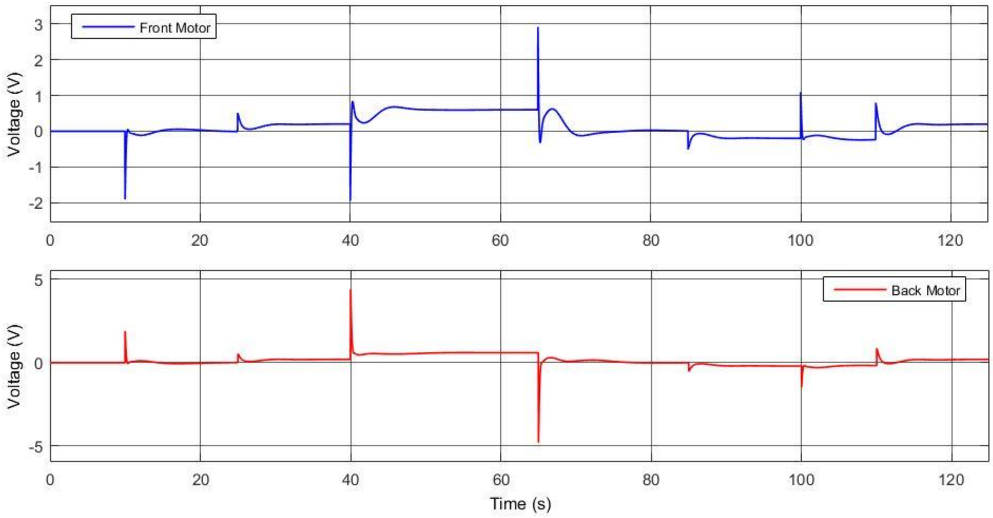

Figure 10. *Input voltages for MRAC method*

4.3. SMC Results

For the SMC design proposed in this paper, the following weighting matrices and parameter are selected as follows,

$$Q(x)=\begin{bmatrix}40&0&0&0&0&0&0&0\\0&2&0&0&0&0&0&0\\0&0&200&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&10&0&0&0\\0&0&0&0&0&10&0&0\\0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&10\end{bmatrix},\quad R=\begin{bmatrix}0.5&0\\0&0.5\end{bmatrix},\qquad k=4$$

Figure 11 illustrate the system time responses, namely elevation, travel and pitch angles, to the desired step commands. The SDRE based SMC is capable of controlling the helicopter dynamics as well. The proposed control approach is capable of controlling the helicopter dynamics. The desired elevation and travel commands are followed successfully. It should be noted that the time responses of the system can be changed by using different control parameters. For instance, the time response for the travel angle could be better shaped with different parameters.

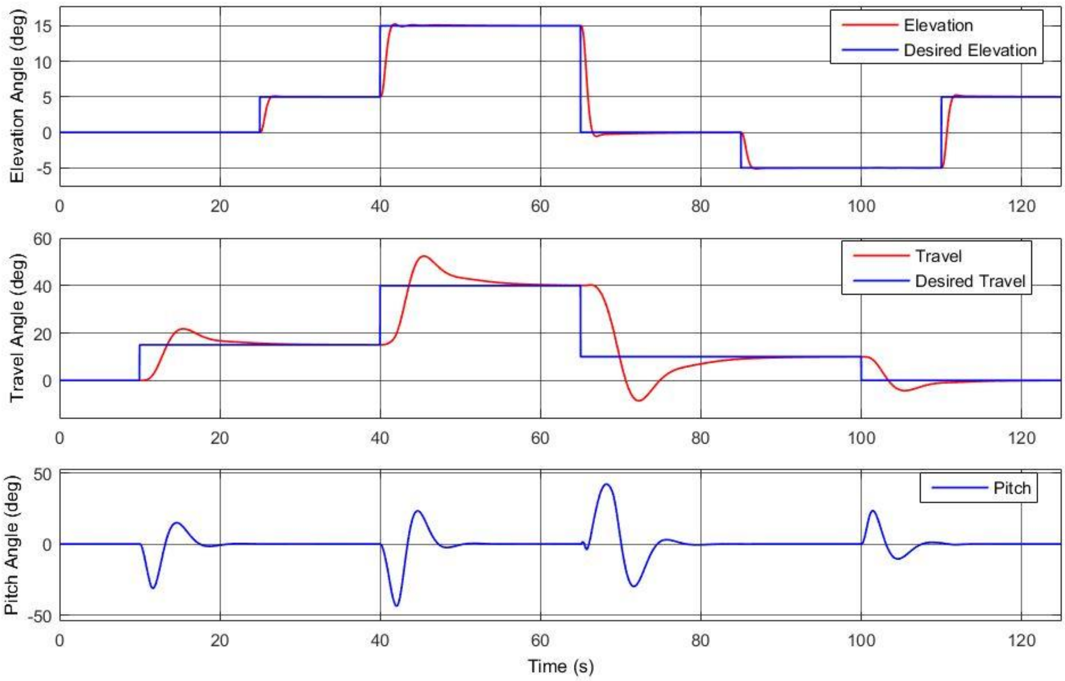

Figure 11. *System Responses of 3 DOF Helicopter for SMC method*

Figure 12, on the other hand, give the control input signal obtained from the proposed SMC design. Unlike the previous control actions, the control signals are again between ±10 Volts which are compatible with the motors input voltages.

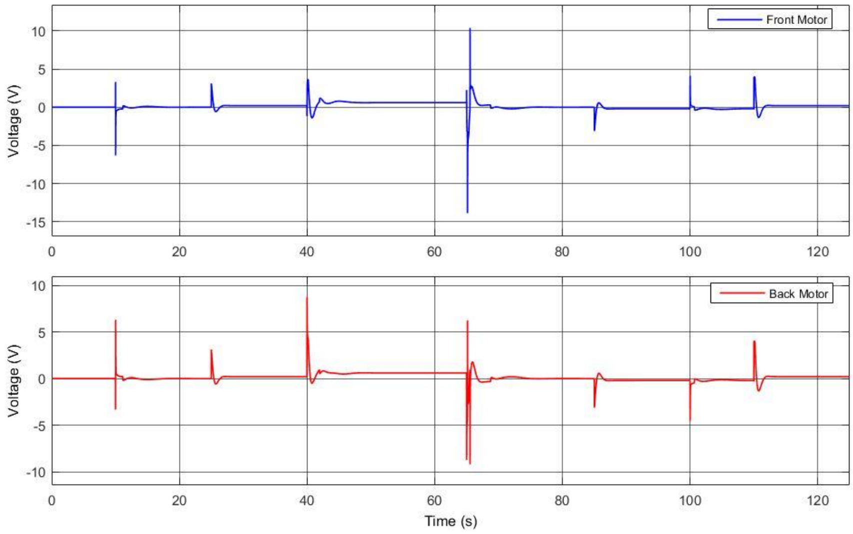

Figure 12. *Input voltages for SMC method*

5. CONCLUSION

This paper introduces 3 popular nonlinear controller design methodologies in a systematic manner. The control methods studied in this paper are “State Dependent Riccati Equation based Optimal Control”, “Model Reference Adaptive Control” and “Sliding Mode Control”. The designed control approaches are applied to a 3 DOF Helicopter dynamics for tracking purposes. The 3 DOF experimental helicopter setup has a state-dependent nonlinear dynamic feature. SDRE technique and robust controllers are synthesized to get much better control performance and robustness against nonlinearity and parametric uncertainties. The simulation results clearly show that these three nonlinear controller methods are sufficient enough to achieve the goal which is decent performance characteristic.

CONFLICT OF INTEREST

No conflict of interest was declared by the authors

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