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The New Diffusion Tensor and the Equatorial Anomaly Altitudes of F-Region

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Abstract

In this study, the difference ($\Delta D = D_{(\omega \neq 0)} - D_{(\omega = 0)}$) of the classical diffusion tensor (D_0, D_1, D_2 for both steady-state and unsteady-case) for F- region of the ionosphere at the equatorial region is investigated by taking (**B**=B_0**z)**, the geometry of the Earth's magnetic field and by neglecting the neutral winds velocity (U). The measurements were made in the height (280,300,340,390 and 410 km) where the equatorial anomaly was observed predominantly. The magnitudes of the difference of classical diffusion tensor for electrons are $\Delta D_0 > \Delta D_1 > \Delta D_2$ for all seasons with respect to both 12.00LT and 24.00LT at F-region. However, ΔD_1 and ΔD_2 are bigger during nighttime than daytime and show a behavior reverse to the change with latitude of electron density in the magnetic equator. It is possible to say that the behavior of them can be the result of electromagnetic drift and dynamo effect.
Keywords: Ionosphere, Diffusion Tensor, Equatorial F- Region

1. INTRODUCTION

The ionosphere is defined as a part of the upper atmosphere of the Earth, which stretches from 50 km to about 1000 km [1, 2]. This region is filled with ionized gas called plasma. Ionosphere is a function of electron density as vertical but it is horizontally very complicated due to the event called as the *equatorial anomaly* composing of low latitudes, equatorial region, middle and high latitudes. Equatorial anomaly that is and unexpected situation reaches on both sides of the magnetic equator between 17ºS-17ºN latitudes [1, 3, 4, 5]. Ionospheric plasma events could be analyzed using macroscopic momentum transport equations not only by considering the plasma as a multi-constituent fluid but also by treating the plasma as a single conducting fluid. There is force of pressure gradients that are associated with the existence of either density gradients or temperature gradients, or both in flooded plasma. This motion of the electrons, induced by pressure gradients, is called diffusion [6]. The diffusion subject has been studied by a lot of authors both theoretically and empirically in the ionosphere plasma [3, 7-13]. Actually, the diffusion studies holds up until the beginning of the 20th century in ionospheric plasma. Mariani, F.(1956) obtained diffusion equations including the thermal diffusion theoretically in 1956 after Ferraro's approximations in non-isothermal ionosphere. Fiala, V. (1963) conducted a study which examined the non-potential component of the electric field diffusion of inhomogeneity which drifts due to the motion of the neutral component of the ionospheric plasma or external electric field. The findings of his work showed that when considering the non-potential electric field, it depended weakly on the drift velocity of an inhomogeneity and the parameters related to the ionosphere was omitted for the ambipolar diffusion. The same researcher published a detailed paper on diffusion in anisotropic ionosphere and resolved the momentum and continuity equation by using some approximations and did some corrections in the diffusion equation in 1967. In 2011, Pavlov, A. V. and Pavlova, N. M. showed that the general expression for thermal diffusion and diffusion correction factors in an ion would be simplified by using Grad's 13-moment approximation in multicomponent partially ionized plasma. Besides, Bohm-Type Coefficient of Diffusion was used by Dominguez, H. J. Quantum Mechanical methods in plasma [14]. One of the recent studies on the diffusion subject in the ionospheric plasma is the one which was conducted by Sagir et al., 2014. They suggested a new solution that established a relationship between the electrical conductivity and diffusion equation and it applied to ionospheric plasma for mid-latitudes. The vast majority of authors studied the diffusion coefficients of not only electrons but also minor ions as the frequency independent for mid-latitudes in the ionosphere plasma, and their findings showed that electrons and a number of minor ions resulted from a temperature gradient in gas or from a relative drift between the major ion gases.

Unlike the above mentioned studies, in this study we considered that diffusion coefficients were actually frequency-dependent ($\omega \neq 0$; this is a driver force as all of the fields, and velocity depends on frequency) "for unsteady-case" in the ionospheric plasma and low-latitudes where the electron density shows a pronounced through centered on the magnetic dip equator. We obtained the basis diffusion coefficients (D_0, D_1 and D_2) when $\omega \neq 0$ and applied in F-region of the low latitude ionosphere for some conditions. It can be said that when $\omega \neq 0$, the magnitude of the basis diffusion coefficients is bigger than $\omega=0$ for considering conditions.

2. The Diffusion Tensor In The Ionospheric Plasma

The behavior of the ionosphere plasma is subject to the equation of state and the general conversion equations for mass, momentum and energy when it is considered as fluid [1, 2 6, 15].

If it does not have impact outside, the transport from place to place of the particles in the ionospheric plasma results from the pressure-gradient (∇P). This force occurs in any part of the plasma density to eliminate inhomogeneity. If $\mathbf{B} \neq 0$, the medium is called as anisotropic. Thus, the ionospheric plasma can be accepted to be anisotropic [16].

If $\mathbf{U}$ and $\mathbf{m}$ are the velocity and mass of the electron, then the force acting on the electron is as follows:

$$m\frac{d\mathbf{U}}{dt} = -e(\mathbf{U}\times\mathbf{B}) - m\nu\mathbf{U} \qquad (2.1)$$

where $\nu = \nu_{ei} + \nu_{en}$, and,

$$\nu_{ei} = N\left[59 + 4.18\log\left(\frac{T_e^3}{N}\right)\right]\times 10^{-6} T_e^{-3/2}\,[m.k.s]$$

and $\nu_{en} = 5.4\times 10^{-16} N_n T_e^{1/2}\,[m.k.s]$

are the electron-ion and electron-neutral collision frequencies. If the velocity and fields vary as $e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$ where ω is angular wave frequency and ω_c is electron angular gyro frequency as follows:

$$\omega_c = \frac{-eB}{m}$$

The z-axis of the coordinate system with its origin located on the ground is vertical upwards (this is only an acceptance). The diffusion tensor of the solution of Eq.(2) is obtained as the diffusion coefficient depending on the real geometry of the earth:

$$\mathbf{\Gamma} = \mu(\mathbf{\Gamma}\times\mathbf{B}) - D\nabla n \qquad (2.2)$$

where, $\mu = \dfrac{-e}{m\nu}$ is the electron mobility,

$$D = \frac{k_b T}{m\nu}$$

is the electron diffusion coefficient, and $\mathbf{\Gamma}$=(n$\mathbf{U}$) is the flux of density. The flux of density in terms of the current density is as follows. Here, the gradient-density vector that depends on the flux of density could be written as follows:

$$D\nabla n = \mu(\mathbf{\Gamma}\times B) \qquad (2.3)$$

From here, the diffusion tensor;

$$D = \begin{bmatrix} D_1 & D_2 & 0 \\ -D_2 & D_1 & 0 \\ 0 & 0 & D_0 \end{bmatrix} \qquad (2.4)$$

where (for steady-state case; $\omega=0$),

$$D_0 = \frac{k_b T}{m\nu},\; D_1 = \frac{\nu^2}{\nu^2+\omega_c^2}D_0 \text{ and } D_2 = \frac{\nu\,\omega_c}{\nu^2+\omega_c^2}D_0$$

For unsteady case; $\omega \neq 0$; the elements of diffusion tensor

$$D_0 = \frac{k_b T}{m(\nu - i\omega)},\; D_1 = \frac{(\nu-i\omega)^2}{(\nu-i\omega)^2+\omega_c^2}D_0,\; D_2 = \frac{(\nu-i\omega)\,\omega_c}{(\nu-i\omega)^2+\omega_c^2}D_0$$

The differences of diffusion coefficients are obtained by;

$$\Delta D_0 = -\frac{k_b T\, Z^2}{m\nu\,(1+Z^2)} + i\frac{k_b T\, Z}{m\nu\,(1+Z^2)} = \Delta D_{0R} + i\Delta D_{0S} \qquad (2.5)$$

$$\Delta D_1 = \left(\frac{k_b T}{m\nu}\right)\left[\frac{(1+Y^2+Z^2)}{(1+Y^2-Z^2)^2+4Z^2} - \frac{1}{1+Y^2}\right] + i\left(\frac{k_b T}{m\nu}\right)\left[\frac{2Z-(1+Y^2-Z^2)Z}{(1+Y^2-Z^2)^2+4Z^2}\right] = \Delta D_{1R} + i\Delta D_{1S} \qquad (2.6)$$

$$\Delta D_2 = \left(\frac{k_b T}{m\nu}\right)\left[\frac{Y(1+Y^2-Z^2)}{(1+Y^2-Z^2)^2+4Z^2} - \frac{Y}{1+Y^2}\right] + i\left(\frac{k_b T}{m\nu}\right)\left[\frac{2YZ}{(1+Y^2-Z^2)^2+4Z^2}\right] = \Delta D_{2R} + i\Delta D_{2S} \qquad (2.7)$$

In which;

$$Y = \frac{\omega_c}{\omega} \quad \text{and} \quad Z = \frac{\nu}{\omega}$$

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Here, R refers to the real part, S refers to the imaginary part, k_b refers to the Boltzmann constant, T refers to the electron temperature, m refers to the electron mass.

3. Numerical Analysis and Results

The general system of transport equations is applied to the low-latitude for ionosphere F2 region. The restriction to this region of the ionosphere enables us to make several simplifying assumptions that significantly reduce the general system of transport equations. It is fully ionized plasma composed of two major ions, electrons, and a number of minors [1,2,4,6,16]. There are various theories explaining equatorial anomaly. The most important one is Martin's theory dragged upwards by diffusion of plasma. Besides, in this theory, the equatorial anomaly extends on both sides of the magnetic equator between 30°S-30°N latitudes [3]. Therefore, this study examined these latitudes.

The difference ($\Delta D = D_{(\omega\neq 0)} - D_{(\omega=0)}$) of the classical diffusion tensor (D_0, D_1 and D_2 for both steady and unsteady-case) at the equatorial F2-region of ionosphere plasma was examined seasonally (both equinox (March 21 and September 23) and during the solstice (June 21 and December 21) by taking ($B=B_0z$) the geometry of Earth's magnetic field for local time (LT) 12.00 and 24.00. The examination was made in the elevation (280, 300, 340, 390 and 410 km) where the equatorial anomaly was observed [1]. The results were obtained for I (dip angle)=55.6°, d (Declination)=3°, R=159 by using Eqs. (2.1)-(2.7). The ionospheric parameters used for calculations were obtained using the IRI (International Reference Ionosphere) model.

The difference of the classical diffusion tensor has a complex mathematical structure as $\omega\neq 0$. It is possible to say that the imaginary part of the difference of diffusion coefficients related to slowing of the average electrons and ions velocity directly depends on the electrons' or ions' mobility. We calculated the magnitude of the tensor elements of the diffusion difference in the accepted conditions and investigated in different seasons (on both equinox and solstice days).

$$\Delta D = \begin{bmatrix} \Delta D_1 & \Delta D_2 & 0 \\ -\Delta D_2 & \Delta D_1 & 0 \\ 0 & 0 & \Delta D_0 \end{bmatrix} \quad (3.1)$$

Scatter of ΔD_0: The difference of diffusion coefficient the parallel to the magnetic field (ΔD_0) was seasonally investigated at the critical altitudes (280, 300, 340, 390 and 410 km) for ionospheric F2-region in the equatorial region. According to a part of Figure 1-4, (ΔD_0) is the order 10^{10}[m²/s] for all seasons (both equinox and solstice days) at both 12.00 and 24.00 LT[4]. The magnitude of ΔD_0 was bigger at 24.00 LT than 12.00 LT for both equinox and solstice days. It had generally a peak between 0^0-20^0 N latitudes at 12 .00 LT. Besides, the diagram of ΔD_0 is similar to March 21-December 21 and June 21- September 23 for both 12.00 LT and 24.00 LT. But, the magnitude of ΔD_0 has markedly been decreasing between 0-30^0 S latitudes for 12.00 and 24.00 LT. It increased sharply having a peak value at 10^0N latitude at 24.00 LT and then it decreased dramatically and it had a minimum value 20^0N at 24.00 LT.

Scatter of ΔD_1: The difference of diffusion constant of the perpendicular to magnetic field (ΔD_1) was investigated seasonally at the critical altitudes (280, 300, 340, 390 and 410 km) for ionospheric F2-region in the equatorial region. According to the **b** part of Figure 1-4, (ΔD_1) the order between 10^4-10^6[m²/s] for all seasons (March 21, June 21, September 23 and December 21) at both 12.00 and 24.00 LT [17]. The magnitude of ΔD_1 was bigger at 24.00 LT than 12.00 LT for all seasons. The magnitude of ΔD_1 had maximum values the biggest one being at 280 km and having a peak at 0^0-5^0N and 12.00 LT latitudes (0^0-15^0N) corresponding to the magnetic equator. But it shows similar electron density changes in the magnetic equator. For 24.00 LT, the diagram of ΔD_1 changes markedly on June 21-March 21. ΔD_1 had a maximum value at 10^0N latitude and suddenly decreased again and had the minimum value at 20^0N. For June 21, there was an anomalous case at 24.00 LT. The magnitude of ΔD_1 showed a sharply peak at 25^0S latitude again 10^0S and it reached the maximum value for 5^0N- 20^0N latitudes. This shows a complex behavior for June 21.

Scatter of ΔD_2: The difference of diffusion constant of the perpendicular magnetic field (ΔD_2) and electric field was investigated seasonally at the critical altitudes (280, 300, 340, 390 and 410 km) for equatorial F-region in ionospheric plasma. According to **c** part of Figure 1-4, (ΔD_2) the order between 10^4-10^6 [m²/s] for all seasons (March 21, June 21, September 23 and December 21) at both 12.00 and 24.00 LT and is smaller than the magnitude of ΔD_1. The magnitude of ΔD_2 is bigger at 24.00 LT than 12.00 LT for all seasons except for June 21. Besides, the magnitude of ΔD_2 for 12.00 LT has maximum values reaching the biggest value at 280 km and between a peak 10^0-5^0S and at the latitudes (0^0-15^0N) corresponding to the magnetic equator. But it shows similar electron density changes in the magnetic equator. For 24.00 LT, the diagram of ΔD_2 changes markedly on June 21-March 21. For September 23 and December 21, the diagram of ΔD_2 was similar to each other, which is the reverse of the magnetic equator. They reached maximum values between 10^0S-20^0N latitudes. But ΔD_2 showed variations strongly for June 21 and March 21 at 24.00 LT especially for June 21. The magnitude of ΔD_2 showed sudden and sharp changes i.e. "sudden increase and decrease" with respect to the latitude. It showed sudden decrease at 10^0N latitude for every altitude on March 21.

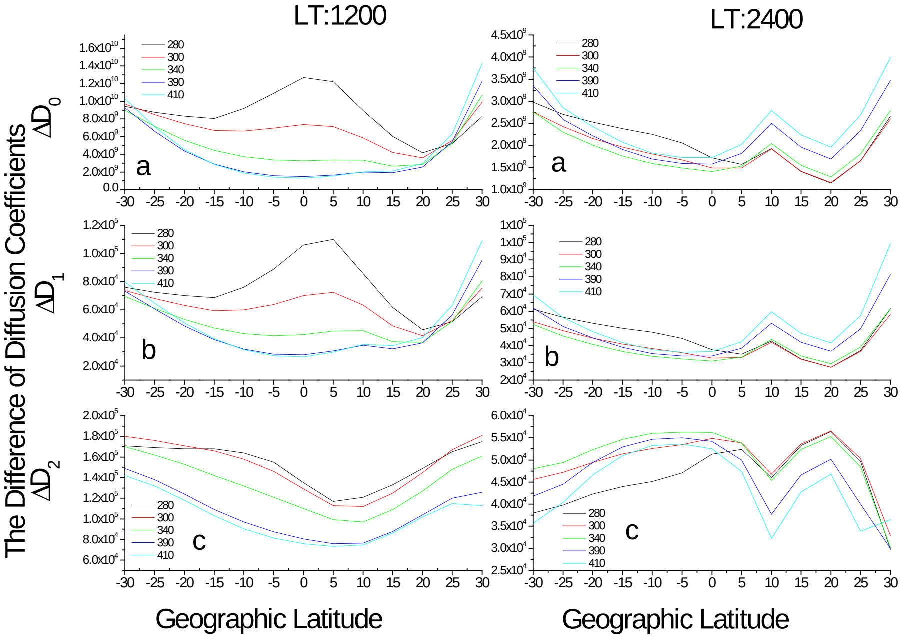

Figure 1. Change of the difference of diffusion with latitude for March 21.

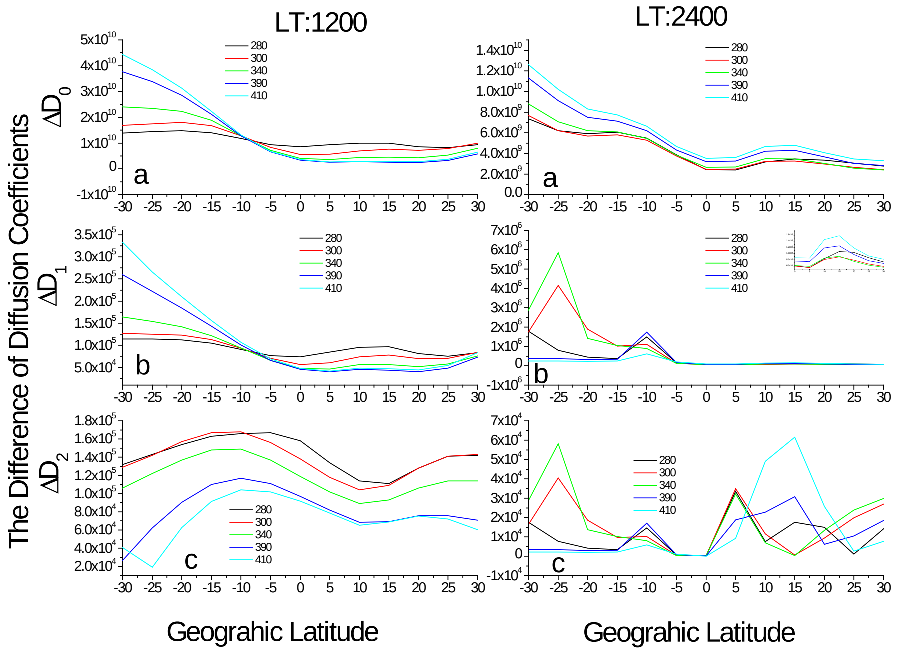

Figure 2. Change of the difference of diffusion with latitude for June 21.

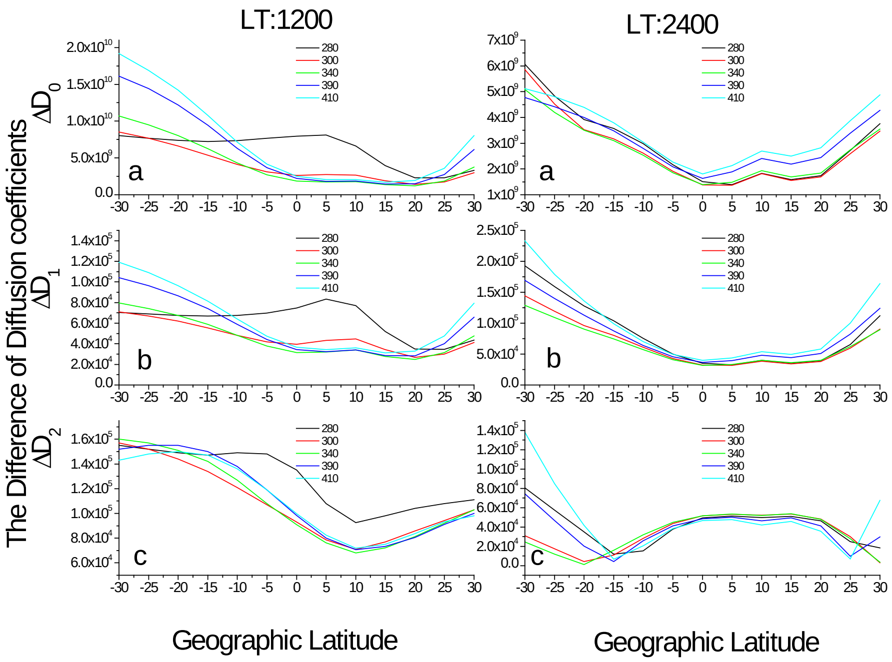

Figure 3. Change of the difference of diffusion with latitude for September 23.

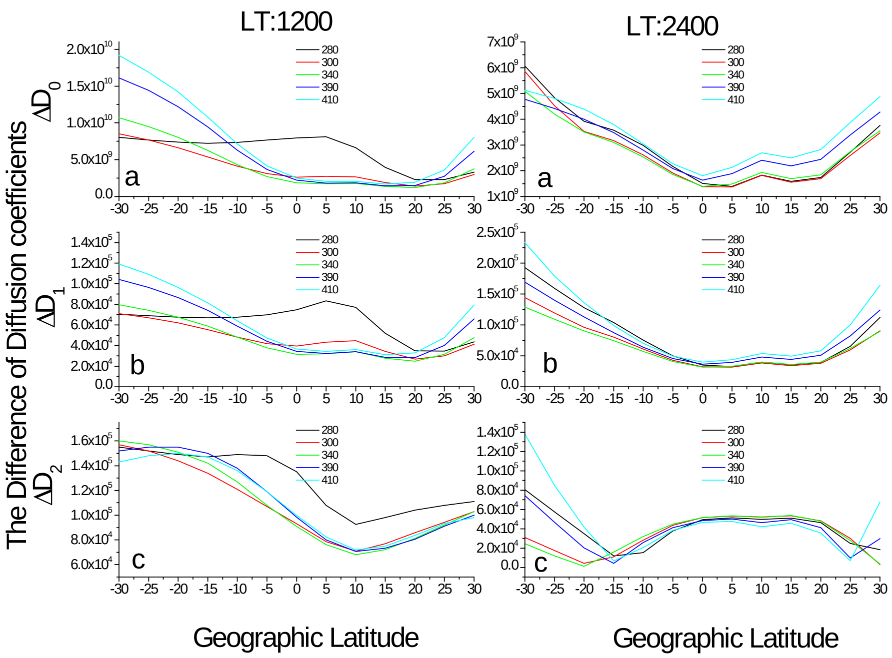

Figure 4. Change of the difference of diffusion with latitude for December 21.

Conductivity is one of the most important properties of ionosphere which plays basic role in ionospheric transport mechanism. Besides, it depends on a lot of parameters such as solar activity, the Earth's magnetic field and collision frequency among particles which are not easy to calculate or measure far from other parameters. Many researchers have examined ionospheric conductivity theoretically by using some approximations in various conditions [17-21]. But, there is a relationship between the diffusion coefficients and conductivity, namely;

$$D_0 = \frac{k_b T_e}{m_e(\nu - i\omega)} = \frac{k_b T_e \sigma_0}{Ne^2(1 - i\omega/\nu)} \text{ and}$$

$$\mu = \sigma_0 / Ne^2 \ .$$

Because of this, it is possible to interpret that when the diffusion coefficients get the maximum value in any condition in ionosphere, then the electrical conductivity and mobility get maximum value and otherwise true such as in Figure 1-4.

When all these findings are evaluated in terms of physical aspect;

- All of diffusion coefficients sharply changed between 0-10^0 geographic latitudes where these latitudes composed the equatorial anomaly for the accepted altitudes and 12.00 LT. The reason of sharp increase/decrease could be thought to have been resulted from conductivity, neutral winds and magnetic field. This case (increase/ decrease) will change the refractive index of medium. In parallel, the behavior of electromagnetic waves sent from the ground to the ionosphere will change in that region such as at reflection height [17-22].

- ΔD_0 is bigger than the other diffusion coefficients as in figure 1-4 for 12.00 LT. In parallel, the longitudinal electrical conductivity and mobility are bigger than the Pedersen and Hall conductivity for 12.00 LT in ionospheric plasma. May be this is derived from the Earth's magnetic field as ΔD_0 does not depend on the magnetic field but it is diffused in the magnetic field direction[13,15,16].

- The trend of ΔD_0 is the same for the accepted conditions (March-21, June-21, September-23 and December-21) and altitudes at 24.00 LT. But, ΔD_1 and ΔD_2 are more complex changes for especially June 21. They had sharp ups and downs for all altitudes and 24.00LT. This abnormal change could result from the Earth's magnetic field or from another anomaly. For ΔD_1 and ΔD_2, we think that this abnormal behavior could be related to the equatorial anomaly [1,13,22].

4. Conclusion

This article has reviewed the difference of classical diffusion tensor for electrons in the low-latitude ionospheric plasma and investigated whether there were any relationships between the diffusion and equatorial anomaly. According to the findings of previous studies, the magnitude order of diffusion coefficients is approximately the same at the considered conditions (10^{10} [m^2/s]) [4].But the difference of this paper is that when $\omega \neq 0$, the diffusion coefficients have a real part and an imaginary structure. The imaginary part of the diffusion coefficients may give rise to the missing of the electron energy; and it is possible that this loss causes decrease in the electrical conductivity [5, 16]. Due to this, all of the diffusion coefficients are interpreted in terms of the electrical conductivities and motilities. This paper gives some clues of the information on the diffusion in the ionospheric plasma, how much the electromagnetic waves have influences at various frequencies sent from the ground to the ionosphere.

The findings indicate that the magnitudes of ΔD_0, ΔD_1 and ΔD_2 have bigger values calculated at 24.00 LT than 12.00 LT for both equinox and solstice days. The values of ΔD_0, ΔD_1 and ΔD_2 value calculated at both 12.00 and 24.00 LT is in the following order 10^{10} [m^2/s], 10^4 -10^6 (m^2/s), 10^4 -10^6 [m^2/s] for all seasons (March 21, June 21, September 23 and December 21), respectively. Finally, the magnitudes of the difference of the classical diffusion tensor for electrons are $\Delta D_0 > \Delta D_1 > \Delta D_2$ at Equatorial F-region for all seasons with respect to both 12.00 LT and 24.00 LT, respectively. However, ΔD_1 and ΔD_2 are bigger during nighttime than in the daytime, and show a behavior unlike the change with the latitude of the electron density in the magnetic equator. It is possible to say that the behavior of these abnormalities may result from electromagnetic drift and dynamo effect. The difference of classical diffusion tensor depends only on the temperature of the electron and collisions frequency of the electron ΔD_0 in Eq.(2.4). However, the other difference diffusion, such as ΔD_1 and ΔD_2 are affected by the Earth's magnetic field as well as the electron temperature and electron collisions.

References

1. Rishbeth, H. and Garriot, O. K. Introduction to Ionospheric Physics; Academic Pres: New York, 1969; 175-186.

2. Schunk, R. and Nagy, A. Ionospheres: physics, plasma physics, and chemistry; Cambridge university press: New York, 2009; 150-200.

3. Maurice J-PST, Schunk R. W. Diffusion and heat flow equations for the mid- latitude topside ionosphere, *Planetary and Space Science*, 1977, 25, 907-920.

4. Sagir, S.; Yesil, A.; Sanac, G.; Unal, I. The characterization of diffusion tensor for mid-latitude ionospheric plasma, *Annals of Geophysics*, 2014, 57(2), A0216.

5. Rishbeth H. A review of Ionospheric F Region Theory, *Proceeding of the IEEE*, 1965, 55, 16-35.

6. Bittencourt, J. A. Fundamentals of plasma physics; Springer Science & Business Media: New York. 2013; 50-140.

7. Johnson F. S, The ion distribution above the Fz-maximum, *Journal of Geophysical Research*, 1960, 65, 577.

8. Mange P, The distribution of minor ions in electrostatic equilibrium in the high atmosphere, *Journal of Geophysical Research*, 1960, 65, 3833.

9. Schunk R. W, and Walker J. C. G, Thermal diffusion in the F2 region of ionosphere, *Planetary and Space Science*, 1970, 18, 535-557.

10. Bauer S. J, The structure of the topside ionosphere, *Journal of Atmospheric Science,* 1962, 12, 276-279.

11. Evans J. V, Julian R. F, Reid W. A. Incoherent scatter measurements of F-region density, temperatures and vertical velocity at Millstone Hill, *M.I.T. Lincoln Lab. Rep.,* 1970, 477.

12. Zhilinskii A. P, Tsendin L.D, Collisional diffusion of a partly-ionized plasma in magnetic field, *Soviet Physics Uspekhi,* 1980, 131, 343-385.

13. Chukwuma V.U, Adekoya B.J, The effects of March 20 2015 solar eclipse on the F2 layer in the mid-latitude, *Advances in Space Research,* 2016, 58, 1720–1731.

14. Domingeuez H. J, Use of Quantum Mechanical Methods to Obtain a Bohm- Type Coefficient of Diffusion, *Acta Polonica,* 2009, A(116), 197-201.

15. Rishbeth H, The ionospheric E-layer and F-layer dynamos - a tutorial review, *Journal of Atmospheric and Solar-Terrestrial Physics,* 1997, 1873-1880.

16. Rishbeth H, On the theory of diffusion in the ionosphere, *Geophysical journal of the Royal Astronomical Society,* 1975, 41, 311-317.

17. Yesil A.,Sagir, S., Ozcan, O, Comparison of maximum electron density predicted by IRI-2001 with that measured over Chilton station, Physical Science (NWSA- New World Sciences Academy) 2009, 4(3), 92-99.

18. Aydogdu, M and Ozcan, O, The possible effects of the magnetic declination on the wave polarization coefficients at the cutoff point. Progress in Electromagnetic Research, PIER, 2001; 30, 179–190.

19. Özcan O, Aydoğdu M, and Ünal İ, Effect of the adiabatic sound speed on the electric conductivities of the Ionospheric plasma, *Balkan Physics Letters,* 1998, 6(3), 204-213.

20. Aydoğdu, Güzel, E, Yeşil, A., Özcan O, Canyılmaz M, Comparison of the Calculated Absorption and the Measured Field Strength of HF Waves Reflected from the Ionosphere, *Il Nuovo Cimento,* 2007, 30(3), 243-253.

21. Özcan O, and Aydoğdu M, Altitudinal Variations of the Conductivity of the ionospheric plasma over Turkey, *Balkan Physics Letters,* 1993, 1(3, 4), 65-71.

22. Katiev A.V, and Biernat H.K, On the effect of geomagnetic field diffusion at the magnetopause, *Planetary and Space Science,* 2002, 50, 811-816.