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Research Article

Scoring open-ended items using the fuzzy topsis method and comparing it with traditional approaches

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Abstract: This study investigates the application of the fuzzy logic method for scoring open-ended items, specifically comparing its effectiveness against traditional scoring methods. Utilizing the fuzzy TOPSIS method within the mathematics domain, this research established seven criteria for evaluating open-ended responses, developed in consultation with three experts. Due to constraints imposed by the pandemic, the study did not proceed with a real-world application; instead, it simulated data for 25 students to compare the rankings derived from traditional and fuzzy logic methods using the MS Excel program. The research produced three distinct rankings using the conventional method and analyzed the correlation between these rankings and those generated by the fuzzy TOPSIS method, employing the Spearman rank correlation coefficient. The findings reveal a significantly positive correlation between the rankings obtained through traditional methods and those acquired via the fuzzy logic approach, suggesting the latter's potential as an effective alternative for evaluating open-ended responses.

1. INTRODUCTION

The word “logic” in Turkish is the Arabic translation of the Greek word *logike*. It denotes both a verbal and mental concept. According to Al-Farabi, the word was derived from *nutk* (to say). Ali Sedad also indicated that *nutk* means both the utterance and the thought (Öner, 1986). As a concept, logic is a science that facilitates one to reach the knowledge of the unknown through the known or a discipline which prevents faulty thinking if one follows the rules. In other words, logic is a branch of science that examines correct and appropriate forms of thinking. The emergence of logic as a science is as old as the existence of mankind. Human beings need to think, reason, and make decisions all the time for different situations they face in their lives. There has been a need for a systematization of intellectual methods so that one can make the correct deduction and decisions (Karataş, 2018). Aristotle (384-322 BC) is the first thinker to examine and establish systematically (Öner, 1986; Paksoy et al., 2013).

According to Aristotle, logic is the science of the ideal laws of thinking (Aristotle, 1989, qtd. in Köz, 2022). Aristotle based his understanding of classical logic on the assumption that right and wrong as concepts are explicitly distinct; he argued that there could be more right or more

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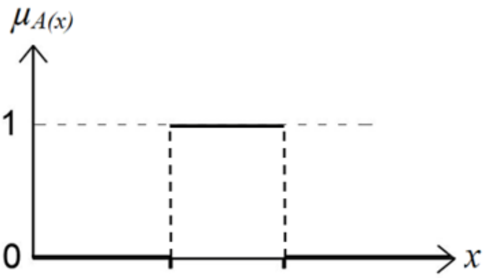
wrong situations; however, because he did not want to engage with fuzziness and thus made logic clear by defuzzying it. as such, Aristotle has established the foundation of classical logic (Erdin, 2007). Reasoning is important in classical logic, and it is its basis (Hasırcı, 2010: Öner, 1986; Taylan, 2008).

Criticism against the reasoning methods of classical logic has set the foundation of modern logic with the advent of symbols in the second half of the 19th century. Bertrand Russell's (1872-1970) contention that classical logic falls short in solving mathematical paradoxes along with his publication of *Principia Mathematica* with Whitehead in 1910 established symbolic (modern) logic (Paksoy et al., 2013). Just like in classical logic, modern logic aims to make inferences from the unknown towards to known. The use of symbolic language in modern logic studies aimed at alleviating the mistakes and shortcomings in language by turning premises and interferences into symbols. Modern logic has developed various inspection methods. These methods take us to the objectivity and univocity of symbolic language by purging the daily language of its polysemy (Eroğlu, 2012).

The critique against binary logic has brought forth the idea that situations between two extreme values should be taken into consideration. This critique also enabled the formation of fuzzy logic. Fuzzy logic as a concept was first coined by L.A. Zadeh in 1965 in his work titled *Fuzzy Sets.* The underlying philosophy of fuzzy logic is based on the assumption that a situation can have a continuous value between 'right' and 'wrong'.In other words, the value could be a reel number between 0 and 1 (Bostan, 2017). Fuzzy set theory emerged because Zadeh thought that the mathematical method of classical logic falls short in dealing with real-world problems (Avcı Öztürk, 2018; Elmas, 2003; Kaptanoğlu & Özok, 2006). The first application of fuzzy logic was in a steam engine designed by Mamdani in 1974. Zadeh introduced the theory of fuzzy logic to the world and Mamdani was the first person to put this theory into practice. Around the same time in Japan, practical application areas of fuzzy logic emerged (Özdağoğlu, 2016; Topçu, 2014).

In classical logic, the membership function for set A could be defined as follows:

Figure 1. *Function graph of membership in classical sets.*



$$\mu_{A(x)} = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$$

As can be seen in the function graph (Figure 1), µA membership function will assume values {0, 1} based on whether x members are in set A. In fuzzy logic, on the other hand, there are different membership functions such as triangular, trapezoidal, S and Z-shaped sigmoid, Cauchy, Gaussian, and monopulse (Baykal & Beyan, 2004; Cheng, 1996; Türe, 2006; Yen & Langari, 1999; Zimmermann, 2001). In practice, the most frequently used ones are the triangular, trapezoidal, curved, and Gaussian membership functions (Armağan, 2008). The core, support, and boundaries forming the membership function for a fuzzy set belonging to a universal set are shown in Figure 2 (Ross, 2010).

Figure 2. *Core, support and boundaries in a fuzzy set.*

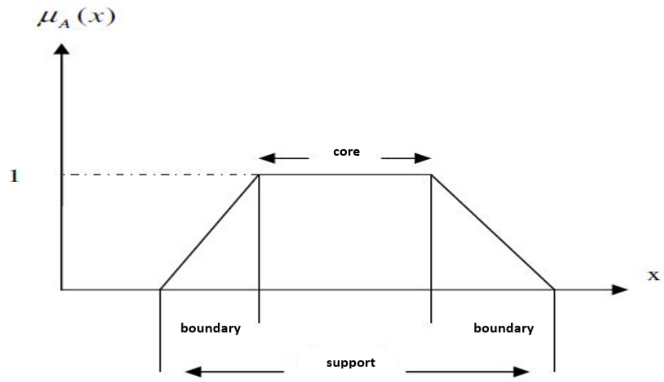


Figure 2 shows that in a fuzzy membership function, the core is a full member of set A in the universal set and contains elements the membership degrees of which are equal to 1. The support, on the other hand, is composed of elements whose membership degree is bigger than 0 in set A. In the fuzzy set A, boundaries indicate the area consisting of the elements, with degrees of membership different from zero, apart from full membership (Ross, 2010).

There are certain advantages and disadvantages of all systems used in decision-making depending on the area they are used. Among the advantages of fuzzy logic are the following: it requires fewer rules and decisions, assessments can be linguistically expressed, more observable variables can be assessed, the output can be related to the input, previously unsolved problems can be solved, quick prototyping is possible, it is more easily designed than traditional systems, it is cheaper, it can be used in the solution of complex problems, and it can be used in unstable and non-linear systems (Baykal & Beyan, 2014; Coşkunırmak, 2010; Elmas, 2003; McNeill & Thro, 1994; Özdağoğlu, 2016). Nevertheless, it has come with disadvantages in practice as well. Rules used in fuzzy logic are highly dependent on people's experience; variables of the membership function are specific to the application and are highly difficult to use in another application; while it is easy and fast to form a prototype it needs more simulation compared to the traditional control systems (Coşkunırmak, 2010; Elmas, 2003; McNeill & Thro, 1994; Özdağoğlu, 2016).

Multi-criteria decision-making methods, whether classical or fuzzy, can be used during decision-making processes. Since having too many criteria would complicate the decision-making process, multi-criteria decision-making methods ease the process and make it more objective (Cakar, 2020). Analytic Hierarchy Process (AHP), Vise Kriterijumska Optimizacija I Kompromisno Resenje (VIKOR), and Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) are some of the multi-criteria decision-making methods. These methods can be made fuzzy if necessary (Chen, 2000; Dündar, Ecer & Özdemir, 2010; Ertuğrul & Karakaşoğlu, 2008; Opricovic & Tzeng, 2004; Zimmermann, 1978).

Traditional approaches are used in making educational decisions, in line with classical logic. When determining student success, different types of tests (multiple choice, open-ended, true/false, matching, etc.) are utilized as a basis for educational decisions. Multiple-choice tests are one of the most frequently used methods to obtain valid and reliable results. While multiple-choice tests have certain advantages (being objective, having high content validity, easy scoring, easy application, etc.), they also have disadvantages when it comes to measuring students' advanced mental skills (such as problem-solving, creative thinking, critical thinking, and reasoning) (Bush, 2001; Klufa, 2018; McMillan, 2017; Miller, Linn & Gronlund, 2009; Popham, 1999; Tekin, 2010; Turgut & Baykul, 2012). To alleviate these disadvantages, open-ended items as well as in-class assessments are also utilised in assessing student success. Open-ended items are advantageous because they promote detailed learning, improve writing skills and alternative thinking, eliminate chance success, aim at improving advanced-level thinking skills, show the possibility of different correct answers as opposed to a single one, and enable students to structure their answers (Badger & Thomas, 1992; Cooney, Sanchez, Leatham & Mewborn, 2004; Geer, 1998; Karakaya, 2022; Öksüz & Güven Demir, 2019).

Ministry of Education (MEB) and Student Selection and Placement Centre (OSYM) (MEB, 2017; ÖSYM, 2017) have carried out trial applications using open-ended items. A total of 15 open-ended items in all fields were tested in an exam designed by OSYM. Answers were put on optic forms, and the scoring was done by a machine to ensure objectivity. In the first semester of the 2017-2018 school year, the Ministry of Education designed a TEOG (transition from primary to secondary education) exam with two open-ended items in Turkish, Mathematics, and Science. Items in this exam were open-ended and required long answers. Students were free to answer them as they liked; an answer sheet was used instead of an optic form, and the scoring was done by expert teachers. MEB prepared a structured answer key for the scoring of these items' answers; objectivity was ensured by asking the expert teachers to use this key when scoring the answers. Assessor-based objectivity has always been an issue when scoring the answers of especially open-ended questions, short-answer items, compositions, projects, and assignments. Using multiple assessors or the assessors scoring each item one by one are some of the methods used to alleviate this problem. Independent of the type of test, students' answers are scored in absolute numbers within the principles of classical logic. Scoring a student's answer to a multiple-choice question as 1 – 0 denotes certainty; scoring their composition 75 out of 100 also denotes certainty. In other words, these scores are certain, meaning they do not belong to a low or high-score group. This scoring takes place by employing the philosophy of classical logic systematised by Aristotle. In fuzzy logic, such concepts as certain and absolute are denoted by truth values, which are shown by membership degrees. These truth values are placed between completely true and completely false. One does not say that above a certain level is true or below a certain level is false. Using linguistic variables in assessment facilitates modeling operations (Elmas, 2011; Sarı, Murat, & Kırabalı, 2005).

Even though there are studies on the use of fuzzy logic in education (Hocalar, 2007; Kaptanoğlu & Özok, 2006; Bakanay, 2009), these studies are limited and none of them has tested fuzzy methods in scoring open-ended questions. It is believed that this present study will be one of the trials of using fuzzy logic methods in the field of assessment and evaluation. This study aimed to score open-ended items by Fuzzy TOPSIS, which is one of the multi-criteria decision-making methods. By doing so, a new method was tested; one in which students' answers did not have a certainty (0-1) and were scored based on different criteria weighted by experts, and one in which the experts scored the answers by linguistic expressions. To this end, scoring was done for the open-ended items developed for the mathematics classes, and students' gradation was compared to the classical method, TOPSIS, and fuzzy TOPSIS.

2. METHOD

In the study classical method, TOPSIS, and fuzzy TOPSIS methods were compared in the scoring of open-ended items. Carried out in the correlation research model, simulative data were used because the actual application was not possible as schools were closed due to COVID-19 restrictions. Moreover, since the study was a trial run to see whether fuzzy logic could be used in scoring open-ended items, only one item was used during the study so that the operations and the logic of gradation could be understood.

2.1. Simulative Design

Due to the global COVID-19 pandemic, schools were closed for face-to-face education and switched to online teaching in the 2020-2021 academic year. Simulated data were designed to exemplify a real-world application, as the study aimed to examine grading based on different methods. Number of students in a classroom may vary in different regions in Turkey. The MEB average for the 2019-2020 academic year was taken into account, and the data set was designed with 25 students (MEB, 2020, p. 24). Sub-criteria were devised to assess the open-ended mathematics item. Three field experts were consulted when devising the criteria and appointing significant weight to them; these experts also worked as scorers for the students' answers. Two of the experts in the study were maths teachers employed at MEB. The third expert was a maths teacher employed at the Evaluation and Assessment Centre at the Provincial Directorate of National Education. The teachers worked at the secondary education level and were included in the study by appropriate sampling (Altun, 2002; Damlar Demirci, 2019; Karadeniz, 2016; Van De Walle et al., 2014). A literature review was conducted when determining the criteria for the scoring of the open-ended mathematics items and different sub-criteria were determined. Then, these criteria were examined based on the separately gathered views of the experts and were reduced to seven, namely, (1) understanding the problem, (2) utilizing what is given in the problem, (3) using operations in the solution of the problem, (4) adapting the formula and the rules to the problem, (5) following the order of operations by making connections between operations, (6) making no mistakes in the operations, and (7) executing the operations clearly and in detail. The established criteria were emailed to the experts so that they could determine the sub-criteria of the scoring of the open-ended mathematics item. The experts assigned the values of "very low," "low," "somehow low," "medium," "somehow high," "high," and "very high," based on their personal views.

The student scores that would constitute the data of the study were randomly created between 1-7, keeping in mind the 7 criteria. During the fuzzification process, these scores were used as "very bad," "bad," "somehow bad," "medium," "somehow good," "good," and "very good" by converting them to linguistic variables. Students' scores and gradation were determined by fuzzy and classical methods by taking into account the scores obtained from students' answers to the items and the weight the experts have given to the sub-criteria.

2.2. Data Analysis

Students' scores for the mathematics item were first calculated according to the classical method. When doing this, the classical TOPSIS method was also used in addition to the classical scoring method. During the scoring, the weights of the sub-criteria were not used as the first method; instead, gradation was done by taking the average of the total scores given by the scorers. In the second method, on the other hand, TOPSIS was used as a multi-criteria decision-making method. The operational steps of the TOPSIS method were realized in the following order (Opricovic & Tzeng, 2004).

Step 1. A decision matrix is established by providing the criteria in the columns and alternatives in the lines.

According to each criterion in the study, scores given to the students are expressed as shown in Formula 1. The (C) in the columns symbolises the criteria, the (A) in the lines symbolise the alternatives, in other words, students, and the (W) symbolises the weight of the criterion.

$$D = \begin{array}{c c} & \begin{array}{cccc} C_1 & C_1 & \dots & C_n \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ \dots \\ A_m \end{array} & \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \dots & \dots & \dots & \dots \\ x_{n1} & x_{n2} & \dots & x_{mn} \end{pmatrix} \end{array} \qquad w = [w_1, w_2, \ldots\ldots, w_n] \tag{1}$$

Step 2. A normalised decision matrix is established.

When forming the normalised decision matrix, the elements in the (D) decision matrix are used and the (r) matrix is formed by applying Formula 2. Each value in the decision matrix is divided by the square root of the sum of the squares of the x_{ij} values in the columns.

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{n} x_{ij}^2}}, \qquad i = 1, 2, \ldots\ldots, m \qquad j = 1, 2, \ldots\ldots, n \tag{2}$$

Step 3. A weighted normalised decision matrix is formed.

The weighted normalised decision matrix (V) is calculated by Formula 3. To carry out this operation, weight values of criteria (w) are first determined. Then, elements on each column of the R matrix are multiplied by the relevant criterion's weight value (w) thereby forming the (V) matrix.

$$v_{ij} = w_j . r_{ij} \qquad i = 1, 2, \ldots\ldots, m \qquad j = 1, 2, \ldots\ldots, n \tag{3}$$

Step 4. A positive ideal solution set and a negative ideal solution set are formed.

To establish ideal solution sets, a positive ideal solution set is formed by selecting the maximums of the weighted evaluation criteria in the (V) matrix, and a negative ideal solution set is formed by selecting the minimums. The minimum value is selected in the positive ideal solution set if the relevant criterion is minimization-oriented, and the maximum value in the negative ideal solution set is selected if it is maximization-oriented. These operations are shown below by Formula 4 and Formula 5, respectively.

$$A^{+} = \{v_1^{+}, v_2^{+}, \ldots\ldots, v_n^{+}\} = \left\{\left(\overset{Maksimum}{i} \; v_{ij} \mid j \in K\right), \left(\overset{Minimum}{i} \; v_{ij} \mid j \in K^{'}\right)\right\} \tag{4}$$

$$A^{-} = \{v_1^{-}, v_2^{-}, \ldots\ldots, v_n^{-}\} = \left\{\left(\overset{Minimum}{i} \; v_{ij} \mid j \in K\right), \left(\overset{Maksimum}{i} \; v_{ij} \mid j \in K^{'}\right)\right\} \tag{5}$$

Step 5. Ideal solution values are calculated.

Euclidean distances are used to find the distances of the evaluation criterion value for each Student (alternative) to the positive and negative ideal solution. Formulas concerning this calculation are given in Formula 6 and Formula 7.

$$D_i^{+} = \sqrt{\sum_{j=}^{n} \left(v_{ij} - v_j^{+}\right)^2} \qquad i = 1, 2, \ldots\ldots, m \qquad j = 1, 2, \ldots\ldots, n \tag{6}$$

$$D_i^{-} = \sqrt{\sum_{j=}^{n} \left(v_{ij} - v_j^{-}\right)^2} \qquad i = 1, 2, \ldots\ldots, m \qquad j = 1, 2, \ldots\ldots, n \tag{7}$$

Step 6. Alternative rankings are done based on ideal solution values.

When calculating each student's closeness to the ideal solution (CC_i), their distance to the positive and negative ideal solutions is used. As can be seen in Formula 8, the distance to the ideal solution is calculated with the ratio of the negative ideal solution to the total distance. This closeness value is between 0 and 1; when CC_i=0 it denotes absolute closeness to the negative ideal solution and when CC_i=1it denotes absolute closeness to the positive ideal solution.

$$C_i = \frac{D_i^{-}}{D_i^{+} + D_i^{-}} \qquad i = 1, 2, \ldots\ldots, m \qquad 0 \le C_i \le 1 \tag{8}$$

In the third method of the study, the following operation steps were carried out in the gradation of students' answers to the open-ended mathematics items with the fuzzy TOPSIS method (Chen, 2000).

Step 1. Decision-makers and criteria are selected.

Three experts were identified as the decision-makers in the study. Based on their expert opinion, the criteria were determined as (1) understanding the problem, (2) utilising what is given in the problem, (3) using operations in the solution of the problem, (4) adapting the formula and the rules to the problem, (5) following the order of operations by making connections between operations, (6) making no mistakes in the operations, and (7) executing the operations clearly and in detail.

Step 2. Appropriate linguistic variables are determined for the significance weights of the criteria; linguistic variables' levels are selected for alternatives according to the criteria.

The linguistic variables the scorers will assign to the criteria and students' answers are presented in Table 1.

Table 1. *Linguistic variables expressing the value weight for the criteria and the alternatives.*

Linguistic Variables for Criteria	Linguistic Variables for Alternatives
Very low (VL)	Very bad (VB)
Low (L)	Bad (B)
Somehow Low (SL)	Somehow Ba (SB)
Medium (M)	Medium (M)
Somehow High (SH)	Somehow Good (SG)
High (H)	Good (G)
Very High (VH)	Very Good (Very Good)

As can be seen in Table 1, seven options were identified for the sub-criteria and the alternatives. While it was thought that using fewer linguistic variables for the criteria and the alternatives would lessen the sensitivity of data, it was also believed that having more linguistic variables would not contribute to the study, either. In this respect, the number of linguistic variables was limited to seven to ensure an optimum sensitivity. The significance weights the scorers have given to the criteria of the scoring of the open-ended mathematics items are presented in Table 2.

Table 2. *Linguistic variables scorers provided for the significance brackets of decision criteria.*

Criteria	1st Scorer	2nd Scorer	3rd Scorer
Understanding the problem	VH	VH	VH
Using what is given in the problem	H	H	VH
Using operations in the solution of the problem	H	SH	VH
Adapting the formula and the rules to the problem	H	SH	H
Following the order of operations by making connections between operations	SH	M	H
Making no mistakes in the operations	SH	L	M
Executing the operations clearly and in detail	M	SH	VH

Formula 9 was used when calculating the significance levels the decision-makers assigned to the criteria. According to this formula, the operation is executed by taking the average of the weights the scorers gave to the criteria.

$$\tilde{w}_j = \frac{1}{K}\left[\tilde{w}_j^1 + \tilde{w}_j^2 + \cdots\cdots \tilde{w}_j^K\right] \tag{9}$$

Step 3. The linguistic variables determined by the decision-makers for the assessment of significance weights and alternatives are converted into triangular fuzzy numbers.

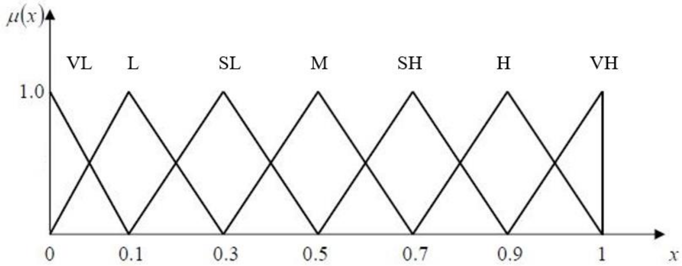
Within the scope of the story, triangular fuzzy numbers are preferred in the conversion of decisions to numbers. Triangular fuzzy number expressions of the criteria's significance weights are presented in Table 3.

Table 3. *Triangular fuzzy numerical expressions indicate the significance weights for the criteria.*

Linguistic Variables for Criteria	Triangular Fuzzy Numbers for Criteria
Very Bad (VB)	(0.0, 0.0, 0.1)
Bad (B)	(0.0, 0.1, 0.3)
Somehow Bad (SB)	(0.1, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.7)
Somehow Good (SG)	(0.5, 0.7, 0.9)
Good (G)	(0.7, 0.9, 0.1)
Very Good (VG)	(0.9, 1.0, 1.0)

The graph of the significance weights in Table 3 is presented in Figure 3.

Figure 3. *Triangular fuzzy numbers show the significance weights for the criteria.*



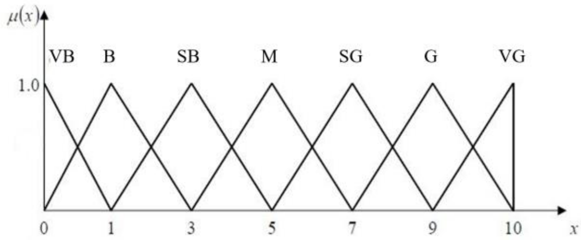
Fuzzy triangular numerical equivalents for the significance weights of alternatives are presented in Table 4.

Table 4. *Triangular fuzzy numbers show the significance weights for the alternatives.*

Linguistic Variables for Alternatives	Triangular Fuzzy Numbers for Alternatives
Very Low (VL)	(0, 0, 1)
Low (L)	(0, 1, 3)
Somehow Low (SL)	(1, 3, 5)
Medium (M)	(3, 5, 7)
Somehow High (SH)	(5, 7, 9)
High (H)	(7, 9, 10)
Very High (VH)	(9, 10, 10)

The graph of the significance weights of triangular fuzzy numbers in Table 4 is presented in Figure 4.

Figure 4. *Triangular fuzzy numbers show the significance weights for the alternatives.*



Values the scorers gave for the alternatives were calculated by Formula 10 and average values were thus obtained.

$$\tilde{x}_{ij} = \frac{1}{K}\left[\tilde{x}_{ij}^{1} + \tilde{x}_{ij}^{2} + \cdots\cdots \tilde{x}_{ij}^{K}\right] \tag{10}$$

Step 4. A fuzzy decision matrix and normalised fuzzy decision matrix are formed.

The fuzzy decision matrix shows the linguistic variables that each scorer assigned to the alternatives according to the criteria. Formula 11 was used for this operation.

$$\tilde{D} = \begin{array}{c} A_1 \\ A_2 \\ \cdots \\ A_m \end{array} \begin{pmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{pmatrix} \tilde{W} = \left[\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n\right] \tag{11}$$

The normalised decision matrix is devised out of the fuzzy decision matrix by dividing the fuzzy numbers in the column of each column to the largest upper limit in this column (Paksoy et al., 2013). Data in this study indicated that the highest fuzzy numbers for the 2nd, 4th, and 7th criteria was 9.7; and this value was used to establish the fuzzy decision matrix. The highest value of other criteria was 10 and it was left as it was. Then, all fuzzy numbers were normalised by dividing them by 10 thereby having the final version of the decision matrix. Formula 12 was used for the normalised fuzzy decision matrix.

$$\tilde{R} = \left[\tilde{r}_{ij}\right]_{mnx} \tag{12}$$

Since there are no negative criteria in this study, the benefit criterion was calculated by Formula 13.

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{c_j^*}, \frac{b_{ij}}{c_j^*}, \frac{c_{ij}}{c_j^*}\right), \; j \in B, c_j^* = \max_i c_{ij}, \tag{13}$$

Step 5. A weighted normalised fuzzy decision matrix is formed.

In this step Formula 14 was used to form the weighted normalised decision matrix by multiplying the normalised decision matrix with the criteria weights.

$$\tilde{V} = \left[\tilde{V}_{ij}\right]_{mxn} \tag{14}$$

In the weighted normalised fuzzy decision matrix, V_{ij} values are positive triangular fuzzy numbers, and their values vary between 0 and 1. Since each criterion has different significance degrees, a weighted normalised fuzzy decision matrix is calculated by Formula 15.

$$\tilde{V}_{ij} = \tilde{r}_{ij} \times \tilde{w}_j \tag{15}$$

Step 6. A fuzzy positive ideal solution and a fuzzy negative ideal solution are identified.

When determining the fuzzy positive and negative ideal solutions in this study, maximum values of the criteria were used for the fuzzy positive ideal solution and minimum values were used for the fuzzy negative ideal solution (Avcı Öztürk, 2018). The fuzzy positive ideal solution set for the normalised fuzzy decision matrix obtained by the triangular fuzzy numbers was calculated by Formula 16, and the negative ideal solution set was calculated by Formula 17.

$$A^* = \left(\tilde{V}_1^*, \tilde{V}_1^*, \ldots, \tilde{V}_n^*\right) \tag{16}$$

$$A^- = \left(\tilde{V}_1^-, \tilde{V}_1^-, \ldots, \tilde{V}_n^-\right) \tag{17}$$

The positive and negative ideal solution sets designed for the sub-criteria by using Formula 16 and Formula 17 are presented below.

$\tilde{A}^*$ = [(1.00, 1.00, 1.00), (1.00, 1.00 , 1.00), (0.97, 0.97, 0.97), (0.80, 0.80, 0.80), (0.87, 0.87, 0.87), (0.63, 0.63, 0.63), (0.87, 0.87, 0.87)]

$\tilde{A}^-$ = [(0.03, 0.03, 0.03) , (0.03, 0.03 , 0.03), (0.00, 0.00, 0.00), (0.01, 0.01, 0.01), (0.02, 0.02, 0.02), (0.00, 0.00, 0.00), (0.02, 0.02, 0.02)]

Step 7. The distance of each alternative first to the fuzzy positive ideal solution and then to the fuzzy negative ideal solution is calculated.

The distance of alternatives to the fuzzy positive ideal solution set was calculated by Formula 18 while their distance to the fuzzy negative ideal solution was calculated by Formula 19.

$$d_i^* = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^*), i = 1, 2, ..., m \tag{18}$$

$$d_i^- = \sum_{j=1}^{n} d\left(\tilde{v}_{ij}, \tilde{v}_j^-\right), i = 1, 2, \ldots, m \tag{19}$$

Formula 18 and Formula 19 show the distance between two fuzzy numbers. This distance is calculated by the Vertex method, which is developed to calculate the distance between fuzzy numbers.

$\tilde{A}$= (m_1, m_2, m_3) and $\tilde{B}$ = (n_1, n_2, n_3) are two fuzzy numbers, and Formula 20 was used to calculate the distance between $\tilde{A}$ and $\tilde{B}$ (Wang and Elhag, 2006; qtd. in Avcı Öztürk, 2018).

$$\mathrm{d}(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{3}[(m_1 - n_1)^2 + (m_2 - n_2)^2 + (m_3 - n_3)^2]} \tag{20}$$

Step 8. Closeness coefficients for each alternative are calculated.

Closeness coefficients were calculated by Formula 21 to rank the alternatives.

$$CC_i = \frac{d_i^-}{d_i^* + d_i^-} \tag{21}$$

Step 9. All alternatives are lined up according to closeness coefficients.

Students are ranked in descending order based on their closeness coefficient values; the student closest to 1 is considered the most successful and the student closest to 0 is considered the least successful.

Student ranking was done after obtaining the scores for the open-ended mathematics item via classical, TOPSIS, and fuzzy TOPSIS methods. To examine the correlation values between ranks Spearman Rank Correlation Coefficient was calculated.

3. RESULTS

Students' score averages and ranking according to the three scorers without using the criterion weights are presented in Table 5.

Table 5. *Ranking of students' scores based on the classical method.*

Rank	Students	Mean	Rank	Students	Mean
1	Student 16	36.3333	13	Student 15	28.6667
2	Student 17	35.3333	15	Student 22	28.0000
3	Student 12	34.0000	16	Student 3	27.3333
4	Student 20	33.3333	16	Student 7	27.3333
5	Student 6	33.0000	18	Student 13	26.6667
6	Student 24	31.0000	18	Student 19	26.6667
7	Student 1	30.6667	20	Student 8	25.6667
7	Student 4	30.6667	21	Student 10	25.3333
9	Student 9	30.0000	22	Student 14	25.0000
10	Student 2	29.3333	23	Student 18	23.6667
11	Student 21	29.0000	24	Student 25	22.6667
11	Student 23	29.0000	25	Student 11	20.3333
13	Student 5	28.6667			

As can be seen in Table 6, the highest mean is 36.33 and the lowest is 20.33. Since criterion weights were not used in the ranking, some students received the same score. When ranking these students, their student numbers were used in ascending order and there is no hierarchy among them. The ranking of the students' scores based on the classical TOPSIS method designed by Hwang and Yoon (1981) according to the closeness coefficient is presented in Table 6.

Table 6. *Students' ranking according to the closeness coefficient for the topsis application.*

Rank	Students	CC_i	Rank	Students	CC_i	Rank	Students	CC_i
1	Student 16	0.6826	10	Student 1	0.5408	19	Student 3	0.4611
2	Student 20	0.6628	11	Student 15	0.5385	20	Student 10	0.4428
3	Student 17	0.6280	12	Student 9	0.5336	21	Student 13	0.4014
4	Student 12	0.6276	13	Student 5	0.5221	22	Student 14	0.3585
5	Student 6	0.5779	14	Student 22	0.5160	23	Student 18	0.3453
6	Student 21	0.5677	15	Student 23	0.5146	24	Student 25	0.3400
7	Student 4	0.5576	16	Student 19	0.5097	25	Student 11	0.3057
8	Student 2	0.5491	17	Student 7	0.5021			
9	Student 24	0.5436	18	Student 8	0.4849			

Mean=0.5086

In Table 6 students' closeness coefficients are presented in descending order. According to this, Student 16 was in first place with 0.68 while Student 11 was last with 0.31. The mean of the class was $\bar{X}$ = 0.51. Students' scores were fuzzified according to the fuzzification steps suggested by Chen (2000). The ranking of students' closeness coefficient values after the operations for the fuzzy TOPSIS are presented in Table 7.

Table 7. *Students' ranking according to closeness coefficients for the fuzzy topsis application.*

Rank	Students	CC_i	Rank	Students	CC_i
1	Student 16	0.5676	13	Student 5	0.4510
2	Student 20	0.5594	15	Student 22	0.4438
3	Student 17	0.5532	16	Student 19	0.4342
4	Student 12	0.5514	16	Student 3	0.4151
5	Student 6	0.5150	18	Student 8	0.4081
6	Student 4	0.4879	18	Student 7	0.4032
7	Student 1	0.4833	20	Student 10	0.4010
7	Student 24	0.4830	21	Student 13	0.3903
9	Student 2	0.4784	22	Student 14	0.3645
10	Student 9	0.4756	23	Student 25	0.3456
11	Student 21	0.4752	24	Student 18	0.3383
11	Student 23	0.4640	25	Student 11	0.3030
13	Student 15	0.4612			

Mean=0.4501

The closeness coefficients in Table 7 show that Student 16 has the highest value with 0.57, which is followed by Student 20 with 0.56. The lowest value of the class, 0.30, belongs to Student 11. The mean of the class is $\bar{X}$ = 0.45.

The student rankings based on their scores obtained via classical, TOPIS, and fuzzy TOPSIS methods are presented in Table 8. Table 8 shows that Student 16 comes first in all methods and Student 11 comes last. While Student 17 comes second in the classical method, the same student comes third when ranked according to the multi-criteria decision-making methods, and Student 20 comes second.

Table 8. *Student rankings based on their scores obtained via classical, topis, and fuzzy topsis methods.*

Rank	Classical	TOPSIS	Fuzzy TOPSIS	Rank	Classical	TOPSIS	Fuzzy TOPSIS
1	Student 16	Student 16	Student 16	14	Student 15	Student 22	Student 5
2	Student 17	Student 20	Student 20	15	Student 22	Student 23	Student 22
3	Student 12	Student 17	Student 17	16	Student 3	Student 19	Student 19
4	Student 20	Student 12	Student 12	17	Student 7	Student 7	Student 3
5	Student 6	Student 6	Student 6	18	Student 13	Student 8	Student 8
6	Student 24	Student 21	Student 4	19	Student 19	Student 3	Student 7
7	Student 1	Student 4	Student 1	20	Student 8	Student 10	Student 10
8	Student 4	Student 2	Student 24	21	Student 10	Student 13	Student 13
9	Student 9	Student 24	Student 2	22	Student 14	Student 14	Student 14
10	Student 2	Student 1	Student 9	23	Student 18	Student 18	Student 25
11	Student 21	Student 15	Student 21	24	Student 25	Student 25	Student 18
12	Student 23	Student 9	Student 23	25	Student 11	Student 11	Student 11
13	Student 5	Student 5	Student 15				

When students' rank differences were examined, it was seen that most students were ranked in similar places no matter the method; the most obvious difference was with Student 21. Student 21 was ranked 6th in the TOPSIS method but was ranked 11th in the classical and fuzzy TOPSIS methods.

Spearman rank correlation coefficient values were obtained to test whether there is a relationship between student rankings and the Classical, TOPSIS, and fuzzy TOPSIS methods; these values can be found in Table 9.

Table 9. *Spearman rank correlation coefficient shows the relationship between student rankings and the employed method.*

Methods	Scoring via Classical Method	Scoring via TOPSIS	Scoring via Fuzzy TOPSIS
Scoring via Classical Method	1.000		
Scoring via TOPSIS	0.958*	1.000	
Scoring via Fuzzy TOPSIS	0.984*	0.975*	1.000

* $p<0.01$; n: 25

Table 9 shows that the highest ratio of similarity when it comes to student rankings was between fuzzy TOPSIS and Classical methods: $r=0.984$ and ($p<0.01$, $r^2=0.968$; n:25). The ratio of similarity between the two multi-criteria decision-making methods – TOPSIS and fuzzy TOPSIS – were found to be r=0.975 ($p<0.01$, $r^2=0.951$; n:25). All ranking methods used in this study have a positive high relationship.

4. DISCUSSION and CONCLUSION

Rigidly defined binary values such as yes/no, fast/slow, and good/bad are not always sufficient when making decisions in life. Some cases may contain qualities that fall under both the good and the bad. In such cases, the human mind makes a complex assessment by taking into account different conditions. Compared to classical logic, fuzzy logic is more compatible with the way humans think and it uses multi-level operations (Elmas, 2003; Yazırdağ, 2018). Fuzzy logic is a system of logic that overlaps with humans' ability to think in uncertain expressions (Ertuğrul, 2006). It indicates that assessment may have intermediate values as opposed to merely right and wrong results (Elmas, 2011; Uygunoğlu & Ünal, 2005). In decision-making, complex assessments are expressed in linguistic expressions. These linguistic expressions contain vagueness and variability (Yazırdağ, 2018). To alleviate this vagueness, linguistic expressions should be defined based on fuzzy sets and values that cannot be expressed clearly should be qualified approximately by using linguistic variables.

With the advancement of mathematical methods, different approaches to decision-making approaches have also emerged. Multi-criteria decision-making methods provide a more objective assessment alternative for the assessors along with classical and fuzzified ones. TOPSIS, which is one of the multi-criteria decision-making methods, is based on identifying the best alternative among the alternatives to be selected. The best alternative should geometrically have the shortest distance to the positive ideal solution and the longest distance to the negative ideal solution (Çakar, 2020; Tzeng & Huamg, 2011). In the Fuzzy TOPSIS method, fuzzy numbers are used to assign weight criteria, and linguistic scales are used in ranking alternatives (Madi, Garibaldi & Wagner, 2017). At the basis of the fuzzy TOPSIS method lies the fact that criteria used by the assessors may have different weights when assessing alternatives. This method eliminates the problems of subjectivity that emerge in making group decisions, and promotes more accurate decision-making (Ecer, 2007). The most significant point here is that different assessors can make different weightings and these weightings along with their numerical equivalents are included in the decision-making process. Fuzzy logic methods are more suitable for selecting the best among alternatives or classifying alternatives rather than a way of scoring. There are exemplary studies in the literature on this. In his 2006 study, Ertuğrul aimed to determine academics' performance by using the fuzzy logic method and categorised the results as "very inadequate," "inadequate," "normal," "successful," and "highly successful." Güler and Yücedağ (2017) developed a decision support system by using the fuzzy logic method to help vocational school students in selecting a field. Areas of the profession in which students may succeed were tried to be predicted by using the Self-concept scale. In their 2013 study, Çiçekli and Karaçizmeli aimed to determine students' ranking by using multiple criteria instead of merely evaluating their success based on their exam scores. A model was designed by using fuzzy AHP and students' rankings were examined. Wimatsari et al. (2013) aimed to help students at Udayan University in their selection of scholarships and determine the scholarship types according to established criteria. To this end, they combined Fuzzy TOPSIS and Fuzzy Multi-Criteria Decision-Making Methods to determine the functionality of scholarship selection. The study tried to determine the selection and rankings of students who would be given a scholarship.

Open-ended items play an important role in assessing advanced thinking skills, especially in in-class assessments; given the need for objectivity in scoring, testing the Fuzzy TOPSIS method's selection and ranking mechanism in the assessment of open-ended items was important. To this end, the criteria to be used in the study and their weights were determined by different experts, rankings were obtained by using both classical and fuzzy methods. Students' scores were not identical in the multi-criteria decision-making methods used in the study while some students received the same score when the classical method was used. This indicates that the classical method makes a less sensitive assessment even though it is easier to use. There was a strong and positive relationship among the rankings done by the Classical, TOPSIS, and Fuzzy TOPSIS methods. Weighted and fuzzified scores based on different criteria can be interpreted as not causing significant changes in the rankings of students compared to the classical method. Similar results have also been obtained in other studies in the literature. In a study conducted by Arslan in 2019, teacher performances were evaluated using fuzzy logic methods and the results were compared. In the study, the correlation value expressing the relationship between scores obtained via fuzzy and classical methods was determined, and it was concluded that there was a positive and high relationship between the two methods. In a study by Yılmaz in 2008, multi-criteria decision-making methods were used for selecting candidates applying for graduate studies. Within the scope of this study, criteria to be included in the assessment of student selection were determined, and the weighting of these criteria by pairwise comparison was carried out. Then, candidates were ranked using the AHP, TOPSIS, and Weighted Product methods, and the results were compared. Nursikuwagu et al. modeled and examined student competencies in vocational schools using the Fuzzy TOPSIS method in 2018. At the end of their study, they declared the Fuzzy TOPSIS as a simpler and more dynamic model that produces effective results in determining competencies compared to the traditional method which uses the average value.

The subjectivity of the scorer in scoring open-ended items affects the validity and reliability of scores (Haladyna, 1997; Nitko &Brookhart, 2014; Royal &Hecker, 2016). To prevent this, analytical or holistic scoring rubrics are used (Karakaya, 2022; Kutlu et al., 2014). When using these graded scoring rubrics, it is assumed that criteria weights are the same for each scorer, and the tools are designed accordingly. One wonders how rankings change when, rather than binary scoring, the weights of criteria change and when scores are considered with their intermediate values. The traditional method is undoubtedly the most common because it is practical for educators. Although there is a scoring key for scoring open-ended questions, evaluators have to evaluate according to these standard scores. Fuzzy logic, unlike classical logic, allows evaluators to weight and score criteria. This study focuses on how to apply fuzzification to the scores obtained from one of the most frequently used tools in the field of educational sciences and examines the resulting outcomes. Studies focusing on the differences of the methods can be done similarly. The results obtained from the study have focused more on ranking than on scoring, based on the preferred methods, and have shown that there were no significant changes in students' rankings. Given the increasing prevalence of fuzzy logic studies in the field of education, there will be a need to know the details of the algorithm, how the fuzzification mechanism differs from traditional methods, and how the selections and rankings yield results. The results of this study are expected to provide a cue for other studies. The study wanted to examine open-ended questions, an important component of assessment and evaluation, since it tested an example of especially the fuzzification process. On the other hand, external variables were kept at a minimum by limiting the scope. In this regard, conducting broader studies with both simulated and real data and examining their results would be beneficial. The fact that the algorithm can be created practically by using any coding language would make it easier for researchers to develop/test models in the future. Similar comparisons can be made not only by TOPSIS but by using other fuzzy methods, and the results of these comparisons can be examined.

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Declaration of Conflicting Interests and Ethics

The authors declare no conflict of interest. This research study complies with research publishing ethics. The scientific and legal responsibility for manuscripts published in IJATE belongs to the authors.

Contribution of Authors

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