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Synchronization of a 4D Hyperchaotic System with Active Disturbance Rejection Control and Its Optimization via Particle Swarm Algorithm

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4D Hiperkaotik Sistemin Aktif Bozucu Reddetme Kontrolü ile Senkronizasyonu ve Parçacık Sürü Algoritması ile Optimizasyonu

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Abstract

In this paper, a synchronization study is proposed by using a 4D hyperchaotic system model to be used in secure data transfer applications. Active Disturbance Rejection Control (ADRC) method is used for synchronization process. To prove the success of ADRC method, it is compared with Proportional-Integral-Derivative (PID) control method. The coefficients of both control methods are optimized with Particle Swarm Optimization (PSO) algorithm. Synchronization system is modelled and tested in Matlab/Simulink environment. ADRC and PID methods are tested in simulation environment for the cases without disturbance and under disturbance. It can be seen from the test results that the ADRC method managed to keep the system synchronous without being affected by any disturbances. On the other hand, it is clearly seen that the PID method cannot maintain the synchronization of system under disturbance effects.

Anahtar Kelimeler: *Chaotic system; Synchronization; ADRC; PID; PSO*

Öz

Bu çalışmada, güvenli veri aktarım uygulamalarında kullanılmak üzere 4 boyutlu hiperkaotik sistem modeli kullanılarak bir senkronizasyon çalışması önerilmektedir. Senkronizasyon işlemi için Aktif Bozucu Reddetme Kontrolü (Active Disturbance Rejection Control (ADRC)) yöntemi kullanılmaktadır. ADRC yönteminin başarısının kanıtlanması için Oransal-İntegral-Türev (Proportional-Integral-Derivative (PID)) kontrol yöntemiyle karşılaştırması yapılmıştır. Her iki kontrol yönteminin katsayıları Parçacık Sürü Optimizasyonu (Particle Swarm Optimization (PSO)) algoritması ile optimize edilmiştir. Senkronizasyon sistemi Matlab/Simulink ortamında modellenip test edilmiştir. ADRC ve PID yöntemleri, bozucunun olmadığı ve bozucunun olduğu durumlar için simülasyon ortamında test edilmektedir. ADRC yönteminin, sistemi herhangi bir bozulmadan etkilenmeden senkron tutmayı başardığı test sonuçlarında görülmektedir. Öte yandan PID yönteminin, bozucu etkiler altında sistemin senkronizasyonunu sağlayamadığı açıkça görülmektedir.

Keywords: *Kaotik sistem; Senkronizasyon; ADRC; PID; PSO*

1. Introduction

Chaotic systems are a fascinating and intricate class of dynamical systems that exhibit a remarkable degree of complexity and unpredictability. These systems are characterized by their sensitivity to initial conditions, meaning that even tiny variations in the starting conditions can lead to vastly different outcomes over time. The behavior of chaotic systems often appears random, yet it is governed by deterministic mathematical equations, making them a captivating subject of study in fields ranging from physics and mathematics to biology (Güven, 2022) economics (Guegan, 2009). Chaotic systems have the intriguing property of being both deterministic and unpredictable, challenging our traditional notions of causality and determinism. They are often described as "butterfly effect" systems, where a small perturbation, akin to the flap of a butterfly's wings, can set off a chain reaction of events with profound and unforeseeable consequences. The study of chaos theory, pioneered by mathematicians and scientists in the latter half of the 20th century, has provided valuable insights into understanding and modeling these systems (Oestreicher, 2022).

Controlling chaotic systems is a formidable and intellectually stimulating endeavor that lies at the intersection of mathematics (Azar and Vaidyanathan, 2015), physics (Fradkov, 2007), engineering (Fradkov and Evans, 2005), and a wide array of other scientific disciplines. Chaos, characterized by its inherent unpredictability and sensitivity to initial conditions, might seem inherently uncontrollable. However, the pursuit of understanding and harnessing chaos has given rise to a captivating field known as "chaos control." In the realm of chaotic systems, chaos control represents a pursuit of

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order amidst apparent disorder. It involves the deliberate manipulation of system parameters or external inputs to steer a chaotic system's behavior toward desired states or trajectories (Boccaletti *et al.*, 2000). This endeavor is driven by the recognition that chaos can sometimes hinder rather than facilitate efficient functioning, whether in electronic circuits, biological systems, or financial markets. The study of chaos control is not only an intellectual challenge but also holds immense practical importance. It finds applications in various domains, from stabilizing the motion of spacecraft and controlling the chaos in heart rhythms to optimizing the performance of complex industrial processes (Schöll and Schuster, 2008). A 4D chaotic system represents a fascinating and highly intricate class of dynamical systems that exhibit extreme sensitivity to initial conditions and an exceptionally complex behavior in four dimensions. These systems are an extension of chaotic systems, which are characterized by unpredictability and irregularity in their trajectories.

Hyperchaotic systems, including 4D variants, take chaos to a whole new level by demonstrating multiple positive Lyapunov exponents, which indicates a more profound level of unpredictability and complexity. Unlike simple chaotic systems, which may exhibit chaotic behavior in three dimensions, 4D hyperchaotic systems add an additional dimension to the mix, making them exceptionally challenging to understand and analyze. The dynamics of these systems are often described by a set of nonlinear differential equations that involve various parameters and nonlinear terms. The study and exploration of 4D hyperchaotic systems have applications in a wide range of fields, including physics, engineering, cryptography, and chaos-based communication systems. These systems have been of particular interest due to their potential for secure communications and their role in generating pseudo-random sequences for encryption purposes. Iskakova et al. proposed a 4D hyperchaotic model (Iskakova *et al.*, 2023). They analyzed the model for integer order and fractional order structures. Matignon stability criteria is used to show the stability of the fractional order system. To prove the success of the proposed system, they implemented a Field Programmable Analog Arrays (FPAA) application. Gong et al. presented a 4D chaotic system with coexisting hidden chaotic attractors (Gong *et al.*, 2020).

In this study, they proposed a linear state feedback controller for Sprott C chaotic system. The novel chaotic system's dynamic properties have been comprehensively examined, including investigations into phase portraits, bifurcation diagrams, Lyapunov exponents, and Poincaré maps. An analog circuit is designed and implemented to verify the proposed 4D chaotic system. Qi and Chen proposed a new 4D chaotic system (Qi and Chen, 2006). The proposed system displays two coexisting double-wing chaotic attractors. Various circuits have been designed to realize the proposed system. Simulation and experimental results are compared. 4D chaotic systems are difficult to control (i.e. synchronization) because they have more states than other systems.

Synchronization of chaotic systems is a captivating phenomenon that has intrigued researchers and engineers alike for decades (Pecora and Carroll, 2015). It represents a fascinating interplay between chaos theory and control theory, offering profound insights into the behavior of complex dynamical systems. The concept of synchronization offers a means to bring order to chaos. Gokyildirim et al. presented a study about secure communication application (Gokyildirim *et al.*, 2023). They proposed a five-term 3D chaotic system for crypting the transferred data. To show the performance of the proposed model, microcontroller-based implementation is realized. Experimental results show the success of the model. Assali presented a study about predefined-time synchronization for chaotic systems (Assali, 2021). He proposed a control method for synchronization of different dimensioned two chaotic systems. Additionally, the adaptive control method against parameter uncertainties is also used in this study. The success of the proposed method is proven by simulation results. Zaqueros-Martinez et al. presented a study about fuzzy synchronization of chaotic systems with hidden attractors (Zaqueros-Martinez *et al.*, 2023). They examined the viability of employing fuzzy control to synchronize chaotic systems featuring hidden attractors. To achieve this, they utilized a specific numerical integration method designed to leverage the oscillatory nature of chaotic systems. A three-dimensional Chua chaotic system is used in this study. The fuzzy controller is used to achieve synchronization between master-slave chaotic systems. When the results are examined, the effect of the proposed system is clearly seen on chaotic systems with hidden attractors. Mirzaei et al. proposed a sliding mode controller for synchronization of chaotic systems with unmodeled dynamics and disturbance (Mirzaei *et al.*, 2023). They used a fixed time sliding mode controller. In this study, the fixed time synchronization problem of the nonlinear memristor chaotic system is also examined. The proposed method is tested for synchronization between the master and slave memristor chaotic systems in simulation. The numerical simulations serve to validate and substantiate the robustness of the theoretical results. The main control methods frequently used in the literature are PID control (Johnson and Moradi, 2005), fractional order PID control (Demirtas *et al.*, 2019; Ilten, 2022a), sliding mode control (Ilten and Demirtas, 2019), fuzzy logic control (Ilten and Demirtas, 2023) and neural network control (Gökçe, 2023; Sarangapani, 2018). When the synchronization studies in the literature are examined, there is a lack of Active Disturbance Rejection Control (ADRC) application. ADRC is a cutting-edge control strategy that has gained significant attention and popularity in the field of control systems engineering (Feng and Guo, 2017; Huang and Xue, 2014). It represents a paradigm shift in the way control systems are designed and implemented, offering a robust and versatile approach to handling complex and dynamic processes in a wide range of applications. ADRC stands out for its ability to effectively reject disturbances and uncertainties in real-time, making it a powerful tool for achieving precise and stable control in challenging and unpredictable environments (Fareh *et al.*, 2021). Zheng et al. presented ADRC based load frequency control study (Zheng *et al.*, 2021). They proposed chaotic fractional-order beetle swarm optimization (CFBSO) algorithm. To show effectiveness, the proposed method (CFBSO-based ADRC) is compared with Proportional-Integral-Derivative (PID) and linear ADRC. The results show the proposed method has smaller undershoot and shorter settling time then the others. Optimization has a very important place in controller design and must be implemented. Optimization is a fundamental concept that permeates virtually every aspect of our lives, from the way we make decisions in our daily routines to the complex systems that drive modern technology and industry. At its core, optimization is the art and science of finding the best possible solution from a set of available options, considering various constraints and objectives. It serves as a powerful tool for improving efficiency, making informed choices, and achieving superior outcomes across a wide range of disciplines (Chong *et al.*, 2023). The ubiquity of optimization can be seen in fields as diverse as mathematics, engineering, economics, biology, and computer science, among many others. In essence, it's about optimizing the use of limited resources, whether they are time, money, materials, or energy, to achieve specific goals. Optimization problems can vary widely in complexity, from simple linear programming tasks to intricate nonlinear, multi-objective, or combinatorial challenges, each demanding specialized techniques and approaches. Genetic algorithm (Wibowo and Jeong, 2013), particle swarm optimization (Çaşka and Uysal, 2021; Ilten, 2022b), response surface methodology (İlten, 2021) etc. are frequently used optimization methods in literature. Particle Swarm Optimization (PSO) is a powerful and versatile optimization technique inspired by the collective behavior of birds and fish in nature. Introduced in the mid-1990s by James Kennedy and Russell Eberhart, PSO has since gained popularity in various fields, including engineering, computer science, finance, and many others. This bio-inspired algorithm is designed to solve complex optimization problems by simulating the social interaction and movement of a group of particles within a multidimensional search space (Kennedy and Eberhart, 1995). PSO's underlying concept draws parallels with the cooperative behavior observed in flocks of birds or schools of fish, where individuals adjust their positions based on their own experiences and the experiences of their peers to find the best solution collectively. In PSO, each potential solution, referred to as a "particle," explores the search space while being guided by its historical best position and the global best position found by the swarm. This dynamic interplay between exploration and exploitation allows PSO to efficiently locate optimal or near-optimal solutions in a wide range of problem domains (Poli *et al.*, 2007).

In this study, ADRC based synchronization is applied for a 4D hyperchaotic system. PSO is used in optimization process of the controller coefficients. The performance of ADRC is tested under disturbances in simulation. The organization of the paper is given as follows. In section 2, a 4D hyperchaotic system equations are defined and phase portraits are presented. Synchronization process of chaotic systems and optimization of controllers are presented. In section 3, simulation results are illustrated. Finally, the discussion and conclusion are given in section 5.

2. Materials and Methods

2.1 4D hyperchaotic system

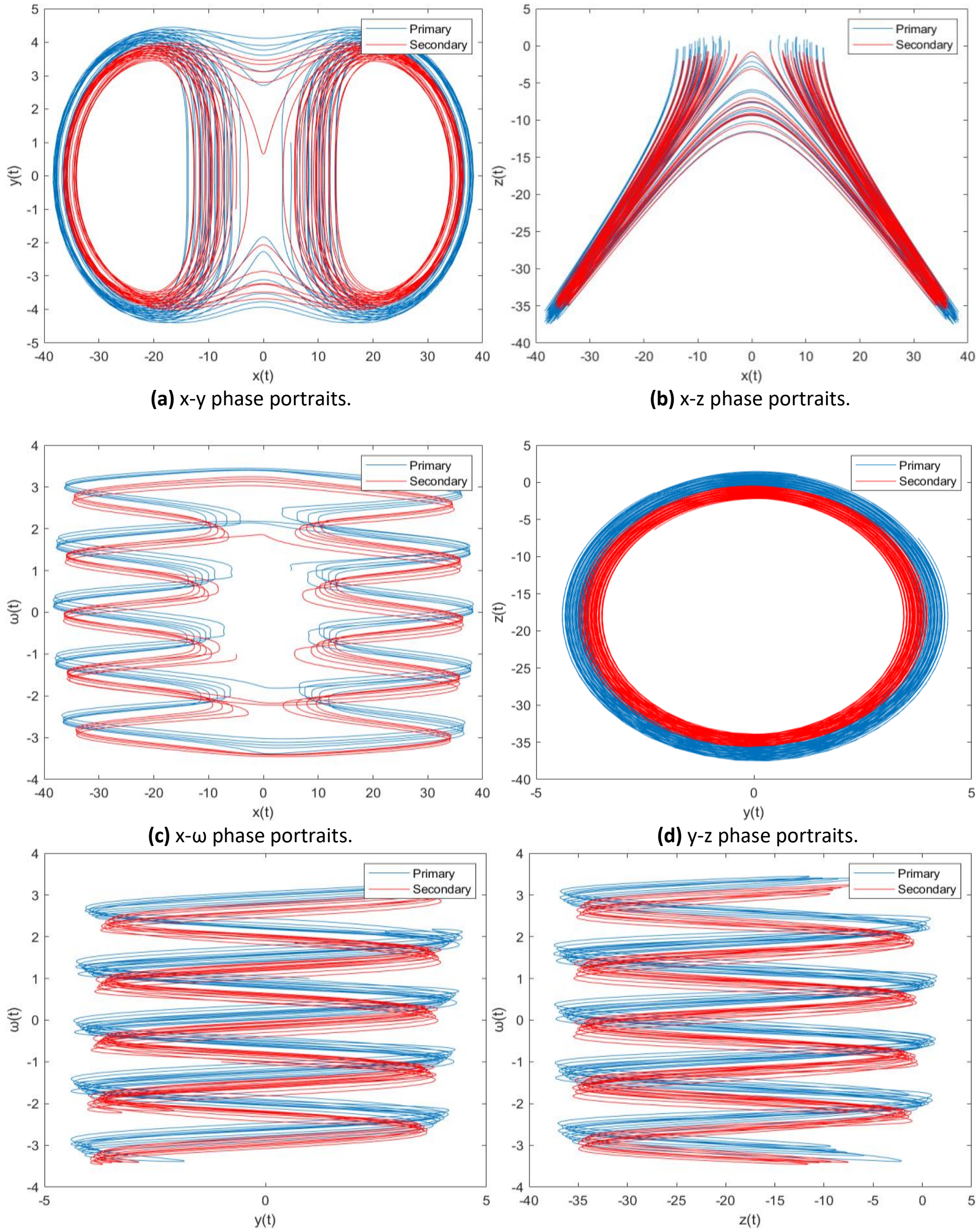
The equations of 4D hyperchaotic system is used in this study is given as follows (Iskakova *et al.*, 2023):

$$\begin{aligned}
\dot{x}(t) &= a \cdot y(t) \cdot z(t) \\
\dot{y}(t) &= b \cdot x(t) - x(t) \cdot z(t) - c \cdot x(t) \\
\dot{z}(t) &= d \cdot x(t) \cdot y(t) - x(t) \cdot \omega(t) \\
\dot{\omega}(t) &= x(t) - e \cdot \omega(t) + y(t)
\end{aligned} \tag{1}$$

The primary and the secondary system equations can be defined as below according to Eq. (1).

$$\begin{aligned}\dot{x}_1(t) &= a\cdot y_1(t)\cdot z_1(t)\\ \dot{y}_1(t) &= b\cdot x_1(t)-x_1(t)\cdot z_1(t)-c\cdot x_1(t)\\ \dot{z}_1(t) &= d\cdot x_1(t)\cdot y_1(t)-x_1(t)\cdot\omega_1(t)\\ \dot{\omega}_1(t) &= x_1(t)-e\cdot\omega_1(t)+y_1(t)\end{aligned} \tag{2}$$

$$\begin{aligned}\dot{x}_2(t) &= a\cdot y_2(t)\cdot z_2(t)\\ \dot{y}_2(t) &= b\cdot x_2(t)-x_2(t)\cdot z_2(t)-c\cdot x_2(t)\\ \dot{z}_2(t) &= d\cdot x_2(t)\cdot y_2(t)-x_2(t)\cdot\omega_2(t)\\ \dot{\omega}_2(t) &= x_2(t)-e\cdot\omega_2(t)+y_2(t)\end{aligned} \tag{3}$$



(a) x-y phase portraits. **(b)** x-z phase portraits. **(c)** x-ω phase portraits. **(d)** y-z phase portraits. **(e)** y-ω phase portraits. **(f)** z-ω phase portraits.

Figure 1. Phase portraits of different state variables.

where the parameters are a=20, b=2, c=20, d=20 and e=0.2. Initial values for primary system is chosen as x_1=5, y_1=1, z_1=1 and ω_1=1. For secondary system the initial conditions are set to x_2=-5, y_2=-1, z_2=-1 and ω_2=-1. According to these parameters, the phase portraits are drawn as given in Figure 1. When the initial conditions are different, the two systems exhibit different behavior, as can be seen in the figures.

2.2 Synchronization

In applications such as secure data transfer, synchronization of the receiver (secondary) system to the transmitter (primary) system is of vital importance. In order to achieve this, the calculation of the error values between the secondary system and the primary system must be minimized by means of a controller.

In synchronization process of the secondary system, error functions (error system) are determined as follows from Eq. (2) and (3).

$$\begin{aligned} \dot{e}_x(t) &= a\cdot y_1(t)\cdot z_1(t) - \left[a\cdot y_2(t)\cdot z_2(t)\right] \\ e_y(t) &= b\cdot x_1(t) - x_1(t)\cdot z_1(t) - c\cdot x_1(t) \\ &\quad - \left[b\cdot x_2(t) - x_2(t)\cdot z_2(t) - c\cdot x_2(t)\right] \\ e_z(t) &= d\cdot x_1(t)\cdot y_1(t) - x_1(t)\cdot \omega_1(t) \\ &\quad - \left[d\cdot x_2(t)\cdot y_2(t) - x_2(t)\cdot \omega_2(t)\right] \\ e_\omega(t) &= x_1(t) - e\cdot \omega_1(t) + y_1(t) \\ &\quad - \left[x_2(t) - e\cdot \omega_2(t) + y_2(t)\right] \end{aligned} \tag{4}$$

where $e_x(t) = x_1(t) - x_2(t)$, $e_y(t) = y_1(t) - y_2(t)$, $e_z(t) = z_1(t) - z_2(t)$, $e_\omega(t) = \omega_1(t) - \omega_2(t)$.

The secondary system is controlled to synchronize to the primary system. Secondary system model equation with controller signals $u_x(t)$, $u_y(t)$, $u_z(t)$ and $u_\omega(t)$ are given in the following equations.

$$\begin{aligned} \dot{x}_2(t) &= a\cdot y_2(t)\cdot z_2(t) + u_x(t) \\ \dot{y}_2(t) &= b\cdot x_2(t) - x_2(t)\cdot z_2(t) - c\cdot x_2(t) + u_y(t) \\ \dot{z}_2(t) &= d\cdot x_2(t)\cdot y_2(t) - x_2(t)\cdot \omega_2(t) + u_z(t) \\ \dot{\omega}_2(t) &= x_2(t) - e\cdot \omega_2(t) + y_2(t) + u_\omega(t) \end{aligned} \tag{5}$$

The synchronization process block diagram of the system is presented in Figure 2. This model is prepared in the Matlab/Simulink (MathWorks, 2023) environment.

In Figure 2, synchronizer (PID or ADRC) is used to minimize the errors between primary and secondary systems. Synchronizer can be activated via switch. Also, disturbance can be added to system via another switch to test the performance of the synchronizer.

2.3 PID control

PID is well-known method in the literature (Johnson and Moradi, 2005). It is used in this study to compare the performance of proposed method which is given in detailed in the next section. PID controller equation is presented in Eq. (6).

$$u(t) = K_P\cdot e(t) + K_I\cdot \int_0^t e(t)dt + K_d\cdot e(t)\frac{d}{dt} \tag{6}$$

where $e(t)$ is the error function. K_P, K_I and K_D are proportional, integral and derivative controller coefficients, respectively.

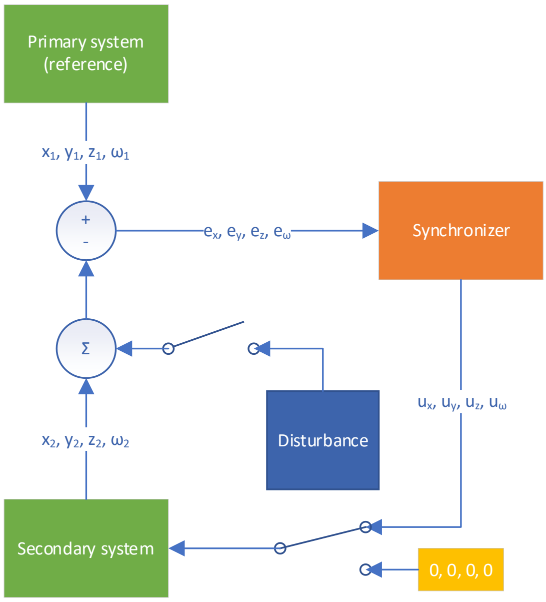


Figure 2. Block diagram of the system.

2.4 Active disturbance rejection control

Linear continuous-time error-based Active Disturbance Rejection Controller (ADRC) is used in this study (Lakomy *et al.*, 2021). The system can be described as follows:

$$\begin{aligned} \dot{e}(t) &= A_n e(t) + b_n\left[\hat{b}(\hat{e},t)u(t) + d(e,\hat{e},u,t)\right], \\ y(t) &= c_n^T e(t) - \omega(t) \end{aligned} \tag{7}$$

The extended state $z = \begin{bmatrix} e^{\mathrm{T}} & d \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{n+1}$ can be defined according to Eq. (7) as below.

$$\begin{aligned} \dot{z}(t) &= A_{n+1}z(t) + b_{n+1}\dot{d}(z,\hat{z},u,t) + d_{n+1}\hat{b}(\hat{e},t)u(t), \\ y(t) &= c_{n+1}^T z(t) - \omega(t) \end{aligned} \tag{8}$$

where $\hat{b}$ is the estimate of input gain. The observation error $\tilde{z}(t) = z(t) - \hat{z}(t)$ and the closed-loop control error $e(t)$ can be defined as below.

$$\begin{aligned} &\dot{\tilde{z}}(t)\left(A_{n+1} - l_{n+1}c_{n+1}\right)\tilde{z}(t) + b_{n+1}\dot{d}(t) - l_{n+1}w(t), \\ &\dot{e}(t) = \left(A_n - b_n k_n\right)e(t) + \left[k_n \quad 1\right]\tilde{z}(t). \end{aligned} \tag{9}$$

In Eq. (9), $l_{n+1} = \left[l_1, l_2, \ldots, l_{n+1}\right]^{\mathrm{T}} \in \mathbb{R}^{n+1}$ can be described with a single parameter ω_o (ω_o>0) as below.

$$l_i = \frac{(n+1)!}{i!(n+1-i)!}\omega_o^2 \text{ for } i\{1,\ldots,n+1\} \quad (10)$$

In Eq. (9), $k_n = [k_1 k_2,\ldots,k_n] \in \mathbb{R}^{1\times n}$ can be write with a single parameter ω_c (ω_c>0) as in Eq. (9).

$$k_i = \frac{n!}{i!(n-i)!}\omega_c^2 \text{ for } i \in \{1,\ldots,n\} \quad (11)$$

In Eq (10) and (11), ω_o is the observer bandwidth and ω_c is the controller bandwidth.

2.5 Optimization of the controllers

PSO algorithm is used for optimization of the controller coefficients. The optimization process is presented in Figure 3. Minimization of integral time absolute error (ITAE) has been taken into account as the performance criteria of the optimization process. ITAE is calculated with summing of four error functions $e_x(t)$, $e_y(t)$, $e_z(t)$ and $e_\omega(t)$. This equation is given below.

$$ITAE = \int_0^t t\left|e_x(t)+e_y(t)+e_z(t)+e_\omega(t)\right|dt \quad (12)$$

The limits of controller coefficients for PID and ADRC are defined as Eq. (13) and (14). These limit values are determined as a result of preliminary studies.

$$PID\begin{cases} 0.1 \le K_P \le 100 \\ 0.0001 \le K_I \le 10 \\ 0.0001 \le K_D \le 10 \end{cases} \quad (13)$$

$$ADRC\begin{cases} 0.8 \le \hat{b} \le 1.2 \\ 1 \le \omega_o \le 150 \\ 1 \le \omega_c \le 20 \end{cases} \quad (14)$$

At the end of optimization process, ITAE values are obtained for PID and ADRC are 32.9720 and 53.9200, respectively. These values are measured in the absence of disturbance and for initial conditions. Although PID looks better when looking at these values, it is explained in Section 4 that ADRC is much better when the performances under disturbance are examined. The optimum values of parameters obtained via PSO for PID and ADRC are given in Table 1.

3. Results

PID and ADRC controllers are compared without disturbance (Test-1) and with disturbance (Test-2) conditions. In Test-1, the system is started with uncontrolled until 5th second.

Table 1. Optimum controller coefficients.

Method	Coefficient	Value
PID	K_P	97.7096
	K_I	0.0068
	K_D	1.3128
ADRC	$\hat{b}$	0.8495
	ω_o	138.0050
	ω_c	19.9958

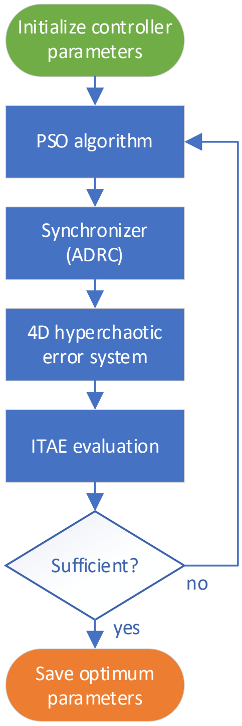
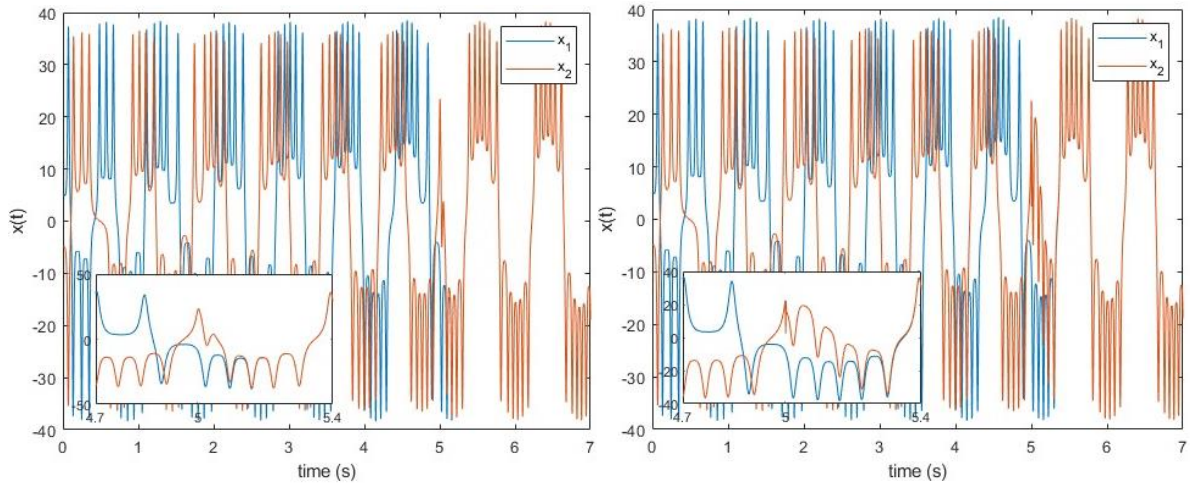


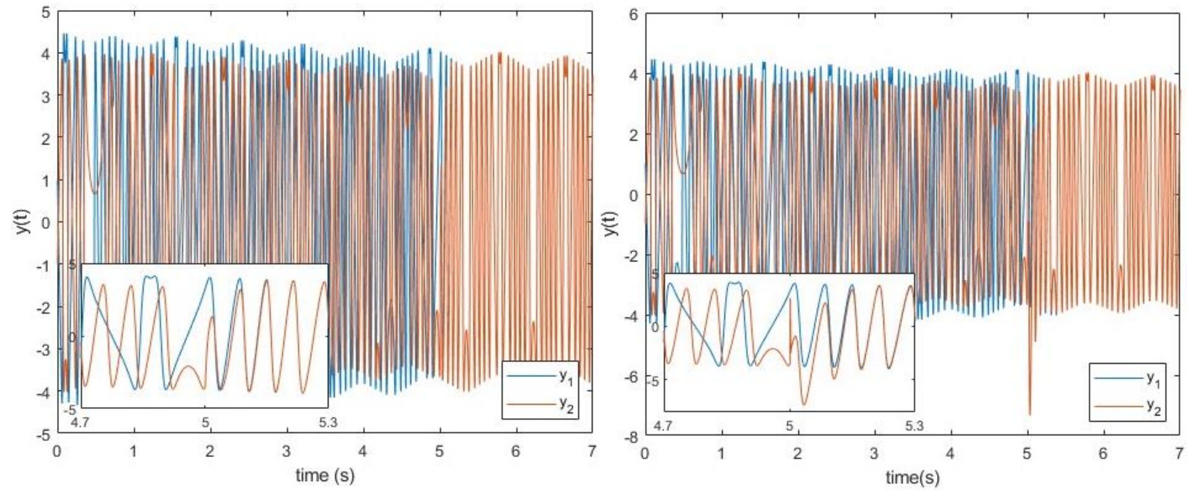
Figure 3. Optimization process.

Then controller is activated. Test-1 runs for 7 seconds. PID and ADRC results are compared for each function (x, y, z, and ω) and illustrated in Figure 4 to Figure 7. When Figure 4, 5, 6 and 7 are examined PID shows better results than ADRC such as low overshoot and shorter settling time. It should be noted that there was no disturbance in Test-1. The real performance of ADRC will be seen in Test-2. In Figure 4, settling times are measured are 5.142 s and 5.286 s for PID and ADRC, respectively. In Figure 5, these values are observed as 5.091 s and 5.154 s. The settling times are 5.162 s and 5.396 s in Figure 6 for PID and ADRC. Finally, in Figure 7, these values are obtained as 5.092 s and 5.389 s. In Test-2, the system is started with no disturbance and uncontrolled. Starting from the 5th second, the controller is activated. Disturbances (d_x, d_y, d_z and d_ω) are enter the system separately for each state (x, y, z, and ω). d_x, d_y, d_z and d_ω are added between 8-9 s, 11-12 s, 14-15 s and 17-18 s, respectively. Test-2 runs for 20 seconds. PID and ADRC results are compared for each function (x, y, z, and ω) and illustrated in Figure 8 to Figure 11.



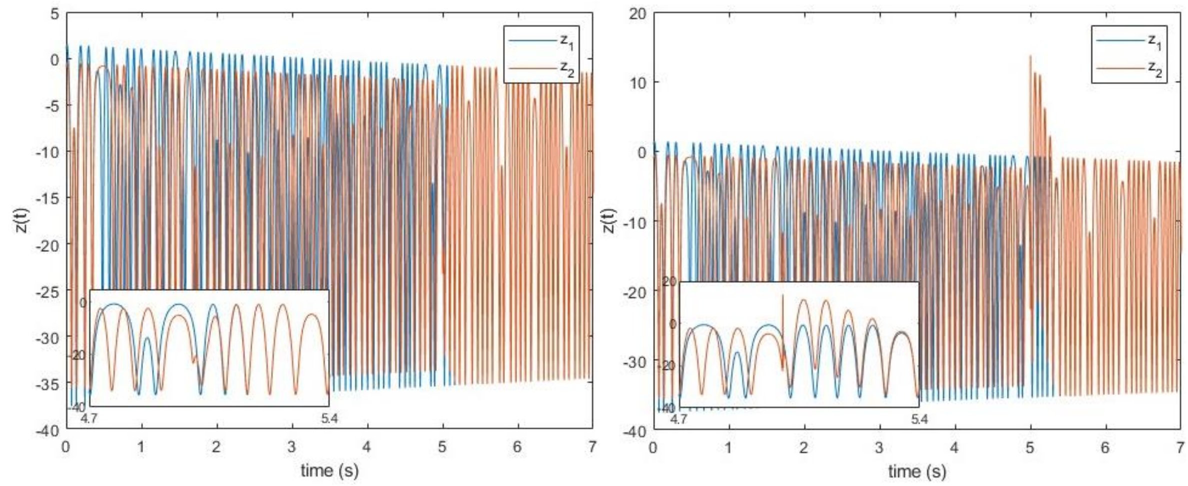
(a) PID. **(b)** ADRC.

Figure 4. Synchronization of $x(t)$ function with PID (a) and ADRC (b).



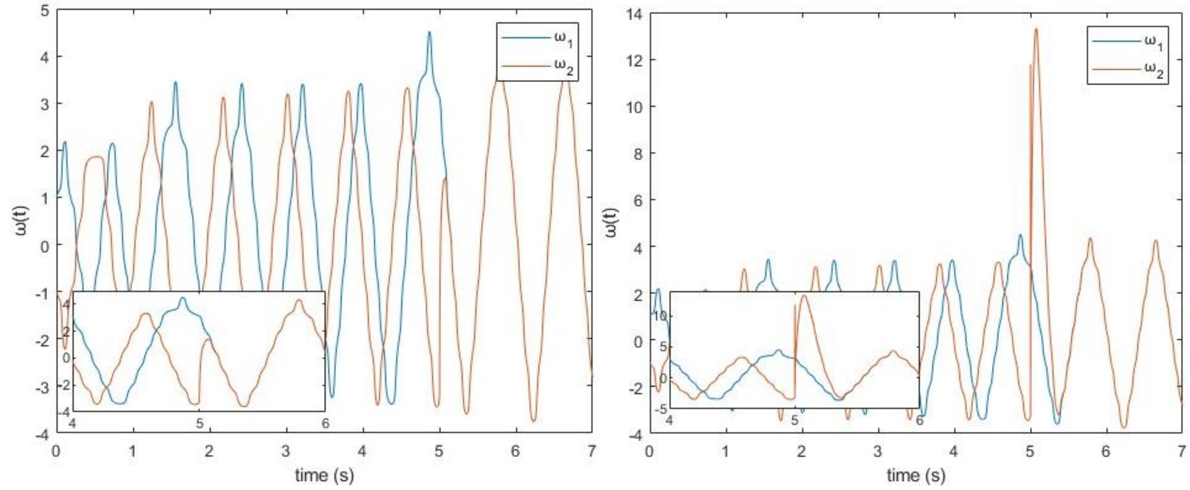
(a) PID. **(b)** ADRC.

Figure 5. Synchronization of $y(t)$ function with PID (a) and ADRC (b).



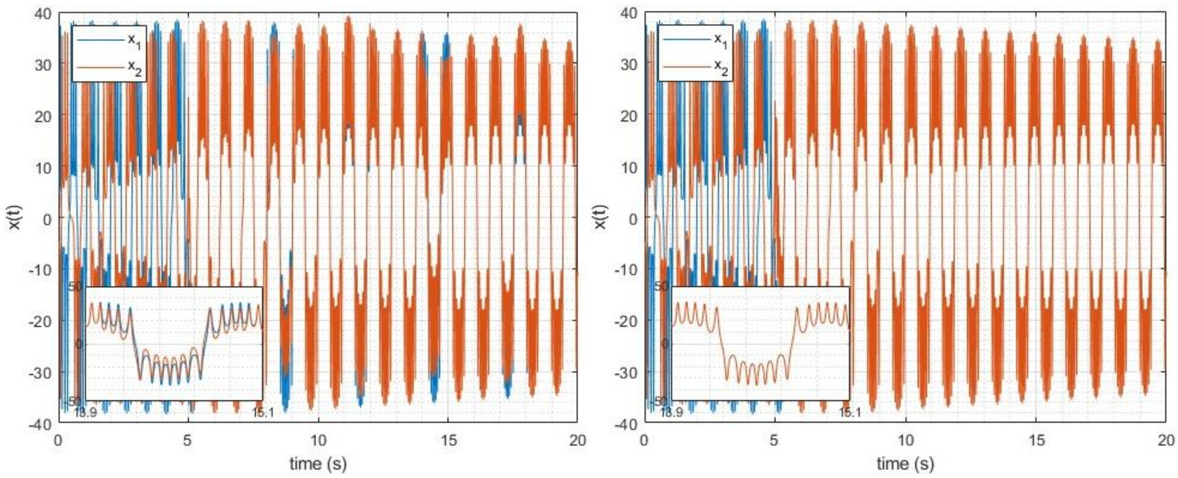
(a) PID. **(b)** ADRC.

Figure 6. Synchronization of $z(t)$ function with PID (a) and ADRC (b).



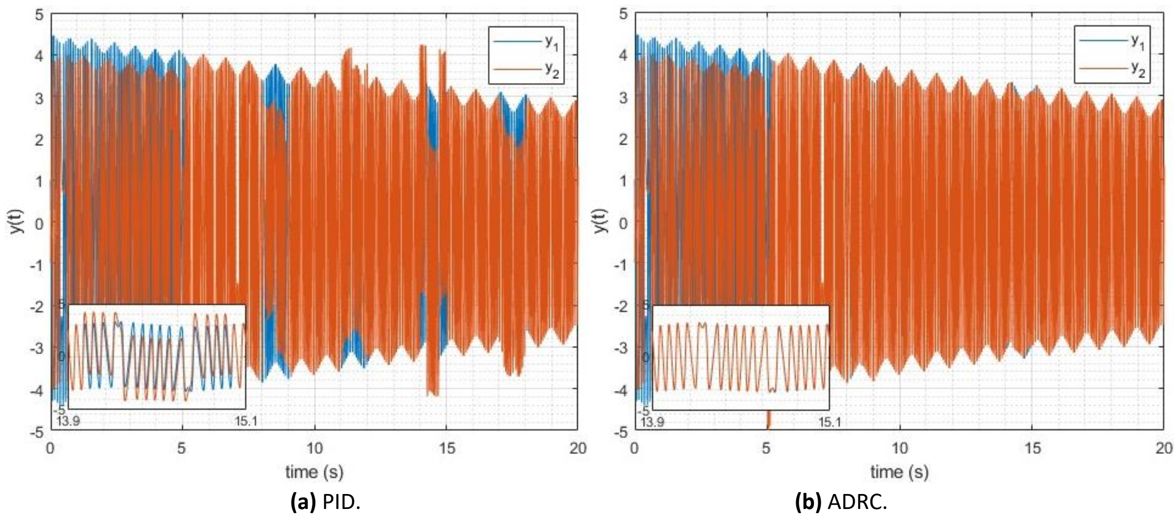
(a) PID. **(b)** ADRC.

Figure 7. Synchronization of $\omega(t)$ function with PID (a) and ADRC (b).



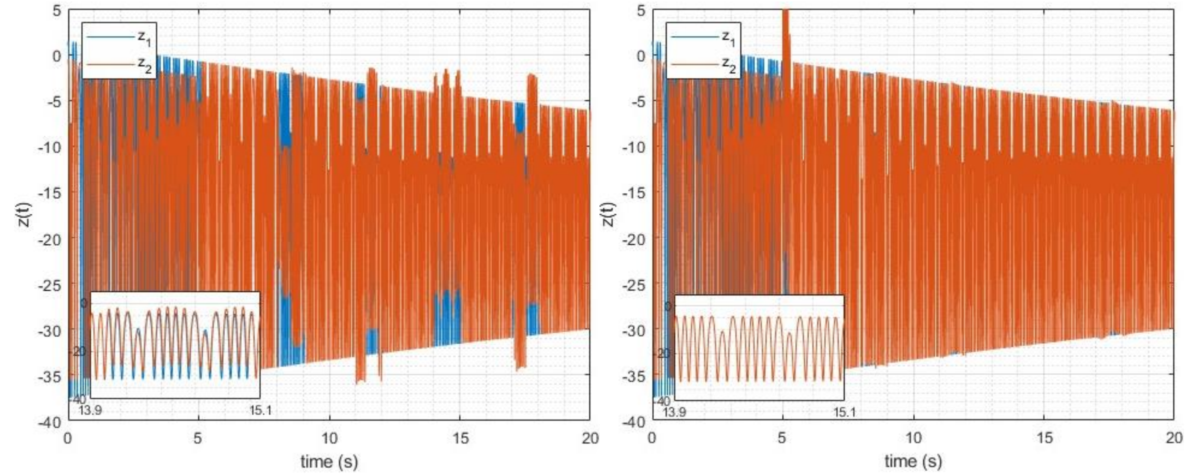
(a) PID. **(b)** ADRC.

Figure 8. Synchronization of $x(t)$ function with PID (a) and ADRC (b) under disturbance.



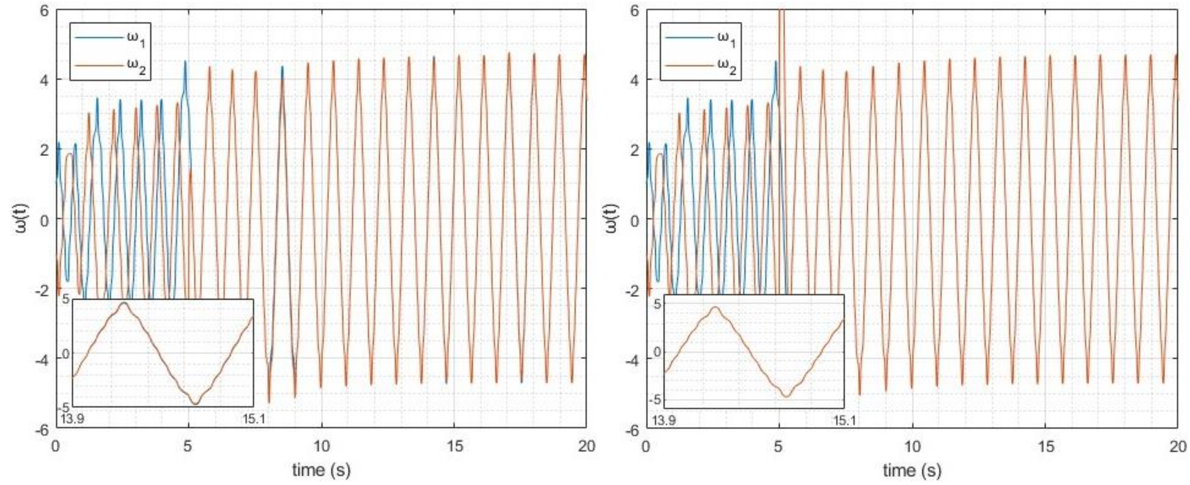
(a) PID. **(b)** ADRC.

Figure 9. Synchronization of $y(t)$ function with PID (a) and ADRC (b) under disturbance.



(a) PID. **(b)** ADRC.

Figure 10. Synchronization of $z(t)$ function with PID (a) and ADRC (b) under disturbance.



(a) PID. **(b)** ADRC.

Figure 11. Synchronization of $\omega(t)$ function with PID (a) and ADRC (b) under disturbance.

When Figure 8, 9, 10 and 11 are examined, it is clearly seen that ADRC is not affected from any disturbances. On contrary to this, large deviations are observed in PID. PID is directly affected by disturbances and cannot maintain the synchronization of system. The only case where PID is not affected by disturbances is synchronization of the ω(t) function. This situation is clearly seen when the zoomed-in sections in Figures 8, 9, 10 and 11 are examined. As a result of all these investigations, it is seen that the synchronization is maintained perfectly with the ADRC method.

4. Discussion and Conclusion

Synchronization of a 4D hyperchaotic system is studied in this paper. ADRC method is proposed as synchronizer unit and compared with PID controller for prove the success. Both controller coefficients are optimized by using PSO algorithm. Synchronization system is modelled and tested in Matlab/Simulink environment. Optimum controllers are compared on this model under no-disturbance and under disturbance tests, separately. In no-disturbance test, PID performs better for start-up process of synchronization. In other cases, especially under disturbance effects, ADRC performs much better. PID is directly affected by any disturbances and cannot maintain the synchronization. On the contrary to this, ADRC almost no affected any disturbances and maintains the synchronization perfectly. According to these results, it has been proven that the ADRC method is quite successful in synchronization of chaotic systems. When compared to synchronization studies in the literature, it is seen that ADRC exhibits similar performance to methods such as fuzzy (Zaqueros-Martinez *et al.*, 2023), sliding-mode (Mirzaei *et al.*, 2023), feedback controller (Gokyildirim *et al.*, 2023). Unlike these studies in the literature, the proposed method is also examined under disturbance effects, in this study.

Declaration of Ethical Standards
The author declares that he complies with all ethical standards.

Credit Authorship Contribution Statement
Author: Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Resources, Data curation, Writing – original draft, Writing – review and editing, Visualization

Declaration of Competing Interest
The author has no conflict of interest to declare regarding the content of this article.

Data Availability
All data generated or analyzed during this study are included in this published article.

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