Int. J. Pure Appl. Sci. 9(2);270-273 (2023)

ijpas@munzur.edu.tr
ISSN: 2149-0910

Research article/Araştırma makalesi
DOI:10.29132/ijpas.1381836

Ultra-Slow-roll Inflation Dynamics in $f(R,\phi,X)$ Gravity

Ali İhsan Keskin
Batman University, Besiri OSB Vocational School, Department of Electrical and Energy, Batman, Türkiye
aliihsan.keskin@batman.edu.tr
Received date:26.10.2023, Accepted date: 21.11.2023

Abstract

In this study, we apply the ultra-slow-roll condition to a model containing mixed terms in the background of $f(R,\phi,X)$ gravity. After the field equations are setting according to the model considered, the slow-roll indices of the inflation field are calculated. It is observed that the ultra-slow-roll inflation field comes to the fore for this case where the phenomenological mixed term is dominant. The inflation observables (the spectral index parameter and the tensor-to-scalar ratio) that occur in the background of gravity in the high-energy era of the universe are calculated, and we determine that the results are in agreement with Planck 2018 data.

Keywords: $f(R,\phi,X)$ gravity, scalar field, ultra-slow-roll inflation, high energy, cosmology

$f(R,\phi,X)$ Kütleçekiminde Ultra Yavaş Dönüşlü Enflasyon Dinamikleri

Öz

Bu çalışmada $f(R,\phi,X)$ yerçekiminin arka planında karışık terimler içeren bir modele ultra yavaş yuvarlanma koşulunu uyguladık. Ele alınan modele göre alan denklemleri oluşturulduktan sonra enflasyon alanının yavaş yuvarlanma endeksleri hesaplandı. Fenomenolojik karma terimin hâkim olduğu bu durumda ultra-yavaş yuvarlanma enflasyon alanının ön plana çıktığı görülmektedir. Evrenin yüksek enerji çağında yerçekiminin arka planında meydana gelen enflasyon gözlemlenebilirleri (spektral indeks parametresi ve tensör/skaler oranı) hesaplandı ve sonuçların Planck 2018 verileriyle uyumlu olduğunu tespit ettik.

Anahtar Kelimeler:$f(R,\phi,X)$ kütleçekim, skaler alan, ultra-yavaş dönme enflasyonu, yüksek enerji, kozmoloji

INTRODUCTION

Despite its enormous success, the typical Big Bang cosmology in General Relativity (GR) has certain well-known problems, like the horizon problem and the flatness problem (Guth 1981; Linde 1982) Adding an accelerating expansion phase in the early stage of cosmic development, before the start of the standard big bang cosmology, is one promising solution to these problems (Bardeen, 1983; Linde 1983). The mechanism to seed the current large-scale structure of the universe and the correctly measured anisotropy in the cosmic microwave background is also provided by the early accelerating expansion, called inflation (Guth 1981; Linde 1982; Linde 1983). The single-field slow-roll models are the most straightforward of all the inflationary models, and research into them is still ongoing (Chowdhury et. al. 2019).

The tensor-to-scalar ratio $\sim r$, one of the inflationary observables, is severely constrained by the recently revealed BICEP/Keck data (Ade et al., 2021) and Planck 2018 observations (Akrami et al. 2020). Several single-field inflationary models, like chaotic inflation and the original version of natural inflation, are already ruled out by the severely limited. In reality, a workable inflationary model must not only predict a tiny enough tensor-to-scalar ratio but also produce enough e-folds to fit within empirical limitations.

In this study, in high energy era of the universe, we applied the ultra-slow-roll condition to a model containing mixed terms in the background of gravity $f(R,\phi,X)$ (Odintsov and Oikonomou, 2019; Hwang and Noh, 2005; Tsujikawa, 2007; Keskin 2018; Korunur, 2016; Bahamonde et al., 2015; Yerzhanov et al., 2021; Wu, 2021; Kaczmarek and Szczesniak, 2020; Salti et al., 2016). We found that the inflation observables that occur in the background of gravity in the high energy era of the universe with an

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exponential scalar potential are in agreement with Planck 2018 data.

In the second title, the field equations of the $f(R,\phi,X)$ gravity theory and the inflationary parameters (the spectral index parameter and tensor-to-scalar ratio parameters) related to the gravity theory are given. In the third title, the calculations of the observation parameters in the ultra-slow-roll field are made within the framework of the model, where the results are compared with Planck 2018 data. The findings are summarized in the conclusion section.

$f(R,\phi,X)$ Gravity And Ultra Slow-Roll Condition Setup

To begin, we have the following gravitational action (Odintsov and Oikonomou, 2019)

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}f(R,\phi,X)\right]. \quad (1)$$

Here, R is Ricci scalar, ϕ shows the scalar field and X is the kinetic term of the scalar field. we consider the following gravity model that includes a mixing term,

$$f(R,\phi,X)=\frac{R}{\kappa^2}-2\alpha X-2V(\phi)-\gamma\phi X, \quad (2)$$

where we take $\alpha=1$ in this study. Note that $\alpha=-1$ corresponds to the phantom type scalar field. However, $\kappa^2=\frac{1}{M_p^2}$ is reduced Planck mass and γ is a free parameter of the model. According to the following FRW metric

$$ds^2=-dt^2+a(t)^2(dx^2+dy^2+dz^2), \quad (3)$$

where $a(t)$ is the scale factor (expansion rate of the universe). From the variation of the action integral given by (1) with respect to the metric tensor and scalar field, respectively, we have the following equations (Odintsov and Oikonomou, 2019)

$$3H^2=\frac{1}{F}(Xf_x+\frac{RF-f}{2}-3H\dot{F}), \quad (4)$$
$$-2\dot{H}-3H^2=\frac{1}{F}(-\frac{RF-f}{2}+\ddot{F}+2H\dot{F}), \quad (5)$$
$$\frac{1}{a^3}(a^3\dot{\phi}f_x)^{\cdot}+f_\phi=0. \quad (6)$$

Herein, the upper dot shows differentiation with respect to the cosmic time and $F=\frac{\partial f}{\partial R}, f_x=\frac{\partial f}{\partial X}$. Also, metric (3) gives $X=-\frac{\dot{\phi}^2}{2}$. According to the model (2) we can write

$$3H^2=\kappa^2(-\alpha X+V(\phi)-\frac{\gamma\phi X}{2}), \quad (7)$$
$$-2\dot{H}-3H^2=\kappa^2(\alpha X-V(\phi)-\frac{\gamma\phi X}{2}), \quad (8)$$
$$(\ddot{\phi}+3H\dot{\phi})(-2\alpha+\gamma\phi)-2V_\phi-\gamma X=0. \quad (9)$$

However, the inflationary parameters are given as follows,

$$\epsilon_1=\frac{\dot{H}}{H^2}, \epsilon_2=\frac{\ddot{\phi}}{H\dot{\phi}}, \ \epsilon_3=\frac{\dot{F}}{2HF}, \ \epsilon_4=\frac{\dot{E}}{2HE}, \quad (10)$$

where $E=-\frac{F}{2X}(Xf_x+2X^2f_{xx})$. The ultra-slow-roll condition (Martin et al., 2013) is associated with the second parameter. This condition is given as follows.

$$\ddot{\phi}=\sigma H\dot{\phi}\ , \quad (11)$$

with $\sigma=-3$. This case states the existence of a flat potential, $V_\phi=0$. Therefore, from eq. (9) we obtain the velocity term,

$$\dot{\phi}=\frac{\sqrt{4V_0 n\phi^{\frac{n-1}{2}}}}{\gamma^{\frac{1}{2}}}, \quad (12)$$

with power-law potential given by (Linde, 1983)

$$V(\phi)=V_0\phi^n\ , \quad (13)$$

where V_0 is a constant parameter. To measure inflation amount we use the number of e-foldings N given by

$$N=\int_{\phi_i}^{\phi_f}\frac{H}{\dot{\phi}}d\phi, \quad (14)$$

where ϕ_i and ϕ_f indicate the initial and the final value of the scalar field. Note that the inflation of the universe occurs at the horizon crossing point, so the value of the initial scalar field can be started at this point, $\phi_i\equiv\phi_c$. However, ϕ_f can be found from the equation $\epsilon_1(\phi_f)\sim 1$. On the other hand, the spectral index parameter and the tensor-to-scalar ratio are given by (Hwang and Noh 2005)

$$n_s=1+2\frac{2\epsilon_1-\epsilon_2+\epsilon_3-\epsilon_4}{1+\epsilon_1}\ , \quad (15)$$

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$$r = 4\left(\frac{\Gamma\left(\frac{3}{2}\right)}{\Gamma\left(\frac{3}{2}+\epsilon_2\right)2^{\epsilon_2}}\frac{\sqrt{3}\dot{\phi}\sqrt{-2\alpha+\gamma\phi}}{\sqrt{2V(\phi)}}\right)^2, \tag{16}$$

which must be calculated at the horizon crossing point.

The Inflation of the Universe from the Ultra-Slow-Roll Perspective

From the slow-roll approximation $\dot{\phi}^2 \ll V(\phi)$ equation (7) and (8) turn into the forms,

$$3H^2 \cong \kappa^2 V(\phi), \tag{17}$$

$$\dot{H} = \kappa^2\left(-\alpha X - \frac{\gamma\phi X}{2}\right). \tag{18}$$

Using eqs. (17), (14) and (12) we obtain the scalar field value at the horizon crossing point as follows,

$$\phi_c = \frac{\left(2\sqrt{27}\gamma^{\frac{1}{2}}n^{\frac{3}{2}}\kappa - 3N\sqrt{12n}\right)^{\frac{2}{3}}}{2^{\frac{2}{3}}\kappa^{\frac{2}{3}}\gamma^{\frac{1}{3}}} \equiv \frac{A^{\frac{2}{3}}}{2^{\frac{2}{3}}\kappa^{\frac{2}{3}}\gamma^{\frac{1}{3}}}. \tag{19}$$

Therefore, the leading terms of the slow-roll parameters are obtained as follows

$$\epsilon_1 = \frac{3n}{\phi}, \epsilon_2 = -3, \epsilon_3 = 0, \epsilon_4 = \frac{\sqrt{12n}}{4\gamma^{\frac{1}{2}}\phi^2}. \tag{20}$$

Here, we assume the condition γ≫α and we will check whether this assumption is true. As a result, inflationary observables were found as follows:

$$n_s = \frac{28A^{\frac{4}{3}} + 602^{\frac{2}{3}}\gamma^{\frac{1}{3}}n\kappa^{\frac{2}{3}}A^{\frac{2}{3}} - 2\sqrt{12n}2^{\frac{4}{3}}\kappa^{\frac{4}{3}}\gamma^{\frac{1}{6}}}{4A^{\frac{4}{3}} + 12n\gamma^{\frac{1}{3}}\kappa^{\frac{2}{3}}2^{\frac{2}{3}}A^{\frac{2}{3}}}, \tag{21}$$

$$r = 4\left(\frac{8\sqrt{6n}\,\Gamma\left(\frac{3}{2}\right)}{\Gamma\left(\frac{-3}{2}\right)}\right)^2. \tag{22}$$

The Planck 2018 data show the following constraint for the inflation phase (Akrami, et al. 2020)

$$n_s = 0.9649 \pm 0.0042, \quad r < 0.064. \tag{23}$$

We proceed by investigating from the point of the phenomenological of our model given by equation (2). Note that the tensor-to-scalar ratio depends only on the power term of the potential. Since it is assumed the condition $\gamma \gg \alpha$ given below eq. (20), the tensor-to-scalar ratio produces the range $0 < n < 0.00029$ for n. So, by using $n = 0.0028$ into n_s given by (21) we obtain $\gamma = 4.6782 \times 10^{10}$, which is coincided with the condition $\gamma \gg \alpha$, where $\alpha = 1$. As a result, the spectral index parameter is found in the range of the observational data (Akrami et al. 2020), as follows,

$$n_s = 0.964. \tag{24}$$

CONCLUSION

In this work, we investigate the ultra-slow-roll condition in the early high-energy era of the universe. For this purpose, we take the phenomenological model of $f(R,\phi,X)$ gravity theory given by (2). We obtained results in which the mixed term is fully dominant, that is, condition $\gamma \gg \alpha$. Under this condition, we calculated the tensor-to-scalar ratio and the spectral index parameter. It is observed that the results are in good agreement with Planck's observation data. On the other hand, we can write the state of equation parameter w from equations (7) and (8). From the expression $w = -1 - \frac{2\dot{H}}{3H^2}$ we obtain the equality $w = \frac{n-1}{n+1}$. If we write the value $n = 0.00028$ into the equality, we obtain $w = -0.99944$, which corresponds to the de Sitter-like phase. This is an expected value for the ultra-slow-roll inflation.

CONFLICT OF INTEREST

The Author report no conflict of interest relevant to this article

RESEARCH AND PUBLICATION ETHICS STATEMENT

The author declares that this study complies with research and publication ethics.

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