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Dynamic Dependence between Oil and Stock Markets: International Evidences with Stochastic Copula Approach

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Petrol ile Borsalar arasındaki Dinamik Bağımlılık: Stokastik Kopula Yaklaşımı ile Uluslararası Bulgular

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Abstract
Modeling the dependency structure between variables has recently received increasing attention in many disciplines, especially in finance and economics. In this study, the dependence between oil prices and stock markets is investigated through the stochastic copula approach, which is a subset of time-varying copulas. This model enables to capture whole dependency between variables dynamically. Unlike time-varying copulas, it regards the latent process and observations in modeling dependency and thus evaluates the dependency structure in a more comprehensive framework. Empirical findings suggest that dependency between oil and stock markets evolve over time. There is a symmetric dependence between oil and the UK stock market, but the relationship between oil and the US stock exchange is measured by upper tail dependence. This indicates that oil and the US stock market are more likely to move together during periods of market uptrend.

Keywords: *Dependency; Stochastic Copula; Dynamic Modeling; Oil Price; International Stock Market.*

Öz
Değişkenler arasındaki bağımlılık yapısının modellenmesi son dönemde öncelikle finans ve ekonomi olmak üzere birçok disiplinde giderek artan bir ilgi kazanmıştır. Bu çalışmada, petrol fiyatları ile hisse senedi piyasaları arasındaki bağımlılık, zamanla değişen kopulaların bir sınıfı olan stokastik kopula yaklaşımıyla araştırılmaktadır. Bu model değişkenler arasındaki tüm bağımlılığı dinamik olarak yakalamayı sağlar. Zamanla değişen kopulalardan farklı olarak bağımlılığı modellemede gözlemlerin yanı sıra gizli süreci de dikkate alır ve böylece bağımlılık yapısını daha kapsamlı bir çerçevede değerlendirir. Deneysel bulgular, petrol ve hisse senedi piyasaları arasındaki bağımlılığın zamanla geliştiğini göstermektedir. Petrol ile Birleşik Krallık borsası arasında simetrik bir bağımlılık vardır ancak petrol ile ABD borsası arasındaki ilişki üst kuyruk bağımlılığı ile ölçülmektedir. Bu, petrol ve ABD borsasının yükseliş trendi dönemlerinde birlikte hareket etme eğiliminin daha yüksek olduğunu göstermektedir.

Anahtar Kelimeler: *Bağımlılık; Stokastik Kapula; Dinamik Modelleme; Petrol Fiyatı; Uluslararası Borsalar*

1. Introduction

There are various dependency structures between variables, and the discovery of these dependencies constitutes one of the main research topics of many fields. Classical approaches used in dependency modeling either require the assumption of normality for the marginals of the relevant variables or are inadequate in modeling various dependency structures such as tail dependency. One of the methods proposed to deal with such issues is copula. According to the theorem of Sklar (1959) who first introduced the existence of copulas, there is a copula function that joins the multivariate distribution function to their univariate marginals. This theorem suggests that there is a copula that can handle various dependency structure between variables with different marginals. Cherubini et al. (2004) states the fundamental theories of copulas and discusses their different applications in various markets.

McNeil, et al. (2005) presents a comprehensive theory of copulas and reveals effective solutions that copulas offer to some problems in financial risk management. Nelsen (2006) outlines the basic theory of copulas, copula construction methods, and implications of dependency measurements, and also provides many examples in various fields. Caillault and Guegan (2005) present a non-parametric method of obtaining lower and upper tail dependencies based on the Bootstap approach. It uses the copula approach to model dependencies in the Asian market. It concludes that there is symmetrical dependence between Thailand and Malaysian markets, and asymmetrical dependence between Thailand and Indonesia and Malaysia and Indonesian markets. Hu

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(2006) investigates the dependency structure among international financial markets with mixed copulas. This proposed approach offers significant flexibility since it can model different dependency structures. Rodriguez (2007) models the dependencies in Asian and Latin American markets using the Markov-Swithing model. It is concluded that the dependence between the relevant markets is asymmetric. These studies are conducted assuming that dependency is constant over time. However, Patton (2006) introduced the time-varying copulas to the literature. In this model, it is assumed that the dependency between variables changes over time. The dependency structure between different financial markets was investigated via time-varying copulas Hu (2010); Haffar and Le Fur (2022); Garcia-Jorcano and Benito (2020); Li and Zeng (2018); Naeem et al. (2021); Rehman et al. (2020), Yang and Hamori (2014). The time-varying copulas, also called the conditional copulas, are estimated by considering only observations. However, in stochastic copula approach, estimations are performed by considering both observations and the latent process, thus utilizing more information in modeling the dependence structure. Therefore, the stochastic copula model evaluates the relationship between variables in a more comprehensive framework. In addition, this approach is recommended as a more suitable model for the dynamics of financial markets, as it assumes that dependence varies over time, like time-varying copulas.

In this study, the dependency structure between oil prices and stock exchange is handled by way of the stochastic copula approach. Autocorrelation and heteroscedasticity problems in each series are overcome by ARMA-GARCH (Autoregressive Moving Average – Generalized Autoregressive Conditional Heteroscedasticity) method and then the dependency structures are investigated with the stochastic copula. It is found that the dependence between oil prices and the UK stock market evolve over time and is symmetrical. It is concluded that the dependence between oil prices and the USA stock market also changes over time, but this relationship is measured by upper tail dependence.

The rest of the paper is organized as follows: the methodology is introduced in Section 2. Data description and empirical findings are presented in Section 3. The results are discussed in Section 4.

2. Methodology

The background of dependency modeling and marginal modeling is given in this section.

2.1 Dependence modeling

There is a need for powerful tools that can overcome many dependency structures, and one of these methods is copula. Since they do not require strict assumptions in dependency modeling, copulas are widely used in many disciplines, especially in finance and econometrics. Moreover, handling asymmetric as well as symmetric dependencies has led to an increased interest in copulas. Let F be the *n*-dimensional joint distribution function and $F_1, \dots, F_n$ be the marginal distribution functions. There is an n-dimensional copula C for all x in R^n and is defined as follows:

$$F(x_1, \dots, x_n) = C\big(F_1(x_1), \dots, F_n(x_n)\big) \tag{1}$$

n-dimensional joint probability density function can be factorized into the product of marginal densities and a copula density:

$$\begin{aligned} f(x_1, \dots, x_n) &= \frac{\partial F(x_1, \dots, x_n)}{\partial x_1, \dots, x_n} \\ &= \frac{\partial C(u_1, \dots, u_n)}{u_1, \dots, u_n} x \prod_{i=1}^{n} \frac{\partial F_i(x_i)}{\partial x_i} \end{aligned}$$

$$f(x_1, \dots, x_n) = c(u_1, \dots, u_n) x \prod_{i=1}^{n} f_i(x_i) \tag{2}$$

Here c represents the copula density function. Eq. (2) indicates that the joint probability density function can be separated as the product of the marginal densities and a copula that captures whole dependencies between related variables. Additionally, the marginals can be chosen independently of each other, and similarly the copula can be determined independently of the marginals. This provides flexibility in modeling the dependency between variables. Kendall Tau and tail dependency for some copula families are presented in Table 1.

Table 1. Kendall Tau and tail dependency coefficients of copulas

Copula	Kendall Tau	Lower Tail	Upper Tail
Normal/Gaussian	$\tau = \frac{2}{\pi} arcsin\,(\theta)$	/	/
Clayton	$\tau = \frac{\theta}{\theta + 2}$	$\lambda_L = 2^{-1/\theta}$	$\lambda_U = 0$
Gumbel	$\tau = 1 - \frac{1}{\theta}$	$\lambda_L = 0$	$\lambda_U = 2 - 2^{1/\theta}$

Notes: Rotated copulas called survival copulas are 180^0 rotation versions of copulas. They are associated with the Gumbel and Clayton families.

For more information on the copulas, Nelsen (2006) and Joe (2014) can be reviewed. The stochastic copula model proposed by Hafner and Manner (2012) is described as follows. Let $(u_{1,t}, u_{2,t})$ for $t = 1, \dots, T$ be bivariate time series and their distribution function is defined by the copula model with dynamic θ parameter given below.

$$\left(u_{1,t}, u_{2,t}\right) \sim C(u_1, u_2 | \theta_t) \tag{3}$$

Where $\theta_t \in \Theta \subset R$ and It is based on the assumption that it the parameter θ_t is driven by an unobserved process. $\theta_t = \Psi(\lambda_t)$ transform is applied to ensure that parameter of the copula remains in its domain. Here, $\Psi: R \rightarrow \Theta$ and transformation Ψ depends on the chosen copula. For information on transformations, the appendix can be viewed. λ_t is the latent process and follows the first order of the Gaussian autoregressive process:

$$\lambda_t = \alpha + \beta\lambda_{t-1} + \kappa\varepsilon_t \qquad |\beta| < 1,\ \kappa > 0 \tag{4}$$

Here ε_t is a Gaussian innovation process. The estimations of parameter are carried out by an independent stochastic process. The model is nonlinear and can be written in its state space representation.

$$(u_{1t}, u_{2t}) | \lambda_t \sim C\left(u_1, u_2 \middle| \Psi(\lambda_t)\right) \tag{5}$$

The state equation is given in Eq. (5) and the transition equation is defined as in Eq. (6):

$$\lambda_t = \alpha + \beta\lambda_{t-1} + \kappa\varepsilon_t \tag{6}$$

The parameters in Eq. (6) are estimated by the ML-EIS method, which is a combination of efficient importance sampling (EIS), one of the Monte Carlo simulation methods, and maximum likelihood. For comprehensive information on the parameter estimation process of the stochastic copula model, Hafner and Manner (2012) can be scrutinized.

2.2 Marginal modeling

There is evidence of autocorrelation and heteroscedasticity in financial time series Bollerslev (1986). ARMA – GARCH models are used to overcome such problems and these models are defined as follows:

$$y_t = \mu + \sum_{i=1}^{p} \varphi_i y_{t-i} + \sum_{j=1}^{q} \gamma_j a_{t-j} + a_t \tag{7}$$

$$a_t = \sigma_t \epsilon_t$$

$$\sigma_t^2 = \omega + \sum_{k=1}^{m} \delta_k \sigma_{t-k}^2 + \sum_{l=1}^{n} \zeta_l a_{t-l}^2 \tag{8}$$

Where φ and γ are the parameters AR and MA, respectively, and μ is the constant. The parameters ω, δ and ζ in the GARCH model represent constant, GARCH and ARCH parameters, respectively. The standardized residuals obtained from ARMA-GARCH models are transformed into uniform variables with PIT (Probability Integral Transform), thus providing the required inputs for copulas in dependency modeling.

3. Empirical Findings

The data consists of daily closing prices of oil and stock markets. The labels are ‘UK’ for the FTSE 100 from United Kingdom, ‘USA’ for the S&P 500 from the United States of America and ‘oil’ for the Brent crude oil. The dataset ranges from January 4, 2016 to December 30, 2022 and includes 1726 daily observations. All data is extracted from Yahoo Finance. The study is carried out with MATLAB and R software languages. The prices of oil and stock exchange are exhibited in Fig.1.

Significant trend patterns appear over the whole sample period for each series, implying that the price series are not stationary. For this reason, return series calculated as Eq. (9) are used instead of the original series. These series satisfy the stationarity condition and offer some important statistical properties.

$$r_t = ln(p_t) - ln(p_{t-1}) \tag{9}$$

Here, p_t is the price of the asset at time t. The descriptive statistics of oil and stock market returns are presented in Table 2

The returns of the interested stock markets and oil are positive, and the mean return of oil with the highest risk is comparatively higher. The unconditional standard deviations of the returns are larger than their means, indicating that the returns are highly volatile. Although the mean return of the US stock market is three times higher than the mean return of the UK stock market, there is a very little difference between the returns of the two stock markets in terms of median returns. It is observed that all return series are left-skewed and excess kurtosis. This implies that the return series are far from normal distribution. The unconditional correlation coefficients are evaluated as 0.29659 for UK and oil and 0.27111 for US and oil. It is possible to refer a positive significant relationship between the relevant variable pairs. The null hypothesis of normal distribution for all return series is rejected by the JB test. ADF test confirms that the series are stationary. The Ljung Box Q test suggests that there is serial correlation in return series at lag 12 excluding oil and also there is serial correlation in the squares of all series at lag 12. The ARCH effect in return series is confirmed by the Lagrange Multiplier test.

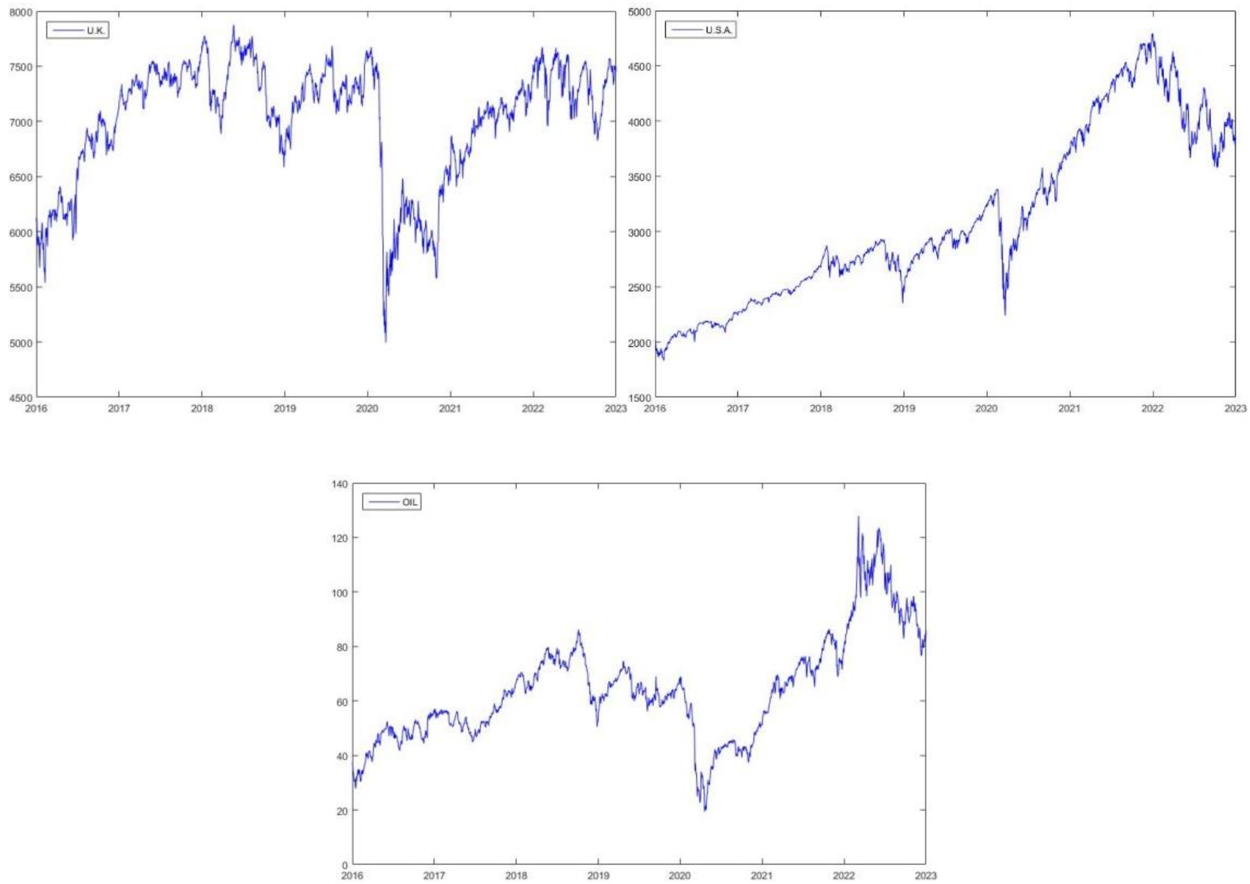

Figure 1. The price series of oil and the stock markets.

Table 2. Descriptive statistics and statistical tests of daily returns

	UK	USA	OIL
Panel A: Descriptive Statistics			
Mean	0.00011	0.00037	0.00048
Std. Dev.	0.01065	0.01231	0.02702
Median	0.00061	0.00070	0.00233
Min	-0.11512	-0.12765	-0.27976
Max	0.08666	0.08968	0.19077
Skewness	-0.89912	-0.83170	-1.17837
Kurtosis	14.25351	15.75640	17.09193
Corr.	0.29659	0.27111	
Panel B: Statistical Tests			
J-B	14876.0***	18093.0***	21454.0***
ADF	-41.8***	-49.3***	-39.8***
Q(12)	47.523***	247.71***	16.993
$Q^2(12)$	892.85***	2304.7***	448.24***
ARCH(12)	423.77***	673.45***	250.63***

Notes: This table exhibits summary statistics for UK and USA stock market returns. The sample period is from January 1, 2016 to December 31, 2022. Correlation refers to the relationship between oil and stock exchange returns. *** indicates the rejection of the null hypotheses of normality, nonstationary, no autocorrelation and homoscedasticty at % 1 significance level.

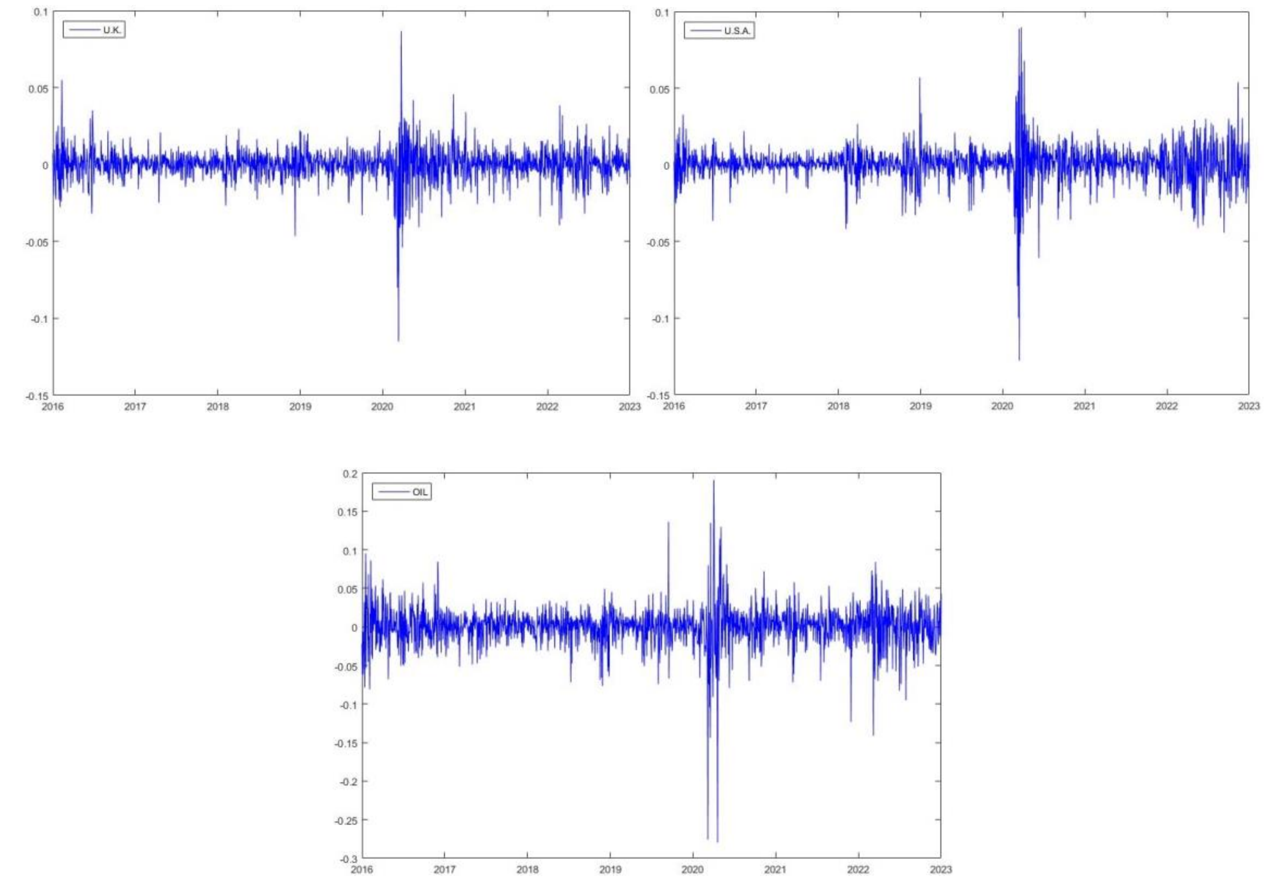

Figure 2. The return series of oil and the stock markets.

These results reveal that the ARMA-GARCH approach[1] is needed in modeling the marginals of return series. The standardized residuals obtained from the ARMA-GARCH model are independent and identically distributed. By applying probability integral transformation to the standardized residuals obtained from ARMA-GARCH models, the uniform inputs required for the copula are produced. The original series of oil and stock markets are exhibited in Fig. 1. It is clear that there are trend patterns in the series, implying that the series are not stationary. Although the upward trend in the UK stock exchange and oil prices ended in the first half of 2018, it has been observed that the upward trend in oil and the USA stock market, which started in 2016, continues until 2022. After the declaration of Covid-19 as an epidemic disease by the World Health Organization, sharp fall occurred in both stock markets and oil prices. The US stock market compensated for these declines more quickly. Common levels between oil and stock markets are presented in Fig. A1 in the Appendix. The return series of the related variables are shown in Fig. 2. These series appear to be stationary around the mean. However, high volatility emerged in the return series during the period of negative events affecting financial systems. It is evident that there is volatility clustering in the series. It can be said that volatility has increased in the oil and stock exchange after the Covid-19 pandemic period. Due to the structure of the stochastic copula approach, the dependency structure between variables is considered in pairs. In this study, the dependency between oil and stock exchange are analysed separately. According to the log likelihood, AIC and BIC criterions, the best fitted copula model is selected. In the stochastic copula model, the β parameter indicates persistency in the dependency, while the κ parameter refers whether the dependency is dynamic or not (Marimoutou and Soury, 2015). The best fitted stochastic copula results for each pair of variables are presented in Table 3. Gumbel, Clayton, Normal, Rotated Clayton and Rotated Gumbel copulas are investigated and the dependence between the UK stock market and oil is best modeled with the normal copula. The β parameter indicates high persistency in the dependence between the UK stock market and oil. Since the κ parameter is not equal to zero, there is a dynamic dependence between the UK stock exchange and oil and the dependency is symmetrical and time-varying. The Gumbel copula for the dependency between oil and US stock market is the best fitted model. Since the value of the β parameter is 0.9234, there is high persistency in the dependency between the relevant markets. The κ parameter being non-zero hints that the dependence between the US stock market and oil evolves over time. Moreover, the relationship between the US stock market and oil is measured by upper tail dependency because Gumbel copula can only

[1] The results of ARMA-GARCH models are available upon request

model upper tail dependence. This means that the US stock market and oil prices tend to move together more during periods of market uptrend than during periods of market downtrend. Since the USA is one of the world's largest oil exporting countries, the rise in oil prices contributes to the USA's financial markets, and this is supported by the results of the stochastic copula. The results of all stochastic copula models estimated for oil and stock exchange are reported in Table A1 and Table A2 in the Appendix. The dependence paths between oil and stock markets are exhibited in Fig. 3.

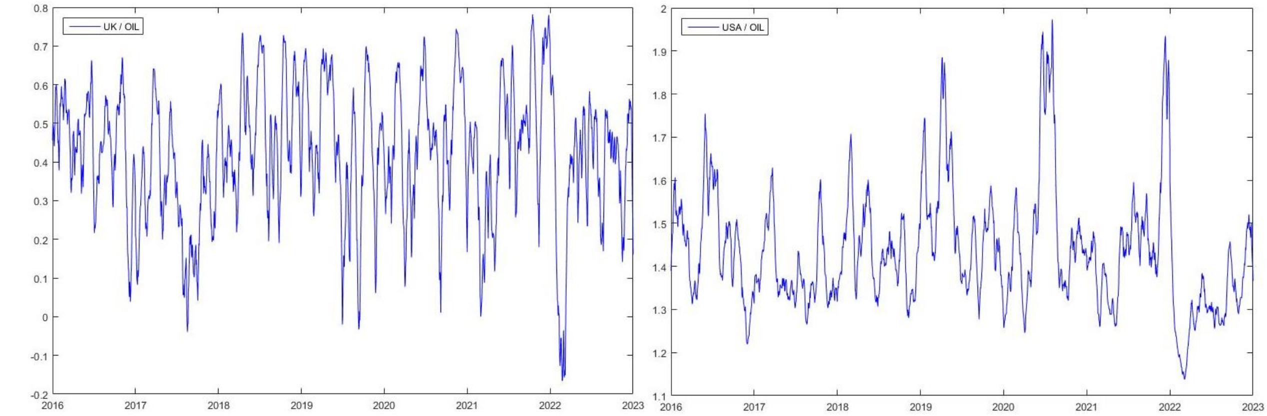

Figure 3. Dependence path of oil and the stock markets.

Table 3. Parameter estimations of the best fitted stochastic copula.

	α S.E.	β S.E.	κ S.E.	Log L	AIC	BIC
OIL – UK	0.0720	0.8563	0.2030	128.0075	-254.015	-248.562
(Normal)	(0.0266)	(0.0500)	(0.0474)			
OIL – USA	-0.0766	0.9234	0.2249	102.4549	-202.910	-197.457
(Gumbel)	(0.0520)	(0.0498)	(0.0999)			

Notes: This table exhibits estimations of the parameter for stochastic copula models. The parameters represent constant term, persistence level and dynamic dependency, respectively.

It is apparent that the dependency between oil and stock exchange changes dramatically over time. While the unconditional correlation coefficient between oil and the UK stock exchange is 0.29659 (<0.05), the average of the conditional correlation coefficient, described as the average of the parameter estimations of the normal copula with only the ρ_t correlation coefficient parameter, is 0.4211. Additionally, it is found out that there is an asymmetric dependence structure between oil and the US stock market through the stochastic copula approach. While well-known relationship coefficients can evaluate unconditional relationships between variables, the stochastic copula approach can model time-varying symmetric and asymmetric dependencies such as tail dependencies between variables. Thus, it provides more information on the dependence structure between the relevant variables and gives more realistic results by considering this dependency structure in a time-dependent manner.

4. Conclusion Remarks

The dependency modeling between variables has become popular recently, particularly in finance and economics. In this study, dependency patterns between oil and stock markets are handled through the stochastic copula approach. The data set consists of FTSE 100 and S&P 500 from international stock markets and Brent crude oil spot prices from oil markets. To ensure the stationarity, log-return series are used instead of price series. Dependency modeling consists of two stages: First, modeling the marginals and then modeling the dependency between the variables.

The marginal distributions of the oil and stock markets are estimated with the ARMA-GARCH approach. The resulting standardized residuals are transformed into uniform inputs. The best fitted stochastic copula models are estimated based on these inputs. Then, the stochastic copula approach is considered to model the dependency structure between oil and stock markets. This approach enables to handle the dependency structure between variables dynamically. The stochastic copula models differ from the time-varying copulas because they consider the unobservable process as well as observations in modeling the dependency. Thus, this approach models the dependency in a broad way by utilizing more information on the variables. Since the stochastic copula approach does not need any assumptions on the marginals of the variables, it can model the dependence between the related variables flexibly. Empirical findings find out that the dependence between oil and stock exchange changes over time. Additionally, there is symmetrical dependence between oil and the UK stock market since the dependence between the two markets is best modeled by the normal copula. Gumbel copula is the best fitted model for the dependency structure between oil and the US stock market, and the relationship between the relevant markets is evaluated by upper tail dependency. This indicates that oil prices and the US stock market are more likely to move together during market uptrends.

Declaration of Ethical Standards

The authors declare that they comply with all ethical standards.

Credit Authorship Contribution Statement

Author-1: Investigation, methodology and software, visualization and writing – original draft and editing.
Author-2: Conceptualization, review, supervision.

Declaration of Competing Interest

The authors have no conflicts of interest to declare regarding the content of this article.

Data Availability Statement

The dataset that support the findings of this study are available from the corresponding author upon request.

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Appendix

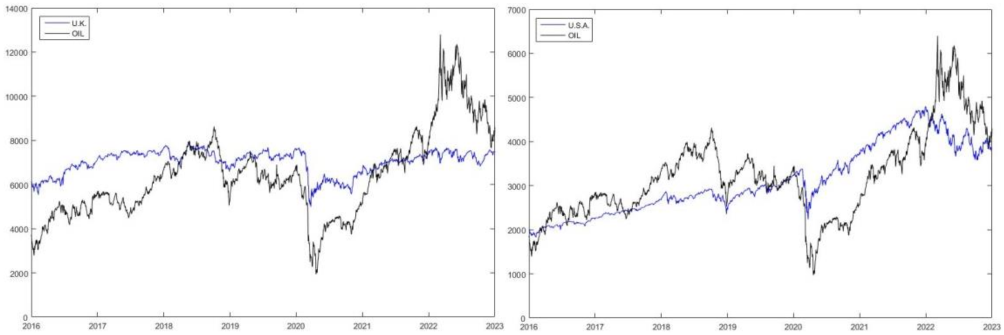

Figure A1. Common levels between oil and the stock markets.

Table A1. Parameter estimations for stochastic copula: UK and OIL

Copula	α S.E.	β S.E.	κ S.E.	Log L	AIC	BIC
Gumbel	-0.0925	0.9023	0.3297	127.6229	-253.246	-247.793
	(0.0352)	(0.0346)	(0.0763)			
Clayton	-0.0593	0.9256	0.3084	92.3139	-182.628	-177.175
	(0.0261)	(0.0346)	(0.0677)			
Normal	0.0720	0.8563	0.2030	128.0075	-254.015	-248.562
	(0.0266)	(0.0500)	(0.0474)			
Rotated Gumbel	-0.1092	0.9118	0.3457	107.8554	-213.711	-208.258
	(0.0390)	(0.0325)	(0.0676)			
Rotated Clayton	-0.0479	0.8837	0.3944	110.6160	-219.232	-213.779
	(0.0164)	(0.0345)	(0.0595)			

Table A2. Parameter estimates for stochastic copula: USA and OIL

Copula	α S.E.	β S.E.	κ S.E.	Log L	AIC	BIC
Gumbel	-0.0766	0.9234	0.2249	102.4549	-202.910	-197.457
	(0.0520)	(0.0313)	(0.0999)			
Clayton	-0.0521	0.9450	0.2327	66.2312	-130.462	-125.009
	(0.0310)	(0.0313)	(0.0849)			
Normal	0.0469	0.8981	0.1409	100.0040	-198.008	-192.555
	(0.0223)	(0.0470)	(0.0436)			
Rotated Gumbel	-0.0805	0.9420	0.2505	74.0597	-146.119	-140.666
	(0.0480)	(0.0332)	(0.0861)			
Rotated Clayton	-0.0616	0.8525	0.3413	89.6505	-177.301	-171.848
	(0.0361)	(0.0726)	(0.1159)			