
ELECTRO HYDRAULIC SUSPENSION SYSTEM DESIGN WITH OPTIMAL STATE DERIVATIVE FEEDBACK CONTROLLER

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ABSTRACT

This paper is concerned with a control design of an electro-hydraulic suspension system. In some practical problems, for instance in the active suspension design, the state derivative signals such as acceleration and velocity are easier to obtain rather than the state variables such as displacement and velocity, since the most commonly used sensors are the accelerometers. Hence, design of an optimal state derivative feedback controller is proposed by employing the linear matrix inequalities framework. In order to demonstrate the effectiveness of the proposed controller, a two-degree-of-freedom quarter vehicle suspension model equipped with an electro hydraulic actuator is preferred. Throughout the numerical simulation studies, bump type road irregularities at different vehicle forward velocities are applied to evaluate the performances of the controller in terms of ride comfort and safety.

Keywords: State derivative feedback, Optimal control, Electro-hydraulic suspension, Linear matrix inequalities

1. INTRODUCTION

The primary concern of a vehicle suspension is to isolate the vehicle body from road induced vibrations to improve the ride comfort. Other functions of the vehicle suspensions are to maintain road holding and to prevent excessive suspension deflection to avoid safety problems and road damage. Enhancement of ride comfort and safety are conflicting demands, since improved ride comfort results in a higher suspension deflection and lightly damped wheel motions [1].

Many types of vehicle suspension systems such as passive [2], semi-active [3] and [4] and active [5], [6] and [7] are currently being employed and studied in both academy and industry. It is widely accepted that active suspensions have a great potential to meet the aforementioned conflicting demands. For instance, it has been shown that suspension performance can be improved significantly by Linear Quadratic (LQ) type optimal controllers when the weighting factors of the cost function are chosen properly [8]. Ulsoy et al. [9] investigated robustness properties of optimal controllers against parameter variations by neglecting actuator dynamics. As an important field of study, preview control strategies with capability of sensing incoming road irregularities are proposed to improve active suspension performance. Look-ahead preview active suspension control problem was formulated as an optimal LQ state feedback design by Abdel-Hady [10]. In order to eliminate steady state errors due to ramp type road inputs or inertial forces/moments caused by maneuvers, an optimal active vehicle suspension having integral constraints was developed by El Madany and Al-Majed [11]. Han et al. designed LQ type feedforward and feedback optimal vibration control law for active vehicle suspension system with input delay [12]. All the LQ type optimal control laws discussed above were designed with a solution of Algebraic Riccati Equations (AREs). However, Linear Matrix Inequalities (LMIs) based optimal controller design has received considerable attention in recent years [13], [14], [15], [16], [17]. An optimal LMI based active suspension controller, which is robust against parameter

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variations and having pole location constraints, has been proposed by Soliman and Bajabaa [18]. Soliman et al. [19] has extended the robust regional pole placement controller with actuator saturation. Aforementioned research studies addressed the active suspension controller design by state feedback or full order observer based optimal controllers. In active suspensions, accelerometers are mostly used due to their simple structure and low operational cost [20]. In order to design state feedback active suspension controllers, displacement and velocity signals has to be measured. However, displacement signals are not possible to obtain accurately, since the signals of accelerometers are noisy and contain offset [21]. Moreover, full order observer based compensators, such as LQG, are sensitive to parameter variations and implementation errors. In the light of aforementioned considerations, the state derivative feedback appears as a promising active suspension controller approach in the following aspects. Firstly, the measured signals are the velocity and acceleration signals which are more available than displacement and velocity signals. Secondly, the order of the closed loop system is not increased, since the state derivative feedback controllers are static and memoryless with no additional state variables.

In the last few decades, the state derivative feedback control strategy has received the considerable interests of researchers. Abdelaziz and Valasek studied the state derivative feedback design of pole assignment and Linear Quadratic Regulator (LQR) problems [22] and [21]. Design of state derivative feedback controllers for uncertain systems and regional pole placement constraints were formulated as convex optimization problem via LMIs framework [23], [24] and [25]. A state derivative feedback controller having regional pole location constraint was applied to an experimental quarter vehicle active suspension system [26]. In order to design an active suspension controller for an integrated suspension system, an L_2 gain state derivative feedback controller has been proposed by Sever and Yazici [27]. Thereafter, the L_2 gain state derivative feedback controller has been extended with robustness property for the systems having polytopic type uncertainties by Yazici and Sever [28]. To the best knowledge of the authors, there is no work employing optimal state derivative feedback LQR controller for active suspension design in the literature. Thus, an LMI based design of optimal state derivative feedback LQR controller and its application to an electro-hydraulic suspension system is introduced.

The rest of the paper is organized as follows: Section 2 provides the modelling of electro-hydraulic vehicle suspension system. In Section 3, solvability conditions of the proposed optimal state derivative feedback LQR controller are formulated by LMIs. Then, effectiveness of the electro-hydraulic active suspension with proposed controller is tested against bump type road irregularities in Section 4. Finally, Section 5 concludes the paper.

Notation: Throughout the text, a fairly standard notation is used. The superscript “T” stands for the transpose of a matrix; \mathbb{R}^n denotes the n dimensional vector space, $\mathbb{R}^{m \times n}$ is the set of all $m \times n$ real matrices; “tr” is the standard trace operator; notation $P > 0$ ($<$, \leq , \geq , 0) means that P is symmetric and positive definite (negative definite, negative semi-definite, positive semi-definite); I and 0 represent identity and zero matrices. In addition, in symmetric block matrices, we use “*” to represent a block matrix which is induced by symmetry. $diag(M_1, M_2, \dots, M_n)$ stands for a diagonal matrix with elements M_1, M_2, \dots, M_n appearing on its diagonal. Matrices if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. MODELLING OF ELECTRO HYDRAULIC VEHICLE SUSPENSION

In this study, a two degree-of-freedom quarter vehicle suspension model equipped with an electro-hydraulic actuator is used to design the active suspension controller. Figure 1 illustrates such a system. In Figure 1, m_s is the sprung mass which corresponds to vehicle body, m_u is the unsprung mass which represents the wheel assembly. $z_s(t)$ and $z_u(t)$ are the vertical displacements of sprung and unsprung masses. $z_r(t)$ is the road irregularity. k_s and c_s are the spring and damping coefficients of vehicle

suspension system. k_t is the coefficient of tire stiffness. Finally, $F_a(t)$ is the active control force generated by the electro-hydraulic actuator.

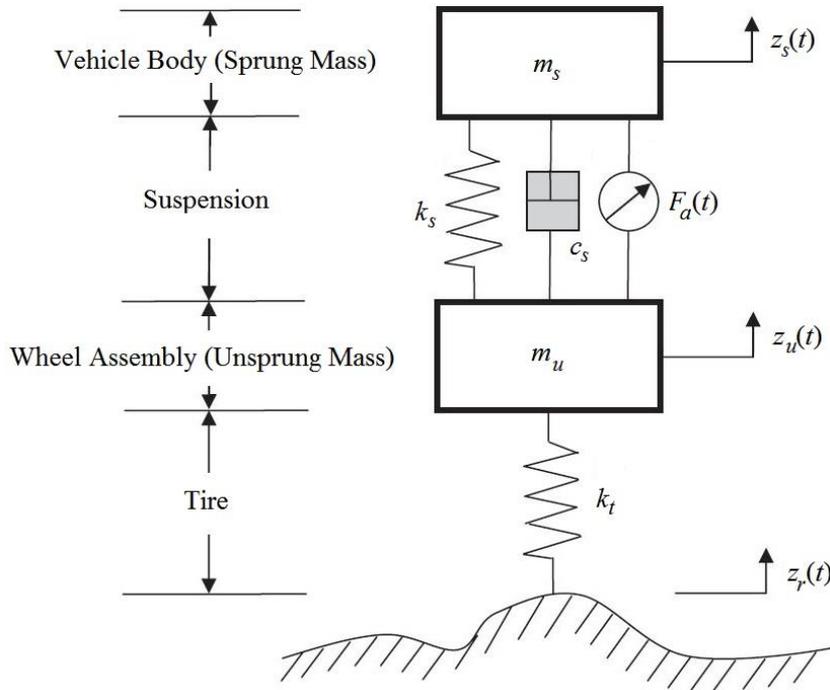


Figure 1. Quarter vehicle suspension model with an electro-hydraulic actuator

In order to generate the active control force, the electro-hydraulic actuator is installed between the sprung and unsprung masses. Hence, active control force is obtained as [29]

$$F_a(t) = A_p P_L(t) \quad (1)$$

where $P_L(t)$ is the load pressure across the piston and A_p is the area of the piston. A four-way valve-piston system has been considered as the electro-hydraulic actuator. Therefore, the load pressure dynamics are given by [5].

$$\frac{V_t}{4\beta_e} \dot{P}_L(t) = -C_{tp} P_L(t) - A_p (\dot{z}_s(t) - \dot{z}_u(t)) + Q_L(t) \quad (2)$$

Here, V_t is the total actuator volume, β_e is the effective bulk modulus, C_{tp} is the total piston leakage coefficient. In (2), the $Q_L(t)$ is the load flow and considered as the control input [5].

Then, equations of motion of the quarter vehicle are given by

$$m_s \ddot{z}_s(t) = -c_s [\dot{z}_s(t) - \dot{z}_u(t)] - k_s [z_s(t) - z_u(t)] + F_a(t) \quad (3)$$

$$m_u \ddot{z}_u(t) = c_s [\dot{z}_s(t) - \dot{z}_u(t)] + k_s [z_s(t) - z_u(t)] - k_t [z_u(t) - z_r(t)] - F_a(t). \quad (4)$$

The state variables, the control input and the exogenous input can be defined as follows:

$$x(t) = [z_s(t) \quad z_u(t) \quad \dot{z}_s(t) \quad \dot{z}_u(t) \quad P_L(t)/\bar{P}_L]^T \tag{5}$$

$$u(t) = Q_L(t) \tag{6}$$

$$w(t) = z_r(t), \tag{7}$$

Notice that, the fifth state of the system is the normalized load pressure. The normalization operation is performed to avoid ill conditioning problem during the controller design and simulation [30]. The state space representation of the electro-hydraulic quarter vehicle suspension system can be described by

$$\dot{x}(t) = Ax(t) + Bu(t) + B_w w(t) \tag{8}$$

where $A \in \mathbb{R}^{n \times n}$ is the state matrix, which is given as

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -k_s/m_s & k_s/m_s & -c_s/m_s & c_s/m_s & A_p \bar{P}_L/m_s \\ k_s/m_u & -(k_s + k_t)/m_u & c_s/m_u & -c_s/m_u & -A_p \bar{P}_L/m_u \\ 0 & 0 & -a_1 A_p / \bar{P}_L & a_1 A_p / \bar{P}_L & -a_2 \end{bmatrix}, \tag{9}$$

$B \in \mathbb{R}^{n \times m}$ is the control input matrix which is given by

$$B = [0 \quad 0 \quad 0 \quad 0 \quad a_1/\bar{P}_L]^T, \tag{10}$$

$B_w \in \mathbb{R}^{n \times p}$ is the disturbance input matrix which is given by

$$B_w = [0 \quad 0 \quad 0 \quad k_t/m_u \quad 0]^T \tag{11}$$

where $a_1 = 4\beta_e/V_t$ and $a_2 = a_1 C_{tp}$. The suspension parameters used throughout the controller design and numerical simulations are given in Table 1 [5] and [29].

Table 1. Electro-hydraulic suspension parameters

Parameter	value	unit	parameter	Value	Unit
m_s	320	[kg]	a_1	4.515×10^{13}	[N/m ⁵]
m_u	40	[kg]	a_2	1	[s ⁻¹]
c_s	1000	[Ns/m]	A_p	3.35×10^{-4}	[m ²]
k_s	18×10^3	[N/m]	\bar{P}_L	1.03425×10^7	[Pa]
k_t	200×10^3	[N/m]			

3. OPTIMAL STATE DERIVATIVE FEEDBACK CONTROLLER

In this section, solvability conditions of the optimal state derivative feedback LQR controller are presented. In order to obtain the optimal active suspension controller, minimization of quadratic cost function is ensured by using convex optimization techniques based on LMI conditions.

Consider the linear time-invariant system given by

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{12}$$

where $x(t) \in \mathbb{R}^n$ is the state vector and $u(t) \in \mathbb{R}^m$ is the control input vector. Our goal is to find an optimal state derivative feedback control in the form of

$$u(t) = -K\dot{x}(t) \tag{13}$$

where $K \in \mathbb{R}^{m \times n}$ is a controller gain matrix. The closed-loop system can be written in the Reciprocal State Space (RSS) [20] framework as follows.

$$x(t) = A^{-1}(I + BK)\dot{x}(t) \tag{14}$$

The quadratic cost function is given in the form of

$$J = \int_0^{\infty} [\dot{z}(t)^T Q \dot{z}(t) + u(t)^T R u(t)] dt. \tag{15}$$

Here, $z(t) \in \mathbb{R}^c$ and $z(t) = C_z x(t)$ is a linear function of the states and the vector of variables to be minimized. $Q \in \mathbb{R}^{n \times n}$ and $R \in \mathbb{R}^{m \times m}$ are performance weight matrices. The following Lemma [21] presents a Lyapunov equation which provides the optimal state derivative feedback controller.

Lemma [21]: *The optimal value of the quadratic cost function (14) converges to*

$$J = x(0)^T P x(0) \tag{16}$$

where $P \in \mathbb{R}^{n \times n}$ is the solution of the Lyapunov equation given by

$$PA^{-1} + A^{-T}P + PA^{-1}BK + K^T B^T A^{-T}P + K^T R K + \bar{Q} = 0. \tag{17}$$

Here, $\bar{Q} = C_z^T Q C_z$.

Proof: Assume a symmetric positive matrix P which satisfies

$$\dot{x}(t)^T (\bar{Q} + K^T R K) \dot{x}(t) = -\frac{d}{dt} (x(t)^T P x(t)). \tag{18}$$

An integration to the both sides of (18) is applied:

$$J = \int_0^{\infty} \dot{x}(t)^T (\bar{Q} + K^T R K) \dot{x}(t) dt = x(t)^T P x(t) \Big|_0^{\infty} = -x(\infty)^T P x(\infty) + x(0)^T P x(0). \tag{19}$$

The quadratic cost function converges to

$$J = x(0)^T P x(0) \tag{20}$$

since the closed loop system is asymptotically stable and the state vector goes to the origin by time goes to the infinity. Rewrite the equation (18) as given below.

$$\dot{x}(t)^T (\bar{Q} + K^T R K) \dot{x}(t) = -(\dot{x}(t)^T P x(t) + x(t)^T P \dot{x}(t)). \tag{21}$$

Substitute the closed loop system (14) in the RSS framework into the (21),

$$\dot{x}(t)^T (\bar{Q} + K^T R K) \dot{x}(t) = -\dot{x}(t)^T (PA^{-1}(I + BK) + (I + BK)^T A^{-T}P) \dot{x}(t). \tag{22}$$

is obtained. The equation (17) can be easily derived by simply arranging the equation (22). This completes the proof. ■

Design of the optimal state derivative feedback controller problem can be cast to the matrix inequality constraint problem by the change of variables. Let us define a new variable $Y = Y^T > P$. Then, substituting Y into (17) allows us to write

$$YA^{-1} + A^{-T}Y + YA^{-1}BK + K^T B^T A^{-T}Y + K^T RK + \bar{Q} < 0. \tag{23}$$

By applying the Schur complement formula [31], (23) is congruent to

$$\begin{bmatrix} YA^{-1} + A^{-T}Y + YA^{-1}BK + K^T B^T A^{-T}Y & C_z^T & K^T \\ * & -Q^{-1} & 0 \\ * & * & R^{-1} \end{bmatrix} < 0. \tag{24}$$

The inequality (24) is not in the LMI form yet due to the multiplication of decision variables Y and K . Pre- and post- multiply the (24) by

$$\text{diag}(S, I, I) \tag{25}$$

where $S = S^T = Y^{-1}$ and

$$\begin{bmatrix} A^{-1}S + SA^{-T} + A^{-1}BW + W^T B^T A^{-T} & SC_z^T & W^T \\ * & -Q^{-1} & 0 \\ * & * & R^{-1} \end{bmatrix} < 0 \tag{26}$$

is obtained. Here, $W = KS$ is a modest variable change operation. Recall that the quadratic cost function (16) has to be minimized by the optimal state derivative feedback control law (13). Then, a new decision variable $M \in \mathbb{R}^{c \times c}$ is introduced to set an upper bound on the cost as follows:

$$M > Y \leftrightarrow \begin{bmatrix} M & I \\ * & S \end{bmatrix} > 0. \tag{27}$$

In the light of the results obtained above, the following theorem presents an LMI based method to design an optimal state derivative feedback LQR controller.

Theorem: For a given values of Q and R , asymptotic stability of the linear time invariant system (12) is ensured with a minimum value of the quadratic cost function (16), if there exists a solution for the following optimization problem

$$\begin{aligned} & \min \text{tr}(M) \\ & \text{s. t. (26) and (27)} \end{aligned}$$

then, the optimal control law can be calculated as $u(t) = -K\dot{x}(t) = WS^{-1}\dot{x}(t)$.

4. NUMERICAL SIMULATION STUDIES

In this section, numerical simulations are studied to demonstrate the effectiveness of the proposed controller against bump type road irregularities at different vehicle forward velocities. All the simulations and computations were accomplished by using MATLAB and Simulink.

Recall that $z(t) = C_z x(t)$ is the vector of variables to be minimized. Then, $z(t)$ is given by

$$z(t) = [z_s(t) \quad z_s(t) - z_u(t) \quad \dot{z}_s \quad \dot{z}_s(t) - \dot{z}_u(t) \quad P_L(t)/\bar{P}_L]^T \tag{28}$$

and

$$C_z = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (29)$$

The performance weight matrices which are used in the controller design are

$$Q = \text{diag}(10^6, 10^{-2}, 10^6, 10^{-2}, 10^{-2}), \quad R = 10. \quad (30)$$

In the light of Theorem, in order to minimize the quadratic cost given by (28)-(30), the proposed controller was calculated by YALMIP parser [32] and SeDuMi solver [33]. Thus, the optimal state derivative feedback LQR control law is computed as

$$u(t) = -K\dot{x}(t) = -[3.3500 \quad -3.3500 \quad -0.0001 \quad 0.0001 \quad 0.0023]\dot{x}(t). \quad (31)$$

In order to analyze vehicle suspension performance with respect to ride comfort, suspension stroke and tire deflection, the road irregularity is considered as an isolated bump in an otherwise smooth road surface. It is very common to use bump type road irregularities in the literature [5] and [34], since it allows analyzing the transient response for severe road conditions.

The bump type road irregularity is given by

$$z_r(t) = \begin{cases} \frac{a}{2} \left(1 - \cos\left(\frac{2\pi V}{\ell} t\right) \right), & 0 \leq t \leq \frac{\ell}{V} \\ 0 & t > \frac{\ell}{V} \end{cases} \quad (32)$$

where $a = 0.1$ m and $\ell = 5$ m [5], [34]. Here, V is the vehicle forward velocity. Figure 2 shows the sprung mass acceleration, suspension deflection, tire deflection and active control force responses for vehicle forward velocity of 45 km/h.

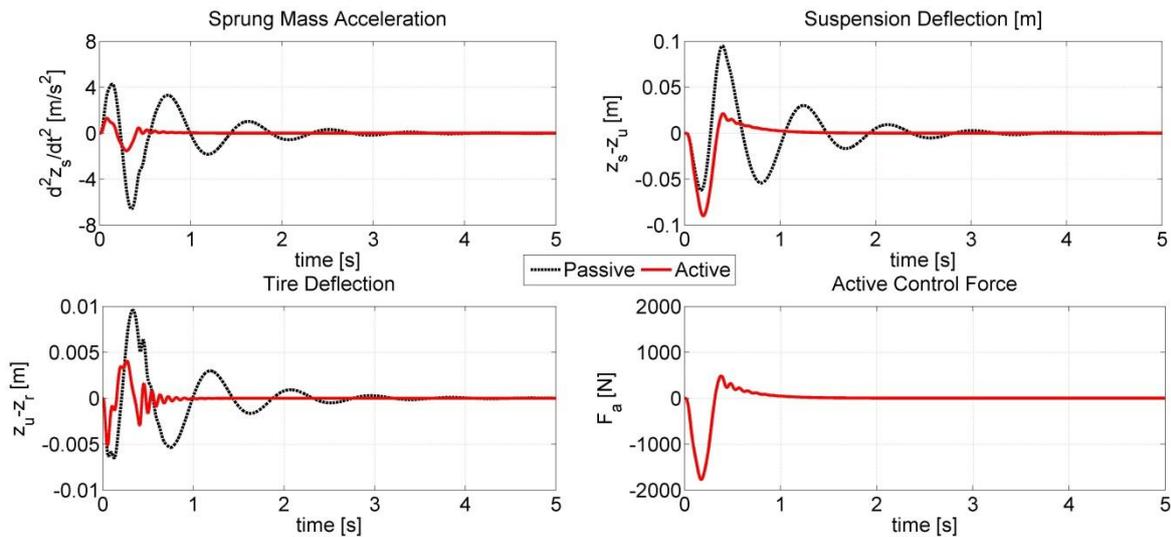


Figure 2. Sprung mass acceleration, suspension deflection, tire deflection and active control force responses.

As can be observed from Figure 2, the proposed controller ensures better ride comfort and road holding performances by the decrease on peak values of sprung mass acceleration and tire deflection. It is clear that lower peak value and shorter settling time is obtained for sprung mass acceleration. Reduction on the tire deflection amplitudes, indicates that dynamic tire load does not exceed the passive suspension case. Hence the road holding ability is slightly improved. Furthermore, the suspension deflection has not been significantly deteriorated. Despite the fact that negative peak is larger for active suspension, absolute value of the positive peak for passive suspension is larger than the negative peak of the proposed active suspension. Consequently, maximum values of the suspension deflection are below 0.1 m for both passive and active suspensions. In order to validate the effectiveness of the proposed controller against wide range of road disturbances, simulations were performed under different vehicle forward velocities as shown in Figure 3.

Figure 3 demonstrates that the proposed controller provides a great enhancement in the ride comfort and the road holding. Moreover, maximum peak values of the suspension deflection have also been mitigated for broad range of vehicle forward velocity. It is generally assumed that suspension deflection must be kept in the range of ± 0.1 m. Hence, it is apparently seen that road damage has been successfully avoided. Besides, active control force demand stays within the range of 2kN, which is adequate for practical implementation. The simulation results validate that improved ride comfort is achieved, and meanwhile the safety requirements are obtained within allowable bounds. Finally, frequency response of the sprung mass acceleration is given in Figure 4.

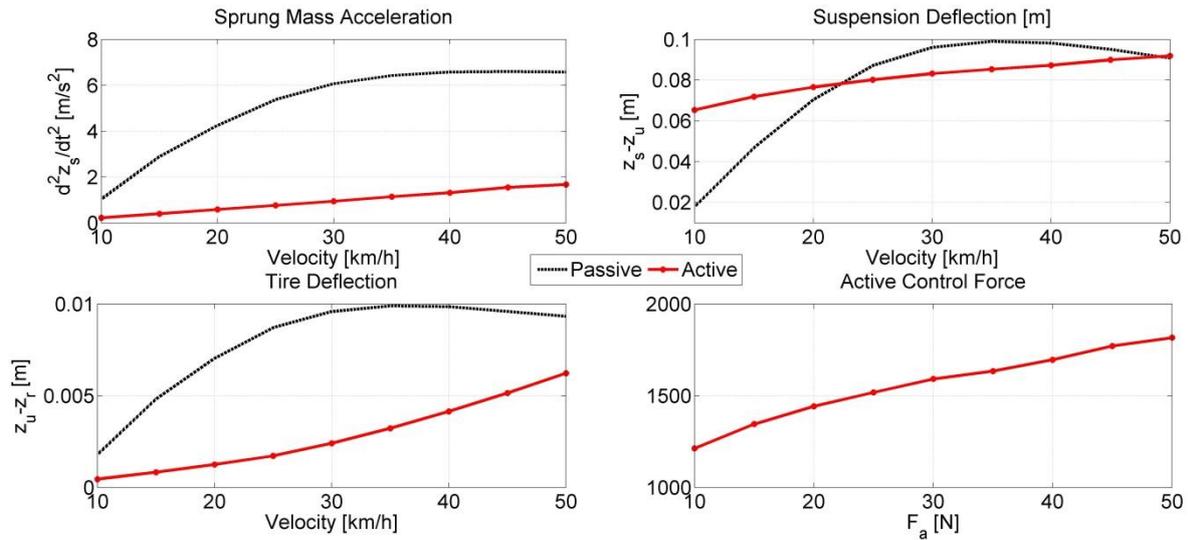


Figure 3. Peak values of sprung mass acceleration, suspension deflection, tire deflection and active control forces for different vehicle forward velocities.

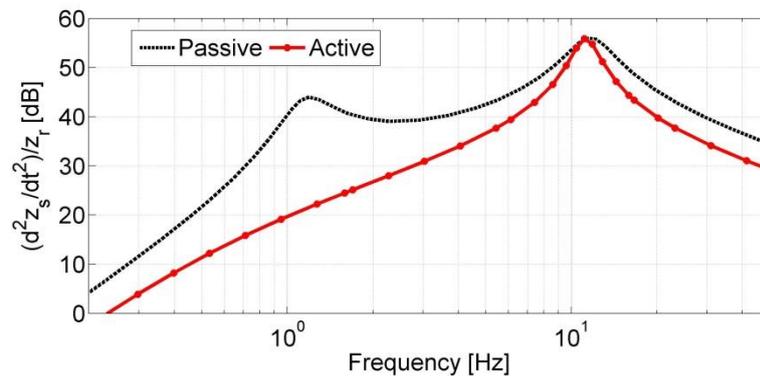


Figure 4. Frequency response of sprung mass acceleration

As expected, the proposed controller yields the significant reduction over a broad frequency range, compared with the passive suspension, which clearly indicates that an improved ride comfort has been guaranteed. Note that least gain is achieved over the frequency range 4-8 Hz which humans are most sensitive to the vertical vibrations according to ISO2631-1 [35].

Remark: In the literature [8]-[12], [18] and [19], LQ type optimal control of active suspensions has been extensively studied. However, in these studies, state derivative feedback type control law has not been considered so far. Simulation results provided through the section show that enhancement of the ride comfort without sacrificing the road holding has been achieved by the proposed optimal state derivative feedback LQR controller. By taking into account that the state derivative signals can be obtained more accurately, the proposed controller is a very promising solution for active suspension systems.

5. CONCLUSIONS

In this study, design of an optimal state derivative feedback LQR controller is proposed for an electro-hydraulic active suspension system design. Main goal of this research is to present an easily realizable optimal control strategy, compared to the previous LQ type optimal controllers found in the literature. Performance of the proposed controller is investigated by bump type road irregularities for different vehicle forward velocities. Simulation results revealed that better ride comfort without a considerable deterioration in safety can be achieved by proposed controller. Extending the proposed method for robustness against parameter uncertainties and gain scheduling capability might be a direction for future work.

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