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COMPARISON OF THE METHODS OF ESTIMATING ERROR VARIANCE, σ^2, IN TWO-WAY CLASSIFICATION WITH INTERACTION

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SUMMARY : *Methods of estimating error variance in non-additive models were discussed. Since the structure of interaction and the causes of non-additivity have influences on error estimation of proposed methods Monte-Carlo study were carried out regarding different types of interaction pattern. Whatever the reasons of non-additivity were considered, the characteristic roots methods gave suitable estimation of error variance.*

Key Words and Phrases: Non-additivity; two-way classification; multiplicative model; 2x2 table difference; characteristic roots; outlier.

INTERAKSİYONLU VE İKİ-YÖNLÜ SINIFLAMADA HATA VARYANSININ (σ^2) TAHMİN METODLARININ KARŞILAŞTIRILMASI

ÖZET : *Additif olmayan modellerde hata varyansının tahmin metodları tartışılmıştır. Hata tahmin metodları interaksiyonun yapısından ve toplanamazlığın sebeplerinden farklı şekilde etkilendiği için. farklı interaksiyon tiplerini içeren örneklerde Monte-Carlo simülasyon çalışması yapılmıştır. Toplanamazlığın sebebi ne olursa olsun hata tahmininde karekteristik kok metodu diğer metotlardan daha iyi sonuç vermiştir.*

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INTRODUCTION

The basic additive model in two-way classification with interaction without replication is,

$$y_{ij} = \mu + \alpha_i + \beta_j + e_{ij} \quad (1)$$

$$i = 1, 2, ..., m \; ; \quad j = 1, 2, ..., n$$

In data analysis if this structure is hold, statistical test and inference becomes easy. In some cases the true model is far from being additive. If there is a indication for interaction effect, a more suitable model assumed to be,

$$y_{ij} = \mu + \alpha_i + \beta_j + \gamma_{ij} + e_{ij} \quad (2)$$

Since the expectation of error variance for model (2) is

$$E(\sigma^2) = \sigma^2 + \Sigma\Sigma\gamma^2{}_{ij} / (m-1)(n-1),$$

the conventional test of hypothesis and inference are not applicaple.

The estimation of error variance, σ^2, for the methods by Tukey, characteristic roots and two by two table differences were compared in respect to different type of interaction. Data were simulated with Monte-Carlo study for each experiment.

METHODS

Test for non-additivity in model (2) was first proposed by Tukey (1949). The model considered by Tukey is

$$y_{ij} = \mu + \sigma_i + \beta_j + \theta.\alpha_i \beta_j + e_{ij.}$$

The sum of square due to remainder for this model is

$$E_{SS} = \Sigma\Sigma (y_{ij} - y_{i.} - y_{.j} + y_{..})^2 - N_{SS}$$

Where N_{SS} is the sum of square for non-additivity,

$$N_{SS} = (\Sigma\Sigma\, y_{ij}\, \alpha_i\beta_j)^2 / \Sigma\, \alpha\, {}_i^2\, \Sigma\, \beta_j^2$$

Then, the error estimation is

$$\sigma^2 = E_{SS} / ((m-1)(n-1) - 1). \quad (3)$$

Yates (1972) showed that Tukey's test was not good for smaller values of $\Sigma\, \alpha_i^2/\sigma^2$ and $\Sigma\, \beta_j^2 / \sigma^2$, whereas the performance of Tukey statistics was better for wide range of these parameters.

Gollob (1968) and Mandel (1969, 1971) developed a model which is called as fanova or multiplicative model.

$$y_{ij} = \mu + \alpha_i + \beta_j + \lambda_1 u_{1i} v_{1j} + e_{ij} \qquad (4)$$

Where λ_1 : Square root of the largest characteristic root of z'z or zz' matrix; u_{1i} and v_{1j} ; Characteristic vectors of z'z matrix corresponding to the $\lambda^2{}_1$; z: residuals matrix which is obtained from,

$$z_{ij} = y_{ij} - y_{i.} - y_{.j} + y_{..}$$

Jhonson and Graybill (1972) presented the likelihood estimation of parameters and the likelihood ratio test of them. If $\lambda_1 = 0$ in model (4), then

$$(\lambda_2{}^2 + \lambda_3{}^2 + ... - \lambda_{n-1}{}^2) / ((m-1)(n-1) - w_1) \qquad m>n \qquad (5)$$

is an unbiased estimate of σ^2, where $w_1 = E(\lambda_1{}^2/\sigma^2)$. If $\lambda_1{}^2 \neq 0$ in model (4), the maximum likelihood estimate of error variance is

$$\sigma^2 = (\lambda_2{}^2 + \lambda_3{}^2 + ... + \lambda_{n-1}{}^2) / mn \qquad (6)$$

Hegeman and Jhonson (1976) suggested that the estimation of σ^2 by (5) is also "good" estimation even if $\lambda_1{}^2 \neq 0$.

Jhonson and Graybill (1972) proposed two by two table differences method to estimate for error variance in model (2). They developed a conservative test of hypothesis that all the 2x2 contrasts obtained from observation were not equal to zero.

Let $\varepsilon_1, \varepsilon_2, ..., \varepsilon_p$ denote two by two contrasts which are zero and one the pxmn constant matrix consist of 0 and $\pm$ 1 as,

$$\mu' \; A = \varepsilon_1, \varepsilon_2, ..., \varepsilon_p$$

then,

$$\sigma^2 = y' \; A^- \; Ay/k \qquad (7)$$

is the estimate of σ^2 with k degree of freedom where A^- : response generalized invers of coefficient matrix A; k = rank (A).

The critical point for determining the significance of estimates of two by two table difference be represented by z_α

$$Z_{\alpha} = 2\sqrt{x_{\alpha}(\hat{\lambda}_2^2 + \hat{\lambda}_3^2 + \ldots + \hat{\lambda}_{n-1}^2)/(1-x_{\alpha})}$$

where x_α is the percentage point of the distribution for $\lambda_1^2/(\lambda_1^2 + \lambda_2^2 + \ldots + \lambda_{n-1}^2)$. Coefficient matrix of A will contain two by two tables differences which were less than Z_α.

RESULT AND DISCUSSION

There types of sources for non-additivity were taken into consideration in the simulation experiments.

1- Tukey's model which assumes the interaction pattern ($\gamma_{ij} = \alpha_i \beta_j$) as a function of block and treatment effects

2- The multiplicative interaction model ($\gamma_{ij} = u_i v_j$) which assumes the sources of non-additivity free from block and treatment effect.

3- Sources of non-additivity could be attributed to the outliers.

Response variables were being generated with respect to defferent models and 4x7 size experiment. In order to generate data for each model, the following values were given for u_i, v_j, α_i, and β_j.

$u_i = \{ 11, -7, 9, -8, -5, 7, -7\}$; $v_j = \{ -10, 7, -12, 15\}$

$\alpha_i = \{8, -9, 10, -8, -1, 10, -10\}$; $\beta_j = \{ -8, 9, -7, 6\}$

The residuals of each Monte-Carlo experiment were randomly drown from independent normal distribution with mean zero and variance, σ^2, of 64 and added to model. The mean of error estimation, was calculaled from different methods with a thousand repetiation.

Result obtained from simulation trials were discussed according to the structure of interaction and outliers. The mean of error estimates , σ^2, relative to the actual error, σ^2, was presented in Table 1.

Table 1. Values of, σ^2 / σ^2, Obtained For Different Model.

Models Methods	Tukey (a_ib_j)	Multiplicative Model (u_iv_j)	Additive 1 outlier	Additive 2 outliers
Tukey (3)	1.0	49.7	15.5	27.2
Chrc.Root (5)	2.8	5.8	0.9	2.8
Chrc. Root (6)	0.6	0.6	0.6	0.6
2x2 Table Dif. (7)	23.1	14.5	1.0	1.0

Conclusion drawn from Table 1. can be summarized as follows:

(i) Whatever the pattern of interaction and the resons of non-additivity were considered, the characteristic roots methods gave suitable estimation of error variance.

(ii) If an interaction exists among all of the levels of main factors as in the Tukey's and multiplicative interaction models, 2x2 table differences method gave quite biased estimation. However, 2x2 table differences methods had very close error estimation, when the interaction was considered as the result of one or two outliers.

(iii) When the Tukey's model is not fitted to the response variable, the error estimate calculated from Tukey's method has been at least 15 times bigger than actual error.

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