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New Exact and Numerical Experiments for the Caudrey-Dodd-Gibbon Equation

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Abstract

In this study, an exact and a numerical method namely direct algebraic method and collocation finite element method are proposed for solving soliton solutions of a special form of fifth-order KdV (fKdV) equation that is of particular importance: Caudrey-Dodd-Gibbon (CDG) equation. For these aims, homogeneous balance method and septic B-spline functions are used for exact and numerical solutions, respectively. Next, it is proved by applying von-Neumann stability analysis that the numerical method is unconditionally stable. The error norms L_2 and L_∞ have been computed to control proficiency and conservation properties of the suggested algorithm. The obtained numerical results have been listed in the tables. The graphs are modelled so that easy visualization of properties of the problem. Also, the obtained results indicate that our method is favourable for solving such problems.

1. Introduction

The fifth-order KdV-type (fKdV) equation has the following form

$$u_t+\alpha u^2u_x+\beta u_xu_{xx}+\gamma uu_{xxx}+u_{xxxxx}=0, \tag{1.1}$$

where α, β and γ are arbitrary positive parameters [1]-[4]. The fKdV equation (1.1) identifies motions of long waves in shallow water under gravity and in a one-dimensional nonlinear lattice [5]-[14], and has many physical applications in fields as diverse as nonlinear optics and quantum mechanics. These parameters greatly modify affect the characteristics of the equation. For example, if $\alpha=180$, $\beta=30$, and $\gamma=30$ are taken the following CDG equation

$$u_t+180u^2u_x+30u_xu_{xx}+30uu_{xxx}+u_{xxxxx}=0, \tag{1.2}$$

is obtained. It is well-known that equation is fully integrable. That means that it has multiple-soliton solutions [15]. The CDG equation owns the Painleve´ property as verified by Weiss in [16]. The equation can be found out to be solved by several methods, among other methods in the literature; Hirota's bilinear method [17] Hirota's direct method [15], Riccati equation method [18], tanh method [19], exp-function method [20, 21], collocation finite element approach [22].

The paper has been designated as follows: Analytical solutions of the equation are shown in Section 2 along with the graphs. In Section 3, construction of the numerical method has been done. Section 4 contains stability analysis of the numerical technique. Test problems taken from the literature have been solved and the obtained results are given in the tabular form as well as plotted graphically in Section 5. The article ends with the conclusions.

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2. Analytical solutions

Here, we implement the direct algebraic method to the converted ODE of the investigated model by employing $u(x,t) = \mathcal{U}(\xi)$, $\xi = x + ct$, which is given by

$$c\mathcal{U}' + \alpha \mathcal{U}^2 \mathcal{U}' + \beta \mathcal{U}' \mathcal{U}'' + \gamma \mathcal{U}\mathcal{U}^{(3)} + \mathcal{U}^{(5)} = 0. \tag{2.1}$$

Applying the homogeneous balance rule along with the method's framework, one gets the next general solutions of the ODE:

$$\mathcal{U}(\xi) = \sum_{i=0}^{n} a_i \phi(\xi)^i = a_2 \phi(\xi)^2 + a_1 \phi(\xi) + a_0, \tag{2.2}$$

where a_0, a_1, a_2 are arbitrary constants to be determined later. Using Eq.(2.2) along with the ODE (2.1) and the employed method's framework, obtain the values of the above-shown parameters as follows:
Set I

$$a_0 \to \frac{2a_2 d}{3}, a_1 \to 0, c \to \frac{2}{3}\left(a_2 \beta d^2 + 36 d^2\right), \alpha \to -\frac{6\left(a_2\beta + 2a_2\gamma + 60\right)}{a_2^2}. \tag{2.3}$$

Set II

$$a_1 \to 0, a_2 \to -\frac{60}{\beta+\gamma}, c \to \frac{-a_0^2\beta^2\gamma - 2a_0^2\beta\gamma^2 - a_0^2\gamma^3 - 80a_0\beta\gamma d - 80a_0\gamma^2 d - 160\beta d^2 - 1360\gamma d^2}{10(\beta+\gamma)}, \tag{2.4}$$

$$\alpha \to \frac{1}{10}\gamma(\beta+\gamma). \tag{2.5}$$

Set III

$$a_0 \to -\frac{40d}{\gamma}, a_1 \to 0, a_2 \to -\frac{60}{\gamma}, c \to \frac{8\left(3\gamma d^2 - 5\beta d^2\right)}{\gamma}, \alpha \to \frac{1}{10}\gamma(\beta+\gamma). \tag{2.6}$$

Thus, the soliton wave solutions of the investigated model are constructed by
for $b<0$, we get

$$u_{\mathrm{I},1} = \frac{1}{3}a_2\left(3b\tan^2\left(\sqrt{b}\left(\frac{2}{3}t\left(a_2\beta d^2 + 36d^2\right) + x\right)\right) + 2d\right), \tag{2.7}$$

$$u_{\mathrm{I},2} = \frac{1}{3}a_2\left(3b\cot^2\left(\sqrt{b}\left(\frac{2}{3}t\left(a_2\beta d^2 + 36d^2\right) + x\right)\right) + 2d\right), \tag{2.8}$$

$$u_{\mathrm{II},1} = a_0 - \frac{60b\tan^2\left(\sqrt{b}\left(\frac{t\left(-a_0^2\beta^2\gamma - 2a_0^2\beta\gamma^2 - a_0^2\gamma^3 - 80a_0\beta\gamma d - 80a_0\gamma^2 d - 160\beta d^2 - 1360\gamma d^2\right)}{10(\beta+\gamma)} + x\right)\right)}{\beta+\gamma}, \tag{2.9}$$

$$u_{\mathrm{II},2} = a_0 - \frac{60b\cot^2\left(\sqrt{b}\left(\frac{t\left(-a_0^2\beta^2\gamma - 2a_0^2\beta\gamma^2 - a_0^2\gamma^3 - 80a_0\beta\gamma d - 80a_0\gamma^2 d - 160\beta d^2 - 1360\gamma d^2\right)}{10(\beta+\gamma)} + x\right)\right)}{\beta+\gamma}, \tag{2.10}$$

$$u_{\mathrm{III},1} = -\frac{20\left(3b\tan^2\left(\sqrt{b}\left(\frac{8t\left(3\gamma d^2 - 5\beta d^2\right)}{\gamma} + x\right)\right) + 2d\right)}{\gamma}, \tag{2.11}$$

$$u_{\mathrm{III},2} = -\frac{20\left(3b\cot^2\left(\sqrt{b}\left(\frac{8t\left(3\gamma d^2 - 5\beta d^2\right)}{\gamma} + x\right)\right) + 2d\right)}{\gamma}. \tag{2.12}$$

For $b>0$, we get

$$u_{\mathrm{I},3} = \frac{1}{3}a_2\left(3b\tan^2\left(\sqrt{b}\left(\frac{2}{3}t\left(a_2\beta d^2 + 36d^2\right) + x\right)\right) + 2d\right), \tag{2.13}$$

$$u_{\mathrm{I},4}=\frac{1}{3}a_2\left(3b\cot^2\left(\sqrt{b}\left(\frac{2}{3}t\left(a_2\beta d^2+36d^2\right)+x\right)\right)+2d\right),$$

$$u_{\mathrm{II},3}=a_0-\frac{60b\tan^2\left(\sqrt{b}\left(\frac{t\left(-a_0^2\beta^2\gamma-2a_0^2\beta\gamma^2-a_0^2\gamma^3-80a_0\beta\gamma d-80a_0\gamma^2 d-160\beta d^2-1360\gamma d^2\right)}{10(\beta+\gamma)}+x\right)\right)}{\beta+\gamma}, \tag{2.14}$$

$$u_{\mathrm{II},4}=a_0-\frac{60b\cot^2\left(\sqrt{b}\left(\frac{t\left(-a_0^2\beta^2\gamma-2a_0^2\beta\gamma^2-a_0^2\gamma^3-80a_0\beta\gamma d-80a_0\gamma^2 d-160\beta d^2-1360\gamma d^2\right)}{10(\beta+\gamma)}+x\right)\right)}{\beta+\gamma}, \tag{2.15}$$

$$u_{\mathrm{III},3}=-\frac{20\left(3b\tan^2\left(\sqrt{b}\left(\frac{8t\left(3\gamma d^2-5\beta d^2\right)}{\gamma}+x\right)\right)+2d\right)}{\gamma}, \tag{2.16}$$

$$u_{\mathrm{III},4}=-\frac{20\left(3b\cot^2\left(\sqrt{b}\left(\frac{8t\left(3\gamma d^2-5\beta d^2\right)}{\gamma}+x\right)\right)+2d\right)}{\gamma}. \tag{2.17}$$

For $b=0$, we get

$$u_{\mathrm{I},5}=a_2\left(\frac{1}{\left(\frac{2}{3}t\left(a_2\beta d^2+36d^2\right)+x\right)^2}+\frac{2d}{3}\right), \tag{2.18}$$

$$u_{\mathrm{II},5}=a_0-\frac{60}{(\beta+\gamma)\left(\frac{t\left(-a_0^2\beta^2\gamma-2a_0^2\beta\gamma^2-a_0^2\gamma^3-80a_0\beta\gamma d-80a_0\gamma^2 d-160\beta d^2-1360\gamma d^2\right)}{10(\beta+\gamma)}+x\right)^2}, \tag{2.19}$$

$$u_{\mathrm{III},5}=\frac{20\left(-\frac{3}{\left(\frac{8t\left(3\gamma d^2-5\beta d^2\right)}{\gamma}+x\right)^2}-2d\right)}{\gamma}. \tag{2.20}$$

The following figures belong to each exact solution family:

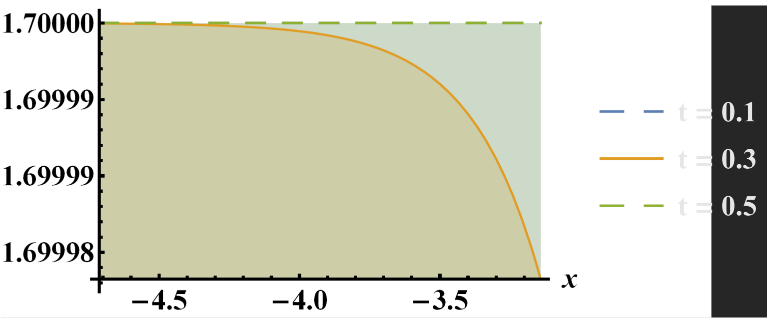

Figure 1: Graph of Set I.

3. Numerical scheme for the model problem

In this section, Eq. (1.2) has been solved by using the septic B-spline collocation method with the following boundary and initial conditions

$$\begin{array}{ll} u(a,t)=0, & u(b,t)=0, \\ u_x(a,t)=0, & u_x(b,t)=0, \\ u_{xx}(a,t)=0, & u_{xx}(b,t)=0, \\ u(x,0)=f(x), & a\leq x\leq b. \end{array} \tag{3.1}$$

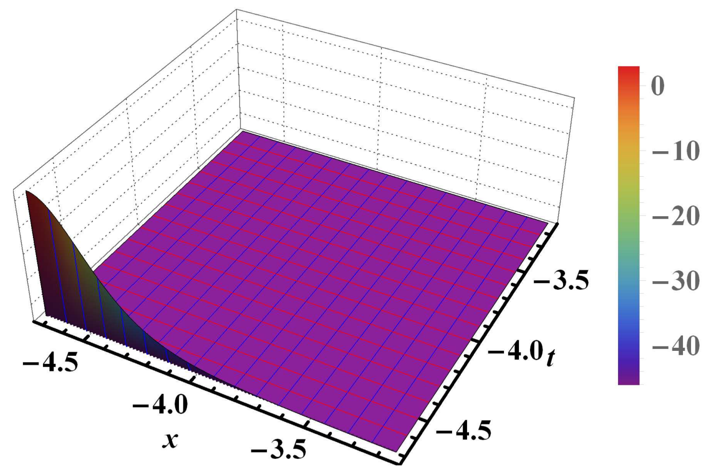

Figure 2: Graph of Set II.

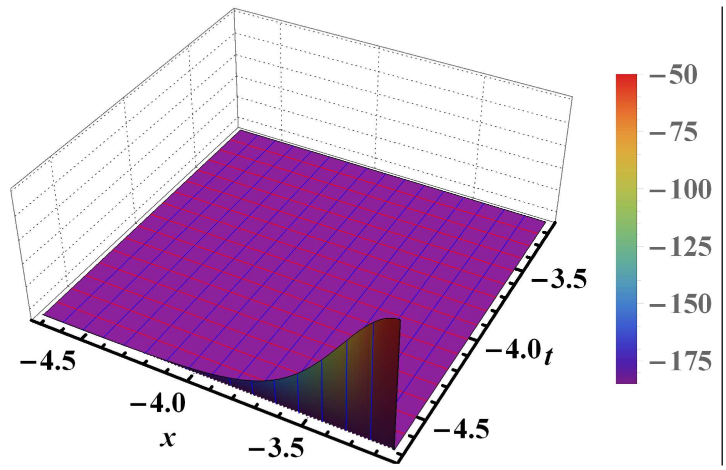

Figure 3: Graph of Set III.

Septic B-spline functions $\phi_m(x)$, $m=-3(1)N+3$, at the nodes x_m are given over the solution interval $[a,\ b]$ by Prenter [23]. In collocation method, $u_{numeric}(x,t)$ corresponding to the $u_{exact}(x,t)$ can be given as a linear combination of septic B-splines as follows [24]

$$u_N(x,t)=\sum_{m=-3}^{N+3}\phi_m(x)\sigma_m(t). \tag{3.2}$$

Implementing the following transformation $h\rho=x-x_m,\quad 0\le\rho\le 1$ to specific region $[x_m,x_{m+1}]$, the region turns to an interval of $[0,1]$ [25]. Thus the septic B-spline functions in the new region $[0,1]$ are obtained as follows:

$$\begin{aligned}
\phi_{m-3}&=1-7\rho+21\rho^2-35\rho^3+35\rho^4-21\rho^5+7\rho^6-\rho^7,\\
\phi_{m-2}&=120-392\rho+504\rho^2-280\rho^3+84\rho^5-42\rho^6+7\rho^7,\\
\phi_{m-1}&=1191-1715\rho+315\rho^2+665\rho^3-315\rho^4-105\rho^5+105\rho^6-21\rho^7,\\
\phi_{m}&=2416-1680\rho+560\rho^4-140\rho^6+35\rho^7,\\
\phi_{m+1}&=1191+1715\rho+315\rho^2-665\rho^3-315\rho^4+105\rho^5+105\rho^6-35\rho^7,\\
\phi_{m+2}&=120+392\rho+504\rho^2+280\rho^3-84\rho^5-42\rho^6+21\rho^7,\\
\phi_{m+3}&=1+7\rho+21\rho^2+35\rho^3+35\rho^4+21\rho^5+7\rho^6-\rho^7,\\
\phi_{m+4}&=\rho^7.
\end{aligned} \tag{3.3}$$

Using the equalities given by (3.2) and (3.3), the following expressions are obtained:

$$\begin{aligned}
u_N(x_m,t)&=\rho_{m-3}+120\rho_{m-2}+1191\rho_{m-1}+2416\rho_m+1191\rho_{m+1}+120\rho_{m+2}+\rho_{m+3},\\
u'_m&=\frac{7}{h}(-\rho_{m-3}-56\rho_{m-2}-245\rho_{m-1}+245\rho_{m+1}+56\rho_{m+2}+\rho_{m+3}),\\
u''_m&=\frac{42}{h^2}(\rho_{m-3}+24\rho_{m-2}+15\rho_{m-1}-80\rho_m+15\rho_{m+1}+24\rho_{m+2}+\rho_{m+3}),\\
u'''_m&=\frac{210}{h^3}(-\rho_{m-3}-8\rho_{m-2}+19\rho_{m-1}-19\rho_{m+1}+8\rho_{m+2}+\rho_{m+3}),\\
u^{iv}_m&=\frac{840}{h^4}(\rho_{m-3}-9\rho_{m-1}+16\rho_m-9\rho_{m+1}+\rho_{m+3}),\\
u^{v}_m&=\frac{2520}{h^5}(-\rho_{m-3}+4\rho_{m-2}-5\rho_{m-1}+5\rho_{m+1}-4\rho_{m+2}+\rho_{m+3}).
\end{aligned} \tag{3.4}$$

Now, putting (3.2) and (3.4) into Eq.(1.2) and simplifying, the following system of ODEs are reached:

$$\begin{array}{l}\dot{\rho}_{m-3}+120\dot{\rho}_{m-2}+1191\dot{\rho}_{m-1}+2416\dot{\rho}_{m}+1191\dot{\rho}_{m+1}+120\dot{\rho}_{m+2}+\dot{\rho}_{m+3}\\+(180Z_{m1}+30Z_{m2})\frac{7}{h}(-\rho_{m-3}-56\rho_{m-2}-245\rho_{m-1}+245\rho_{m+1}+56\rho_{m+2}+\rho_{m+3})\\+30Z_{m3}\frac{210}{h^3}(-\rho_{m-3}-8\rho_{m-2}+19\rho_{m-1}-19\rho_{m+1}+8\rho_{m+2}+\rho_{m+3})\\+\frac{2520}{h^5}(-\rho_{m-3}+4\rho_{m-2}-5\rho_{m-1}+5\rho_{m+1}-4\rho_{m+2}+\rho_{m+3})=0,\end{array} \tag{3.5}$$

where $\dot{\rho}=\frac{d\sigma}{dt}$,

$$\begin{array}{l}Z_{m1}=u^2=(\rho_{m-3}+120\rho_{m-2}+1191\rho_{m-1}+2416\rho_{m}+1191\rho_{m+1}+120\rho_{m+2}+\rho_{m+3})^2,\\Z_{m2}=u_{xx}=\frac{42}{h^2}(\rho_{m-3}+24\rho_{m-2}+15\rho_{m-1}-80\rho_{m}+15\rho_{m+1}+24\rho_{m+2}+\rho_{m+3}),\\Z_{m3}=u=\rho_{m-3}+120\rho_{m-2}+1191\rho_{m-1}+2416\rho_{m}+1191\rho_{m+1}+120\rho_{m+2}+\rho_{m+3}.\end{array}$$

If Crank-Nicolson scheme and forward difference approximation which are defined below is used respectively in Eq.(3.5)

$$\rho_i=\frac{\rho_i^{n+1}+\rho_i^n}{2},\ \dot{\rho}_i=\frac{\rho_i^{n+1}-\rho_i^n}{\Delta t} \tag{3.6}$$

the following iteration equation is obtained

$$\begin{array}{l}\lambda_1\rho_{m-3}^{n+1}+\lambda_2\rho_{m-2}^{n+1}+\lambda_3\rho_{m-1}^{n+1}+\lambda_4\rho_{m}^{n+1}+\lambda_5\rho_{m+1}^{n+1}+\lambda_6\rho_{m+2}^{n+1}+\lambda_7\rho_{m+3}^{n+1}\\=\lambda_7\rho_{m-3}^{n}+\lambda_6\rho_{m-2}^{n}+\lambda_5\rho_{m-1}^{n}+\lambda_4\rho_{m}^{n}+\lambda_3\rho_{m+1}^{n}+\lambda_2\rho_{m+2}^{n}+\lambda_1\rho_{m+3}^{n},\end{array} \tag{3.7}$$

where

$$\begin{array}{l}\lambda_1=[1-E-T-M],\\\lambda_2=[120-56E-8T+4M],\\\lambda_3=[1191-245E+19T-5M],\\\lambda_4=[2416],\\\lambda_5=[1191+245E-19T+5M],\\\lambda_6=[120+56E+8T-4M],\\\lambda_7=[1+E+T+M],\\E=\frac{\varpi}{2}\Delta t,\quad T=\frac{\varkappa}{2}\Delta t,\quad M=\frac{2520}{2h^5}\Delta t,\\\varpi=[180Z_{m1}+30Z_{m2}],\\\varkappa=[\frac{6300}{h^3}Z_{m3}].\end{array} \tag{3.8}$$

By eliminating the unknown parameters $\rho_{-3},\rho_{-2},\rho_{-1},\rho_{N+1},\rho_{N+2}$, and ρ_{N+3} which are not in the solution region of the problem, the system of equations given by (3.7) becomes solvable. This procedure can be easily done using the values of u and boundary conditions, and then the following system

$$R\mathbf{d}^{\mathbf{n+1}}=S\mathbf{d}^{\mathbf{n}} \tag{3.9}$$

is obtained where $d^n=(\rho_0,\rho_1,...,\rho_N)^T$.

4. Stability Analysis

For the stability analysis, Von Neumann technique has been used. In a typical amplitude mode, we can define the magnification factor ξ of the error as follows [26, 27]:

$$\rho_m^n=\xi^n e^{imkh}. \tag{4.1}$$

Using (4.1) into the (3.7),

$$\xi=\frac{\rho_1-i\rho_2}{\rho_1+i\rho_2}, \tag{4.2}$$

is obtained and in which

$$\begin{array}{l}\rho_1=2\cos(3kh)+240\cos(2kh)+2382\cos(kh)+2416,\\\rho_2=(2M+2T+2E)\sin(3kh),\end{array} \tag{4.3}$$

so that $|\xi|=1$, which proves unconditional stability of the linearized numerical scheme for the CDG equation.

5. Numerical Experiments and Discussions

In this section, the proposed scheme is applied for solution of CDG equation for different values of the time and space division and we approximate them using the described scheme. Error norms, namely L_2 and L_∞, are used in order to check the method [28, 29]:

$$L_2 = \left\| u^{exact} - u_N \right\|_2 \simeq \sqrt{h \sum_{j=1}^{N} \left| u_j^{exact} - (u_N)_j \right|^2}, \tag{5.1}$$

and

$$L_\infty = \left\| u^{exact} - u_N \right\|_\infty \simeq \max_j \left| u_j^{exact} - (u_N)_j \right|, \quad j = 1, 2, ..., N. \tag{5.2}$$

The CDG equation has an exact solution of the form [22]

$$u(x,t) = \frac{k^2 \exp(k(x - k^4 t))}{(1 + \exp(k(x - k^4 t)))^2}, \tag{5.3}$$

and the equation will be examined with the boundary-initial condition which is

$$u(x,0) = f(x) = \frac{k^2 \exp(kx)}{(1 + \exp(kx))^2}, \tag{5.4}$$

where $k = 1$ and $u \to 0$ as $x \to \pm\infty$.
To prove accuracy of our numerical algorithm, interval of the problem is chosen as $[-15, 15]$ and up to time $t = 1$. In simulation calculations in terms of compliance comply with the literature, as common values $\Delta t = 0.0004$ and 0.0001 with $h = 0.5$ and 0.05 are chosen. In Tables $(1-3)$, values of the error norms L_2 and L_∞ calculated over these values for time levels and step sizes are presented. So, it can be seen more clearly how the amount of collocation points have an effect on the method. When tables are examined, the calculated error norms L_2 and L_∞ are obtained to be marginally small. It is clear that the minimum L_∞ error norm 2.4892×10^{-5} with the parameters $\Delta t = 0.0001$ and $h = 0.05$. These errors hardly change as time progresses. Moreover, it can be said from the tables that the values of the error norms are compatible with the exact solution and the numerical solution, and the method is quite efficient. Two and three dimensional forms of bell-shaped solitary wave solutions produced from $t = 0$ to $t = 1$ are clearly seen in Figure (4). Besides, the contour line for the movement of the individual wave is plotted in Figure (4). It can be indicated that the wave maintains its amplitude and shape as time passes from these figures. Also, error distribution is shown at $t = 1$ for different values of h and Δt in Figure (5).

Table 1: Error norms for $k = 0.01$ and different values of h and Δt.

	$\Delta t = 0.0004, h = 0.5$		$\Delta t = 0.0001, h = 0.05$	
t	L_2	L_∞	L_2	L_∞
0.1	.0000494593	.0000249293	.0000414945	.0000270235
0.2	.0000532876	.0000252706	.0000465528	.0000248927
0.3	.0000532946	.0000257885	.0000497414	.0000271132
0.4	.0000537765	.0000249308	.0000557418	.0000303556
0.5	.0000544981	.0000255910	.0000617223	.0000337686
0.6	.0000582073	.0000249191	.0000619409	.0000441115
0.7	.0000563601	.0000249129	.0000679581	.0000468114
0.8	.0000553124	.0000249021	.0000803376	.0000475394
0.9	.0000559314	.0000256542	.0000912949	.0000595731
1.0	.0000587193	.0000256739	.0001058028	.0000597128

6. Conclusion

In this study, two important goals have been executed: Generating the direct algebraic method for obtaining exact solutions of the CDG equation and based on septic B-spline approximation, a collocation method has been introduced and performed for the numerical solution of CDG equation by taking into consideration different parameter values of test problem. The von Neumann method has been applied rigorously to check stability of the numerical scheme and the method has been proved to be unconditionally stable. The algorithm is run with a single solitary wave motion whose exact solution is known to perform

Table 2: The error norms for $k = 0.01$, $t = 0.0001$ and various values of h.

h	L_2	L_∞
0.25	.0000419227	.0000276440
0.1	.0000337090	.0000230659
0.01	.0000339219	.0000366059
0.05	.0000317705	.0000242181
0.025	.0000335113	.0000341147
1.0	.0000510913	.0000283967

Table 3: The error norms for$k = 0.01$,$h = 0.1$ and various values of Δt.

Δt	L_2	L_∞
0.04	.0000431579	.0000492631
0.02	.0000405068	.0000491365
0.01	.0000380355	.0000489782
0.001	.0000266854	.0000293648
0.005	.0000323331	.0000310227
0.0025	.0000298689	.0000306730
0.00125	.0000274591	.0000298586

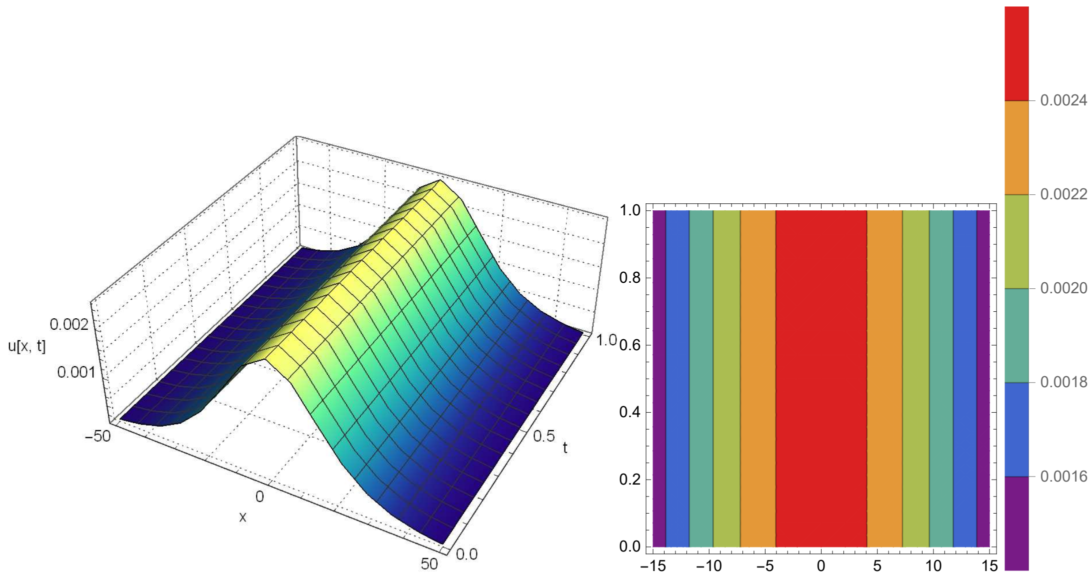

Figure 4: Motion of single solitary wave and its contour line for $\Delta t = 0.0004$ and $h = 0.5$.

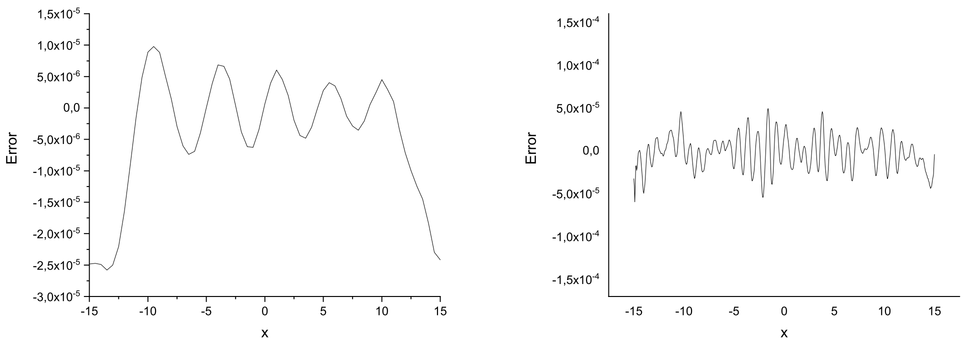

Figure 5: Error distributions at $t = 1$ for the parameters with $h = 0.05$; $\Delta t = 0.0004$; $h = 0.05$ and $\Delta t = 0.0001$.

numerical experiments. The obtained solutions from both methods are plotted graphically to check the dynamical behavior of the solutions. The reliability and efficiency of the numerical method have been evaluated using L_2 and L_∞ error norms and it can be seen that the obtained results are quite good. Finally, it is said that the approach applied in this study can be easily applied to other nonlinear evolutions and good results can be achieved.

Declarations

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