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Research Article

Mathematical Thinking and Problem Solving Performance: The Mediating Role of Hope

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Abstract

The purpose of the present study is to provide a mediation model by exploring the relationships among mathematical thinking, hope and problem solving performance in mathematics. The participants of the present study were composed of 311 preservice mathematics teachers. In this mixed method research, these preservice mathematics teachers were conducted to the scales of Dispositional Hope and Mathematical Thinking, and Problem Solving Test. Based on the quantitative data analysis, it was observed that statistically significant relationships among the variables of hope, mathematical thinking and problem solving performance. The quantitative data collected through these instruments were also analyzed by mediation analysis techniques. The qualitative data gathered through semi-structured interviews were analyzed by content analysis technique. These data encouraged the findings acquired from quantitative data analysis. The qualitative findings confirmed the findings of the mediator role of hope. The qualitative findings encouraged the relationships among the variables and the role of mediator.

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Introduction

In general view, problem as a challenge situation can be described as "a situation for which one does not have a ready solution" (Henderson & Pingry, 1953 p. 248), and "the situation is unfamiliar in some sense to the individual and a clear path from the problem conditions to the solution is not apparent" (Grouws 1996, p. 72). In this respect, a circumstance can be accepted as a problem in case that the individuals are aware of this situation, not able to directly progress through the solution, willing and settled to get rid of the situation (Lester, 1980). In other words, a problem solver is an individual who is not able to find the solution for the problem at the first glance and having motivation and tend to be resistant to search for problem solution. In this kind of situations having challenges,

individuals face with in every parts of real life. In this respect, problem solving as one of the important mathematical skills needing for mathematics learning and removing these obstacles by finding a path for the challenge situation can affect individuals' daily life. The individuals equipped with problem solving skills and engaging in problem solving tasks in schools tend to be more successful in facing with and getting rid of problems in daily life outside from the classrooms, and reason scientifically (Irwanto, et al., 2018). In problems, the individuals engage in examination of new ways and paths to reach a solution to remove the challenge benefiting from experiences and previously acquired knowledge and skills (Woolfolk, 1993). In this respect, because problem solving necessitates to begin with what individuals know and how they reason mathematically, it is important to make individuals benefit from their mathematical thinking using their previous knowledge (van de Walle, 1994) with the help of reflective thinking (Köseoğlu, et al., 2017). Previous research emphasize the relationship between thinking and problem solving (Kolodner, et al., 1985; Rogoff, 1990; Schoenfeld, 1992).

In problem solving performance, it is important that the solvers insist on finding the solution path and reaching accurate result. At that point, it can be necessary to be hopeful. Because hope describes a general inclination in making conscious efforts to reach an end or perform a goal, it is in relationship with problem solving (Chang, 1998; Snyder et al., 1991). Traditional theories explain hope as a kind of emotion encouraging reaching the target, performing daily life events, facing with life conditions and the power of identity (Menninger, 1959; Peterson & Seligman, 2004). On the other hand, based on the cognitive context developed by cognitive behaviorist and clinical applications, hope is explained as the thoughts about alternative strategies and engaging in them; agency and pathways (Snyder et al., 1998). At that point, it can be stated that thinking is in connection with the hope in order to find alternatives to insist on reaching the goal necessitating for problem solving. The pathways illustrate the improvement of the actions to meet the goal effectively and refers to the thoughts about the alternative ways that can be exemplified as "I will find a way in order to do that" (Snyder et al., 2002). In other words, pathways describe the ways that the individuals plan to reach determined goals (e.g., particular strategies that can be used to acquire a desired result). On the other hand, agency can be explained by the perception of individuals' goal-directed determination such as self-efficacy (Snyder et al., 1991).

In the light of the given explanations, hope describes the positive expectations about reaching the goal (Snyder et al., 2006). Hence, hope refers to the perceptions that the individuals conceptualize their goals, produce alternatives to reach these goals and insist on using these alternatives by being motivated (Snyder et al., 2003). Previous research show that individuals having high level of hope tend to identify and use alternative strategies to reach their goals successfully, to be resistant to face with obstacles and to accept these obstacles as challenger rather than failure (Arnau et al., 2007; Aspinwall & Leaf, 2002; Snyder et al., 1991).

1.1. The Rationale of the Study

Attachment theory explains the effects of individuals' early attachment to primary caregivers on their whole remaining life (Bowlby, 1982). The variables of hope and thinking can be explored and conceptualized based on attachment theory describing the tendency and perception of setting and sustaining strong emotional ties with primary caregivers (Bowlby, 1973).

The previous research emphasize the connection between attachment and hope by stating that "hope flourishes when the child establishes a strong bond to one or more caregivers during this infant to toddler stage…. Instilling hope in children is based, in part, on their perceived security. Secure early attachments relate to a sense of empowerment and goal-directed thought." (Snyder et al., 1997: 12). Hope encourages the individuals' thinking based on this point of view (Quick, et al., 1996). Previous research show that hope is positively related to success and problem solving performance (Chang, 1998; Oettingen & Gollwitzer, 2002; Snyder et al., 1991). With this motivation, it can be stated that hope has cognitive aspect so it can encourage thinking about alternatives to face with obstacles (Quick et al., 1996). In that respect, Chang (1998) has emphasized and suggested the necessity of studies examining the relationship between hope and problem solving performance in particular areas in detail.

In the light of the explanations, the present study focused on the examination of the effects of hope and thinking on problem solving performance by being specified in the area of mathematics. This examination was performed by paying attention on preservice mathematics teachers in the present study. Preservice years have critical importance on lifelong and professional development (Arnett, 2000). Also, in that period, they might leave their homes and families, and begin to communicate with people having different characteristics from various cultures so they can feel confused (Arnett, 2000; Benn, et al.,

2005). Through this critical duration and actions, attachment styles gain importance (Parade et al., 2010). Moreover, necessary knowledge and skills needing for performing teaching profession are acquired through preservice years. Hence, preservice teachers are expected to have high level of problem solving performance and mathematics knowledge in order to help their students acquire the skills of mathematical thinking and problem solving in the future in teaching real classrooms. In this respect, it can be stated that it is important to help preservice mathematics teachers acquire high level of hope and mathematical thinking to represent problem solving performance successfully by using mathematical thinking effectively.

1.2. The Purpose of the Present Study

The purpose of the present study is to explore the relationship between the mathematical thinking and problem solving performance. Moreover, the aim of this present study is to examine whether hope has mediating roles between preservice mathematics teachers' (PMT) mathematical thinking levels and mathematical problem solving performance. Figure 1 illustrates the theoretical model of these explained variables. For this purpose, the following questions were formulated:

- What are the relationships among the variables referring to preservice mathematics teachers' levels of mathematical thinking, hope and problem solving performance?
- Is hope mediators in the relationship among mathematical thinking and problem solving performance?
- What are the PMT's views about the relationships among the variables?

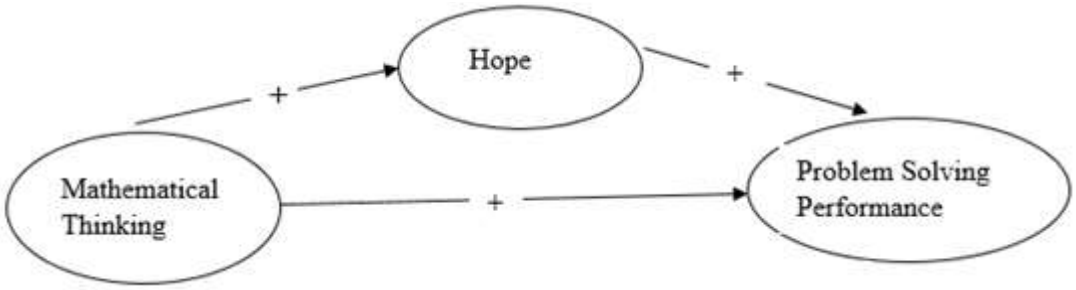

Figure 1. Hypothesized model

2. Method

This quantitative dominant mixed method study was designed by sequential explanatory mixed method research design (Sullivan, 2009). In the quantitative methods, the

quantitative data was benefited from in order to represent the relationships among variables and predictors of problem solving performance. Afterwards, by the qualitative method, the qualitative data were used in order to interpret and make clear quantitative data so that detailed and holistic picture of the findings can be provided (Creswell, 2013).

2.1. Participants

Through the enactment of the quantitative methods, 311 (198 females and 113 males) volunteer preservice mathematics teachers (PMTs). These participants were enrolled in elementary mathematics education program in a university in southern part of Turkey participated in the present study. Approximately, 64% of the participants were female and 36% of them were male. The participants' age range was between 18-22 years old. Then, the qualitative methods of the study was conducted to volunteer 30 PMT selected by randomly sampling technique among the PMTs being administered the instruments. Qualitative and quantitative data collected through the present study were limited to the honesty of the participants' responses to the instruments and interview questions.

2.2. Instruments

2.2.1 Dispositional hope scale (DHS)

This instrument was developed by a group of researchers including Synder and the colleagues (1991) with the aim of exploring the level of dispositional hope of individuals who are 15 years old and over. The instrument of DHS is designed as a 8-point Likert-type measurement material including 12 items (4 items for pathways, 4 items for agency, 4 items for filler). The total score that can be acquired from this scale is in range between 8 and 64 points. The studies of preparing Turkish adaptation form of the DHS scale was performed by Tarhan and Bacanlı (2015). Cronbach's alpha coefficient value was determined as .84 and the value for test-retest reliability coefficient was identified as .86. In the present study, Cronbach's alpha coefficient value was estimated as .86.

2.2.2. Mathematical thinking scale (MTS)

This instrument was developed by Ersoy and Başer (2013) with the aim of assessing the PMTs' levels of mathematical reasoning and mathematical learning by cognitive dimension. The scale was composed of four sub-dimensions; high-level thinking tendency, reasoning, mathematical thinking skill, and problem solving. This instrument was designed as a 5-point Likert-type (5= Completely Agree to 1= Not Completely Agree) including 32

items. Cronbach's alpha coefficient value was determined as .78 in the previous study (Ersoy & Başer, 2013) and as .81 in the present study.

2.2.3. Problem solving test (PST)

A problem solving test was prepared by the researcher by selecting ten mathematical problems from the book of "Problem Solving Strategies for Efficient and Elegant Solutions: A Resource for the Mathematics Teachers" (Posamentier & Krulik 1998). The selection was performed based on the criteria of being solved by more than one strategy. Afterwards, these problems were translated and adapted into Turkish. These adapted form was analyzed and evaluated by two experts except from the researcher having doctorate degree in mathematics education and Turkish education. Based on their views, the revised form of the test was prepared and conducted to the participants. The solutions of the PMTs were scored based on 4-point scale (1= incorrect solution and result, 2= partially correct solution and incorrect result, 3= correct solution and result, 4= correct solutions by more than one strategy and correct result).

2.2.4. Semi-structured interview protocol

Three interview questions and one probing question (see Appendix) were prepared in order to detail the views of the PMTs about the relationships and the value of these connections among the variables examined in the present study. These questions were prepared by exploring the literature related to these explained variables. Then, expert opinion from two experts except from the researchers having doctorate degree in mathematics education and educational psychology. Afterwards, three PMTs, not participating in the present study was conducted to the interview questions. Based on their views about the clarity, coherence and convenience of the interview questions, the questions were revised and the final version of the interview protocol were used in conducting interviews to the participants.

2.3. Data Collection and Data Analysis

The instruments of Dispositional Hope Scale (DHS), Mathematical Thinking Scale (MTS) and Problem Solving Test (PST) were conducted to the PMTs through the academic year of 2020. Moreover, the demographic information about their grade level, age and gender was collected. The administration of DHS and MTS lasted approximately seventy minutes. Then, they solved the problems. While solving problems, they were wanted to solve

the problems by more than one strategy if possible. The solution process completed in approximately forty minutes. Afterwards, semi-structured interviews were conducted to 30 randomly selected PMTs to acquire the detailed information to answer research questions of the present study. In other words, the information about the PMT's views the connection of variables to problem solving performance was collected through interviews. The interviews conducted to the participants of the present study lasted approximately 40 minutes. These meetings were made with the researcher and 30 PMTs. The interviews were recorded via audio-recording and then these audio-recording were transcribed verbatim.

Initially, with the aim of the analysis of the quantitative data collected through the instruments except for interview protocol, descriptive statistics analysis and Pearson's correlation method were used. Moreover, in order for the quantitative data analysis, an approach by Ordinary Least Squares Regression, and the technique of Bootstrapping were performed. Using Mahalanobis distance, 5 outliers were got off from the data set since these scores of these outliers exceeded the critical value of chi-square. The analyses were performed the data collected from 306 PMT. In order to identify normal univariate distribution, kurtosis and skewness values were found and it was observed that these values were in acceptable ranges in the region of -1.5 to +1.5 (see Table 1). Hence, it could be stated that the data distributed normally based on the suggestion made by Tabachnick and Fidell (2013).

A multiple mediation model including "simultaneous mediation by multiple variables" (Preacher & Hayes, 2008, p. 880), was benefited from in order to test the hypothesized model exploring mediating role of hope in the relationship among mathematical thinking and problem solving performance in the present study. Based on the suggestions and explanations of Hayes (2009) "if zero is not between the lower and upper bound, then the analyst can claim that the indirect effect is not zero with ci% confidence." (p. 412). In addition, a contrast test was performed with the aim of identifying particular indirect effects of performed by "Multiple Mediation Model 4" using PROCESS Macro 3 via IBM SPSS 24.0 (Hayes, 2017) with statistical significance p value of .05. Based on Pearson correlation value, it was observed that there was not multicollinearity in the set of the data since the binary correlation values between the variables were less than .90 as suggested by Cokluk et al., (2012).

In order to examine the qualitative data collected through transcripts of semi-structured interviews conducted to 30 randomly selected PMTs, content analysis was used. The content analysis was performed through six stages proposed by Marshall and Rossman (1999). Firstly, the researcher organized the interview data transcribed verbatim. Secondly, the researcher and an academician having doctorate degree in mathematics education independently identified the codes and explored the patterns on the data. Thirdly, the researcher and the academician came together and compared their list of codes. They discussed about their lists of codes until reaching consensus so that they formed a list of common codes. The codes on the list was designed based on 90% consistency level among the researcher and the academician. The remaining codes were eliminated. Fourthly, they analyzed the data based on this list in an iterative process independently. Moreover, by this way, the evidence for the reliability and validity of the present study was provided by investigator triangulation. Furthermore, another academician in mathematics education area evaluated the qualitative data analysis process considering coherence and consistency in order to provide evidence for reliability and validity. Moreover, member checking strategy was used by talking about the interpretations made based on the PMTs' responses to the interview questions (Lincoln & Guba, 1985). Fifthly, similar codes came together and themes were formed. For example, based on the PMTs' explanations provided in the first question about the meaning of problem solving, the themes of relational thinking, scientific thinking, critical thinking and being adaptable were identified. Among these themes, the codes of focusing on previous knowledge, using prior related knowledge, and connecting to culture and other disciplines were determined for the theme of relational understanding. Lastly, the report was formed.

3. Results

3.1. Descriptive Statistics and Correlations

When the coefficient values calculated by Pearson's correlation are examined, it has been observed that there are statistically significant relationships among the variables. Table 1 represents the correlational matrix and descriptive values for the variables.

Table 1. Descriptive Statistics and Pearson's Correlations

Variables	1	2	3
Mathematical Thinking	-		
Hope	.573**	-	

Problem Solving Performance	.567**	.723*	-
Mean	96.25	48.54	20.94
Standard Deviation	18.75	7.04	3.38
Skewness	-.85	-.97	-.02
Kurtosis	1.04	1.18	.79

Note. *p< .05, **p< .01

Table 1 illustrates that problem solving performance is correlated with mathematical thinking (r= .567, p< .01) positively and hope (r= .19, p< .01) positively. Hope is correlated with mathematical thinking (r= .573, p< .01) positively.

3. 2. Mediation Model Analysis

The results acquired related to the mediating role of hope in the relationship among mathematical thinking and problem solving performance are illustrated on Figure 2.

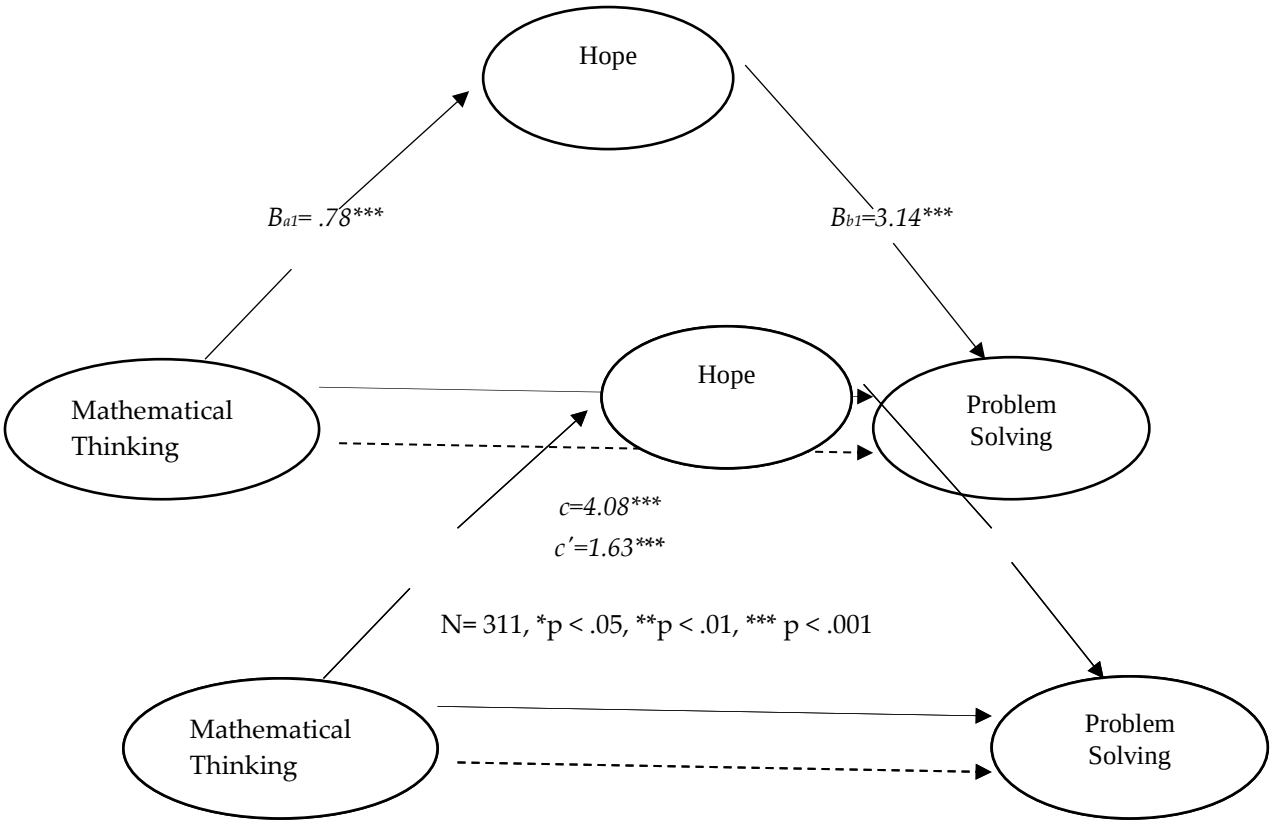

Figure 2. Mediation of Hope between Mathematical Thinking and Problem Solving

As illustrated in Figure 2 representing the mediating role of hope in the relationship among mathematical thinking and problem solving performance, the total effect of mathematical thinking on problem solving is statistically significant (c = 4.08, SE = .34, t = 11.99, p < .001) (step 1). The direct effect of mathematical thinking on hope (B = .78, SE= .06, t=

12.18, p < .001) is statistically significant (step 2). As well as, the direct effect of hope (B = 3.14, SE= .25, t= 12.74, p < .001) is statistically significant on problem solving (step 3). When mathematical thinking and mediating variable of hope has been taken into consideration and utilized simultaneously (Step 4), the relationship among mathematical thinking and problem solving has decreased; but the significance value has remained at the same level and statistically significant coefficient value has acquired (c'= 1.63, SE= .34, t = 4.86, p< .001). In the light of these results, hope has partially mediated role in the relationship among mathematical thinking and problem solving. According to the results, this mediational model is significant (F(2, 303) = 191.29, R2 = .56, p <.001) and it explains 56% of the total variance on problem solving performance in the context of mathematics.

The findings belonged to the comparisons of total, direct and specific indirect effects of problem solving performance on mathematics achievement through eye tracking measurements are illustrated in Table 2.

Table 2. The findings on the mediating role of hope in the relationship between mathematical thinking and problem solving

			Product of Coefficients		Bootstrapping 95% BCa Confidence Interval	
Effects	Point Estimate	SE	*t*	*p*	Lower	Upper
Indirect	2.4482	.3923	-	-	1.7314	3.2878
Total	4.0821	.3403	11.9952	.0000	3.4125	4.7518
Direct	1.6339	.3355	4.8696	.0000	.9737	2.2942

Note: *N*= 311, *k* = 5000, *p < .05, ** p < .01, ***p < .001, *BCa*: Bias corrected and accelerated 5000 bootstrap samples

In order to examine the indirect effects, bootstrapping with 10000 bootstrap samples have been utilized within 95% confidence interval. The results which are the bias corrected and accelerated show that the mediation effect of the variables of hope is statistically significant based on the values of BootLLCI and BootULCI within the expected range as represented on Table 2. The results illustrated in Table 2 emphasize that the indirect effect of mathematical thinking through hope on problem solving is statistically significant (point estimate= 2.4482 and 95% BCa CI [1.7314, 3.2878].

3.3. Qualitative Findings

In order to collect qualitative data to provide detailed data and make the connections among variables explored in the present study clear, the interviews were performed and the sample of the explanations were provided as follows:

> *I think that problem solving is like using adapted form of previous knowledge in new situation. Hence, it necessitates to think scientifically. I need to think critically and identify necessary previous knowledge, relate it to new situation by making necessary changes, and to find a path for solution reasoning by induction, reduction or creatively.*

By this explanation, the PMT provided evidence for the relationship between mathematical thinking and problem solving performance by *imitating problem solving process into scientific thinking process emphasizing the relational thinking*.

Another PMT made explanation about the relationship between hope and problem solving as follows:

> *When I thing and believe that if I continue and insist on exploring the solution and alternative strategies, I can solve the problem. A case become a problem when it challenges me. Hence, it is important not to give up exploring solution when it is not solved easily.*

As it is observed in this explanation, this explanation provided the relationship between hope and problem solving. Moreover, a different PMT made explanation as follows:

> *When I face with the problem, I initially feel nervous about how to find the solution and to overcome this case. Then, I begin to think that I am able to solve it because I will be a mathematics teachers and teach my students to solve the problems. I know that the students can ask the problem that can force me when I become a teacher. Hence, I need to believe that I can solve it if I am decided and insist on exploring strategies. Maybe, finding solution for this kind of problems takes longer time than for others but I can solve this problem since I have necessary knowledge and make reasoning mathematically. I believe in myself to have the power to face with math.*

By this explanation, the PMT provided evidence for the statistically significant positive connection of hope to mathematical thinking and problem solving performance stating *the belief in himself and to be decided*.

4. Discussion and Conclusion

In the present study on the possible causes of problem solving performance as an important factor affecting the academic achievement of the PMTs and mathematical skills, it was aimed to investigate whether hope had mediator roles in the relationship among mathematical thinking and problem solving performance. In the light of the findings, it was observed that the mediation of hope in association between mathematical thinking and

problem solving were statistically significant in the proposed and tested model. Moreover, qualitative data collected by interviews and analyzed with the aim of acquiring information about the PMT's views about the relationship among hope, mathematical thinking and problem solving encouraged the quantitative findings reported in the present study. When the literature about the variables explained in the present study is explored, this study differentiates from other research by reporting the direct relationship of problem solving performance and thinking with hope in a particular context, mathematics. In this respect, it is hoped that these research can make important contributions to the literature and guide further research in especially psychology of mathematics education. The present research is also different from the previous research by detailing the relationships among the variables in the present study by providing qualitative data.

In the present study, it is observed that there is positive statistically significant relationship among mathematical thinking and mathematical problem solving performance. This finding is in line with the result of previous research (Karakoca, 2011). In this previous study, it is stated that the preservice teachers having high level of performance in mathematics and solving mathematics problems are likely to have high level of mathematical thinking. Moreover, similar suggestions have been provided based on the findings of the previous research (Tasdemir, 2008) in the context of science. Mathematical thinking is critically necessitated for problem solving. Van de Walle (1994) emphasize this necessity by explaining that "problem solving places the focus of the students' attention on ideas and sense making" (p. 39). Hence, in the literature, many research have examined problem solving and mathematical thinking together (Cai, 2002; Polya, 1997). With this motivation, Baroody (2003) describes problem solving emphasizing its connection to mathematical thinking. In solving problems and finding alternative strategies for the solution, because mathematical thinking refers to dynamic process enhancing the comprehension of complex structures benefiting from the situations that have been overcome and coped with (Mason, et al., 2010), the PMTs are expected to use their previous knowledge and skills by criticizing. In this respect, it can be stated that mathematical thinking is an important predictor for problem solving performance as observed based on the qualitative and quantitative findings of the present study.

In line with the findings of previous research in the literature (Chang 1998; Snyder et al. 1991), the findings of the present study have showed that hope is an important predictor

for problem solving performance. Hope referring to insistency and thought of finding alternatives and engaging in them can affect the performance of individuals in solving problems. Chang (1998) found that the higher level of hope individuals had, the more positively problem orientation they represented. On the other hand, the individuals having low level hope tended to represent negative problem orientation and abstain solving problems. This finding of the present study is also parallel to the finding of research suggesting the relationship between problem solving and hope performed by Snyder and his colleagues (1991). As a result, it can be stated that these research finding support the predictive effect of hope on problem solving performance examined in the present study. In the present study, it is observed that hope has the mediator role in the relationship among mathematical thinking and problem solving performance. In other words, hope that can be predicted by mathematical thinking has effect on problem solving performance in the context of mathematics. This finding can be encouraged with the help of the view that secure early attachments increase the sense of empowerment and goal oriented actions and thoughts in the attachment theories (Snyder, 1997). In this respect, this present study can make important contribution to the literature by differentiating from other research and examining the connection of hope to problem solving performance and thinking mathematics as a more specific dimension as suggested by Chang (1998).

4. 1. Limitations and Implications

In the present study, there are some limitations. The present study included the PMTs selected using convenience sampling strategy. Also, the causality among the variables could not be determined because of the cross-sectional of the collected data. Further research can be performed by different participants selected by random sampling strategy and the causality can be explored by experimental and longitudinal research. Moreover, further research can detail the relationship between the variables explained in the present study by using alternative measurement tools or different data collection strategies. In addition, the present study can help the teachers be aware of the effects of feelings and thoughts about the knowledge and skills on the problem solving performances. Hence, they can help their students to increase hope with the aim of representing problem solving performance successfully. In this respect, the PMTs can be provided to increase their level of hope so that they can use their potential of mathematical thinking more effectively. Moreover, the teachers should help their students insist on solving problems by feeling the self confidence

about the ability to reach the result because of the nature of the problem by being inspired from the suggestion that "problems are not stop signs, they are guidelines" made by Robert Schuller.

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Since the data of this study were collected before 2020, there is no ethics committee approval.

Author Contribution Statement

Tuğba UYGUN: *Conceptualization, literature review, methodology, implementation, data analysis, organization and writing.*

Rahime DERE: *Conceptualization, literature review, data collection and writing.*

Elif ERTEM AKBAŞ: *Conceptualization, literature review, data collection and data analysis.*

References

Arnau, R. C., Rosen, D. H., Finch, J. F., Rhudy, J. L. & Fortunato, V. J. (2007). Longitudinal effects of hope on depression and anxiety: A latent variable analysis. *Journal of Personality, 75*(1), 1-21.

Arnett, J. J. (2000). Emerging adulthood: A theory of development from the late teens through the twenties. *American Psychologist*, 469-480.

Aspinwall, L. G. & Leaf, S. L. (2002). In search of the unique aspects of hope: Pinning our hopes on positive emotions, future-oriented thinking, hard times, and other people. *Psychological Inquiry, 13*, 276-288.

Baroody, A. J. (2003). The development of adaptive expertise and flexibility: The integration of conceptual and procedural knowledge. In A. J. Baroody and A. Dowker (Eds.), *The development of arithmetic concepts and skills* (pp. 1–34). Lawrence Erlbaum Associates.

Benn, L., Harvey, J. E, Gilbert, P., & Irons, C. (2005). Social rank, interpersonal trust and recall of parental rearing in relation to homesickness. *Personality and Individual Differences, 8*(38), 1813–1822.

Bowlby, J. (1982). *Attachment and loss, Vol. 1, Attachment*. Basic Books.

Bowlby, J. (1973). *Attachment and loss, Vol. 2, Separation*. Basic Books.

Cai, J., Moyer, J. C., & Wang, N. (1999). Parental roles in students' learning of mathematics: An exploratory study. *Research in Middle Level Education Quarterly, 22*(3), 1–18.

Chang, E. C. (1998). Hope, problem-solving ability, and coping in a college student population: Some implications for theory and practice. *Journal of Clinical Psychology, 54*, 953–962.

Creswell, J. W. (2013). *Research design qualitative, quantitative, and mixed method approaches*. Sage Publications.

Cokluk, O., Sekercioglu, G., & Buyukozturk, S. (2012). *Sosyal bilimler için çok değişkenli istatistik: SPSS ve LISREL uygulamaları (Vol. 2) [Multivariate statistics for social sciences: SPSS and LISREL applications]*. Pegem Akademi.

Ersoy, E., & Baser, N. (2012). Matematiksel dusunme olceginin gelistirilmesi [The development of mathematical thinking scale]. *Kastamonu Egitim Dergisi, 21*(49), 1471-1486.

Grouws, D. A. (1996). Critical issues in problem solving instruction in mathematics. In D. Zhang, T. Sawada, and J. P. Becker (Eds.), *Proceedings of the China-Japan-U.S. seminar on mathematical education* (pp. 70-93). Carbondale, IL: Board of Trustees of Southern Illinois University.

Henderson, K. B. & Pingry, R. E. (1953). Problem solving in mathematics. In H. F. Fehr (Ed.), *The learning of mathematics: Its theory and practice (21st Yearbook of the National Council of Teachers of Mathematics)* (pp. 228-270). National Council of Teachers of Mathematics.

Irwanto, Saputro, A. D., Rohaeti, E., & Prodjosantoso, A. K. (2018). Promoting critical thinking and problem solving skills of preservice elementary teachers through process-oriented guided-inquiry learning (POGIL). *International Journal of Instruction, 11*(4), 777-794.

Karakoca, A. (2011). *Altıncı sınıf öğrencilerinin problem çözmede matematiksel düşünmeyi kullanma durumları [Using of mathematical thinking of sixth grade students in problem-solving].* Unpublished master thesis. Eskişehir Osmangazi University, Turkey.

Kolodner, J., Simpson, R., & Sycara-Cyransky, K. A. (1985). *Process Model of Case-Based Reasoning in Problem Solving*. Proceedings of IJCAI-85, Los Angeles, CA, 1985, 284-290.

Köseoğlu, E., Demirci, F., Demir, B. and Özyürek, C. (2017). 7. sınıf öğrencilerinin problem çözmeye yönelik yansıtıcı düşünme becerilerinin incelenmesi: Ordu ili örneği. *International e-Journal of Educational Studies (IEJES), 1*(1), 60-68.

Lincoln, Y. S., & Guba, E. G. (1990). Judging the quality of case study reports. *Qualitative Studies in Education 3*(1), 53-59.

Marshall, C., & Rossman, G. B. (1989). *Designing qualitative research*. Sage.

Mason, J., Burton, L., & Stacey, K. (2010). *Thinking mathematically*. Pearson Education Limited.

Menninger, K. (1939). The academic lecture on hope. *The American Journal of Psychiatry, 109*, 481-49.

Oettingen, G., & Gollwitzer P. M. (2002). Turning hope thoughts into goal-directed behavior. *Psychological Inquiry, 13,* 304-7.

Parade, S. H., Leerkes, E. M., & Blankson, A. N. (2010). Attachment to parents, social anxiety, and close relationships of female students over the transition to college. *Journal of Youth and Adolescence, 39*(2), 127-137.

Peterson, C., & Seligman, M. E. P. (2004). Introduction to a manual of the sanities. In *Character strengths and virtues: A handbook and classification* (pp. 3–32). American Psychological Association and Oxford University Press.

Polya, G. (1990). *How to solve it: A new aspect of mathematical method*.

Posamentier, A.S., & S. Krulik. 1998. *Problem-solving strategies for efficient and elegant solutions: A resource for the mathematics teacher*. Corwin Press.

Quick, J. D., Nelson, D., Matuszek, P., Whittington, J. L. & Quick, J. C. (1996). Social support, secure attachments, and health. In C. L. Cooper, *Handbook of Stress, Medicine and Health*. CRC Press.

Rogoff, B. (1990). *Apprenticeship in thinking: Cognitive development in social context*. Oxford University Press.

Schoenfeld, A. H. (1992): Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In: Grouws, D. A. (Ed.): *Handbook of Research on Mathematics, Teaching and Learning* (NCTM). – New York, 334–370.

Snyder, C. R., Harris, C., Anderson, J. R., Holleran, S. A., Irving, L. M., Sigmon, S. T., et al. (1991). The will and theways: Development and validation of an individual-differences measure of hope. *Journal of Personality and Social Psychology, 60*, 570–585.

Snyder, C. R., Lapointe, A. B., Crowson, J. J. Jr., & Early, S. (1998). Preferences of high- and low-hope people for self-referential input. *Cognition and Emotion, 12*, 807–823.

Snyder, C. R., Lehman, K. A., Kluck, B., & Monsson, Y. (2006). Hope for rehabilitation and vice versa. *Rehabilitation Psychology, 51*, 89-112.

Snyder, C. R., Lopez, S. J., Shorey, H. S., Rand, K. L. & Feldman, D. B. (2003). Hope theory, measurements, and applications to school psychology. *Psychology Quarterly, 18*, 122-139.

Snyder, C. R., McDermott, D., Cook, W., & Rapoff, M. A. (1997). *Hope for the journey: Helping children through good times and bad*. Boulder, CO: Westview.

Snyder, C. R., Rand, K. L., & Sigmon, D. R. (2002). Hope theory: A member of the positive psychology family. In C. R. Snyder, & S. Lopez (Eds.), *Handbook of positive psychology* (pp. 257–266). OxfordUniversity Press.

Sookman D., & Pinard, G. (1997).Vulnerability and response to unpredictability, Newness and Change in Obsessive Compulsive Disorder. *Congress of European Association of Behavioral and Cognitive Therapies*. Venezia (Italy).

Sullivan, L. E. (2009). The glossary of the social and behavioral sciences. Sage.

Tabachnick, B.G. & Fidell, L.S. (1996). *Using Multivariate Statistics, 3rd ed.*, CSU, Harper-Collins, Northridge, CA.

Tarhan, S. & Bacanlı, H. (2015). Sürekli umut ölçeği'nin Türkçe'ye uyarlanması: Geçerlik ve güvenirlik çalışması [Adaptation of dispositional hope scale into Turkish: Validity and reliability study]. *The Journal of Happiness and Well-being 3*(1), 1-14.

Taşdemir, A. (2008). *Matematiksel düşünme becerilerinin ilköğretim öğrencilerinin fen ve teknoloji dersindeki akademik başarilari, problem çözme becerileri ve tutumlari üzerine etkileri [The effects of mathematical thinking skills on primary school students' academic achievements, problem-solving skills and attitudes in science and technology courses]*. Unpublished doctoral dissertation, Gazi University, Turkey.

Van de Walle, J. A. (1994). *Elementary school mathematics teaching developmentally.* Longman.

Woolfolk, A. E. (1993). *Educational psychology*. Allyn and Bacon.

Appendix

Semi-structured Interview Questions

1. What do the problem and problem solving mean to you?

2. How do you enact your mathematical problem solving performance successfully?

3. Which feelings have the power of affecting problem solving performance in mathematics? How? (Probing questions: Do you think that the thought and perception that I am able to reach the solution affect problem solving performance? How?)