

Deflection Prediction for Reinforced Concrete Beams Through Different Effective Moment of Inertia Expressions

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Abstract — The effective moment of inertia expressions proposed by Branson and Bischoff are examined by comparing the deflection estimates from these two approaches to the measured deflection values of reinforced concrete beams with high reinforcement ratios ($0.024 < \rho < 0.034$). It was found out that both methods yield to deflection estimates in close agreement with the actual values and the method proposed by Bischoff

bending deformations of heavily-reinforced concrete beams. Furthermore, the restrained shrinkage cracking was found to

cause the deflection response of a concrete beam to be much weaker than the responses estimated by the effective moment of inertia expressions. Finally, the cracking moment estimates from the methods given in ACI 318-05, Eurocode 2 and TS 500 are compared to the experimental cracking moments of reinforced concrete beams. The cracking moment estimates based on the modulus of rupture expression in Eurocode 2 were found to be in closest agreement with the experimental values.

Index Terms — Effective moment of inertia; Serviceability; Deflection; Reinforced concrete; Tension stiffening; In-plane bending.

I. INTRODUCTION

The in-plane bending stiffness (EI_x) of a beam is the product of two variables: (1) the in-plane second moment of area (the in-plane moment of inertia I_x), reflecting the cross-sectional resistance to loading; and (2) the modulus of elasticity (E), reflecting the material resistance to loading. In concrete beams, both variables are subject to change during the course of loading. The variation in the modulus of elasticity with the increasing load is caused by the inelastic stress-strain behavior of concrete beyond the elastic limits, while the variation in the moment of inertia is associated with the cracking of concrete due to the tensile strains greater than the cracking strain of concrete. The cracked zones in a concrete beam are ineffective in resisting stresses originating from applied loads and moments. Therefore, cracking of concrete decreases the resistance of a concrete beam to loading, leading to greater deformations in the beam. The decrease in the second moment of area of a concrete beam during the course of loading is

taken into account by the effective moment of inertia approach, which is summarized in the following discussion.

When the maximum moment (M_a) in a beam does not exceed the cracking moment (M_{cr}), the beam is in the uncracked condition. The uncracked moment of inertia of a beam with no compression reinforcement is obtained from the following equation:

$$I_{ucr} = \frac{1}{12} \cdot b \cdot h^3 + b \cdot h \cdot \left(y' - \frac{h}{2} \right)^2 + (n-1) \cdot A_s \cdot (d - y')^2 \quad (1)$$

where b and h are the width and height of the beam, respectively; y' is the depth of the centroid of the transformed uncracked cross-section from the compression face; n is the modular ratio of steel to concrete; A_s is the total cross-sectional area of the longitudinal reinforcement; and d is the effective depth of the tension reinforcement. With the exception of beams with heavy reinforcement, the gross moment of inertia (I_g) gives close values to the uncracked moment of inertia (I_{ucr}). I_g , which neglects the contribution of the reinforcement, is obtained from the following equation:

$$I_g = \frac{1}{12} \cdot b \cdot h^3 \quad (2)$$

When the in-plane bending moment (M) at a cross-section of the beam reaches M_{cr} , vertical flexural cracks form in the outermost layers of the tension zone. These cracks propagate upwards, as M increases. The section becomes fully-cracked, when the flexural cracks reach the neutral axis, rendering the entire tension zone ineffective in resisting the bending moment. The moment of inertia of the section in the fully-cracked condition is determined from the following equation:

$$I_{cr} = \frac{1}{12} \cdot b \cdot c^3 + n \cdot A_s \cdot (d - c)^2 \quad (3)$$

where c is the neutral axis depth of the fully-cracked section. Equation (3) assumes that the concrete in the compression zone has a linear elastic behavior up to the yielding of the tension reinforcement.

The overall moment of inertia of a concrete beam decreases gradually from the uncracked moment of inertia (I_{ucr}) to the fully-cracked moment of inertia (I_{cr}), as flexural cracks form

at discrete locations along the span. The concrete in the uncracked portions of the beam between the discrete cracks contributes to resisting tensile stresses, due to the bond between concrete and reinforcement. The tensile contribution of the concrete between the cracks is known as tension-stiffening. Tension-stiffening decreases as the applied load (or moment) increases and more cracks form along the span. The decrease in the tension-stiffening of concrete with the increasing load leads to the gradual decrease in the moment of inertia of the beam. This gradual decrease is taken into consideration by the effective moment of inertia approach. The following effective moment of inertia expression was originally proposed by Branson (1965):

$$I_e = \left(\frac{M_{cr}}{M_a} \right)^m \cdot I_{ucr} + \left[1 - \left(\frac{M_{cr}}{M_a} \right)^m \right] \cdot I_{cr} \quad (4)$$

where M_{cr} is the cracking moment and M_a is the maximum moment in the beam.

Branson's effective moment of inertia expression (Equation 4), which averages the moments of inertia of the uncracked and fully-cracked portions of a concrete beam, is adopted by ACI 318-05 (ACI 2005), AASHTO LRFD (AASHTO 2005), CSA A23.3-04 (CSA 2004), AS 3600 (SAA 1994) and TS 500 (TS 2000) in the immediate deflection calculations of concrete beams. All of these codes set the value of m to 3 to obtain an average moment of inertia for the entire span of a beam. Previously, Al-Shaikh and Al-Zaid (1993) found out that the value of m decreases as the reinforcement ratio (ρ) of a concrete beam increases. Accordingly, they proposed the following equation for m :

$$m = 3 - 0.8 \cdot \rho \quad (5)$$

Furthermore, Al-Zaid et al. (1991) experimentally showed that the power m in the effective moment of inertia expression is affected by the loading conditions of a beam and the load level (M_a/M_{cr}).

The effective moment of inertia expression proposed by Branson (1965) was developed empirically based on the test results of simply-supported rectangular reinforced concrete beams with reinforcement ratios between 1% and 2%. Branson's expression accurately estimates the moments of inertia of concrete beams with medium to high reinforcement ratios ($\rho > 1\%$). Nonetheless, different studies [Scanlon et al. (2001), Gilbert (1999), Gilbert (2006)] indicated that the expression constantly overestimates the moments of inertia of reinforced concrete beams with low reinforcement ratios ($\rho < 1\%$), which causes underestimation of the deflections. Bischoff (2005) found out that the misestimation of the moments of inertia and deflections of lightly-reinforced concrete beams by the Branson's approach is caused by the overestimation of the tension stiffening of concrete. According to the analytical study carried out by Bischoff (2005), the tension-stiffening component in Branson's method depends on the applied load level (M_a/M_{cr}) and on the ratio of the gross moment of inertia to the cracked moment of inertia (I_g/I_{cr}) of the beam, which varies inversely with the

reinforcement ratio (ρ). Branson's expression provides accurate estimates for reinforced concrete beams with reinforcement ratios greater than 1%, which corresponds to an I_g/I_{cr} ratio of 3. For lower reinforcement ratios ($I_g/I_{cr} > 3$), the member response estimated by Branson's approach is stiffer than the actual response, resulting in the underprediction of the deflections.

The deflection calculations in the European structural concrete codes, Eurocode 2 (CEN 2002) and BS 8110-2 (BS 1985), are based on the determination of the curvatures and deflections of a concrete beam corresponding to its uncracked and fully-cracked conditions. Eurocode 2 (CEN 2002) states in Section 7.4.3 that "Members which are expected to crack should behave in a manner intermediate between the uncracked and fully cracked conditions". The code requires the calculation of a deflection value which is a weighted average of the uncracked and fully-cracked deflections of the member. Based on Equation (7.18) given in Eurocode 2 (CEN 2002), the following equation is used for the calculation of the deflections (δ) of a reinforced concrete beam loaded at a level causing the beam to crack:

$$\delta = \left[1 - \beta \cdot \left(\frac{M_{cr}}{M_a} \right)^2 \right] \cdot \delta_{II} + \beta \cdot \left(\frac{M_{cr}}{M_a} \right)^2 \cdot \delta_I \quad (6)$$

where δ_{II} and δ_I are the deflection values corresponding to the fully cracked and uncracked conditions of the beam, respectively; and β is a coefficient accounting for the duration of loading or of repeated loading on the average strain. β is taken 1.0 for a single short-term loading (immediate deflections) and 0.5 for sustained loads (long-term deflections) or many cycles of repeated loading. The long-term deflections of reinforced concrete beams are out of scope of this study. Therefore, β is taken 1.0 in the present study.

Eurocode 2 (CEN 2002) uses the concept of averaging the flexibilities of the uncracked and cracked portions of the beam rather than averaging the stiffnesses. The tension-stiffening model setting the stage for Equation (6) is presented in CEB-FIP Model Code (CEB 1990), where the application of the model to the axial response of reinforced concrete tension members is depicted. Bischoff (2005) presented the application of the method to the in-plane bending behavior of reinforced concrete beams and developed the following effective moment of inertia expression, which is a weighted average of the flexibilities of the uncracked and cracked portions of a reinforced concrete beam:

$$\frac{1}{I_e} = \left(\frac{M_{cr}}{M_a} \right)^m \cdot \frac{1}{I_{ucr}} + \left[1 - \left(\frac{M_{cr}}{M_a} \right)^m \right] \cdot \frac{1}{I_{cr}} \quad (7)$$

A value of 2 was proposed for the power m in Equation (7), based on the deflection equation given in Eurocode 2. The use of $m=2$ assures that the tension-stiffening contribution in the model is only dependant on the applied load level (M_a/M_{cr}), as explained by Bischoff (2005) and Bischoff (2007), in detail. Consequently, the tension-stiffening model becomes

independent from the gross-to-uncracked moment of inertia ratio (I_g/I_{cr}) and the reinforcement ratio (ρ) of the beam.

Gilbert (2006) and Bischoff and Scanlon (2007) compared the experimental results of the beams and slabs with different reinforcement ratios to the analytical deflection estimates obtained from the two approaches [Branson (1965) and Bischoff (2005)]. It was shown that the analytical estimates produced by the effective moment of inertia expression proposed by Bischoff (2005) are in closer agreement with the experimental results, particularly for the lightly-reinforced concrete beams. Although both approaches generate satisfactory load-deflection (or moment-curvature) curves at medium to high reinforcement ratios ($\rho > 1\%$), Bischoff's approach provides a better agreement with the measured values at low reinforcement ratios ($\rho < 1\%$). The load-deflection response estimated by Branson approach was found to be much stiffer than the actual response of a lightly-reinforced concrete beam.

To explain the differences between the effective moment of inertia expressions proposed by Branson (1965) and Bischoff (2005), Bischoff and Scanlon (2007) and Bischoff (2007) used spring models (Fig. 1). Accordingly, Branson's approach models the uncracked and cracked portions of a concrete beam as springs in parallel, while Bischoff's approach models them as springs in series. In the springs-in-parallel model (Fig. 1b) the stiffnesses of the uncracked and cracked portions are averaged, while in springs-in-series model the flexibilities are averaged. The tension-stiffening approach used by Bischoff (2005) indicates that the weighted flexibilities of the uncracked and cracked portions should be averaged to obtain the overall material response of a cracked concrete beam. A cracked portion and an uncracked portion of a concrete beam, neighboring each other, resist approximately the same bending moment and their curvatures are integrated when assessing the deflections of the member. Consequently, the springs-in-series model used in Bischoff's approach represents the in-plane bending behavior of a cracked reinforced concrete beam more appropriately.

II. RESEARCH SIGNIFICANCE

There are a limited number of studies in the literature comparing the actual deflections of reinforced concrete beams to the deflection estimates from the effective moment of inertia methods proposed by Branson (1965) and Bischoff (2005). The present study aims at contributing to the topic by comparing the estimates from the methods to the experimental in-plane bending deflections of the heavily-reinforced concrete beams tested by Kalkan (2009). Although both methods were found out to provide satisfactory deflection estimates for reinforced concrete beams with medium to heavy reinforcement

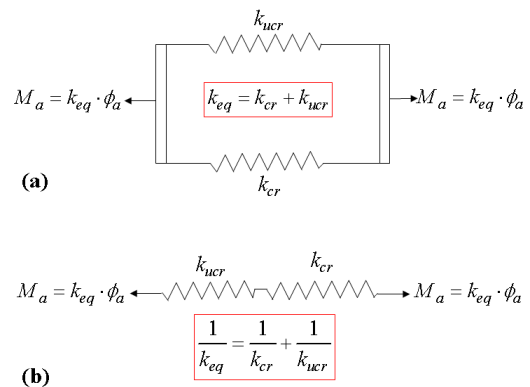


Fig. 1 – Spring Model for (a) Branson's effective moment of inertia; (b) Bischoff's effective moment of inertia

ratios and Bischoff's method was found out to produce deflection estimates superior to the ones of Branson's method in the case of lightly-reinforced concrete beams [Bischoff (2005), Gilbert (2006), Bischoff and Scanlon (2007)], the present study depicts that the method proposed by Bischoff (2005) provides a slightly better correlation with the experimental results even in the case of heavily-reinforced concrete beams.

III. APPLICATION OF BRANSON'S AND BISCHOFF'S METHODS TO THE EXPERIMENTAL DATA

In the present study, the analytical load-deflection estimates obtained from Branson's and Bischoff's approaches are compared to the experimental load-deflection curves of reinforced concrete beams tested by Kalkan (2009). The nominal dimensions, material properties and cross-sectional details of the specimens tested by Kalkan (2009) are presented in Table 1. Reinforcing steel with an average yield strength of 62.0 ksi (426.6 MPa) was used in all of the specimens, except the Specimens B22-2 and B18-2, whose longitudinal reinforcement had an average yield strength of 52.1 ksi (358.5 MPa). Two 2x6-W2.5xW3.5 Welded Wire Reinforcement Sheets, one on each side of the longitudinal reinforcement, constituted the shear reinforcement of the specimens tested by Kalkan (2009). The beams, loaded with a single concentrated load at midspan, had simple support conditions in and out of plane at the ends (Fig. 2).

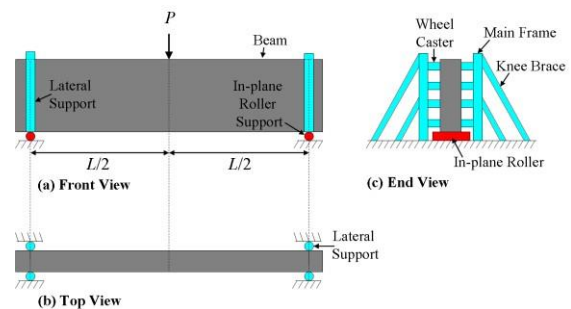


Fig. 2 – Experimental Setup used by Kalkan (2009)

Table 1 – Details of the specimens tested by Kalkan (2009)

Specimen	Nominal Dimensions Width x Height x Length <i>b x h x L</i> in. (mm)	Effective Depth, <i>d</i> in. (mm)	Number and Sizes of the Tension Rebars	Percent Reinforcement Ratio, $\rho\%$	Compressive Strength of Concrete, f'_c ksi (MPa)	Modulus of Elasticity of Concrete, <i>E</i> ksi (MPa)
B36L-1	3 x 36 x 468 (76 x 914 x 11890)	30.5 (774.7)	4#8 (4Ø25)	2.9	7.90 (54.47)	4300 (29650)
B36L-2	3 x 36 x 468 (76 x 914 x 11890)	30.5 (774.7)	4#8 (4Ø25)	2.9	7.94 (54.74)	4500 (31000)
B44-1	3 x 44 x 468 (76 x 1118 x 11890)	37.5 (952.5)	4#8 (4Ø25)	2.4	8.47 (58.40)	4450 (30700)
B44-2	3 x 44 x 468 (76 x 1118 x 11890)	37.5 (952.5)	4#8 (4Ø25)	2.4	8.54 (58.88)	4450 (30700)
B44-3	3 x 44 x 468 (76 x 1118 x 11890)	37.5 (952.5)	4#8 (4Ø25)	2.4	8.56 (59.02)	4550 (31400)
B36	2.5 x 36 x 240 (64 x 914 x 6100)	31.1 (789.9)	3#9 (3Ø29)	3.3	12.78 (88.11)	5850 (40350)
B30	2.5 x 30 x 240 (64 x 762 x 6100)	25.5 (647.7)	3#8 (3Ø25)	3.2	12.22 (84.25)	5950 (41000)
B22-1	1.5 x 22 x 144 (38 x 559 x 3660)	18.7 (474.9)	3#5 & 1#3 (3Ø16 & 1Ø10)	3.2	11.73 (80.87)	5200 (35800)
B22-2	1.5 x 22 x 144 (38 x 559 x 3660)	18.7 (474.9)	3#5 & 1#3 (3Ø16 & 1Ø10)	3.2	11.00 (75.83)	4850 (33400)
B18-1	1.5 x 18 x 144 (38 x 457 x 3660)	15.3 (388.6)	3#5 (3Ø16)	3.4	11.46 (79.01)	5000 (34450)
B18-2	1.5 x 18 x 144 (38 x 457 x 3660)	15.3 (388.6)	3#5 (3Ø16)	3.4	11.32 (78.05)	5000 (34450)

The value of cracking moment (M_{cr}) used in Branson's and Bischoff's effective moment of inertia expressions (Equations 4 and 7) significantly affects the analytical load-deflection curves of a beam. Previously, Yost et al. (2003) experimentally found out that the cracking moment estimates from the empirical relationships given in ACI 318-05 (ACI 2005) are 20-40% greater than the actual cracking moments of reinforced concrete beams, which influences the analytical deflection estimates to a major extent. Therefore, in the present study the experimental cracking moments of the specimens, obtained from the experimental load and deflection data, were used in the effective moment of inertia expressions when obtaining the analytical load-deflection curves. Structural concrete codes include equations for the evaluation of the cracking moments of concrete beams. These equations are helpful, particularly in the design of beams. In Table 2, the cracking moment estimates obtained from the equations given in ACI 318-05 (ACI 2005), Eurocode 2 (CEN 2002), and TS 500 (TS 2000) are compared to the experimental cracking moments of the specimens. All three groups of analytical estimates are obtained from the following cracking moment equation with different modulus of rupture (f_r) expressions:

$$M_{cr} = \frac{f_r \cdot I_g}{y_t} \quad (8)$$

where y_t is the vertical distance of the extreme tension fibers from the neutral axis. ACI 318-05 (ACI 2005) presents the following empirical expression for the

calculation of the modulus of rupture (f_r), which is used in Equation (8):

$$f_r = 7.5 \cdot \sqrt{f'_c} \quad (9)$$

where f'_c is the mean compressive strength of concrete in ksi, obtained from the cylinder tests. According to Section 3.1.9 of Eurocode 2 (CEN 2002), the mean flexural tensile strength (modulus of rupture) of concrete is obtained from the following equation:

$$f_r = \max \left[f_{ctm}, \left(1.6 - \frac{h}{1000} \right) \cdot f_{ctm} \right] \quad (10)$$

where f_{ctm} is the mean axial tensile strength of concrete, obtained from Equation (11); and h is the beam height in millimeters.

$$f_{ctm} = 2.12 \cdot \ln \left[1 + \left(\frac{f'_c}{10} \right)^{0.7} \right] \quad (11)$$

where f'_c is the mean compressive strength of concrete in MPa, obtained from the cylinder tests. Finally, the modulus of rupture of concrete is obtained from the following equation according to Section 13.2.2 of TS 500 (TS 2000):

$$f_r = 2.5 \cdot \left(\frac{f_{ctk}}{1.5} \right) \quad (12)$$

Table 2 – Cracking moments of the specimens tested by Kalkan (2009)

Specimen	Experimental Cracking Moment, M_{crex} in.kip (mm.MN)	Estimated Cracking Moment, M_{cre} in.kip (mm.MN)			Experimental-to-Estimated Cracking Moment Ratio, M_{crex}/M_{cre}		
		ACI 318-05	Eurocode 2	TS 500	ACI 318-05	Eurocode 2	TS 500
B36L-1	330 (37.3)	420 (47.4)	361 (40.7)	393 (44.4)	0.79	0.91	0.84
B36L-2	330 (37.3)	423 (47.8)	364 (41.1)	396 (44.8)	0.78	0.91	0.83
B44-1	566 (63.9)	626 (70.7)	536 (60.6)	586 (66.3)	0.90	1.06	0.97
B44-2	472 (53.3)	631 (71.3)	540 (61.0)	591 (66.8)	0.75	0.87	0.80
B44-3	460 (51.9)	636 (71.8)	544 (61.5)	595 (67.2)	0.72	0.85	0.77
B36	300 (33.9)	438 (49.5)	363 (41.0)	410 (46.4)	0.68	0.83	0.73
B30	240 (27.1)	300 (33.9)	250 (28.2)	281 (31.8)	0.80	0.96	0.85
B22-1	98 (11.0)	94 (10.6)	82 (9.3)	88 (10.0)	1.04	1.20	1.11
B22-2	95 (10.7)	88 (10.0)	77 (8.7)	83 (9.3)	1.08	1.23	1.14
B18-1	72 (8.1)	63 (7.1)	60 (6.8)	59 (6.7)	1.14	1.20	1.22
Sample Mean					0.87	1.00	0.93
Standard Deviation					0.16	0.16	0.17
% Coefficient of Variation					18.79	15.74	18.67

where f_{ctk} is the characteristic axial tensile strength of concrete, obtained from

$$f_{ctk} = 0.35 \cdot \sqrt{f_{ck}} \quad (13)$$

where f_{ck} is the characteristic compressive strength of concrete. Unlike Eurocode 2 (CEN 2002), the cracking moment equation in TS 500 (TS 2000) is expressed in terms of the characteristic strength values of concrete (f_{ctk} and f_{ck}). The compressive strength values presented in Table 2 are the mean compressive strength values (f'_c), obtained experimentally. Although f'_c and f_{ck} are different, f'_c values of the specimens were used in Equation (12) due to the lack of the equations in TS 500 applicable to the mean strength values.

Table 2 shows that the Eurocode 2 method produced the cracking moment estimates in closest agreement with the experimental cracking moments of the specimens. The experimental-to-estimated cracking moment ratios corresponding to the Eurocode 2 method were in the range of 0.83-1.23 with a sample mean of 1.00 and a coefficient of variation of 15.74 %. The ACI 318-05 and TS 500 methods generally overestimated the cracking moments of the specimens. The TS 500 method had a better agreement with the experimental values compared to the ACI 318-05 method.

In Figures 3-12, the analytical load-deflection curves obtained from Branson’s and Bischoff’s methods are compared to the experimental load-vertical deflection curves of the specimens. The figures also include the analytical lines corresponding to the uncracked and fully cracked responses of the specimens. As shown in Table 1, the specimens had high reinforcement ratios, in the range of

0.024-0.034. The figures depict that both methods provide close agreement with the measured deflection values of the specimens with the exception of Beams B36, B30, B22-1 and B22-2. This close agreement complies with the findings of Gilbert (2006) and Bischoff and Scanlon (2007), who found out that both methods provide good correlation with the test results at high reinforcement ratios. Figures 3-7 and Figure 12 also indicate that the method proposed by Bischoff (2005) provides a slightly better correlation with the experimental measurements as compared to the method proposed by Branson (1965), even at high reinforcement ratios.

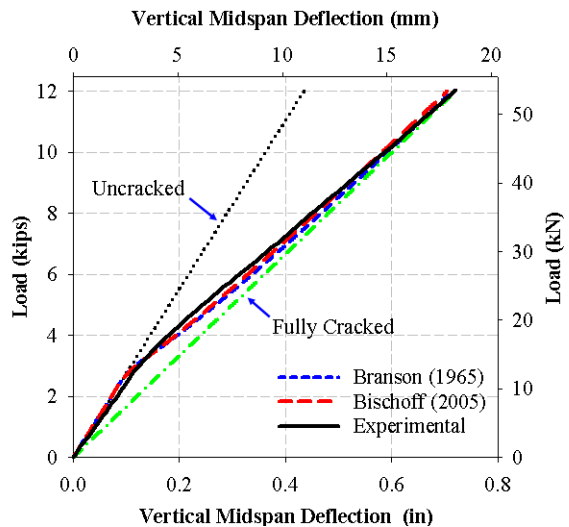


Fig. 3 - Specimen B36L-1 tested by Kalkan (2009)

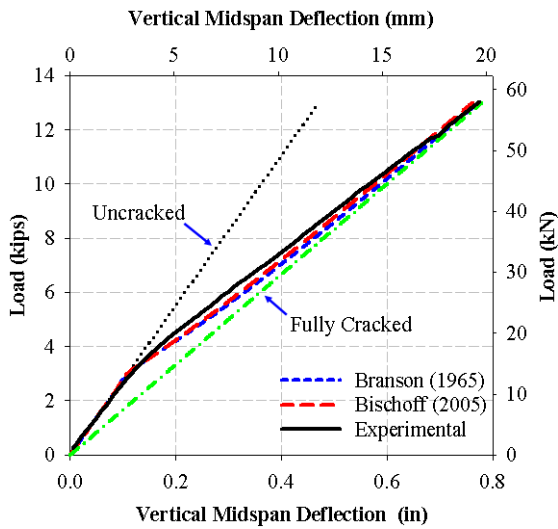


Fig. 4 - Specimen B36L-2 tested by Kalkan (2009)

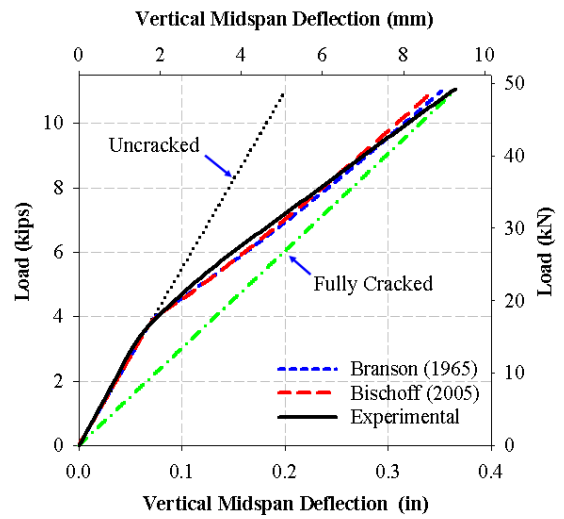


Fig. 7 - Specimen B44-3 tested by Kalkan (2009)

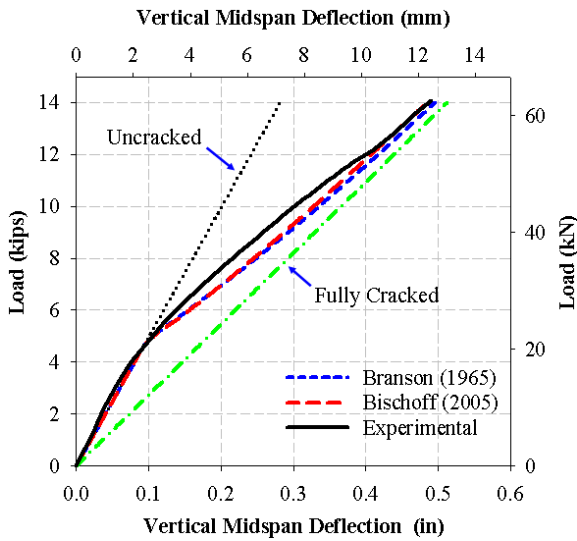


Fig. 5 - Specimen B44-1 tested by Kalkan (2009)

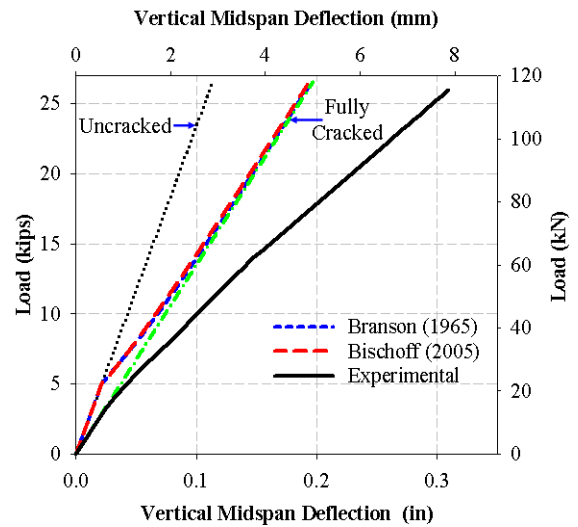


Fig. 8 - Specimen B36 tested by Kalkan (2009)

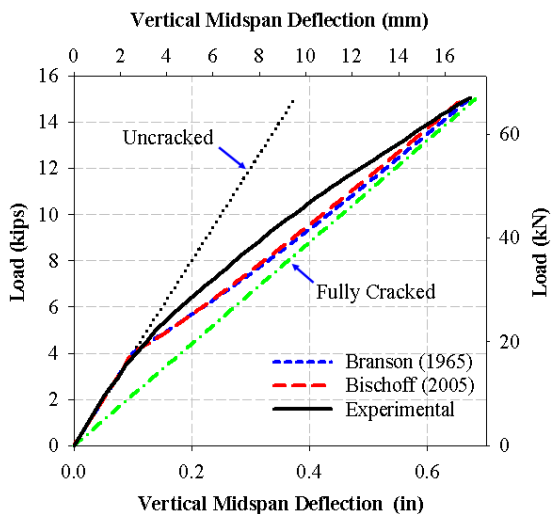


Fig. 6 - Specimen B44-2 tested by Kalkan (2009)

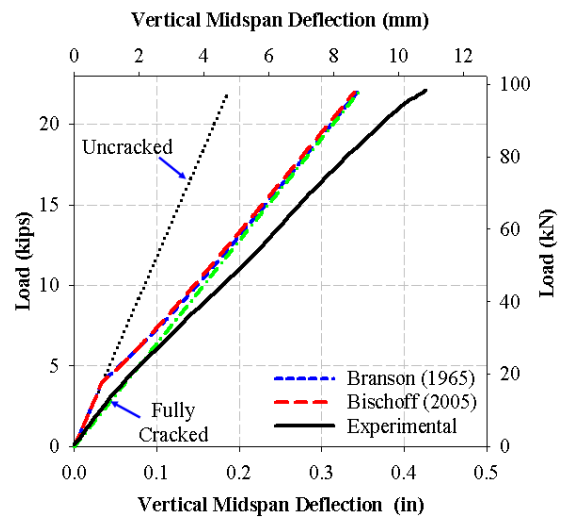


Fig. 9 - Specimen B30 tested by Kalkan (2009)

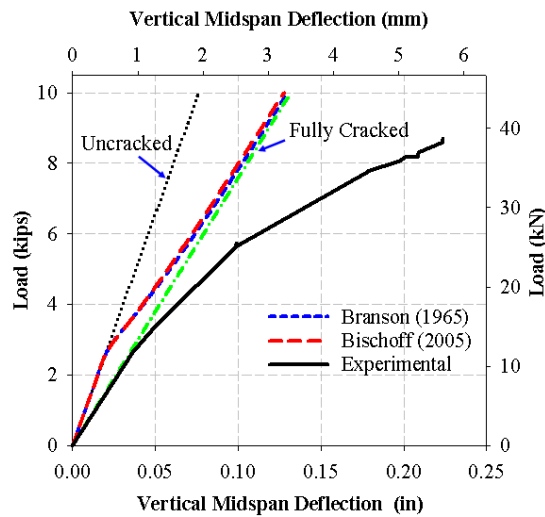


Fig. 10 - Specimen B22-1 tested by Kalkan (2009)

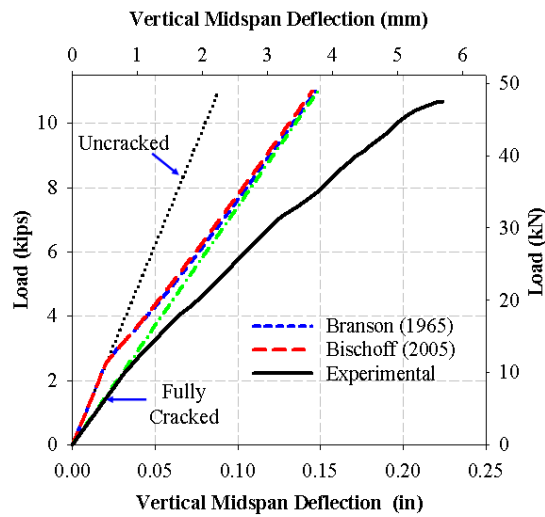


Fig. 11 - Specimen B22-2 tested by Kalkan (2009)

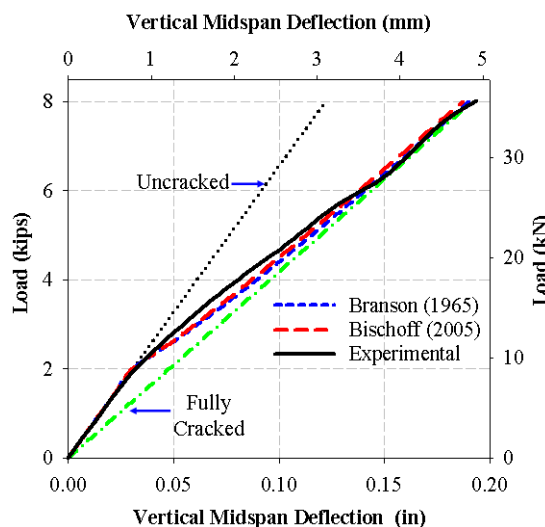


Fig. 12 - Specimen B18-2 tested by Kalkan (2009)

The experimental load-deflection curves of Specimens B36, B30, B22-1 and B22-2 are not in good agreement with the analytical curves from Branson's and Bischoff's methods (Figures 8-11). Both methods estimated responses which are much stiffer than the actual responses of these specimens. According to Kalkan (2009), the discrepancy between the analytical and experimental curves of these specimens was caused by the presence of restrained shrinkage cracks in these beams. Due to the presence of shrinkage cracks, the specimens did not reach the uncracked beam response even at the initial stages of loading. The initial portions of the experimental load-deflection curves of Specimens B36, B30, B22-1 and B22-2 are coincident with the analytical lines corresponding to the fully cracked response, substantiating the influence of the restrained shrinkage cracks on the in-plane bending behavior of concrete beams.

IV. CONCLUSIONS

Based on the study presented in this paper, the following conclusions are drawn:

1. The methods proposed by Branson (1965) and Bischoff (2005) closely estimate the load-deflection behavior of reinforced concrete beams with medium to high reinforcement ratios ($\rho > 1\%$).
2. The method proposed by Bischoff (2005) provides a slightly better correlation with the actual load-deflection curves of reinforced concrete beams with medium to high reinforcement ratios.
3. Restrained shrinkage cracking of concrete has a significant influence on the in-plane bending behavior of reinforced concrete beams. The actual response of a reinforced concrete beam with shrinkage cracks is significantly weaker than the responses estimated by Branson's and Bischoff's methods. The initial linear part of the load-deflection curve of a reinforced concrete beam with major restrained shrinkage cracking overlaps with the analytical line corresponding to the fully cracked response rather than the line corresponding to the uncracked response.
4. The effective moment of inertia and the analytical load-deflection curves corresponding to it are highly dependant on the cracking moment used in the effective moment of inertia expression. Therefore, experimental cracking moments, if known, of beams should be used in the effective moment of inertia calculations for a more accurate comparison of different analytical methods.
5. The cracking moment estimates based on the modulus of rupture expression in Eurocode 2 (CEN 2002) are in closer agreement with the actual cracking moments of the specimens, compared to the methods given in ACI 318-05 (ACI 2005) and TS 500 (TS 2000).

6. The method of ACI 318-05 (ACI 2005) generally overestimates the cracking moments of reinforced concrete beams.

V. FUTURE RESEARCH

In the present study, the major influence of restrained shrinkage cracking of concrete on the in-plane bending deformations of reinforced concrete beams was revealed with the help of the experimental data obtained by Kalkan (2009). Further studies are needed to investigate the degree of agreement of the deflection estimates from the effective moment of inertia expressions accounting for the restrained shrinkage cracking with the measured deflection values of the specimens tested by Kalkan (2009).

VI. NOTATION

A_s	:	Total cross-sectional area of the longitudinal reinforcement
b	:	Beam width
c	:	Neutral-axis depth of the fully-cracked section
d	:	Effective depth of the tension reinforcement
E	:	Modulus of elasticity
E_c	:	Modulus of elasticity of concrete
E_s	:	Modulus of elasticity of steel
EI_x	:	In-plane bending stiffness
f'_c	:	Mean compressive strength of concrete, obtained from the cylinder tests
f_{ck}	:	Characteristic compressive strength of concrete
f_{ctk}	:	Characteristic axial tensile strength of concrete
f_{ctm}	:	Mean tensile strength of concrete
f_r	:	Modulus of rupture
h	:	Beam height
I_{cr}	:	Cracked transformed moment of inertia
I_e	:	Effective moment of inertia
I_g	:	Gross moment of inertia
I_x	:	In-plane moment of inertia
I_{ucr}	:	Uncracked transformed moment of inertia
M_a	:	Maximum moment along the beam span
M_{cr}	:	Cracking moment
n	:	Modular ratio of steel to concrete ($=E_s/E_c$)
y_t	:	Vertical distance of the extreme tension layer from the neutral axis
y'	:	Depth of the centroid of the uncracked transformed cross-section of the beam from the compression face
δ	:	Deflection

δ_I	:	Deflection corresponding to the uncracked condition of a reinforced concrete beam
δ_{II}	:	Deflection corresponding to the fully cracked condition of a reinforced concrete beam
ρ	:	Reinforcement ratio

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