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ARAŞTIRMA MAKALESİ **RESEARCH ARTICLE**

Stochastic Copula Approach for Modeling Dependency: Evidence from Commodity and Exchange Rate Markets

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Abstract

In this study, the dependency structure between commodity prices and exchange rates of BRICS countries is modeled by the stochastic copula which is a particular class of time-varying copulas. This model is a nonlinear and its parameter follows an unobservable stochastic process. Since this approach regards both the observations and the latent process, it enables to be handled the dependency in a more flexible and comprehensive way.

The data set includes daily closing prices between January 2015 and December 2022, and they are extracted from Yahoo finance website. RStudio and MATLAB programs are used to analyze the data. It is found that there is a time-varying symmetrical dependence between oil prices and the exchange rates of BRICS countries. It should not be ignored that there is an upper tail dependence for oil and BRL and oil and RUB, and a lower tail dependence for oil and exchange rates of other BRICS countries. On the other hand, there is a time-varying symmetrical dependence between gold and the exchange rates of other BRICS countries while the relationship between gold and BRL is mostly measured by the upper tail dependence. Finally, it is suggested that dependency between gold and oil prices are dynamic and symmetric, but the upper tail dependency should be taken into account to measure the effect of asymmetry. The findings have important implications for policy makers and investors.

Keywords: Dependence modeling, Stochastic copula, BRICS countries, Commodity prices, Exchange rates.

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Bağımlılık Modellemede Stokastik Kopula Yaklaşımı: Emtia ve Döviz Kuru Piyasasından Bulgular

Özet

Bu çalışmada, BRICS ülkelerinin emtia fiyatları ile döviz kurları arasındaki bağımlılık yapısı, zamanla değişen kopulaların özel bir sınıfı olan stokastik kopula ile modellenmiştir. Bu model doğrusal olmayan bir modeldir ve parametresi gözlemlenemeyen bir stokastik süreci takip etmektedir. Bu yaklaşım hem gözlemleri hem de gizli süreci dikkate aldığından bağımlılığın daha esnek ve kapsamlı bir şekilde ele alınmasına olanak sağlar.

Bu çalışmadaki veri seti Ocak 2015 ile Aralık 2022 arası günlük kapanış fiyatlarını kapsamakta olup Yahoo finans websitesinden alınmıştır. Verilerin analizi için RStudio ve MATLAB programları kullanılmıştır. Petrol fiyatları ile BRICS ülkelerinin döviz kurları arasında zamanla değişen simetrik bir bağımlılık olduğu tespit edilmiştir. Petrol ve BRL ile petrol ve RUB için üst kuyruk bağımlılığı, diğer BRICS ülkelerinin ise petrol ve döviz kurları için alt kuyruk bağımlılığının olduğu göz ardı edilmemelidir. Öte yandan altın ile diğer BRICS ülkelerinin döviz kurları arasında zamanla değişen simetrik bir bağımlılık mevcutken, altın ile BRL arasındaki ilişki çoğunlukla üst kuyruk bağımlılığı ile ölçülmektedir. Son olarak altın ve petrol fiyatları arasındaki bağımlılığın dinamik ve simetrik olduğu ancak asimetrinin etkisini ölçmek için üst kuyruk bağımlılığının da dikkate alınması gerektiği ileri sürülmektedir. Bulguların politika yapıcılar ve yatırımcılar için önemli çıkarımlar sunmaktadır.

Anahtar sözcükler: Bağımlılık Modelleme, Stokastik kopula, BRICS Ülkeleri, Emtia Fiyatları, Döviz Kuru.

1. Introduction

Modeling of the dependence structure between variables has become popular in finance and economics in recent years. These variables are mostly far from the normal distribution due to the fact that they exhibit skewness and excess kurtosis. For this reason, methods that require the assumption of normality are insufficient to model the dependence between financial and economic variables and therefore, flexible approaches that do not need strict assumptions such as normality are explored.

One of the alternative methods in modeling the dependency is copulas. The existence of copulas was first introduced by Sklar's theorem (1959) and has received increasing attention, especially in economics and finance lately. This theorem claims that the multivariate distribution function can be decomposed into univariate marginals and a copula function that completely captures dependence structure between the interested variables. Thus, it enables to model the dependence between the variables independently of the marginal distributions. Moreover, the most important benefits of copula functions are the ability to handle various dependency structures such as asymmetric and tail dependency as well as symmetric dependencies. Nelsen (2007) and Joe (2014) present comprehensive information on the copula theory. Copula can describe the dependency structures of markets under extreme conditions (Cherubini et.al., 2004; Penzer et. al., 2012; Wu and Lin, 2014).

In this paper, the dependence structure between commodity prices and exchange rates of the BRICS countries is modeled by the stochastic copula which is a particular class of time-varying copulas. This process consists of two stages: modeling marginals first and then dependency. Since the return series are serially correlated and the evidence of heteroscedasticity, ARMA-GARCH (Auroregressive Moving Average – Generalized Autoregressive Conditional Heteroscedasticity) approach is performed to overcome these issues. The standardized residuals from the marginal models are converted to uniforms by the probability integral transform and used as input variables for the copulas. Dependency modeling is covered in three parts: oil and exchange rates of BRICS countries, gold and exchange rates of BRICS countries, and gold and oil. The main contributions of this study are as follows: to the best of our knowledge, it is the first study to handle the dynamic of dependence structure between commodity prices and exchange rates of BRICS countries through a stochastic copula model. This approach enables the dependency between the related variables to be studied in a more flexible and comprehensive framework. Moreover, there are very few studies in the literature that model dependency using the stochastic copula approach although it offers outstanding results in modeling dynamic dependency. It is expected that this paper will contribute to the gap in this field. Finally, the sample includes turmoil periods such as the Covid-19 pandemic and the Russia-Ukraine war and it is aimed to present the dynamics of dependency in such periods.

The remainder of the paper is as follows: In the Section 2, the literature review is explained. Section 3 describes the methodology. The definition of the data and empirical findings are presented in Section 4. The results are discussed in section 5.

2. Literature Review

Reboredo (2012) analyzes the co-movement of oil prices and exchange rates through the copula approach and finds that the dependency of the relevant variables is generally weak, that the dependency has increased significantly after the global financial crisis, and that there is no extreme market dependence between oil prices and exchange rates.

Wu et al. (2012) investigate the dependence structure between oil and exchange rate markets using the copula-GARCH method. They applies portfolio risk estimation to verify the effectiveness of the method.

Aloui et al. (2013) discuss the dependence between oil prices and exchange rates using the copula-GARCH approach. It is concluded that there is a significant and symmetrical dependence between all exchange rates of interest and oil. It is associated with the increase in oil prices and the depreciation of the dollar.

Yang and Hamori (2013) examine the dependence between gold and exchange rate markets. It has been determined that the conditional upper tail dependencies of gold and GBP and gold and JPY pairs are higher than other pairs. It is found that the dependency structure between gold and exchange rates is asymmetric.

Sebai and Naoui (2015) perform the copula-DCC-MGARCH approach to the relationship between oil prices and exchange rates. Before the global financial crisis, oil and exchange rates were independent, but after the crisis, there was evidence of a positive dependence between the related variables.

Kayalar et al. (2017) investigates the dependency between crude oil prices and stock market indices, as well as exchange rates, by using the copula approach according to developed/developing and oil importer/exporter classes. They conclude that while the exchange rates and stock markets of most oil exporting countries exhibit higher oil dependency, the markets of developing oil importing countries are less sensitive to price fluctuations.

Albulescu et al. (2018) analyzes the dependence between international exchange rates with time-invariant and time-varying copulas. They concludes that although the dependence between the JPY and other exchange rates is weak, there is a positive and symmetrical dependence between all international exchange rates.

Fenech and Vosgha (2019) explore the conditional dependencies between oil prices and Gulf Corporation Council stock market indices with time-varying copula models. They conclude that portfolios composed of oil and some GCC stocks are less likely to be affected by symmetrical risk.

He and hamori (2019) analyze the dependence between WTI oil prices and the exchange rates of BRICS countries using copula models. They suggest that there is a negative dependence and a significant tail dependency between oil prices and exchange rates.

Kumar et al. (2019) employ the dependence-switching copula approach to model the dependency between exchange rates and stock markets of BRICS countries. They suggested that for all countries except Russia, the dependency and tail dependency are symmetrical during the negative correlation regime, while the dependencies are asymmetrical but the tail dependency is symmetrical during the positive correlation regime. They reveal that the dependencies computed based on the R-vine copulas are optimal for calculating the VaR of the portfolio.

Meng et al. (2020) research the impact of downward and upward global oil price fluctuations on China's commodity sectors through both static and dynamic copulas. They suggest that the degree of downward spillover effect on the crude oil price is greater than the upward effect, thus asymmetric effects exist.

3. Methodology

This section presents the theoretical background of the proposed stochastic copula and marginal models.

3.1. Stochastic copula

The stochastic copula approach introduced by Hafner and Manner (2012) is defined as a particular class of time-varying copulas. In this model, parameters are estimated by considering not only data/observations as in Patton (2006) but also the latent stochastic process. Thus, the dynamics of the dependency structure enables to be handled in a more comprehensive and flexible way.

Let $(u_{1,t}, u_{2,t})$ for $t = 1, \dots, T$ be bivariate time series and their distribution function is defined by the copula model with dynamic θ parameter given below.

$$(u_{1,t}, u_{2,t}) \sim C(u_1, u_2|\theta_t) \tag{1}$$

Where $\theta_t \in \Theta \subset R$. It is assumed that parameter θ_t is driven by an unobservable stochastic process. $\theta_t = \Psi(\lambda_t)$ transform is applied to ensure that the copula parameter remains in its domain. Here, $\Psi: R \rightarrow \Theta$ and transformation Ψ depends on the chosen copula. For information on transformations, the appendix can be viewed.

λ_t is the unobservable underlying process and follows the first order of the Gaussian autoregressive process:

$$\lambda_t = \alpha + \beta\lambda_{t-1} + \kappa\varepsilon_t \qquad |\beta| < 1, \qquad \kappa > 0 \tag{2}$$

Here ε_t is a Gaussian innovation process. Parameter estimates are performed by an independent stochastic process. The model described above is nonlinear and can be written in its state space representation.

$$(u_{1t}, u_{2t})|\lambda_t \sim C\big(u_1, u_2 \big| \Psi(\lambda_t)\big) \tag{3}$$

The state equation is given in Eq. (3) and the transition equation is defined as in Eq. (4):

$$\lambda_t = \alpha + \beta\lambda_{t-1} + \kappa\varepsilon_t \tag{4}$$

For comprehensive information on the parameter estimation process of the stochastic copula model, Hafner and Manner (2012) can be scrutinized.

3.1.1. Tail dependency

Let (U_1, U_2) be two variables with uniform distribution. The lower tail dependency coefficient λ_l defined in [0,1] is stated as follows:

$$\begin{aligned}\lambda_l &= \lim_{u\to 0^+} P(U_1 \le u | U_2 \le u) \\ &= \lim_{u\to 0^+} \frac{C(u,u)}{u}\end{aligned} \tag{5}$$

Similarly, the upper tail dependency λ_u defined in [0,1] is expressed as follows:

$$\begin{aligned}\lambda_u &= \lim_{u\to 1^-} P(U_1 > u | U_2 > u) \\ &= \lim_{u\to 1^-} \frac{C^*(u,u)}{u}\end{aligned} \tag{6}$$

Here C^* is the survival copula. If the lower and upper tails are equal to each other, there is symmetric tail dependence between the two variables, otherwise there is asymmetric tail dependence. If $\lambda_l = 0$ and $\lambda_u = 0$, the two variables are asymptotically independent in the lower and upper tails.

In this study, normal, Gumbel, Clayton, Frank, Rotated Gumbel and Rotated Clayton copula models are considered as stochastic copula models. The normal copula models symmetric dependence. While Gumbel and Rotated Clayton copula models model upper tail dependencies, Clayton and Rotated Gumbel copulas overcome lower tail dependencies.

3.2. Marginal models

There is evidence of autocorrelation and heteroscedasticity in financial time series (Bollerslev, 1986). ARMA (Autoregressive Moving Average) – GARCH (Generalized Autoregressive Conditional Heteroscedasticity) models are used to overcome such problems and these models are defined as follows:

$$y_t = \mu + \sum_{i=1}^{p} \varphi_i y_{t-i} + \sum_{j=1}^{q} \gamma_j a_{t-j} + a_t \tag{7}$$

$$a_t = \sigma_t \epsilon_t$$

$$\sigma_t^2 = \omega + \sum_{k=1}^{m} \delta_k \sigma_{t-k}^2 + \sum_{l=1}^{n} \zeta_l a_{t-l}^2 \tag{8}$$

Where φ and γ are the parameters AR and MA, respectively, and μ is the constant. The parameters ω, δ and ζ in the GARCH model represent constant, GARCH and ARCH parameters, respectively.

4. Data and empirical results

In this paper, gold and oil prices from the commodity market and exchange rates of the BRICS countries against the dollar are considered. The dataset ranges from January 2015 to December 2022 and includes 2059 daily closing prices. All price series are extracted from Yahoo Finance. The labels are ‘OIL’ for the Brent crude oil, ‘GOLD’ for the Gold spot price, ‘BRL’ for the Brazilian real against US dollars, ‘RUB’ for the Russian ruble against US dollars, ‘INR’ for the Indian rupee against US dollars, ‘CNY’ for the Chinese yuan against US dollars and ‘ZAR’ for the South Africa rand against US dollars.

Figure 1. Original series of interested variables.

The sampling period is regarded because it includes crisis periods affecting international markets such as the covid-19 pandemic, the Russia-Saudi Arabia oil price war and the Russia-Ukraine wars.

Table 1. Descriptive statistics for the return series.

	COMMODITIES		EXCHANGE RATES				
	OIL	GOLD	BRL	RUB	INR	CNY	ZAR
Mean	0.0002	0.0001	-0.0003	-0.0001	-0.0001	-0.0001	-0.0001
Std. Dev.	0.0268	0.0087	0.0109	0.0157	0.0039	0.0025	0.0101
Skewness	-0.9600	-0.2329	-0.0619	-1.5116	-0.1973	-0.1758	-0.2461
Kurtosis	14.9550	3.3587	2.0595	37.4014	1.7336	5.5430	0.9530
J-B stat	**19540.0**	**989.5**	**366.79**	**120990.0**	**272.46**	**2653.3**	**99.3**
J-B p-value	**<0.001**	**<0.001**	**<0.001**	**<0.001**	**<0.001**	**<0.001**	**<0.001**
ADF stat	**-44.9**	**-44.6**	**-48.3**	**-44.6**	**-51.6**	**-46.1**	**-45.2**
ADF p-value	**<0.001**	**<0.001**	**<0.001**	**<0.001**	**<0.001**	**<0.001**	**<0.001**
L-B Q Stat	14.494	14.607	15.663	**47.036**	**51.555**	**23.907**	8.3916
L-B Q p-value	0.2703	0.2636	0.2072	**<0.001**	**<0.001**	**0.0209**	0.7538
L-B Q^2	**475.38**	**163.57**	**95.612**	**1292.3**	**207.13**	**192.08**	**103.92**
L-B Q^2 p-value	**<0.001**	**<0.001**	**<0.001**	**<0.001**	**<0.001**	**<0.001**	**<0.001**
Arch LM stat	**249.87**	**93.903**	**63.311**	**532.63**	**108.58**	**111.94**	**75.457**
Arch LM p-value	**<0.001**	**<0.001**	**<0.001**	**<0.001**	**<0.001**	**<0.001**	**<0.001**

The price series of the interested variables are demonstrated in Fig. 1. The trend pattern appears for all price series and this indicates that the price series are far from stationary assumption. For this reason, return series calculated as in Eq. (9) are utilized instead of price series for dependency analysis.

$$r_t = \ln(p_t) - \ln(p_{t-1}) \tag{9}$$

Here, p_t and r_t represent the price and return of the related financial asset at time t, respectively. The return series exhibit in Fig. 2. It is clear that there is evidence of volatility clustering for all series, implying the ARCH effect in the series. Descriptive statistics of the return series are presented in Table 1. The means for all return series are larger than the standard deviations, indicating that there is no trend form in the return series. Since the return series are left-skewed, the probability of loss is greater than the probability of gain for the all series. Although greater in oil and RUB exchange rates, excess kurtosis and heavy-tailedness are present in all series. The null hypothesis of normality of the return series is rejected by the Jarque-Bera test. Methods based on the assumption of normality are insufficient in modeling the dependence of related variables. The ADF (Augmented Dickey Fuller) test confirms the stationarity of the return series. It is propounded by means of Ljung-Box test that there is no serial correlation in the returns of oil, gold, BRL and ZAR, while there is a serial correlation in the returns of RUB, INR and CNY. However, it is revealed that the squares of the return series are serially related, and the existence of the ARCH effect is indicated by the ARCH LM test. These results support that ARMA-GARCH models are suitable for return series.

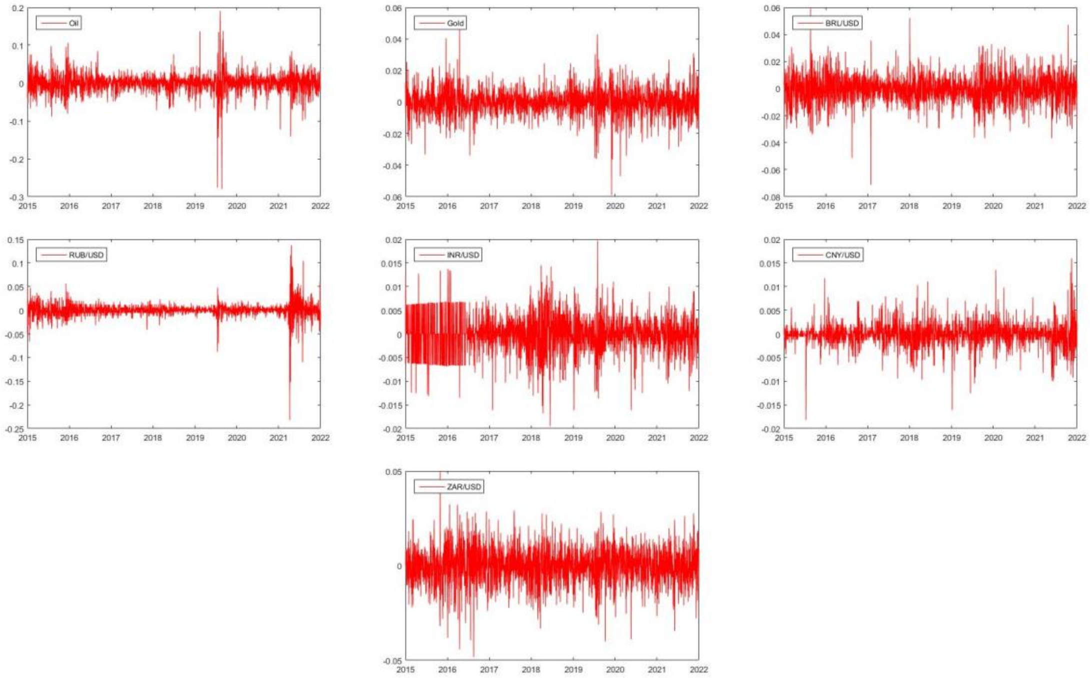

Figure 2. Return series of interested variables.

Table 2. Parameter estimates of the marginal distributions.

	COMMODITIES		EXCHANGE RATES				
	OIL	GOLD	BRL	RUB	INR	CNY	ZAR
μ	**0.0013** **(0.0004)**	0.0001 (0.4530)	-0.0002 (0.0711)	0.0001 (0.3066)	-0.0001 (0.9054)	-0.0001 (0.7911)	**-0.0004** **(0.0000)**
φ_1	**0.5245** **(0.0327)**	**0.2218** **(0.0000)**	**-1.4434** **(0.0000)**	**-0.5939** **(0.0001)**	/	**1.2229** **(0.0000)**	**-0.8541** **(0.0000)**
φ_2	/	**-0.9823** **(0.0000)**	**-0.7581** **(0.0000)**	/	/	**-0.2354** **(0.0000)**	**0.9059** **(0.0000)**
φ_3	/	/	**0.1498** **(0.0000)**	/	/	/	**0.9353** **(0.0000)**
γ_1	**-0.5569** **(0.0200)**	**-0.2308** **(0.000)**	**1.3640** **(0.0000)**	**0.5284** **(0.0014)**	**-0.1342** **(0.0000)**	**-1.3009** **(0.0000)**	**0.8520** **(0.0000)**
γ_2	/	**0.9899** **(0.000)**	**0.6281** **(0.0000)**	/	/	**0.3188** **(0.0000)**	**-0.9215** **(0.0000)**
γ_3	/	/	**-0.2356** **(0.0000)**	/	/	/	**-0.9158** **(0.0000)**
ω	**0.0001** **(0.0021)**	0.0001 (0.7119)	**0.0001** **(0.0258)**	0.0001 (0.5707)	0.0001 (0.3404)	0.0001 (0.9332)	0.0001 (0.0851)
ζ	**0.0986** **(0.0000)**	0.0334 (0.0702)	**0.0497** **(0.0000)**	**0.1251** **(0.0162)**	**0.0607** **(0.0000)**	**0.0795** **(0.0000)**	**0.0399** **(0.0000)**
δ	**0.8821** **(0.0000)**	**0.9554** **(0.0000)**	**0.9362** **(0.0000)**	**0.8709** **(0.0000)**	**0.9301** **(0.0000)**	**0.9193** **(0.0000)**	**0.9378** **(0.0000)**

Table 2. Parameter estimates of the marginal distributions (continuing).

	COMMODITIES		EXCHANGE RATES				
	OIL	GOLD	BRL	RUB	INR	CNY	ZAR
η	/	**0.9465 (0.0000)**	/	/	/	/	**0.8598 (0.0000)**
ν	**4.9730 (0.0000)**	**5.1529 (0.0024)**	**8.8933 (0.0000)**	**6.6807 (0.0000)**	**4.6528 (0.0000)**	**3.4309 (0.0000)**	**17.8680 (0.0043)**
LL	4958.932	6998.013	6492.744	6591.496	8624.244	9767.053	6591.459
AIC	-4.812	-6.791	-6.299	-6.398	-8.375	-9.483	-6.394
BIC	-4.793	-6.763	-6.269	-6.379	-.8.358	-9.458	-6.361

Notes: This table show estimated parameters for marginals. The values are parameter estimations and p-values, respectively.

Table 2 reports the parameter estimations of the marginals. Except for some constant parameters, parameters are significant for all of return series. It is concluded that for gold and all exchange rates except ZAR, models that innovations are Student t distribution are the best suitable, while for gold and ZAR, models with skewed Student t distribution of innovations are the most appropriate. Since the degrees of freedom ν range from 3.43 to 17.86, all return series are heavy-tailed.

Table 3. Parameter estimations of the best fitted SCAR copula: Brent Oil.

	Copula family	α S.E.	β S.E.	κ S.E.	Log L
OIL – BRL	Normal	0.0126 -	0.9380 -	0.0420 -	43.5814
OIL – RUB	Normal	0.0035 (0.0029)	0.9917 (0.0064)	0.0360 (0.0124)	213.8812
OIL – INR	Normal	0.0023 (0.0033)	0.9782 (0.0281)	0.0341 (0.0287)	19.1544
OIL – CNY	Normal	0.0049 -	0.9627 -	0.0153 -	19.7292
OIL - ZAR	Normal	0.0519 (0.0344)	0.7710 (0.1433)	0.1617 (0.0697)	53.7539

Notes: This table presents the parameter estimations of SCAR models. The parameters indicate constant term, persistence level and dynamic dependency, respectively.

The standardized residuals obtained from the estimated marginal models are converted to uniform inputs by probability integral transformation. Table 3 reports the parameter estimations of the stochastic copula that best models the dependence between oil and the exchange rates of the BRICS countries. All results of stochastic copulas for the dependence of commodity prices and the exchange rates of the BRICS countries are presented in Tables A1-A10 in the Appendix. In the model, the parameter β indicates the persistence in dependence and the parameter κ refers whether the dependency is dynamic or not. Due to the nature of the stochastic copula approach, dependencies are handled in pairs. The dependency structure of oil and exchange rates of the BRICS countries is best modeled by the Gaussian copula. The fact that the parameter value κ for each pair is different from zero indicates that there is a dynamic aspect in the dependency.

In terms of persistence in dependency, relationship between oil and ZAR is relatively low, while the other pairs exhibit high persistence in dependency, except for the oil-ZAR pair.

Table 4. Parameter estimations of the best fitted SCAR copula: Gold.

	Copula family	α S.E.	β S.E.	κ S.E.	Log L
GOLD – BRL	Gumbel	-0.0494 -	0.9782 -	0.1905 -	49.7480
GOLD – RUB	Normal	0.0068 (0.0063)	0.9370 (0.0496)	0.0811 (0.0428)	19.9272
GOLD – INR	Normal	0.0041 (0.0030)	0.9785 (0.0139)	0.0496 (0.0185)	58.9407
GOLD – CNY	Normal	0.0058 (0.0035)	0.9792 (0.0104)	0.0611 (0.0162)	118.0010
GOLD - ZAR	Normal	0.0058 (0.0035)	0.9792 (0.0104)	0.0611 (0.0162)	118.0010

Notes: This table presents the parameter estimations of SCAR models. The parameters indicate constant term, persistence level and dynamic dependency, respectively.

However, Clayton, Gumbel and their rotated versions from Archimedean copulas are more informative about the extreme market dependence commonly occurred between financial variables. This information is valuable for investors and policy makers at the decision-making stage. For asymmetrical dependencies between oil-BRL and oil-RUB, Gumbel copula, which only models the upper tail dependence, should be considered. Clayton copula is the best fitted asymmetrical copula model for petrol-INR and petrol-CNY.

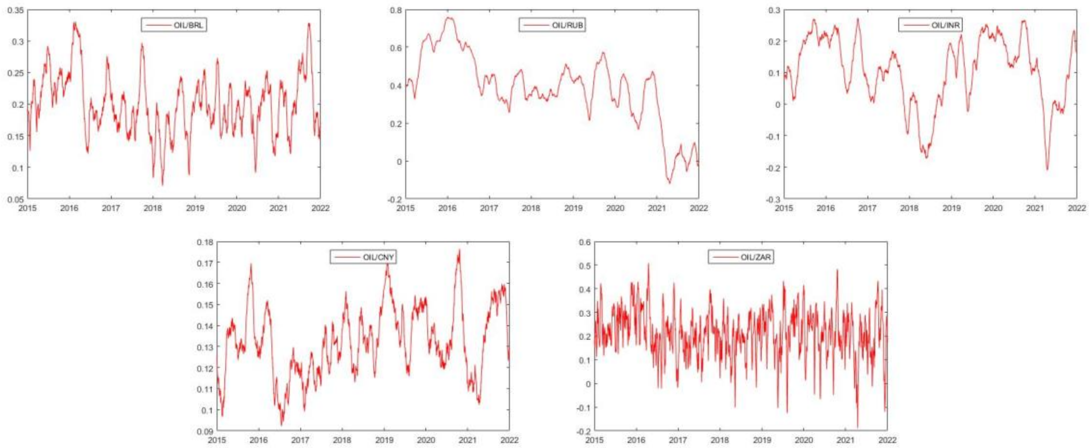

Figure 3. Dependence paths of oil and exchange rates.

The dependency of oil and exchange rates of BRICS countries is presented in Fig. 3. It is apparent that the dependence between oil and the exchange rates of the BRICS countries evolves over time. The dependency between the related variables is symmetrical and time-varying. It is notable to consider the lower tail dependencies for these variable pairs. Finally, there is a lower tail asymmetric dependence between oil and ZAR since the rotated Gumbel copula is selected as the best model for this pair.

Russia and Brazil are two of the world's largest oil producers. The increase in oil prices leads to the appreciation of the RUB and BRL currencies, which is supported by the upper tail dependency. On the other hand, the relationship between oil prices and other exchange rates excluding BRL and RUB is measured by the lower tail dependency. This indicates that oil and related exchange rates are more prone to comovement during downturns in markets. Table 4 summarizes the stochastic copula results for gold prices and the exchange rates of the BRICS countries. For all pairs, the β parameter value indicates high persistence in dependence, while the value of κ parameter refers that the dependency changes during time. The dependency between gold and BRL based on the log-likelihood criterion is best modeled through Gumbel copula. That is, the relations between gold and BRL is measured by the upper tail dependence. The dependency between gold and all exchange rates except BRL is best modeled by the normal copula, and there is a time-varying symmetric dependence between the relevant variables. However, Gumbel copula appears to be the best model for all pairs in terms of tail dependence. This means that gold and exchange rates of BRICS countries are more likely to move together during periods of market uptrend.

Table 5. Parameter estimates for SCAR copula: Gold and Brent oil.

Copula	α S.E.	β S.E.	κ S.E.	Log L
Normal	0.0055 (0.0056)	0.9345 (0.0562)	0.0810 (0.0468)	20.3147
Gumbel	-0.0179 -	0.9940 -	0.1232 -	17.8122
Clayton	-0.0188 -	0.9928 -	0.1144 -	14.6375
Frank	0.0049 (0.0055)	0.9894 (0.0109)	0.1154 (0.0723)	
Rotated Clayton	0.0217 -	0.9658 -	0.2223 -	-112.8307
Rotated Gumbel	-0.1147 -	0.9617 -	0.2280 -	14.5577

Notes: This table presents the parameter estimations of SCAR models. The parameters indicate constant term, persistence level and dynamic dependency, respectively.

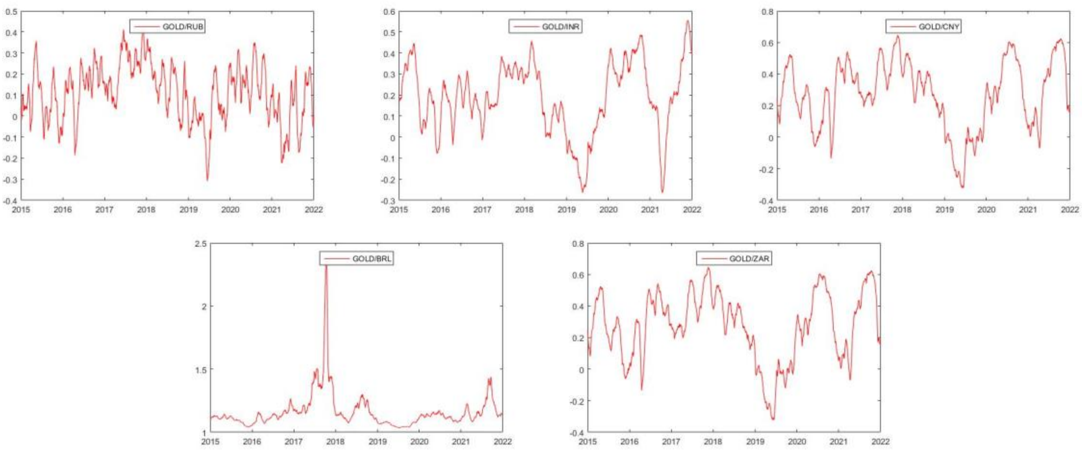

Figure 4. Dependence paths of gold and exchange rates.

The dependence paths between gold and exchange rates are depicted in Fig. 4. It is clear that the relations between exchange rates and gold, which are explained by symmetrical dependence, change both positively and negatively over time. This suggests that modeling the dependence between gold and the exchange rates of BRICS countries in a static way can be misleading. For the dependence between gold and oil prices, the Gaussian copula is the best fitted model. It provides the inference that the dependence between the related variables is symmetrical and evolves over time. However, the information on the tail dependency between gold and oil prices should not be ignored. Gumbel copula best models the extreme dependence between gold and oil and refers that the dependence between the two variables is explained by the upper tail dependence.

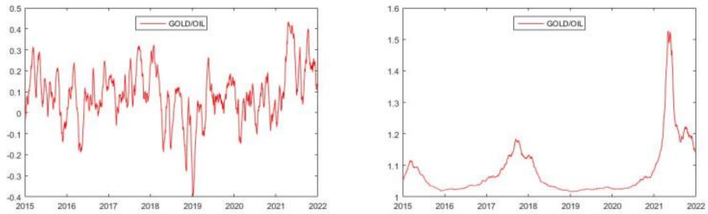

Figure 5. Symmetric and tail dependency between gold and oil, respectively.

The dependence paths of symmetric and tail dependence between gold and oil prices given by Gaussian and Gumbel copula, respectively, are demonstrated in Fig. 5. It reveals that oil and gold prices are both negatively and positively correlated over time. All the results of the dependency structures between the commodity and the exchange rates of the BRICS countries have important implications for both investors and policy makers.

5. Conclusion Remarks

Research on the dependency between economic and financial variables has become very popular in recent years. In this study, the dependency structure between commodity prices and exchange rates of BRICS countries is handled by the stochastic copula approach, which is a particular class of time-varying copulas. This method models the dependency between the variables by considering both the observations and the latent process, thus allowing to deal with the dependency structures in a more comprehensive and flexible way. Among the commodity products, oil and gold markets are investigated and the exchange rate values of BRICS countries against the dollar are used. The research consists of two steps. In the first step, the marginals of each variable are modeled with the ARMA-GARCH approach to overcome the issues of autocorrelation and heteroscedasticity in the return series. It is found that the models with innovations which is skewed t distribution for gold and ZAR are suitable, while the models with t distribution of innovations for oil and other exchange rates except ZAR are the best fitted and all return series are heavy-tailed. In the second step, the dependence structures between the interested commodities and the exchange rates of the BRICS countries are modeled via the stochastic copula approach. The remarkable results of the research are as follows: It is found out that the dependence between oil prices and the exchange rates of BRICS countries is symmetrical and time-varying. While the relationship between gold and BRL is mostly measured by the upper tail dependence, there is a symmetrical and time-varying dependency between gold and other exchange rates except BRL. When the relations between the exchange rates of the BRICS countries and gold are evaluated in terms of tail dependency, the exchange rates of gold and BRICS countries are upper tail dependent, that is, these markets tend to move together more likely during the period of rising markets. The findings have important implications for investors and policy makers.

References

Albulescu, C. T., Aubin, C., Goyeau, D., & Tiwari, A. K. (2018). Extreme co-movements and dependencies among major international exchange rates: A copula approach. *The Quarterly Review of Economics and Finance*, *69*, 56-69.

Aloui, R., Aïssa, M. S. B., & Nguyen, D. K. (2013). Conditional dependence structure between oil prices and exchange rates: a copula-GARCH approach. *Journal of International Money and Finance*, *32,* 719-738.

Bollerslev T (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of econometrics*, 31(3), 307-327.

Cherubini, U., Luciano, E., & Vecchiato, W. (2004). *Copula methods in finance*. John Wiley & Sons.

Fenech, J. P., & Vosgha, H. (2019). Oil price and Gulf Corporation Council stock indices: New evidence from time-varying copula models. *Economic Modelling*, *77,* 81-91.

Hafner C M and Manner H (2012). Dynamic stochastic copula models: Estimation, inference and applications. *Journal of Applied Econometrics*, 27(2): 269-295.

He, Y., & Hamori, S. (2019). Conditional dependence between oil prices and exchange rates in BRICS countries: An application of the copula-GARCH model. *Journal of Risk and Financial Management*, *12*(2), 99.

Joe, H. (2014). *Dependence modeling with copulas*. CRC Press, New York.

Kayalar, D. E., Küçüközmen, C. C., & Selcuk-Kestel, A. S. (2017). The impact of crude oil prices on financial market indicators: copula approach. *Energy Economics*, *61*, 162-173.

Kumar, S., Tiwari, A. K., Chauhan, Y., & Ji, Q. (2019). Dependence structure between the BRICS foreign exchange and stock markets using the dependence-switching copula approach. *International Review of Financial Analysis*, *63,* 273-284.

Meng, J., Nie, H., Mo, B., & Jiang, Y. (2020). Risk spillover effects from global crude oil market to China’s commodity sectors. *Energy, 202,* 117208.

Nelsen R B (2007). *An introduction to copulas*. Springer Science & Business Media, New York.

Patton A J (2006). Modelling asymmetric exchange rate dependence. *International economic review*, 47(2): 527-556.

Penzer, J., Schmid, F., & Schmidt, R. (2012). Measuring large comovements in financial markets. *Quantitative Finance*, *12*(7), 1037-1049.

Reboredo, J. C. (2012). Modelling oil price and exchange rate co-movements. *Journal of Policy Modeling*, *34*(3), 419-440.

Sebai, S., & Naoui, K. (2015). A study of the interactive relationship between oil price and exchange rate: A copula approach and a DCC-MGARCH model. *The Journal of Economic Asymmetries*, *12*(2), 173-189.

Sklar, A. 1959. Functions de Répartition à n dimensions et leurs Marges. *Publications Institut de Statistique de l’Université de Paris* 8: 229–31.

Wu, C. C., & Lin, Z. Y. (2014). An economic evaluation of stock–bond return comovements with copula-based GARCH models. *Quantitative Finance*, *14*(7), 1283-1296.

Wu, C. C., Chung, H., & Chang, Y. H. (2012). The economic value of co-movement between oil price and exchange rate using copula-based GARCH models. *Energy Economics*, *34*(1), 270-282.

Yang, L., & Hamori, S. (2014). Gold prices and exchange rates: a time-varying copula analysis. *Applied Financial Economics*, *24*(1), 41-50.

Appendix

Table A1. Parameter estimates for SCAR copula: Brent oil and BRL/USD exchange rate.

Copula	α S.E.	β S.E.	κ S.E.	Log L
Normal	0.0126 -	0.9380 -	0.0420 -	43.5814
Gumbel	-0.1081 (0.1227)	0.9477 (0.0587)	0.1681 (0.1353)	38.6676
Clayton	-0.0550 (0.0632)	0.9690 (0.0344)	0.1317 (0.1157)	31.6331
Frank	0.0083 (0.0129)	0.9932 (0.0108)	0.0493 (0.0521)	41.8289
Rotated Clayton	0.0076 -	0.9794 -	0.2179 -	8.0521
Rotated Gumbel	-0.0656 (0.0621)	0.9723 (0.0256)	0.1421 (0.1007)	32.1236

Table A2. Parameter estimates for SCAR copula: Brent oil and RUB/USD exchange rate.

Copula	α S.E.	β S.E.	κ S.E.	Log L
Normal	0.0035 (0.0029)	0.9917 (0.0064)	0.0360 (0.0124)	213.8812
Gumbel	-0.0056 -	0.9957 -	0.0812 -	196.1713
Clayton	-0.0056 -	0.9933 -	0.0965 -	167.7098
Frank	0.0142 (0.0106)	0.9947 (0.0038)	0.1882 (0.0599)	210.0190
Rotated Clayton	-0.0015 -	0.9753 -	0.2103 -	143.6485
Rotated Gumbel	-0.0441 -	0.9595 -	0.2208 -	181.3634

Table A3. Parameter estimates for SCAR copula: Brent oil and INR/USD exchange rate.

Copula	α S.E.	β S.E.	κ S.E.	Log L
Normal	0.0023 (0.0033)	0.9782 (0.0281)	0.0341 (0.0287)	19.1544
Gumbel	-0.1662 (0.2164)	0.9466 (0.0669)	0.2933 (0.2416)	8.3299
Clayton	-0.0170 (0.0180)	0.9937 (0.0065)	0.1077 (0.0583)	15.4691
Frank	0.0108 (0.0103)	0.9840 (0.0136)	0.1605 (0.0905)	18.5937
Rotated Clayton	-0.0557 (0.0100)	0.9919 (0.0017)	0.2593 (0.0503)	2.8212
Rotated Gumbel	-0.0753 (0.0184)	0.9757 (0.0059)	0.2421 (0.0420)	13.6347

Table A4. Parameter estimates for SCAR copula: Brent oil and CNY/USD exchange rate.

Copula	α S.E.	β S.E.	κ S.E.	Log L
Normal	0.0049 -	0.9627 -	0.0153 -	19.7292
Gumbel	-0.1667 (0.1481)	0.9455 (0.0448)	0.2924 (0.2626)	11.8728
Clayton	-0.1580 (0.0075)	0.9264 (0.0045)	0.0039 (0.0011)	16.5800
Frank	1.0000 -	-0.2917 -	0.0421 -	17.8081
Rotated Clayton	-0.0493 -	0.9952 -	0.2519 -	0.3888
Rotated Gumbel	-0.2828 (0.5121)	0.8998 (0.1824)	0.0424 (0.4623)	13.9987

Table A5. Parameter estimates for SCAR copula: Brent oil and ZAR/USD exchange rate.

Copula	α S.E.	β S.E.	κ S.E.	Log L
Normal	0.0519 (0.0344)	0.7710 (0.1433)	0.1617 (0.0697)	53.7539
Gumbel	-0.5000 -	0.7668 -	0.5919 -	43.7450
Clayton	-0.0103 (0.0094)	0.9937 (0.0053)	0.0586 (0.0323)	41.5504
Frank	0.3102 (0.2440)	0.7755 (0.1685)	0.9013 (0.5176)	52.1377
Rotated Clayton	0.0073 -	0.9805 -	0.2185 -	-4.2132
Rotated Gumbel	-0.0671 (0.0769)	0.9705 (0.0333)	0.1746 (0.1256)	44.5053

Table A6. Parameter estimates for SCAR copula: Gold and BRL/USD exchange rate.

Copula	α S.E.	β S.E.	κ S.E.	Log L
Normal	0.0035 (0.0096)	0.9823 (0.0460)	0.0326 (0.0548)	45.1280
Gumbel	-0.0494 -	0.9782 -	0.1905 -	49.7480
Clayton	-0.0274 -	0.9859 -	0.1488 -	30.9831
Frank	0.0117 (0.0085)	0.9903 (0.0064)	0.1368 (0.0528)	45.2845
Rotated Clayton	0.0186 -	0.9685 -	0.2197 -	-34.2566
Rotated Gumbel	-0.0900 -	0.9647 -	0.2260 -	39.4066

Table A7. Parameter estimates for SCAR copula: Gold and RUB/USD exchange rate.

Copula	α S.E.	β S.E.	κ S.E.	Log L
Normal	0.0068 (0.0063)	0.9370 (0.0496)	0.0811 (0.0428)	19.9272
Gumbel	-0.0113 -	0.9963 -	0.0906 -	17.1315
Clayton	-0.0112 -	0.9960 -	0.0912 -	10.6415
Frank	0.0398 (0.0397)	0.9408 (0.0521)	0.4443 (0.2720)	19.5813
Rotated Clayton	0.0190 -	0.9673 -	0.2227 -	-92.2317
Rotated Gumbel	-0.1054 (0.0081)	0.9668 (0.0001)	0.2368 (0.0001)	10.4650

Table A8. Parameter estimates for SCAR copula: Gold and INR/USD exchange rate.

Copula	α S.E.	β S.E.	κ S.E.	Log L
Normal	0.0041 (0.0030)	0.9785 (0.0139)	0.0496 (0.0185)	58.9407
Gumbel	-0.0230 (0.0073)	0.9896 (0.0029)	0.1475 (0.0243)	48.3196
Clayton	-0.0456 (0.0163)	0.9781 (0.0077)	0.1938 (0.0365)	37.0884
Frank	0.0162 (0.0106)	0.9855 (0.0087)	0.2252 (0.0758)	47.6704
Rotated Clayton	0.0109 -	0.9727 -	0.2224 -	-19.5923
Rotated Gumbel	-0.0451 (0.0464)	0.9817 (0.0181)	0.1857 (0.1018)	45.3825

Table A9. Parameter estimates for SCAR copula: Gold and CNY/USD exchange rate.

Copula	α S.E.	β S.E.	κ S.E.	Log L
Normal	0.0058 (0.0035)	0.9792 (0.0104)	0.0611 (0.0162)	118.0010
Gumbel	-0.0243 (0.0105)	0.9868 (0.0047)	0.1375 (0.0334)	103.1338
Clayton	-0.0110 -	0.9926 -	0.0964 -	81.9256
Frank	0.0217 (0.0145)	0.9865 (0.0071)	0.2864 (0.0794)	111.1514
Rotated Clayton	0.0137 -	0.9720 -	0.2185 -	37.9628
Rotated Gumbel	-0.0215 (0.0092)	0.9885 (0.0043)	0.1311 (0.0339)	96.2854

Table A10. Parameter estimates for SCAR copula: Gold and ZAR/USD exchange rate.

Copula	α S.E.	β S.E.	κ S.E.	Log L
Normal	0.0058 (0.0035)	0.9792 (0.0104)	0.0611 (0.0162)	118.0010
Gumbel	-0.0243 (0.0105)	0.9868 (0.0047)	0.1375 (0.0334)	103.1338
Clayton	-0.0110 -	0.9926 -	0.0964 -	81.9256
Frank	0.0217 (0.0145)	0.9865 (0.0071)	0.2864 (0.0794)	111.1514
Rotated Clayton	0.0137 -	0.9720 -	0.2185 -	37.9628
Rotated Gumbel	-0.0215 (0.0092)	0.9885 (0.0043)	0.1311 (0.0339)	96.2854

Table A11. Transformation function by copula model.

Copula	Transformation function $\Psi(x)$
Normal	$\frac{exp(2x)-1}{exp(2x)+1}$
Gumbel	$exp(x)+1$
Clayton	$exp(x)$
Frank	x