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Chebyshev Polynomial Solution for the SIR Model of COVID 19

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ABSTRACT

In this study, we deal with solving numerically initial value problem of a mathematical model of COVID-19 pandemic in Turkey. This model is a SIR model consisting of a nonlinear system of differential equations. In order to solve these equations, a collocation approach based on the Chebyshev polynomials is used. Chebyshev polynomials are orthonormal polynomials and the orthonormality reduces the computation cost of the method as an advantage. Another advantage is that the present method does not require any discretization of the domain. So the method is easy to implement. The main idea of the method is to convert the model to a system of nonlinear algebraic equations. For this we write the approximate solution of the system and its first derivative as the truncated series of Chebyshev polynomials with unknown coefficients in matrix forms and then utilizing the collocation points, the SIR model is converted to a system of the nonlinear equations. The obtained system is solved for the unknown coefficients of the assumed Chebyshev polynomial solution by MATLAB, and so the approximate solution is obtained. In order to check the robustness of the method, residual error of the solution is reviewed. The results show that the method is efficient and accurate.

Covid 19 için SIR modelinin Chebyshev Polinom Çözümleri

ÖZ

Bu çalışma, Türkiye'deki COVID-19 salgınına yönelik matematiksel modelin başlangıç değer probleminin sayısal olarak çözümü ile ilgilidir. Bu model, nonlineer denklem sisteminden oluşan SIR modelidir. Bu denklemleri çözmek için Chebyshev polinomlarına dayanan bir kollokasyon yöntemi kullanılmıştır. Chebyshev polinomları ortonormal polinomlardır ve yöntemin bir avantajı olarak, ortonormallik metodun hesaplama maliyetini düşürmektedir. Bir diğer avantaj ise mevcut yöntem, alan ayrıştırması gerektirmemektedir. Dolayısıyla yöntemin uygulanması kolaydır. Metodun ana düşüncesi modeli nonlineer cebirsel denklemlere dönüştürmektir. Bunun için sistemin yaklaşık çözümü ve birinci türevi, matris formlarında katsayıları bilinmeyen Chebyshev polinomlarının kesik serisi olarak yazılmaktadır ve sonra kollokasyon noktalarından yararlanarak, SIR modeli nonlineer denklemler sistemine dönüştürülmektedir. Elde edilen sistem Chebshev polinomlarının bilinmeyen katsayıları için Matlab kullanılarak çözülür ve böylece yaklaşık çözüm elde edilir. Metodun doğruluğunu kontrol etmek için çözümün artık mutlak hatası incelenmiştir. Sonuçlar mevcut metodun etkili ve doğru olduğunu göstermektedir.

Anahtar Kelimeler: SIR modeli, kollokasyon metodu, COVID-19 modellemesi, hata analizi, matematiksel modelleme

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1. Introduction

Until present, there have been various disease and pandemics that human being has struggled with and millions of people have lost their lives due to these diseases. In the last century, the world experienced major outbreaks such as 1918 H1N1 Influenza, Asian Influenza, Hong Kong flu, HIV/AIDS epidemic etc. which were represented as a threat for the human health [1-4].

By the 21st century, in December 2019, a novel epidemic was appeared in Wuhan, Hubei Province of China. Although the symptoms of this disease such as fever, fatigue, cough, headache and sore throat were similar to influenza, it was understood that the disease was caused by the severe acute respiratory syndrome coronavirus 2 (SARS-COV2) and the disease was named as Corona Virus (Covid-19) by World Health Organization (WHO) and declared as a global pandemic [3,4]. The disease was so rapid that in a couple months, the number of Corona Virus positive patients spreads up to 4 milllion. Now as of August 2023, there are globally more than 700 million confirmed cases of Covid-19 and more than 6 million deaths reported to WHO while 17 million confirmed cases are from Turkey [3].

Since December 2019, the world has experienced not only health consequences but also devastating impacts on economy, politics and social life. Therefore the additional solutions such as wearing masks, social-distancing, distance education etc. were very essential as well as the medical treatment to control the pandemic and everyone from the younger to the elder tried to contribute to this. In this sense, also mathematical models have made contributions to find out the dynamic pattern of the disease, to investigate forecasting tools, to predict disease transmission, recovery and other paramaters and analyze Covid-19 outcomes of different populations.

One of the basic epidemic model for Covid-19 spread is the Susceptible-Infected-Recovered (SIR) model which is firstly developed by Kermack and McKendrick [5] and is discussed by many researchers [5-14]. This model divides the population into three groups: The susceptible individuals, the infected individuals and the removed individuals. Their descriptions are given in the Table 1.

Table 1. Descriptions of the variables and the parameters

Notation	Description
S	Susceptible population represents individuals which are possible to be exposed to Covid-19
I	Infected population represents individuals which are infected by Covid-19
R	Removed population represents which are recovered or died from Covid-19
α	The transmission rate from the susceptible individuals to the infected individuals
μ	The recovery rate from the infected individuals to the removed individuals

The aim of this study is to use a collocation method based on Chebyshev polynomials in order to examine the flow of the Covid-19 disease according to the data obtained from the Turkish Ministry of Health. For this we consider the SIR epidemic model for Covid-19 and we note that this problem has not been solved by the collocation method based on Chebyshev polynomials so far.

The SIR epidemic model can be given mathematically by the set of nonlinear ordinary differential equations

$$\begin{aligned} \frac{dS(t)}{dt} &= -\frac{\alpha}{P}S(t)I(t), \\ \frac{dI(t)}{dt} &= \frac{\alpha}{P}S(t)I(t) - \mu I(t), \qquad 0 \le t \le L \qquad (1) \\ \frac{dR(t)}{dt} &= \mu I(t), \end{aligned}$$

subject to the initial conditions

$$S(0) = S_0, \;\; I(0) = I_0, \;\; R(0) = R_0 \qquad (2)$$

where $P = S(t) + I(t) + R(t)$ is the population size which is sufficiently large and remains constant as the assumption of the model.

2. Chebyshev Polynomials

Chebyshev polynomials of the first kind $T^*(t)$ are nth degree polynomials defined by $T_n^*(cos\theta) = \cos(n\theta)$ [15-17] and constructed from the recurrence relation

$$T_{n+1}^*(t) = 2tT_n^*(t) - T_{n-1}^*(t), \;\; n \geq 1$$

with initial values $T_0^*(t) = 1$ and $T_1^*(t) = t$.
As one of the basic properties, these polynomials constitute an orthogonal set $\{T_n^*(t), n = 0, 1,2, \dots\}$ on the interval $[-1,1]$ with respect to the weight function $w^*(t) = \frac{1}{\sqrt{1-t^2}}$, that is

$$\int_{-1}^{1} w^*(t)T_n^*(t)\,T_m^*(t)dx = \begin{cases} 0, & n \neq m, \\ \gamma_n, & n = m, \end{cases}$$

where $\gamma_n = \pi$ if $n = 0$ and $\gamma_n = \frac{\pi}{2}$ if $n \neq 0$. This implies that Chebyshev polynomials will be orthonormal if each polynomial is normalized by dividing $\sqrt{\gamma_n}$, $n = 0,1,2, \dots$. When the interval of interest $[a, b]$ is different than $[-1,1]$, the Chebyshev polynomials are shifted to $[a, b]$ by a suitable transformation. In this study the shifted Chebyshev polynomials under the transformation $T_n(t) = T_n^*(\frac{2}{L}t - 1)$ are orthogonal over the interval $[0, L]$. So, all properties of Chebyshev Polynomials will be transformed to their shifted forms. For instance, any square integrable function $f(t)$ defined on $[0, L]$ can be expressed as

$$f(t) = \sum_{n=0}^{\infty} a_n T_n(t)$$

where $a_n = \int_0^L f(t)\,T_n(t)\,w_n(t)dt$ and $w_n(t) = w_n^*\left(\frac{2}{L}t - 1\right)$. For an approximation, we truncate the series as $f(t) \approx f_N(t) = \sum_{n=0}^{N-1} a_n T_n(t)$, where N is a positive integer.

3. Implementation of Chebyshev Polynomials

Assume that the solutions of the model (1) and (2) is written in terms of the Chebyshev polynomials as

$$\begin{aligned} S_N(t) &= \sum_{n=0}^{N-1} a_n T_n(t) = \boldsymbol{A}^T \emptyset(\boldsymbol{t}) \\ I_N(t) &= \sum_{n=0}^{N-1} b_n T_n(t) = \boldsymbol{B}^T \emptyset(\boldsymbol{t}) \\ R_N(t) &= \sum_{n=0}^{N-1} c_n T_n(t) = \boldsymbol{C}^T \emptyset(\boldsymbol{t}) \end{aligned} \tag{3}$$

where $a_n, b_{n,}\, c_n$ are the unknown Chebyshev coefficients; T_n are the shifted first kind Chebyshev polynomials for $n = 0,1,2, \dots$ and a positive integer N; $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}$ and $\emptyset(\boldsymbol{t})$ are $N \times 1$ matrices of the form

$$\boldsymbol{A} = [a_0\; a_1\; a_2\; \dots\; a_{N-1}]^T, \qquad \boldsymbol{B} = [b_0\; b_1\; b_2\; \dots\; b_{N-1}]^T$$
$$\boldsymbol{C} = [c_0\; c_1\; c_2\; \dots\; c_{N-1}]^T, \qquad \emptyset(\boldsymbol{t}) = [T_0(t)\; T_1(t)\; T_2(t) \dots T_{N-1}(t)]^T$$

Our aim is to obtain these unknown coefficients a_n, b_n, c_n, $n = 0,1,2,3, \dots, N-1$ to get the Chebyshev polynomial solutions of the model (1) and (2), explicitly. For this, firstly utilizing the $N \times N$ operational matrix for the derivative $\boldsymbol{D}$ of the form [18]

$$\boldsymbol{D} = [d_{ij}] = \frac{2}{L}\begin{cases} 2(i-1), & if\ i-j\ is\ odd\ and\ i > j > 1 \\ \sqrt{2}(i-1), & if\ i-j\ is\ odd\ and\ i > j = 1 \\ 0, & otherwise \end{cases}$$

the derivative of $\emptyset(\boldsymbol{t})$ is represented in matrix form as $\frac{d}{dt}\emptyset(\boldsymbol{t}) = \boldsymbol{D}\emptyset(\boldsymbol{t})$, where $\emptyset(\boldsymbol{t})$ is $N \times 1$ matrix of the shifted Chebyshev polynomials of the first kind. By means of the operational matix $\boldsymbol{D}$, the derivatives of

$S_N(t), I_N(t), R_N(t)$ can be expressed as

$$\begin{aligned} S'_N(t) &= \sum_{n=0}^{N-1} a_n\, T'_n(t) = \boldsymbol{A^T D}\, \boldsymbol{\emptyset(t)} \\ I'_N(t) &= \sum_{n=0}^{N-1} b_n\, T'_n(t) = \boldsymbol{B^T D}\, \boldsymbol{\emptyset(t)} \\ R'_N(t) &= \sum_{n=0}^{N-1} c_n\, T'_n(t) = \boldsymbol{C^T D}\, \boldsymbol{\emptyset(t)} \end{aligned} \tag{4}$$

Then $S_N(t), I_N(t), R_N(t)$ and their derivatives are substituted into (1) and (2). Finally, replacing the first roots t_i , $i = 1,2,3, \ldots N-1$, of the N-th shifted Chebyshev polynomials of the first kind instead of t, as

$$\begin{aligned} A^T D\emptyset(t_i) &= -\frac{\alpha}{P} A^T \emptyset(t_i)[\emptyset(t_i)]^T B \\ B^T D\emptyset(t_i) &= \frac{\alpha}{P} A^T \emptyset(t_i)[\emptyset(t_i)]^T B - \mu B^T \emptyset(t_i) \\ C^T D\emptyset(t_i) &= \mu B^T \emptyset(t_i) \end{aligned} \tag{5}$$

$$A^T \emptyset(0) = S_0, \quad B^T \emptyset(0) = I_0, \quad C^T \emptyset(0) = R_0 \tag{6}$$

will provide $3(N-1)$ equations to solve. These equations together with the initial conditions in (6) will form a system of $3N$ equations which can be solved by MATLAB for the unknown coefficients a_n, b_n, c_n for $n = 0,1,2,3, \ldots N-1$.

4. Numerical Experiments

In this section we illustrate the implementation of Chebyshev polynomial solutions method to solve SIR model for the spread of Covid 19. For this firstly the parameters and the initial conditions are determined from the available COVID-19 data in the official source of the Turkey Ministry of Health [19]. Since April 4, 2020 is the first reported date in [19], the number of susceptible people and the number of the people who are infected and removed in Turkey on April 4, 2020 are taken as the initial values of S_0, I_0 and R_0. If the size of the population is assumed to be 84,000,000, the number of infected people is 3013 and the number of removed people is 378 (daily number of deaths is 76 and the number of recovery is 302), the number of susceptible individuals become 83,996,609. Since the recommended quarantine period is 14 days if the individuals are suspected to be exposed to COVID-19 [20], the transmission rate is estimated as $\alpha = 1/14$ $(1/\text{days}) \approx 0.0714$ [14,21]. Since the total number of infected people on April 4 is 23934 while the total number of removed people is 1287 (501 total deaths and 786 total recovered patients), the recovery rate is obtained as $\mu = 1287/23934 \approx 0.0538$. So these values are as represented in Table 2.

Table 2. The values of the parameters and the initial conditions

Data	Values [19]
S_0	83,996,609
I_0	3013
R_0	378
μ	0.0538
α	0.0714

Considering these conditions and the parameters, the model (1) and (2) can be expressed as follows

$$\begin{aligned} \frac{dS(t)}{dt} &= -8.5034 \times 10^{-10} S(t)I(t), \\ \frac{dI(t)}{dt} &= 8.5034 \times 10^{-10} S(t)I(t) - 0.0538I(t), \qquad 0 \le t \le 60 \\ \frac{dR(t)}{dt} &= 0.0538I(t), \end{aligned} \tag{7}$$

$$S(0) = 83{,}996{,}609, \; I(0) = 3013, \; R(0) = 378 \tag{8}$$

Here, the length of the interval is taken as $L = 60$. Letting $N = 6$ corresponds to a fifth order polynomial approximation as

$$S_5(t) = \sum_{n=0}^{5} a_n . T_n(t) = \boldsymbol{A^T} \emptyset(\boldsymbol{t}), \quad I_5(t) = \sum_{n=0}^{5} b_n . T_n(t) = \boldsymbol{B^T} \emptyset(\boldsymbol{t}), \quad R_5(t) = \sum_{n=0}^{5} c_n . T_n(t) = \boldsymbol{C^T} \emptyset(\boldsymbol{t}) \tag{9}$$

where

$$\boldsymbol{A} = [a_0 \ a_1 \ a_2 \ a_3 \ a_4 \ a_5]^T, \quad \boldsymbol{B} = [b_0 \ b_1 \ b_2 \ b_3 \ b_4 \ b_5]^T, \quad \boldsymbol{C} = [c_0 \ c_1 \ c_2 \ c_3 \ c_4 \ c_5]^T, \quad \emptyset(\boldsymbol{t}) = [T_0(t) \ T_1(t) \ T_2(t) \ T_3(t) \ T_4(t) \ T_5(t)]^T.$$

By means of the 6×6 operational matrix of the derivative D of the form

$$\boldsymbol{D} = \frac{2}{60} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 \\ 3\sqrt{2} & 0 & 6 & 0 & 0 & 0 \\ 0 & 8 & 0 & 8 & 0 & 0 \\ 5\sqrt{2} & 0 & 10 & 0 & 10 & 0 \end{bmatrix}$$

the derivative of $S_5(t), I_5(t), R_5(t)$ can be expressed as

$$S_5'(t) = \boldsymbol{A^T D} \emptyset(\boldsymbol{t}), \quad I_5'(t) = \boldsymbol{B^T D} \emptyset(\boldsymbol{t}), \quad R_5'(t) = \boldsymbol{C^T D} \emptyset(\boldsymbol{t}). \tag{10}$$

Substituting the equations (9) and (10) into the model (7) and replacing the roots

$t_1 = 4.0192, \quad t_2 = 15, \quad t_3 = 30, \quad t_4 = 45, \quad t_5 = 55.9808.$

of the sixth order shifted Chebyshev polynomials as collocation points $t_i, i = 1,2,3,4,5$ instead of t gives 15 equations of the form

$$A^T D\emptyset(t_i) = -8.5034 \times 10^{-10} A^T \emptyset(t_i)[\emptyset(t_i)]^T B$$
$$B^T D\emptyset(t_i) = 8.5034 \times 10^{-10} A^T \emptyset(t_i)[\emptyset(t_i)]^T B - 0.0538 B^T \emptyset(t_i)$$
$$C^T D\emptyset(t_i) = 0.0538 B^T \emptyset(t_i), \quad i = 1,2,3,4,5.$$

Together with the initial conditions

$$\boldsymbol{A^T} \emptyset(\boldsymbol{0}) = 83{,}996{,}609, \quad \boldsymbol{B^T} \emptyset(\boldsymbol{0}) = 3013, \quad \boldsymbol{C^T} \emptyset(\boldsymbol{0}) = 378,$$

a system of 18 equations is constructed to solve for the unknown coefficients $a_n, b_n \ c_n$ for $n = 0,1,2,3,4,5$ by a written algorithm including *fsolve* function in MATLAB. Then we obtain the Chebyshev polynomials solutions as

$$S_5(t) = 83996609 - 215.2289t - 1.8949t^2 - 0.0115t^3 - (4.0096 \times 10^{-5})t^4 - (2.9475 \times 10^{-7})t^5,$$
$$I_5(t) = 3013 + 53.1936t + 0.4681t^2 + 0.0028t^3 + (9.9190 \times 10^{-6})t^4 + (7.1973 \times 10^{-8})t^5,$$
$$R_5(t) = 378 + 162.0353t + 1.4268t^2 + 0.0086t^3 + (3.0177 \times 10^{-5})t^4 + (2.2278 \times 10^{-7})t^5.$$

Due to the lack of exact solution, the obtained approximate solutions $S_5(t), I_5(t), R_5(t)$ are compared with the results of Pell-Lucas Collocation method (PLCM) [13] and the fourth order Runga Kutta method (RK4). The results are presented in the Figure 1. We would like to note that both the present method (CPM) and PLCM in [13] use fifth order polynomial approximations while Runge-Kutta method uses an iteration with step size $\Delta t = 0.1$ As can be seen that all the numerical results in the Figure 1 coincide.

On the other hand, the Figure 1 shows the change in the susceptible, the infected and the removed population over time interval $[0,60]$, respectively.

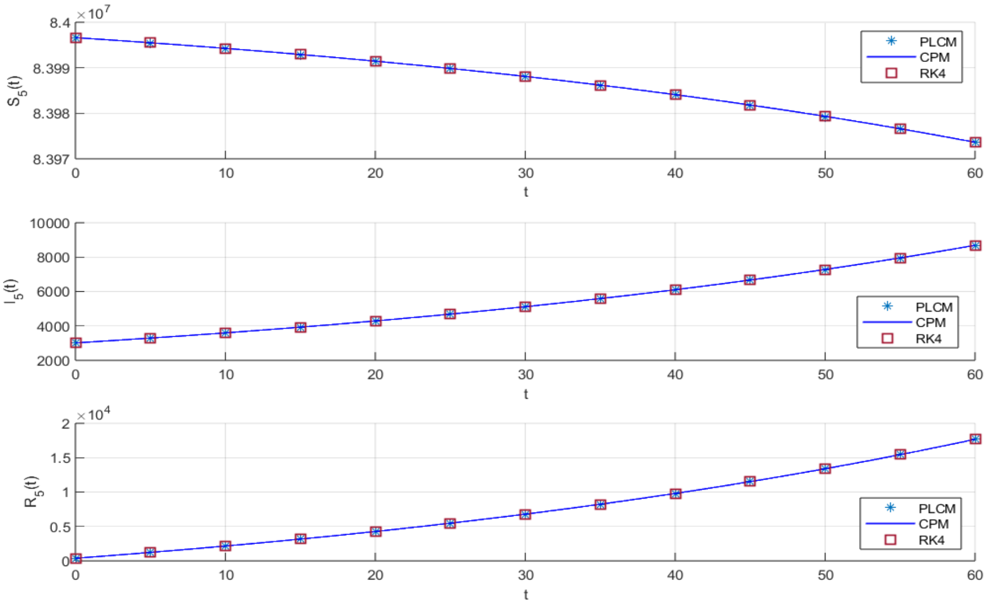

Figure 1. Comparison of the approximate solutions $S_5(t), I_5(t), R_5(t)$ obtained by the present method with Pell-Lucas Collocation method and fourth-order Runge Kutta method.

The number of susceptible individuals is decreased from 83,996,609 individuals to 83,973,800 individuals in two-month time period while the number of infected individuals is increased from 3013 individuals to 8685 individuals and the number of removed individuals is increased from 378 individuals to 17,619 individuals. Although, in the Figure 1, the infected population $I_5(t)$ and removed population $R_5(t)$ seem to be increasing at a close rate, we observe that the number of removed individuals grows faster than the infected population on the time interval [0,60], by the Figure 2.

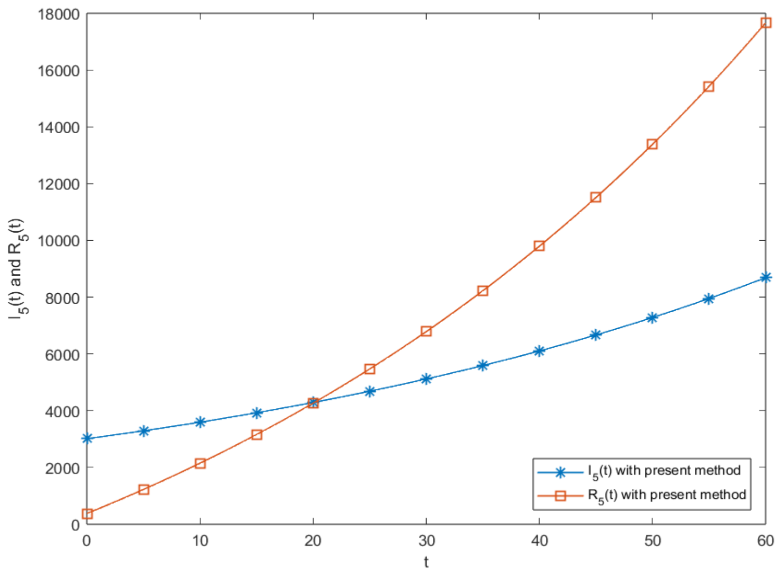

Figure 2. Growth rate comparison of the Infected $I_5(t)$ and Removed $R_5(t)$ populations

5. Error Analysis

In this section, in order to check the robustness and efficiency of the current method, we discuss the absolute errors and the absolute residual errors of the model (1) and (2). The maximum absolute errors related to the functions S_N, I_N, R_N compute how the approximations achieve accurate results through the maximum of the differences between the N-th and $(N-1)$-st degree polynomial approximations as follows:

$$maxE_{S_N},= \max_{t_j\in[0,60]} \left|S_N(t_j) - S_{N-1}(t_j)\right|,$$
$$maxE_{I_N},= \max_{t_j\in[0,60]} \left|I_N(t_j) - I_{N-1}(t_j)\right|, \quad (11)$$
$$maxE_{R_N},= \max_{t_j\in[0,60]} \left|R_N(t_j) - R_{N-1}(t_j)\right|,$$

where S_N, I_N, R_N are approximate solutions of the model (1) and (2) for all $t_j \in [0,60]$. Table 3 shows the maximum of the absolute errors $maxE_{S_N}$, $maxE_{I_N}$ and $maxE_{R_N}$ of the current method for all $t_j \in [0,60]$ when $N = 8$ and $N = 10$ which corresponds to seventh and nineth degree polynomial approximations. As seen in the Table 3, the maximum absolute errors for S_N, I_N and R_N are decreasing and getting closer to zero when the degree of the Chebyshev polynomials N is increasing. One can say that the current method is highly accurate.

Table 3. The maximum absolute errors of the current method for N=8 and N=10.

$maxE_{S_8}$	$maxE_{S_{10}}$	$maxE_{I_8}$	$maxE_{I_{10}}$	$maxE_{R_8}$	$maxE_{R_{10}}$
6.0603e-05	1.4901e-08	1.3926e-05	6.0754e-10	4.6951e-05	1.4992e-08

The absolute residual errors ReS_N, ReI_N, ReR_N of the model (1) and (2) compute how much the approximate solutions S_N, I_N, R_N satisfy the differential equations by

$$ReS_N = \left|\frac{dS_N(t)}{dt} + \frac{\alpha}{P} S_N(t) I_N(t)\right|,$$
$$ReI_N = \left|\frac{dI_N(t)}{dt} - \frac{\alpha}{P} S_N(t) I_N(t) + \mu I_N(t)\right|, \quad (12)$$
$$ReR_N = \left|\frac{dR_N(t)}{dt} - \mu I_N(t)\right|$$

where the parameters α, μ, P are considered as given in Table 2. It is known that the closer the absolute residual errors are to zero, the better the approximations are. In literature, there is not much study on the polynomial solutions of SIR model for Covid 19 pandemic in Turkey, so far. Yüzbaşı and Yıldırım obtained Pell-Lucas solutions of the model (1) and (2) and presents the residual errors of the Pell-Lucas solutions when $N = 10$ in [13]. So, setting $N = 8$ and $N = 10$, Table 4a, 4b and 4c present the absolute residual errors for susceptible, infected and removed populations and compare the residual errors of the current method and Pell-Lucas Collocation method (PLCM) in [13], respectively. As observed in the Table 4, the maximum absolute residual errors of $S_N(t)$, for $t \in [0,60]$ are $O(10^{-6})$ and $O(10^{-10})$ when $N = 8$ and $N = 10$, respectively, while the maximum absolute residual error obtained by PLCM is $O(10^{-8})$.

Table 4. Comparison of absolute residual errors for susceptible individuals

t	ReS_8 *Present Method*	ReS_{10} *Present Method*	ReS_{10} *PLCM [13]*
0	1.3795e-06	3.8620e-10	1.7821e-08
10	1.9682e-07	8.3868e-12	1.0382e-10
20	1.2282e-08	3.2757e-11	1.0065e-11
30	5.1442e-09	3.8460e-11	1.7963e-14
40	1.4451e-08	3.4894e-11	5.4862e-12
50	1.9413e-07	9.5066e-12	2.3212e-11
60	1.3512e-06	4.6717e-10	6.2135e-14

The Table 5 shows that the maximum absolute residual errors for $I_N(t)$, for $t \in [0,60]$ are $O(10^{-7})$ and $O(10^{-10})$ when $N = 8$ and $N = 10$, respectively, while the maximum absolute residual error obtained by PLCM is $O(10^{-8})$.

Table 5. Comparison of absolute residual errors for infected individuals

t	ReI_8 Present Method	ReI_{10} Present Method	ReI_{10} PLCM [13]
0	2.5844e-07	3.9938e-10	1.5858e-08
10	3.6411e-08	8.6413e-12	9.2552e-11
20	9.2450e-09	3.3787e-11	9.0502e-12
30	9.3203e-10	3.9684e-11	5.9179e-14
40	2.5777e-09	3.5924e-11	4.9293e-12
50	3.3723e-08	9.7608e-12	2.1138e-11
60	2.3012e-07	4.8035e-10	3.22511e-14

As one can see in the Table 6, the maximum absolute residual errors of $R_N(t)$, for $t \in [0,60]$ are $O(10^{-6})$ and $O(10^{-11})$ when $N = 8$ and $N = 10$, respectively, while the maximum absolute residual error obtained by PLCM is $O(10^{-9})$.

Table 6. Comparison of Absolute Residual Errors for Removed Individuals

t	ReR_8 Present Method	ReR_{10} Present Method	ReR_{10} PLCM [13]
0	1.1211e-06	1.3161e-11	1.9418e-09
10	7.1870e-08	2.3383e-13	1.1024e-11
20	1.0024e-08	1.0080e-12	1.0284e-12
30	1.2456e-07	1.2472e-12	5.2996e-14
40	8.6769e-08	1.0040e-12	5.4701e-13
50	1.6041e-07	2.2559e-13	2.2316e-12
60	1.1211e-06	1.3148e-11	4.9546e-14

One can say that the current method is as efficient as PLCM when $N = 10$ and considering the lower degree polynomial approximation, it is still powerful.

6. Conclusion

This study present an efficient method based on the Chebyshev polynomials to solve SIR model of Covid-19 pandemic in Turkey. This method transforms the system of nonlinear ordinary differential equations into a system of algebraic equations which can be solved by Matlab. The solutions of the model and their derivatives are approximated by the truncated series of Chebyshev polynomials. The robustness of the method is indicated by means of the absolute errors and the absolute residual errors. We observe that the Chebyshev polynomials method is highly accurate. As an advantage, the method uses the powerful properties of the Chebyshev polynomials such as the orthonormality property shifted to any interval [0, L]. This is also a factor which reduces the computational cost. Another advantage of the method is that it does not require any discretization of the interval, and this advantage facilitates the implementation of the method. In this sense, as a future study, the method can be applied to other models of the Covid-19.

Conflict of Interest

The author declares that there is no conflict of interest.

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